

Atividade 2 - Propriedades da probabilidade e probabilidade condicional
Paulo Ricardo Seganfredo Campana

1.

- a) $\{RR, RG, RB, GR, GG, GB, BR, BG, BB\}$
b) $\{RG, RB, GR, GB, BR, BG\}$

2.

- a) $A \cup B \cup C$
b) $A - B - C$
c) $(A \cup B \cup C) - (A \cap B) - (A \cap C) - (B \cap C)$
d) $(A \cup B \cup C)^c$
e) $(A \cup B) - C$

3.

$$\begin{aligned}P(\bigcup A_i) &= \sum P(A_i) \\P(a, b) &= P(a) + P(b) = 0.5 \\P(b, c) &= P(b) + P(c) = 0.8 \\P(a, b) &= P(a) + P(c) = 0.7 \\P(c) &= P(b) + 0.2 \\2 \cdot P(b) + 0.2 &= 0.8\end{aligned}$$

$$\begin{aligned}P(\{b\}) &= 0.3 \\P(\{c\}) &= 0.5 \\P(\{a\}) &= 0.2 \\P(\{a, b\}) &= 0.5 \\P(\{b, c\}) &= 0.8 \\P(\{a, c\}) &= 0.7 \\P(\{a, b, c\}) &= 1 \\P(\{\emptyset\}) &= 0\end{aligned}$$

4.

$$\text{Se } B \subset A, P(A \cap B) = P(B) = 0.6$$

$$\text{Se } A \cup B = \Omega$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\Omega) = 0.7 + 0.6 - P(A \cap B)$$

$$1 = 0.7 + 0.6 - P(A \cap B)$$

$$P(A \cap B) = 0.3$$

$$0.3 \leq P(A \cap B) \leq 0.6$$

5.

a)

$$P(A \cap B^c) = P(A) \cdot P(B^c)$$

$$P(A - B) = P(A) \cdot (1 - P(B))$$

$$P(A - B) = P(A) - P(A \cap B) \quad \blacksquare$$

b) Simétrico à letra a, pois a interseção e a multiplicação são comutativas.

c)

$$P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$$

$$P((A \cup B)^c) = (1 - P(A)) \cdot (1 - P(B))$$

$$1 - P(A \cup B) = 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Propriedade 7 da probabilidade. \(\blacksquare\)

6.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.4 + p - P(A \cap B)$$

$$0.3 = p - P(A \cap B)$$

$$P(A \cap B) = p - 0.3$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$p - 0.3 = 0.4 \cdot p$$

$$0.6p = 0.3$$

$$p = 0.5$$

$$\frac{P(C \cap D)}{P(D)} = 0.4 \text{ e } \frac{P(C \cap D)}{P(C)} = 0.5$$

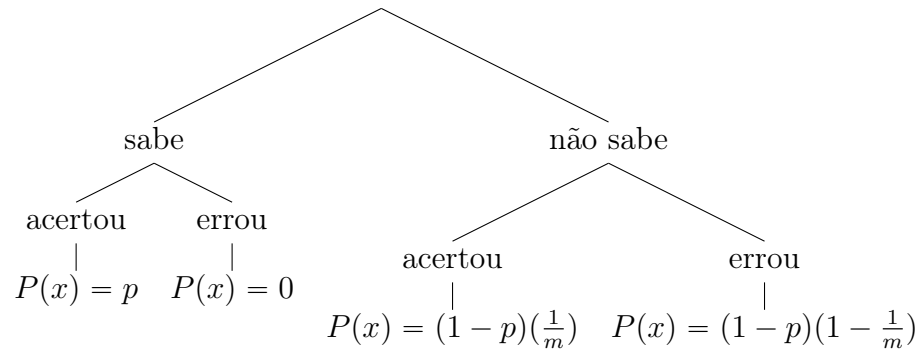
$$P(D) \cdot 0.4 = P(C \cap D) = P(C) \cdot 0.5$$

$$\frac{0.4}{0.5} = \frac{P(C)}{P(D)}$$

$$0.8 = \frac{P(C)}{P(D)}$$

$$P(D) \geq P(C)$$

8.



a)

$$P(\text{sabe}|\text{acertou}) = \frac{P(\text{sabe} \cap \text{acertou})}{P(\text{acertou})}$$

$$P(\text{sabe}|\text{acertou}) = \frac{\frac{p}{p + \frac{1-p}{m}}}{\frac{p}{p + \frac{1-p}{m}}} = 1$$

b)

Quando o número de alternativas aumenta ao infinito, a chance de chutar uma questão e acertar diminui a zero.

$$\lim_{m \rightarrow \infty} \frac{\frac{p}{p + \frac{1-p}{m}}}{\frac{p}{p + \frac{1-p}{m}}} = \frac{p}{p} = 1$$

Quando a chance de saber a resposta uma questão diminui, as questões respondidas corretas serão de maioria as questões chutadas.

$$\lim_{p \rightarrow 0} \frac{\frac{p}{p + \frac{1-p}{m}}}{\frac{p}{p + \frac{1-p}{m}}} = \frac{0}{0 + \frac{1}{m}} = 0$$