UNIVERSIDADE FEDERAL DA PARAÍBA
Introdução à Álgebra Linear
Segunda Lista de Exercícios
Paulo Ricardo Seganfredo Campana

Questão 1.

a)

Sendo $u = (x_1, y_1)$ e $v = (x_2, y_2)$

$T(u + v) = T(x_1 + x_2, y_1 + y_2) = (x_1 + x_2 + 2y_1 + 2y_2, 3x_1 + 3x_2 - y_1 - y_2)$

$(x_1 + 2y_1, 3x_1 - y_1) + (x_2 + 2y_2, 3x_2 - y_2) = T(x_1, y_1) + T(x_2, y_2) = T(u) + T(v)$

$T(\alpha u) = T(\alpha x_1, \alpha y_1) = (\alpha x_1 + 2\alpha y_1, 3\alpha x_1 - \alpha y_1) = \alpha(x_1 + 2y_1, 3x_1 - y_1)$

$\alpha T(x_1, y_1) = \alpha T(u)$ $\blacksquare$

b)

Sendo $u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2)$

$T(u + v) = T(x_1 + x_2, y_1 + y_2, z_1, z_2) = (2x_1 + 2x_2 - y_1 - y_2 + z_1 + z_2, y_1 + y_2 - 4z_1 - 4z_2)$

$(2x_1 - y_1 + z_1, y_1 - 4z_1) + (2x_2 - y_2 + z_2, y_2 - 4z_2) = T(x_1, y_1, z_1) + T(x_2, y_2, z_2) = T(u) + T(v)$

$T(\alpha u) = T(\alpha x_1, \alpha y_1, \alpha z_1) = (2\alpha x_1 - \alpha y_1 + \alpha z_1, \alpha y_1 - 4\alpha z_1) = \alpha(2x_1 - y_1 + z_1, y_1 - 4z_1)$

$\alpha T(x_1, y_1, z_1) = \alpha T(u)$ $\blacksquare$

c)

$T(A + A') = (A + A') \cdot B = AB + A'B = T(A) + T(A')$

$T(\alpha A) = (\alpha A) \cdot B = \alpha(AB) = \alpha T(A)$ $\blacksquare$

d)

$T(A + B) = tr(A + B) = (a_{11} + b_{11} + a_{22} + b_{22} + ... + a_{nn} + b_{nn})$

$(a_{11} + a_{22} + ... + a_{nn}) + (b_{11} + b_{22} + ... + b_{nn}) = tr(A) + tr(B) = T(A) + T(B)$

$T(\alpha A) = tr(\alpha A) = (\alpha a_{11} + \alpha a_{22} + ... + \alpha a_{nn}) = \alpha(a_{11} + a_{22} + ... + a_{nn}) = \alpha tr(A) = \alpha T(A)$ $\blacksquare$

e)

Não é transformação pois $T(0) = T(0 + 0x + 0x^2) = (1, x, x^2)$ ou seja, $T(0) \neq 0$ $\blacksquare$

Questão 2.

$T(1,1,1) = (2,-1,4) \qquad T(1,1,0) = (3,0,1) \qquad T(1,0,0) = (-1,5,1)$

$(x,y,z) = a(1,1,1) + b(1,1,0) + c(1,0,0) = (a+b+c, a+b, a)$

$$\begin{cases} a+b+c = x \\ a+b = y \\ a = z \end{cases} \qquad a = z, \qquad b = y-z, \qquad c = x-y$$

$T(x,y,z) = zT(1,1,1) + (y-z)T(1,1,0) + (x-y)T(1,0,0)$

$z(2,-1,4) + (y-z)(3,0,1) + (x-y)T(-1,5,1)$

$(2z,-z,4z) + (3y-3z,0,y-z) + (-x+y,5x-5y,x-y) = (-x+4y-z, 5x-5y-z, x+3z)$

$T(2,4,-1) = (15,-9,-1)$

Questão 3.

$T(2v_1 - 4v_2 + 5v_3) = T(2v_1) + T(-4v_2) + T(5v_3) = 2T(v_1) - 4T(v_2) + 5T(v_3)$

$2(1,-1,2) - 4(0,3,2) + 5(-3,1,2) = (2,-2,4) + (0,-12,-8) + (-15,5,10) = (-13,-9,6)$

Questão 4.

n^n pois a transformação T pode levar qualquer v_i em qualquer v_j incluindo $i = j$, ou seja, cada vetor v_i pode ser mapeado a n diferentes vetores, e existem n vetores v_i, portanto existem n^n transformações.

Questão 5.

a)

$Im(T) = \{(a,b,c); a = (4x+y-2z-3t), b = (2x+y+z-4t), c = (6x-9z+9t)\}$

$\{(4x+y-2z-3t, 2x+y+z-4t, 6x-9z+9t) \forall x,y,z,t \in \mathbb{R}\}$

$\{x(4,2,6) + y(1,1,0) + z(-2,1,-9) + t(-3,-4,9) \forall x,y,z,t \in \mathbb{R}\}$

$[(4,2,6),(1,1,0),(-2,1,-9),(-3,-4,9)]$

$$\begin{bmatrix} 4 & 2 & 6 \\ 1 & 1 & 0 \\ -1 & 1 & -9 \\ -3 & -4 & 9 \end{bmatrix} \quad \begin{bmatrix} 4 & 2 & 6 \\ 0 & 2 & -9 \\ 0 & -7 & 36 \\ 0 & -1 & 9 \end{bmatrix} \quad \begin{bmatrix} 4 & 2 & 6 \\ 0 & -1 & 9 \\ 0 & 0 & 9 \\ 0 & 0 & -27 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 24 \\ 0 & 1 & -9 \\ 0 & 0 & 9 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$Im(T) = [(1,0,0),(0,1,0),(0,0,1)] = \mathbb{R}^3$, os três vetores estão dentro de $Im(T)$

b)

$N(T) \implies (4x+y-2z-3t, 2x+y+z-4t, 6x-9z+9t) = 0$

$$\begin{bmatrix} 4 & 1 & -2 & -3 & 0 \\ 2 & 1 & 1 & -4 & 0 \\ 6 & 0 & -9 & 9 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 1 & -4 & 0 \\ 0 & 1 & 4 & -5 & 0 \\ 0 & -1 & -4 & 9 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 1 & -4 & 0 \\ 0 & 1 & 4 & -5 & 0 \\ 0 & 0 & 0 & 9 & 0 \end{bmatrix} \Longrightarrow t = 0$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 0 & -3 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix} \quad \begin{bmatrix} 8 & 3 & 0 & 0 \\ 0 & 1 & 4 & 0 \end{bmatrix} \quad \begin{cases} 8x + 3y = 0 \\ y + 4z = 0 \end{cases} \Longrightarrow [(8\alpha, -3\alpha, 12\alpha, 0)]$$

$$dimN + dimIm = dim\mathbb{R}^4 \qquad 1 + 3 = 4 \quad \checkmark$$

c)

Apenas $u = (3, -8, 2, 0)$ pois $(12 - 8 - 4 - 0, 6 - 8 + 2 + 0, 18 - 18 + 0) = (0, 0, 0)$

Questão 6.

$$T(P_2) = T(a + bx + cx^2) = x(a + bx + cx^2) = ax + bx^2 + cx^3$$

$N(T) \Longrightarrow ax + bx^2 + cx^3 = 0$ possui única solução trivial $(a, b, c) = (0, 0, 0) \qquad N(T) = 0$

$$Im(T) \Longrightarrow \{ax + bx^2 + cx^3\} = \{a(x) + b(x^2) + c(x^3)\} = [(x, x^2, x^3)]$$

Boldrini - Capítulo 5

Questão 3.

$$(x, y, z) = a(1, 0, 0) + b(0, 1, 0) + c(0, 0, 1) = (a, b, c) \qquad x = a, \quad y = b, \quad z = c$$

$$T(x, y, z) = xT(1, 0, 0) + yT(0, 1, 0) + zT(0, 0, 1) =$$

$$x(2, 0) + y(1, 1) + z(0, -1) = (2x + y, y - z) = (3, 2)$$

$$\begin{cases} 2x + y = 3 \\ y - z = 2 \end{cases} \qquad z = 1 - 2x, \quad y = 3 - 2x \qquad v = (x, 3 - 2x, 1 - 2x)$$

Questão 4.

a)

$$(x, y) = a(1, 1) + b(0, -2) = (a, a - 2b) \qquad x = a, \quad y = x - 2b, \quad b = \frac{x - y}{2}$$

$$T(x, y) = xT(1, 1) + (\frac{x - y}{2})T(0, -2) = x(3, 2, 1) + (\frac{x - y}{2})(0, 1, 0) = (3x, \frac{5x - y}{2}, x)$$

b)

$$T(1, 0) = \left(3, \frac{5}{2}, 1\right) \qquad T(0, 1) = \left(0, -\frac{1}{2}, 0\right)$$

c)

$$(x, y, z) = a(3, 2, 1) + b(0, 1, 0) + c(0, 0, 1) = (3a, 2a + b, a + c)$$

$$a = \frac{x}{3}, \quad b = y - \frac{2x}{3}, \quad c = z - \frac{x}{3}$$

$$S(x,y,z) = \frac{x}{3}S(3,2,1) + (y - \frac{2x}{3})S(0,1,0) + (z - \frac{x}{3})S(0,0,1)$$

$$\frac{x}{3}(1,1) + (y - \frac{2x}{3})(0,-2) + (z - \frac{x}{3})(0,0) = (\frac{x}{3}, -2y + \frac{5x}{3})$$

d)

$$P(x,y) = S(T(x,y)) = S(3x, \frac{5x-y}{2}, x) = (x,y)$$

Questão 5.

$$T(x,y) = (y,x)$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Questão 10.

$$R = S(T) \qquad T = S^{-1}(R)$$

$$S^{-1} = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 1 & 1 \\ -7 & -3 & -5 \end{bmatrix} \qquad T = \begin{bmatrix} 8 & 0 & 9 \\ 4 & 0 & 4 \\ -13 & 2 & -15 \end{bmatrix}$$

Questão 11.

a)

$$(x,y) = a(1,-1) + b(0,2) = (a, 2b-a) \qquad a = x, \quad b = \frac{x+y}{2}$$

$$[T]_\beta = [T]^\alpha_\beta [V]_\alpha = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ \frac{x+y}{2} \end{bmatrix} = \begin{bmatrix} x \\ \frac{3x+y}{2} \\ \frac{-x-y}{2} \end{bmatrix}$$

$$T = x(1,0,-1) + (\frac{3x+y}{2})(0,1,2) + (\frac{-x-y}{2})(1,2,0) = (\frac{x-y}{2}, \frac{x-y}{2}, 2x-y)$$

b)

$$S(1,-1) = (-2,2,1) = a(1,0,-1) + b(0,1,2) + c(1,2,0) = (a+c, b+2c, 2b-a)$$

$$\begin{cases} a + c = -2 \\ b + 2c = 2 \\ 2b - a = 1 \end{cases} \qquad a = \frac{-11}{3}, \quad b = \frac{-4}{3}, \quad c = \frac{5}{3}$$

$$S(1,-1) = (4,-2,0) = a(1,0,-1) + b(0,1,2) + c(1,2,0) = (a+c, b+2c, 2b-a)$$

$$\begin{cases} a + c = 4 \\ b + 2c = -2 \\ 2b - a = 0 \end{cases} \qquad a = \frac{20}{3}, \quad b = \frac{10}{3}, \quad c = \frac{-8}{3}$$

$$[S]_\beta^\alpha = \begin{bmatrix} \frac{-11}{3} & \frac{20}{3} \\ \frac{-4}{3} & \frac{10}{3} \\ \frac{5}{3} & \frac{-8}{3} \end{bmatrix}$$

c)

$$[T]_\gamma = [T]_\gamma^\alpha [T]_\alpha = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ \frac{x+y}{2} \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ \frac{x+y}{2} \end{bmatrix}$$

$\gamma = \{(1,1,1),(x,y,z),(-1,-1,2)\}$

Questão 14.

a)

$$T(1,0,0,0) = (1,0) \quad T(0,1,0,0) = (0,1) \quad T(0,0,1,0) = (0,1) \quad T(0,0,0,1) = (1,0)$$

$$(1,0) = 1(1,0) + 0(0,1) \quad (0,1) = 0(1,0) + 1(0,1)$$

$$[T]_\alpha^\beta = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

b)

$$S(x,y) = \begin{bmatrix} 2x+y & x-y \\ -x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$x = y \quad x = 0 \quad y = 1$ impossível.

Questão 19.

a) $z = 0 \quad x = y \quad [(1,1,0)]$

b) $dimN(T) + dimIm(T) = dim\mathbb{R}^3 \qquad 1 + dimIm(T) = 3 \qquad dimIm(T) = 2$

c) não pois $Im(T) \neq \mathbb{R}^3$

d) Núcleo é uma reta no $\mathbb{R}^3$ e Imagem é um plano no $\mathbb{R}^3$

Questão 20.

a) $T(x,y,z) = (y,z)$

b) não existe pois $dim\mathbb{R}^3 > dim\mathbb{R}^2$

c) $L(x,y,z) = (0,0)$

d) $M(x,y) = (0,0)$

e) $H(x, y, z) = (0, 0, 0)$

Questão 22.

$$D(a_3x^3 + a_2x^2 + a_1x + a_0) = (6a_3x + 2a_2)$$

$$D(a + b) = (6(a_3 + b_3)x + 2(a_2 + b_2)) = (6a_3x + 2a_2) + (6b_3x + 2b_2) = D(a) + D(b)$$

$$D(\alpha a) = (6\alpha a_3x + 2\alpha a_2) = \alpha(6a_3x + 2a_2) = \alpha D(a)$$

$$N(D) = (0 + 0 + a_1x + a_0) = \{x, 1\}$$