UNIVERSIDADE FEDERAL DA PARAÍBA
Introdução à Álgebra Linear
Terceira Lista
Paulo Ricardo Seganfredo Campana

Boldrini

Questão 1.

a) $\langle v,\ w\rangle = x_1y_1 + x_2y_2 + x_3y_3$ $\qquad v = (x_1, x_2, x_3), w = (y_1, y_2, y_3)$

$p1 : \langle v,\ v\rangle = x_1^2 + x_2^2 + x_3^2 \geq 0$

pois são números reais ao quadrado e somente 0 quando $v = (0, 0, 0)$

$p2 : \langle v + w,\ u\rangle = (x_1 + y_1)z_1 + (x_2 + y_2)z_2 + (x_3 + y_3)z_3 =$

$= x_1z_1 + y_1z_1 + x_2z_2 + y_2z_2 + x_3z_3 + y_3z_3 = \langle v,\ u\rangle + \langle w,\ u\rangle$

$p3 : \langle v,\ w\rangle = x_1y_1 + x_2y_2 + x_3y_3 = y_1x_1 + y_2x_2 + y_3x_3 = \langle w,\ v\rangle$

$p4 : \langle \alpha v,\ w\rangle = \alpha x_1y_1 + \alpha x_2y_2 + \alpha x_3y_3 = \alpha(x_1y_1 + x_2y_2 + x_3y_3) = \alpha \langle v,\ w\rangle$ ■

b) $\langle v_1,\ v_2\rangle = 2x_1x_2 - x_1y_2 - x_2y_1 + y_1y_2$ $\qquad v_1 = (x_1, y_1), v_2 = (x_2, y_2)$

$p1 : \langle v_1,\ v_1\rangle = 2x_1^2 - 2x_1y_1 + y_1^2 = x_1^2 + (x_1 - y_1)^2 \geq 0$

pois são números reais ao quadrado e somente 0 quando $v_1 = (0, 0)$

$p2 : \langle v_1 + v_2,\ v_3\rangle = 2(x_1 + x_2)x_3 - (x_1 + x_2)y_3 - x_3(y_1 + y_2) + (y_1 + y_2)y_3 =$

$= 2x_1x_3 + 2x_2x_3 - x_1y_3 - x_2y_3 - x_3y_1 - x_3y_2 + y_1y_3 + y_2y_3 = \langle v_1,\ v_3\rangle + \langle v_2,\ v_3\rangle$

$p3 : \langle v_1,\ v_2\rangle = 2x_1x_2 - x_1y_2 - x_2y_1 + y_1y_2 = 2x_2x_1 - x_2y_1 - x_1y_2 + y_2y_1 = \langle v_2,\ v_1\rangle$

$p4 : \langle \alpha v_1,\ v_2\rangle = \alpha 2x_1x_2 - \alpha x_1y_2 - \alpha x_2y_1 + \alpha y_1y_2 = \alpha(2x_1x_2 - x_1y_2 - x_2y_1 + y_1y_2) = \alpha \langle v_1,\ v_2\rangle$ ■

c) $\langle f_1,\ f_2\rangle = \displaystyle\int_0^1 f_1(t)f_2(t)dt$

$p1 : \langle f_1,\ f_1\rangle = \displaystyle\int_0^1 f_1^2(t)dt \geq 0$

pois $f_1^2(t)$ é uma função positiva então sua área é positiva, e somente 0 quando $f_1(t) = 0$

$p2 : \langle f_1 + f_2,\ f_3\rangle = \displaystyle\int_0^1 (f_1(t) + f_2(t))f_3(t)dt =$

$= \displaystyle\int_0^1 f_1(t)f_3(t)dt + \int_0^1 f_2(t)f_3(t)dt = \langle f_1,\ f_3\rangle + \langle f_2,\ f_3\rangle$

$p3 : \langle f_1,\ f_2\rangle = \displaystyle\int_0^1 f_1(t)f_2(t)dt = \int_0^1 f_2(t)f_1(t)dt = \langle f_2,\ f_1\rangle$

$p4 : \langle \alpha f_1,\ f_2\rangle = \displaystyle\int_0^1 \alpha f_1(t)f_2(t)dt = \alpha \int_0^1 f_1(t)f_2(t)dt = \alpha \langle f_1,\ f_2\rangle$ ■

Questão 2. $f(v_1, v_2) = 2x_1x_2 + x_1y_2 + x_2y_1 + y_1y_2 \quad v_1 = (x_1, y_1), v_2 = (x_2, y_2)$

$p1 : f(v_1, v_1) = 2x_1^2 + 2x_1y_1 + y_1^2 = x_1^2 + (x_1 + y_1)^2 \geq 0$

pois são números reais ao quadrado e somente 0 quando $v_1 = (0, 0)$

$p2 : f(v_1 + v_2, v_3) = 2(x_1 + x_2)x_3 + (x_1 + x_2)y_3 + x_3(y_1 + y_2) + (y_1 + y_2)y_3 =$

$= 2x_1x_3 + 2x_2x_3 + x_1y_3 + x_2y_3 + x_3y_1 + x_3y_2 + y_1y_3 + y_2y_3 = f(v_1, v_3) + f(v_2, v_3)$

$p3 : f(v_1, v_2) = 2x_1x_2 + x_1y_2 + x_2y_1 + y_1y_2 = 2x_2x_1 + x_2y_1 + x_1y_2 + y_2y_1 = f(v_2, v_1)$

$p4 : f(\alpha v_1, v_2) = 2\alpha x_1x_2 + \alpha x_1y_2 + \alpha x_2y_1 + \alpha y_1y_2 = \alpha(2x_1x_2 + x_1y_2 + x_2y_1 + y_1y_2) = \alpha f(v_1, v_2) \blacksquare$

Questão 3. Desigualdade triangular

$||v + w||^2 = \langle v + w,\ v + w\rangle = \langle v,\ v\rangle + 2\langle v,\ w\rangle + \langle w,\ w\rangle$

$\langle v,\ v\rangle + 2\langle v,\ w\rangle + \langle w,\ w\rangle \leq ||v||^2 + 2||v||||w|| + ||w||^2 = (||v|| + ||w||)^2$

$||v + w||^2 \leq (||v|| + ||w||)^2 \longrightarrow ||v + w|| \leq ||v|| + ||w||$

Questão 4. $\beta = \{(1, 2), (2, 1)\}$

$$v_1' = (1, 2) \quad u_1 = \frac{(1,2)}{||(1,2)||} = \frac{(1,2)}{\sqrt{5}} = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$v_2' = (2, 1) - \frac{\langle (2,1),\ (1,2)\rangle}{\langle (1,2),\ (1,2)\rangle}(1, 2) = (2, 1) - \frac{4}{5}(1, 2) = \left(\frac{6}{5}, -\frac{3}{5}\right)$$

$$u_2 = \frac{\left(\frac{6}{5}, -\frac{3}{5}\right)}{\left|\left|\left(\frac{6}{5}, -\frac{3}{5}\right)\right|\right|} = \frac{\left(\frac{6}{5}, -\frac{3}{5}\right)}{\frac{3}{\sqrt{5}}} = \left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$$

$$\beta' = \left\{\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)\right\}$$

Questão 5. $\beta = \{(1, 1, 0), (1, 0, 1), (0, 2, 0)\}$

$$v_1' = (1, 1, 0) \quad u_1 = \frac{(1,1,0)}{||(1,1,0)||} = \frac{(1,1,0)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$v_2' = (1, 0, 1) - \frac{\langle (1,0,1),\ (1,1,0)\rangle}{\langle (1,1,0),\ (1,1,0)\rangle}(1, 1, 0) = (1, 0, 1) - \frac{1}{2}(1, 1, 0) = \left(\frac{1}{2}, -\frac{1}{2}, 1\right)$$

$$u_2 = \frac{\left(\frac{1}{2}, -\frac{1}{2}, 1\right)}{\left|\left|\left(\frac{1}{2}, -\frac{1}{2}, 1\right)\right|\right|} = \frac{\left(\frac{1}{2}, -\frac{1}{2}, 1\right)}{\sqrt{\frac{3}{2}}} = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$v_3' = (0, 2, 0) - \frac{\left\langle (0,2,0),\ \left(\frac{1}{2}, -\frac{1}{2}, 1\right)\right\rangle}{\left\langle \left(\frac{1}{2}, -\frac{1}{2}, 1\right),\ \left(\frac{1}{2}, -\frac{1}{2}, 1\right)\right\rangle}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) - \frac{\langle (0,2,0),\ (1,1,0)\rangle}{\langle (1,1,0),\ (1,1,0)\rangle}(1, 1, 0)$$

$$(0, 2, 0) - \frac{-1}{\frac{3}{2}}\left(\frac{1}{2}, -\frac{1}{2}, 1\right) - \frac{2}{2}(1, 1, 0) = (0, 2, 0) + \left(\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}\right) - (1, 1, 0) = \left(-\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

$$u_3 = \frac{\left(-\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)}{\left|\left|\left(-\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)\right|\right|} = \frac{\left(-\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)}{\frac{2}{\sqrt{3}}} = \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\beta' = \left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$$

Questão 6. $\beta = \{(-1,1),(1,1)\}$ $\qquad \langle v_1,\ v_2\rangle = 2x_1x_2 + x_1y_2 + x_2y_1 + y_1y_2$

$$v_1' = (-1,1) \quad u_1 = \frac{(-1,1)}{||(-1,1)||} = \frac{(-1,1)}{1} = (-1,1)$$

$$v_2' = (1,1) - \frac{\langle(1,1),\ (-1,1)\rangle}{\langle(-1,1),\ (-1,1)\rangle}(-1,1) = (1,1) - \frac{-1}{1}(-1,1) = (0,2)$$

$$u_2 = \frac{(0,2)}{||(0,2)||} = \frac{(0,2)}{\sqrt{4}} = (0,1)$$

$$\beta' = \{(-1,1),(0,1)\}$$

Questão 7. $V = \{(x,y,z) \in \mathbb{R}^3; x - y + z = 0\}$

$$x - y + z = 0 \quad y = x + z \quad v = (x, x+z, z) = x(1,1,0) + y(0,1,1) = [(1,1,0),(0,1,1)]$$

$$v_1' = (1,1,0) \quad u_1 = \frac{(1,1,0)}{||(1,1,0)||} = \frac{(1,1,0)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$$v_2' = (0,1,1) - \frac{\langle(0,1,1),\ (1,1,0)\rangle}{\langle(1,1,0),\ (1,1,0)\rangle}(1,1,0) = (0,1,1) - \frac{1}{2}(1,1,0) = \left(-\frac{1}{2}, \frac{1}{2}, 1\right)$$

$$u_2 = \frac{\left(-\frac{1}{2}, \frac{1}{2}, 1\right)}{\left|\left|\left(-\frac{1}{2}, \frac{1}{2}, 1\right)\right|\right|} = \frac{\left(-\frac{1}{2}, \frac{1}{2}, 1\right)}{\sqrt{\frac{3}{2}}} = \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$\beta' = \left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{\sqrt{2}}{\sqrt{3}}\right)\right\}$$

Questão 8. $W = [(1,0,1),(1,1,0)] = (a+b, b, a)$

a) $\langle v,\ w\rangle$ **usual**

$$W^\perp = \{v \in \mathbb{R}^3; \langle v,\ (1,0,1)\rangle = 0 \wedge \langle v,\ (1,1,0)\rangle = 0\} \qquad v = (x,y,z)$$

$$\langle(x,y,z),\ (1,0,1)\rangle = x + z = 0 \quad \langle(x,y,z),\ (1,1,0)\rangle = x + y = 0$$

$$\begin{cases} x + z = 0 \longrightarrow z = -x \\ x + y = 0 \longrightarrow y = -x \end{cases} \quad v = (x, -x, -x)$$

$$W^\perp = [(1,-1,-1)]$$

b) $\langle (x,y,z),\ (x',y',z')\rangle = 2xx' + yy' + zz'$

$$\langle (x,y,z),\ ((1,0,1))\rangle = 2x + z = 0 \quad \langle x,y,z,\ (1,1,0)\rangle = 2x + y = 0$$

$$\begin{cases} 2x + z = 0 \longrightarrow z = -2x \\ 2x + y = 0 \longrightarrow z = -2x \end{cases} \quad v = (x, -2x, -2x)$$

$$W^{\perp} = [(1,-2,-2)]$$

Questão 9. $T(x,y,z) = (z, x-y, -z)$

a) $\langle v,\ w\rangle\ usual$

$$W = N(T) = \{(x,y,z) \in \mathbb{R}^3; (z, x-y, -z) = 0\}$$

$$\begin{cases} z = 0 \\ x - y = 0 \longrightarrow x = y \\ -z = 0 \end{cases} \quad W = (x,x,0) = [(1,1,0)]$$

$$\langle (a,b,c),\ (1,1,0)\rangle = a + b = 0 \longrightarrow b = -a \quad v = (a,-a,c) \qquad W^{\perp} = [(1,-1,0),(0,0,1)]$$

$$v_1' = (1,-1,0) \quad u_1 = \frac{(1,-1,0)}{||(1,-1,0)||} = \frac{(1,-1,0)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$v_2' = (0,0,1) - \frac{\langle (0,0,1),\ (1,-1,0)\rangle}{\langle (1,-1,0),\ (1,-1,0)\rangle}(1,-1,0) = (0,0,1) - \frac{0}{2}(1,-1,0) = (0,0,1)$$

$$\beta' = \left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), (0,0,1)\right\}$$

b) $\langle (x,y,z),\ (x',y',z')\rangle = 2xx' + yy' + 4zz'$

$$\langle (a,b,c),\ (1,1,0)\rangle = 2a + b \longrightarrow b = -2a \quad v = (a,-2a,c) \qquad W^{\perp} = [(1,-2,0),(0,0,1)]$$

$$v_1' = (1,-2,0) \quad u_1 = \frac{(1,-2,0)}{||(1,-2,0)||} = \frac{(1,-2,0)}{\sqrt{6}} = \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, 0\right)$$

$$v_2' = (0,0,1) - \frac{\langle (0,0,1),\ (1,-2,0)\rangle}{\langle (1,-2,0),\ (1,-2,0)\rangle}(1,-2,0) = (0,0,1) - \frac{0}{6}(1,-2,0) = (0,0,1)$$

$$\beta' = \left\{\left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, 0\right), (0,0,1)\right\}$$

Questão 10. $W = [(1,0,0),(0,1,1),(1,-1,-1)] = [(1,0,0),(0,1,1)]$

a)

$$\langle (x,y,z),\ (1,0,0)\rangle = x = 0$$

$$\langle (x,y,z),\ (0,1,1)\rangle = y + z = 0 \longrightarrow z = -y$$

$$W^{\perp} = (0,1,-1)$$

b) $Im(T) = W \quad N(T) = W^{\perp}$

$T(0,1,-1) = (0,0,0) \quad T(1,0,0) = (1,0,0) \quad T(0,1,1) = (0,1,1)$

$(x,y,z) = a(0,1,-1) + b(1,0,0) + c(0,1,1) = (b, a+c, c-a)$

$$\begin{cases} x = b \\ a + c = y \longrightarrow a = y - c \\ c - a = z \longrightarrow c + c - y = z \longrightarrow c = \dfrac{y+z}{2} \longrightarrow a = \dfrac{y-z}{2} \end{cases}$$

$$T(x,y,z) = \frac{y-z}{2}(0,0,0) + x(1,0,0) + \frac{y+z}{2}(0,1,1) = \left(x, \frac{y+z}{2}, \frac{y+z}{2}\right)$$

Questão 11. $\langle (x,y,z),\ (x',y',z') \rangle = xx' + 5yy' + 2zz'$

a)

$p1 : \langle (x,y,z),\ (x,y,z) \rangle = x^2 + 5y^2 + 2z^2 \geq 0$

pois são números reais ao quadrado e somente 0 quando $(x,y,z) = (0,0,0)$

$p2 : \langle (x+x', y+y', z+z'),\ (x'' + y'' + z'') \rangle = (x+x')x'' + 5(y+y')y'' + 2(z+z')z'' =$

$= xx'' + x'x'' + 5yy'' + 5y'y'' + 2zz'' + 2z'z'' = \langle (x,y,z),\ (x'',y'',z'') \rangle + \langle (x',y',z'),\ (x'',y'',z'') \rangle$

$p3 : \langle (x,y,z),\ (x',y',z') \rangle = xx' + 5yy' + 2zz' = x'x + 5y'y + 2z'z = \langle (x',y',z'),\ (x,y,z) \rangle$

$p4 : \langle \alpha(x,y,z),\ (x',y',z') \rangle = \alpha xx' + 5\alpha yy' + 2\alpha zz' = \alpha(xx' + 5yy' + 2zz') = \alpha \langle (x,y,z),\ (x',y',z') \rangle$ ■

b)

A base $\{(1,0,0),(0,1,0),(0,0,1)\}$ já é ortogonal, basta normaliza-la

$$u_1 = \frac{(1,0,0)}{||(1,0,0)||} = \frac{(1,0,0)}{1} = (1,0,0)$$

$$u_2 = \frac{(0,1,0)}{||(0,1,0)||} = \frac{(0,1,0)}{\sqrt{5}} = \left(0, \frac{1}{\sqrt{5}}, 0\right)$$

$$u_3 = \frac{(0,0,1)}{||(0,0,1)||} = \frac{(0,0,1)}{\sqrt{2}} = \left(0, 0, \frac{1}{\sqrt{2}}\right)$$

$$\beta' = \left\{(1,0,0), \left(0, \frac{1}{\sqrt{5}}, 0\right), \left(0, 0, \frac{1}{\sqrt{2}}\right)\right\}$$

Questão 12. $\langle f,\ g \rangle = \int_{-1}^{1} f(t)g(t)dt$

a) Sim, como visto na Questão 1. letra c)

b) $W = [(1), (1-t)]$

$v_1' = (1)$

$$v_2' = (1-t) - \frac{\langle (1-t),\ (1) \rangle}{\langle (1),\ (1) \rangle}(1) = (1-t) - \frac{2}{2}(1) = (-t)$$

$\beta' = \{(1), (-t)\}$

Questão 13. $S = [(1,0,1),(1,1,0),(2,1,1)] = [(1,0,1),(1,1,0)]$

a)

$$\langle (x,y,z),\ (1,0,1)\rangle = x + z = 0 \longrightarrow z = -x$$
$$\langle (x,y,z),\ (1,1,0)\rangle = x + y = 0 \longrightarrow y = -x$$
$$S^{\perp} = (x,-x,-x) = [(1,-1,-1)]$$

b)

$$\langle (1,0,1),\ (1,1,0)\rangle = 1 \neq 0$$

Não há base ortogonal para S pois S não é subespaço de $\mathbb{R}^3$

c)

Para S:

$$v_1' = (1,0,1)$$
$$v_2' = (1,1,0) - \frac{\langle (1,1,0),\ (1,0,1)\rangle}{\langle (1,0,1),\ (1,0,1)\rangle}(1,0,1) = (1,1,0) - \frac{1}{2}(1,0,1) = \left(\frac{1}{2}, 1, -\frac{1}{2}\right)$$
$$\beta' = \left\{(1,0,1), \left(\frac{1}{2}, 1, -\frac{1}{2}\right)\right\}$$

Para $S^{\perp}$:

$$v' = (1,-1,-1)$$
$$\beta' = \{(1,-1,-1)\}$$

Questão 14.

a) $2 + 5 = 7$

b) Sim, para o caso 2x2:

$$tr(AB) = tr\left(\begin{vmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{vmatrix}\right) = ae + bg + cf + dh$$
$$tr(BA) = tr\left(\begin{vmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{vmatrix}\right) = ae + cf + bg + hd$$
$$tr(AB) = tr(BA)$$

c) Sim, pois transpor uma matriz não altera os elementos da diagonal principal

d) Não, para o caso 2x2:

$$tr(A) = (tr(A^{-1}))^{-1}$$
$$a + d = \left(\frac{1}{ad - bc}(d + a)\right)^{-1}$$
$$a + d = \frac{ad - bc}{a + d}$$

Que apenas é valido quando $-bc = a^2 + d^2 + ad$

e) Não, para o caso 2x2:

$tr(AB) = tr(A)tr(B)$

$ae + bg + cf + dh = (a + d)(e + h)$

$ae + bg + cf + dh = ae + ah + de + dh$

Que apenas é valido quando $bg + cf = ah + de$

Questão 15.

a) $\langle A,\ B\rangle = tr(B^tA)$

$$p1 : \langle A,\ A\rangle = tr(A^tA) = tr\left(\begin{vmatrix} a & c \\ b & d \end{vmatrix}\begin{vmatrix} a & b \\ c & d \end{vmatrix}\right) = tr\left(\begin{vmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{vmatrix}\right) = a^2+b^2+c^2+d^2 \geq 0$$

pois são números reais ao quadrado e somente 0 quando $A = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$

$p2 : \langle A + B,\ C\rangle = tr(C^t(A + B)) = tr(C^tA + C^tB) = tr(C^tA) + tr(C^tB) =$

$= \langle A,\ C\rangle + \langle B,\ C\rangle$ pela propriedade do traço: $tr(A + B) = tr(A) + tr(B)$

$p3 : \langle A,\ B\rangle = \langle B,\ A\rangle$ pela propriedade do traço: $tr(AB) = tr(BA)$

$p4 : \langle \alpha A,\ B\rangle = \alpha \langle A,\ B\rangle$ pela propriedade do traço: $tr(\alpha A) = \alpha tr(A)$ ■

b) $\beta = \left\{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}\right\}$

$$v_1' = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad u_1 = \frac{1}{\sqrt{2}}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{vmatrix}$$

$$v_2' = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix}$$

$$u_2 = \frac{1}{\sqrt{\frac{3}{2}}}\begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{6}} \end{vmatrix}$$

$$v_3' = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix}, \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} \right\rangle}\begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \frac{0}{\frac{3}{2}}\begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} - \frac{2}{2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

$$u_3 = \frac{1}{\sqrt{1}} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

$$v_4' = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \right\rangle} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix}, \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} \right\rangle} \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} - \frac{\left\langle \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle}{\left\langle \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\rangle} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - \frac{1}{1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - \frac{1}{\frac{3}{2}} \begin{vmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{vmatrix} - \frac{2}{2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} -\frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{vmatrix}$$

$$u_4 = \frac{1}{\frac{1}{\sqrt{3}}} \begin{vmatrix} -\frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} \end{vmatrix} = \begin{vmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \end{vmatrix}$$

$$\beta' = \left\{ \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{vmatrix}, \begin{vmatrix} \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{6}} \end{vmatrix}, \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \end{vmatrix} \right\}$$

Questão 16. $A = (-1, 3) \quad B = (5, 2) \quad F = (3, 2)$

$AB = (5, 2) - (-1, 3) = (6, -1)$

$\mathbb{T} = \langle (6, -1),\ (3, 2) \rangle = 16$

Questão 17. $\langle (x_1, y_1),\ (x_2, y_2) \rangle = |x_2 - x_1| + |y_2 - y_1|$

Não define produto interno pois não satisfaz a propriedade 4:

$p4 : \langle \alpha(x_1, y_1),\ (x_2, y_2) \rangle = |x_2 - \alpha x_1| + |y_2 - \alpha y_1| =$

$\alpha \left(\left| \frac{x_2}{\alpha} - x_1 \right| + \left| \frac{y_2}{\alpha} - y_1 \right| \right) = \alpha \left\langle (x_1, y_1),\ \left(\frac{x_2}{\alpha}, \frac{y_2}{\alpha} \right) \right\rangle \neq \alpha \langle (x_1, y_1),\ (x_2, y_2) \rangle$

Questão 18.

$O = (0, 0, 0) \qquad OA = A - O = (1, 1, 0) \qquad ||OA|| = \sqrt{2}$

$A = (1, 1, 0) \qquad AB = B - A = (-1, 0, 1) \qquad ||AB|| = \sqrt{2} \qquad v = 1m/s$

$B = (0, 1, 1) \qquad BO = B - O = (0, -1, -1) \qquad ||BO|| = \sqrt{2}$

$OA : 0s \to 1s = \langle (1, 1, 0),\ (1, 1, 1) \rangle (1 - 0) = 2$

$OA : 1s \to \sqrt{2}s = \langle (1, 1, 0),\ (1, 1, -1) \rangle (\sqrt{2} - 1) = 2\sqrt{2} - 2$

$AB : \sqrt{2}s \to 2s = \langle (-1, 0, 1),\ (1, 1, -1) \rangle (2 - \sqrt{2}) = 2\sqrt{2} - 4$

$AB : 2s \to 2\sqrt{2}s = \langle (-1, 0, 1),\ (1, -1, 1) \rangle (2\sqrt{2} - 2) = 0$

$BO : 2\sqrt{2}s \to 3s = \langle (0, -1, -1),\ (1, -1, 1) \rangle (3 - 2\sqrt{2}) = 0$

$BO : 3s \to 4s = \langle (0, -1, -1),\ (-1, 1, 1) \rangle (4 - 3) = -2$

$BO: 4s \to 3\sqrt{2}s = \langle (0,-1,-1),\ (-1,-1,-1) \rangle\,(3\sqrt{2}-4) = 6\sqrt{2}-8$

$\mathbb{T} = \dfrac{10\sqrt{2}-14}{\sqrt{2}} = 10 - 7\sqrt{2}$

Questão 19. $\langle A,\ B \rangle = ae + 2bf + 3cg + dh$ $\qquad A = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}, B = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$

$\langle A,\ B \rangle = 1 + 2 + 12 + x = 15 + x$

$\langle A,\ B \rangle = \cos\theta ||A||||B||$ $\qquad \cos 90^\circ = 0$

$15 + x = 0 \longrightarrow x = -15$