Universidade Federal da Paraíba

Probabilidade II

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Questão 1.

a)

$$S_X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

X	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b)

$$S_X = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36\}$$

X	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Questão 2.

a)

$$\sum_{x=0}^{\infty} c2^{-x} = 1 \qquad \sum_{x=0}^{\infty} \left(\frac{1}{2}\right)^x = \frac{1}{c} \qquad \frac{1}{1 - \frac{1}{2}} = \frac{1}{c} \qquad \frac{1}{2} = \frac{1}{c} \qquad c = \frac{1}{2}$$

b)

$$P(X \leqslant 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$\frac{1}{2}\left(2^{-0} + 2^{-1} + 2^{-2}\right) \qquad \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) \qquad \frac{7}{8}$$

c)

$$P(X > 5) = 1 - P(X \leqslant 5) = P(X = 0) + P(X = 1) + P(X = 2) + ... + P(X = 5)$$

$$1 - \frac{1}{2}\left(2^{-0} + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5}\right)$$

$$1 - \frac{1}{2}\left(\frac{32}{32} + \frac{16}{32} + \frac{8}{32} + \frac{4}{32} + \frac{2}{32} + \frac{1}{32}\right) \qquad 1 - \frac{63}{64} \qquad \frac{1}{64}$$

d)

$$P(X \text{ ser ímpar}) = P(X = 2k + 1) = \frac{1}{2}(2^{-1} + 2^{-3} + 2^{-5} ...)$$

$$\frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{2k+1} \qquad \frac{1}{4}\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^{2k} \qquad \frac{1}{4}\sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^{k} \qquad \frac{1}{4}\cdot\frac{1}{1 - \frac{1}{4}} \qquad \frac{1}{3}$$

Questão 3.

$$F_X(x) = \int_{-\infty}^{x} 2t^{-3}\mathbb{I}_{(1,\infty)}(x)dt \qquad \int_1^x 2t^{-3}dt \qquad -t^{-2}\Big|_1^x \qquad (-x^{-2}+1)\mathbb{I}_{(1,\infty)}(x)$$

$$\int_{0.5}^{2} 2x^{-3}\mathbb{I}_{(1,\infty)}(x)dx \qquad \int_1^2 2x^{-3}dx \qquad -x^{-2}\Big|_1^2 \qquad -\frac{1}{4}+1 \qquad \frac{3}{4}$$

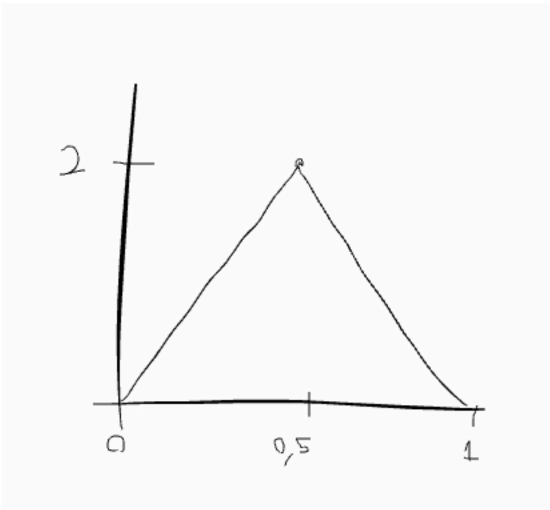
Questão 4.

a)

$$\int_0^{0.5} cxdx + \int_{0.5}^{1} c - cxdx = 1 \qquad \frac{cx^2}{2}\Big|_0^{0.5} + c\left(x - \frac{x^2}{2}\right)\Big|_{0.5}^{1} = 1$$

$$\frac{c}{8} + c\left(\frac{1}{2} - \frac{3}{8}\right) = 1 \qquad \frac{c}{8} + \frac{c}{8} = 1 \qquad \frac{c}{4} = 1 \qquad c = 4$$

b)



c)

$$4\int_{0.8}^{1} 1 - xdx \qquad 4\left(x - \frac{x^2}{2}\right) \qquad 4(0.5 - 0.8 + 0.32) \qquad 4(0.02) \qquad 0.08$$

Para $P(0.25 < x < 0.75)$, percebe-se que é um intervalo simétrico em uma função simétrica em torno de $x = 0.5$, portanto basta calcular $2P(0.25 < x < 0.5)$.

$$2 \cdot 4\int_{0.25}^{0.5} xdx \qquad 8\frac{x^2}{2}\Big|_{0.25}^{0.5} \qquad 1 - \frac{1}{4} \qquad \frac{3}{4}$$

Questão 5.

a)

$$\int_{\mathbb{R}} cx^2\mathbb{I}_{(-1,1)}(x)dx = 1 \qquad \int_{-1}^{1} cx^2dx = 1 \qquad c\frac{x^3}{3}\Big|_{-1}^{1} = 1 \qquad \frac{c}{3} + \frac{c}{3} = 1 \qquad c = \frac{3}{2}$$

b)

$$P\left(|X| > \frac{1}{2}\right) = P\left(X < -\frac{1}{2}\right) + P\left(X > \frac{1}{2}\right)$$

$$\int_{-1}^{-0.5} \frac{3}{2}x^2dx + \int_{0.5}^{1} \frac{3}{2}x^2dx \qquad \left.\frac{x^3}{2}\right|_{-1}^{-0.5} + \left.\frac{x^3}{2}\right|_{0.5}^{1}$$

$$\left(-\frac{1}{16}+\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{16}\right) \qquad \frac{7}{16}+\frac{7}{16} \qquad \frac{7}{8}$$

Questão 6.

a)

$$\int_{\mathbb{R}} \lambda e^{-\lambda x}\mathbb{I}_{(0,\infty)}(x)dx = 1 \qquad \lambda\int_0^\infty e^{-\lambda x}dx = 1$$

$$\lambda\left.\left(\frac{-e^{-\lambda x}}{\lambda}\right)\right|_0^\infty = 1 \qquad \lambda\left(0+\frac{1}{\lambda}\right) = 1 \qquad 1 = 1 \qquad \text{é função densidade se } \lambda > 0$$

b)

$$F_X(x) = \int_{-\infty}^{x} \lambda e^{-\lambda t}\mathbb{I}_{(0,\infty)}dt \qquad \lambda\int_0^x e^{-\lambda t}dt$$

$$\lambda\left.\left(\frac{-e^{-\lambda t}}{\lambda}\right)\right|_0^x \qquad \left.-e^{-\lambda t}\right|_0^x \qquad -e^{-\lambda x}+e^0 \qquad 1-e^{-\lambda x}$$

c)

$$P(X \geqslant 6) = 1 - P(X < 6)$$

$$1-(1-e^{-6\lambda}) \qquad e^{-6\lambda}$$

Questão 7.

a)

$$\Gamma(\alpha+1) = \int_0^\infty t^\alpha e^{-t}dt \qquad \begin{cases} u = t^\alpha \implies du = \alpha t^{\alpha-1}dt \\ v = -e^{-t} \implies dv = e^{-t}dt \end{cases}$$

$$\left.(-e^{-t}t^\alpha)\right|_0^\infty + \alpha\int_0^\infty t^{\alpha-1}e^{-t} \qquad \alpha\Gamma(\alpha)$$

b)

$$\Gamma(1) = \int_0^\infty t^{1-1}e^{-t}dt \qquad \int_0^\infty e^{-t}dt \qquad \left.-e^{-t}\right|_0^\infty \qquad 0+1 \qquad \Gamma(1) = 1$$

c)

$$\Gamma(n+1) = n\Gamma(n)$$

$$\Gamma(n+1) = n(n-1)\Gamma(n-1)$$

$$\Gamma(n+1) = n(n-1)(n-2)\Gamma(n-2)$$

$$\Gamma(n+1) = ...$$

$$\Gamma(n+1) = n(n-1)(n-2)\cdot ... \cdot 2 \cdot 1$$

$$\Gamma(n+1) = n!$$

d)

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^\infty t^{-\frac{1}{2}} e^{-t} dt \qquad t = u^2/2 \qquad dt = udu$$

$$\int_0^\infty \left(\frac{u^2}{2}\right)^{-\frac{1}{2}} e^{-\frac{u^2}{2}} udu \qquad \sqrt{2}\int_0^\infty u^{-1} e^{-\frac{u^2}{2}} udu \qquad \sqrt{2}\int_0^\infty e^{-\frac{u^2}{2}} du$$

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \qquad \Phi(\infty) = \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 1 \qquad \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = \frac{1}{2}$$

$$\sqrt{2}\sqrt{2\pi}\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du \qquad 2\sqrt{\pi}\cdot\frac{1}{2} \qquad \sqrt{\pi}$$

Questão 8.

$$\int_{-\infty}^\infty \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \mathbb{I}_{(0,\infty)}(x) dx = 1$$

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty x^{\alpha-1} e^{-\beta x} dx = 1 \qquad t = \beta x \qquad dt = \beta dx$$

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty \left(\frac{t}{\beta}\right)^{\alpha-1} e^{-t} \frac{1}{\beta} dt = 1$$

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{\beta} \frac{1}{\beta^{\alpha-1}} \int_0^\infty t^{\alpha-1} e^{-t} dt = 1$$

$$\frac{1}{\Gamma(\alpha)} \Gamma(\alpha) = 1$$

$$1 = 1$$