Atividade 1

Probabilidade Condicional

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Considere um espaço de probabilidade $(\Omega, \mathcal{A}, \mathrm{P})$ e assuma que todos os eventos mencionados embaixo pertencem à σ-álgebra $\mathcal{A}$.

Note que se os eventos D_i são disjuntos, então $C \cap D_i$ também são disjuntos e podemos aplicar o terceiro axioma de probabilidade em $\mathrm{P}\big(\bigcup_i (C \cap D_i)\big)$.

a) Mostre que se os eventos D_i são disjuntos e $\mathrm{P}\Big(C \,\Big|\, D_i\Big) = p$, para todo i, então $\mathrm{P}\Big(C \,\Big|\, \bigcup_i D_i\Big) = p$.

$$\mathrm{P}\Big(C \,\Big|\, D_i\Big) = \frac{\mathrm{P}(C \cap D_i)}{\mathrm{P}(D_i)} = p$$

$$\mathrm{P}(D_i) = \frac{1}{p}\mathrm{P}(C \cap D_i)$$

$$\mathrm{P}\left(C \,\middle|\, \bigcup_i D_i\right) = \frac{\mathrm{P}\big(C \cap \bigcup_i D_i\big)}{\mathrm{P}\big(\bigcup_i D_i\big)} = \frac{\mathrm{P}\big(\bigcup_i (C \cap D_i)\big)}{\mathrm{P}\big(\bigcup_i D_i\big)} = \frac{\sum_i \mathrm{P}(C \cap D_i)}{\sum_i \mathrm{P}(D_i)} = \frac{\sum_i \mathrm{P}(C \cap D_i)}{\sum_i \frac{1}{p}\mathrm{P}(C \cap D_i)} = p$$

b) Mostre que se os eventos C_i são disjuntos, então $\mathrm{P}\Big(\bigcup_i C_i \,\Big|\, D\Big) = \sum_i \mathrm{P}\Big(C_i \,\Big|\, D\Big)$.

$$\mathrm{P}\left(\bigcup_i C_i \,\middle|\, D\right) = \frac{\mathrm{P}\big(D \cap \bigcup_i C_i\big)}{\mathrm{P}(D)} = \frac{\mathrm{P}\big(\bigcup_i (C_i \cap D)\big)}{\mathrm{P}(D)} = \frac{\sum_i \mathrm{P}(C_i \cap D)}{\mathrm{P}(D)} = \sum_i \mathrm{P}\Big(C_i \,\Big|\, D\Big)$$

c) Mostre que se os eventos E_i são disjuntos e $\bigcup_i E_i = \Omega$, então $\mathrm{P}\left(C \,\middle|\, D\right) = \sum_i \mathrm{P}\left(E_i \,\middle|\, D\right)\mathrm{P}\left(C \,\middle|\, E_i \cap D\right)$.

$$\sum_i \mathrm{P}\left(E_i \,\middle|\, D\right)\mathrm{P}\left(C \,\middle|\, E_i \cap D\right) = \sum_i \frac{\mathrm{P}(E_i \cap D)}{\mathrm{P}(D)} \frac{\mathrm{P}(C \cap E_i \cap D)}{\mathrm{P}(E_i \cap D)} =$$

$$\sum_i \frac{\mathrm{P}(C \cap E_i \cap D)}{\mathrm{P}(D)} = \sum_i \mathrm{P}\left(C \cap E_i \,\middle|\, D\right) =$$

$$\mathrm{P}\left(\bigcup_i (C \cap E_i) \,\middle|\, D\right) = \mathrm{P}\left(C \cap \bigcup_i E_i \,\middle|\, D\right) = \mathrm{P}\left(C \cap \Omega \,\middle|\, D\right) = \mathrm{P}\left(C \,\middle|\, D\right)$$

d) Mostre que se os eventos C_i são disjuntos e $\mathrm{P}\left(A \,\middle|\, C_i\right) = \mathrm{P}\left(B \,\middle|\, C_i\right)$, para todo i, então $\mathrm{P}\left(A \,\middle|\, \bigcup_i C_i\right) = \mathrm{P}\left(B \,\middle|\, \bigcup_i C_i\right)$.

$$\mathrm{P}\left(A \,\middle|\, C_i\right) = \mathrm{P}\left(B \,\middle|\, C_i\right) \implies \frac{\mathrm{P}(A \cap C_i)}{\mathrm{P}(C_i)} = \frac{\mathrm{P}(B \cap C_i)}{\mathrm{P}(C_i)} \implies \mathrm{P}(A \cap C_i) = \mathrm{P}(B \cap C_i)$$

$$\mathrm{P}\left(A \,\middle|\, \bigcup_i C_i\right) = \frac{\mathrm{P}\left(A \cap \bigcup_i C_i\right)}{\mathrm{P}\left(\bigcup_i C_i\right)} = \frac{\mathrm{P}\left(\bigcup_i (A \cap C_i)\right)}{\mathrm{P}\left(\bigcup_i C_i\right)} = \frac{\sum_i \mathrm{P}(A \cap C_i)}{\sum_i \mathrm{P}(C_i)}$$

$$\frac{\sum_i \mathrm{P}(B \cap C_i)}{\sum_i \mathrm{P}(C_i)} = \frac{\mathrm{P}\left(\bigcup_i (B \cap C_i)\right)}{\mathrm{P}\left(\bigcup_i C_i\right)} = \frac{\mathrm{P}\left(B \cap \bigcup_i C_i\right)}{\mathrm{P}\left(\bigcup_i C_i\right)} = \mathrm{P}\left(B \,\middle|\, \bigcup_i C_i\right)$$