UNIVERSIDADE FEDERAL DA PARAÍBA
Cálculo II
Terceira prova
Paulo Ricardo Seganfredo Campana - 20210044220

Questão 1.

i) $z = \arcsin(x - y), \quad x = s^2 + t^2, \quad y = 1 - 2st$

$\dfrac{\partial z}{\partial s}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \frac{1}{\sqrt{1-(x-y)^2}} \cdot 2s + \frac{1}{\sqrt{1-(x-y)^2}} \cdot (-1)(-2t) =$$

$$x - y = s^2 + t^2 + 2st - 1 = (s+t)^2 - 1$$

$$\frac{\partial z}{\partial s} = \frac{2s + 2t}{\sqrt{1 - ((s+t)^2 - 1)^2}}$$

$$1 - ((s+t)^2 - 1)^2 = 1 - ((s+t)^4 + 1 - 2(s+t)^2) =$$

$$-(s+t)^4 + 2(s+t)^2 = (s+t)^2(2 - (s+t)^2)$$

$$\frac{\partial z}{\partial s} = \frac{2(s+t)}{\sqrt{(s+t)^2(2-(s+t)^2)}} = \frac{2(s+t)}{(s+t)\sqrt{2-(s+t)^2}} = \frac{2}{\sqrt{2-(s+t)^2}}$$

$\dfrac{\partial z}{\partial t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{1}{\sqrt{1-(x-y)^2}} \cdot 2t + \frac{1}{\sqrt{1-(x-y)^2}} \cdot (-1)(-2s)$$

$$= \frac{2}{\sqrt{2-(s+t)^2}}$$

ii) $z = \tan\left(\dfrac{x}{y}\right), \quad x = 2s + 3t, \quad y = 3s - 2t$

$\dfrac{\partial z}{\partial s}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = \sec^2\left(\frac{x}{y}\right) \cdot \frac{1}{y} \cdot 2 + \sec^2\left(\frac{x}{y}\right)\left(-\frac{1}{y^2}\right) \cdot x \cdot 3 = \sec^2\left(\frac{x}{y}\right)\left(\frac{2}{y} - \frac{3x}{y^2}\right)$$

$$= \sec^2\left(\frac{2s+3t}{3s-2t}\right)\left(\frac{2(3s-2t) - 3(2s+3t)}{(3s-2t)^2}\right) = \sec^2\left(\frac{2s+3t}{3s-2t}\right)\left(\frac{-13t}{(3s-2t)^2}\right)$$

$\dfrac{\partial z}{\partial t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \sec^2\left(\frac{x}{y}\right) \cdot \frac{1}{y} \cdot 3 + \sec^2\left(\frac{x}{y}\right)\left(-\frac{1}{y^2}\right) \cdot x \cdot (-2) = \sec^2\left(\frac{x}{y}\right)\left(\frac{3}{y} + \frac{2x}{y^2}\right)$$

$$= \sec^2\left(\frac{2s+3t}{3s-2t}\right)\left(\frac{3(3s-2t) + 2(2s+3t)}{(3s-2t)^2}\right) = \sec^2\left(\frac{2s+3t}{3s-2t}\right)\left(\frac{13s}{(3s-2t)^2}\right)$$

Questão 2. $f(x,y) = 4x - 3x^3 - 2xy^2$

i)

$$\frac{\partial f}{\partial x} = 4 - 9x^2 - 2y^2 = 0 \longrightarrow 9x^2 + 2y^2 = 4$$

$$\frac{\partial f}{\partial y} = -4xy = 0 \longrightarrow xy = 0 \longrightarrow x = 0 \text{ ou } y = 0$$

$$\text{se } x = 0 \longrightarrow 2y^2 = 4 \longrightarrow y^2 = 2 \longrightarrow y = \pm\sqrt{2}$$

$$\text{se } y = 0 \longrightarrow 9x^2 = 4 \longrightarrow x^2 = \frac{9}{4} \longrightarrow x = \pm\frac{3}{2}$$

ii)

$$f_{xx} = -18x$$

$$f_{xy} = -4y \qquad D = \begin{vmatrix} -18x & -4y \\ -4y & -4x \end{vmatrix} \qquad detD = 72x^2 - 16y^2$$

$$f_{yy} = -4x$$

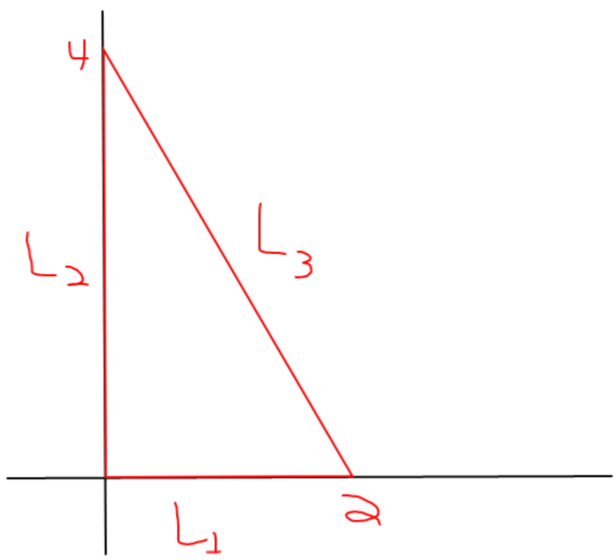
em $(0, \pm\sqrt{2})$, $\quad detD < 0$, $\quad$ ponto de sela

em $\left(\pm\frac{2}{3}, 0\right)$, $\quad detD > 0$

para $\left(\frac{2}{3}, 0\right)$, $\quad f_{xx} < 0$, $\quad$ ponto de máximo

para $\left(-\frac{2}{3}, 0\right)$, $\quad f_{xx} > 0$, $\quad$ ponto de mínimo

Questão 3. $f(x,y) = x + y - xy$



$L_1 : y = 0, \quad 0 \leq x \leq 2, \quad f(x,0) = x$

$$f(0,0) = 0$$

$$f(2,0) = 2$$

$L_2 : x = 0, \quad 0 \leq y \leq 4, \quad f(0,y) = y$

$$f(0,0) = 0$$

$$f(0,4) = 4$$

$L_3 : y = 4 - 2x, \quad f(x, 4-2x) = x + 4 - 2x - 4x + 2x^2 = 2x^2 - 5x + 4$

$$\frac{d}{dx}(2x^2 - 5x + 4) = 4x - 5 = 0 \longrightarrow 4x = 5 \longrightarrow x = \frac{5}{4}$$

$$f\left(\frac{5}{4}, 4 - \frac{10}{4}\right) = f\left(\frac{5}{4}, \frac{6}{4}\right) = \frac{5}{4} + \frac{6}{4} - \frac{30}{16} = \frac{7}{8}$$

$(0,0)$ é ponto de mínimo

$(0,4)$ é ponto de máximo

Questão 4. $g(x,y,z) = y^2 - xz = 9$

$d^2 = x^2 + y^2 + z^2 = f(x,y,z)$

$\nabla f = \lambda \nabla g$

$$\begin{cases} 2x = \lambda(-z) \longrightarrow z = -\dfrac{2x}{\lambda} \\ 2y = \lambda 2y \longrightarrow \lambda = 1 \\ 2z = \lambda(-x) \longrightarrow 2\dfrac{-2x}{\lambda} = -x\lambda \longrightarrow -4x = -x\lambda^2 \longrightarrow \lambda = \pm 2 \end{cases}$$

para $\lambda = 2$:

$$\begin{cases} 2x = -2z \longrightarrow z = -x \\ 2y = 4y \longrightarrow y = 0 \\ 2z = -2x \end{cases}$$

$g(x, 0, -x) = -x^2 = 9, \quad x \notin \mathbb{R}$

para $\lambda = -2$:

$$\begin{cases} 2x = 2z \longrightarrow z = x \\ 2y = 4y \longrightarrow y = 0 \\ 2z = 2x \end{cases}$$

$g(x, 0, x) = x^2 = 9, \quad x = \pm 3$

para $\lambda = 1$:

$$\begin{cases} 2x = -z \longrightarrow z = -2x \\ 2y = 2y \\ 2z = -x \longrightarrow -4x = -x \longrightarrow x = 0, \quad z = 0 \end{cases}$$

$g(0, y, 0) = y^2 = 9 \longrightarrow y = \pm 3$

$f(\pm 3, 0, \pm 3) = 18$

$f(0, \pm 3, 0) = 9$

portanto os pontos mais próximos a origem são $(0, 3, 0)$ e $(0, -3, 0)$

Questão 5.

$f(x, y, z) = x^2 + y^2 + z^2$

$g(x, y, z) = x + 2y + z = 3$

$h(x, y, z) = x - y = 4$

$\nabla f = \lambda \nabla g + \mu \nabla h$

$$\begin{cases} 2x = \lambda + \mu \\ 2y = 2\lambda - \mu \\ 2z = \lambda \end{cases} \longrightarrow \begin{cases} 2x = 2z + \mu \\ 2y = 4z - \mu \\ \end{cases} \longrightarrow 2x + 2y = 6z \longrightarrow x + y = 3z$$

$$\begin{aligned} g(x, y, z) &= x + 2y + z = 3 \\ &3z + y + z = 3 \\ &y = 3 - 4z \end{aligned}$$

$$\begin{aligned} h(x, y, z) &= x - y = 4 \\ &x - 3 + 4z = 4 \\ &x = 7 - 4z \end{aligned}$$

$$\begin{aligned} g(x, y, z) &= x + 2y + z = 3 \\ &7 - 4z + 6 - 8z + z = 3 \\ &-11z = -10 \\ &z = \frac{10}{11} \end{aligned}$$

$x = 7 - \dfrac{40}{11} = \dfrac{37}{11}$

$y = 3 - \dfrac{40}{11} = -\dfrac{7}{11}$

$\left(\dfrac{37}{11}, -\dfrac{7}{11}, \dfrac{10}{11}\right)$ é ponto de mínimo

$$\frac{3}{4} \sum_{n=1}^{\infty} \frac{1}{16^n}$$