Universidade Federal da Paraíba
Probabilidade II - Primeira prova
Paulo Ricardo Seganfredo Campana

Questão 1.

a)

$$F'(x) = f(x)$$

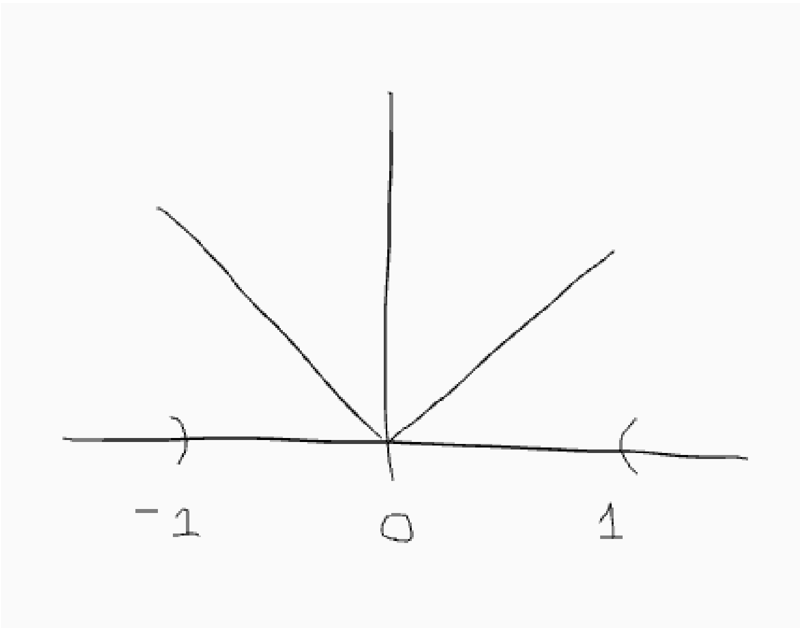
$$\frac{d}{dx}(0) = 0$$

$$\frac{d}{dx}\left(-\frac{1}{2}(x^2 - 1)\right) = -x$$

$$\frac{d}{dx}\left(\frac{1}{2}(x^2 + 1)\right) = x$$

$$\frac{d}{dx}(1) = 0$$

$$f(x) = \begin{cases} -x, \text{ se } -1 < x < 0, \\ x, \quad \text{se } 0 < x < 1, \\ 0, \quad \text{caso contrário.} \end{cases}$$



$$S_X = [-1, 1]$$

b)

Os pontos -1 e 1, pois as derivadas laterais são diferentes.

Para $x = -1^-$ a derivada no ponto é 0
Para $x = -1^+$ a derivada no ponto é 1
Para $x = 1^-$ a derivada no ponto é 1
Para $x = 1^+$ a derivada no ponto é 0

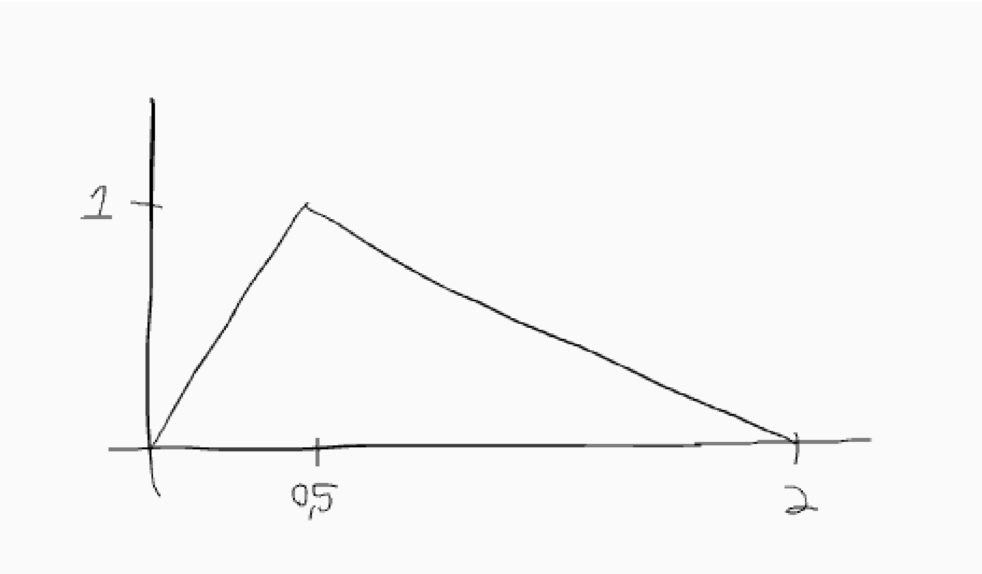
c)

$$P\left(-\frac{1}{2} < x < \frac{1}{2}\right) = P\left(-\frac{1}{2} < x < 0\right) + P\left(0 < x < \frac{1}{2}\right)$$

$$\int_{-\frac{1}{2}}^{0} -xdx + \int_{0}^{\frac{1}{2}} xdx \qquad \left(-\frac{x^2}{2}\right)\Bigg|_{-\frac{1}{2}}^{0} + \left(\frac{x^2}{2}\right)\Bigg|_{0}^{\frac{1}{2}} \qquad \left(0+\frac{1}{8}\right)+\left(\frac{1}{8}-0\right) \qquad \frac{1}{4}$$

Questão 2.

a)



$f(x) \geqslant 0$ Sim.

$$\int_{0}^{\frac{1}{2}} 2xdx + \int_{\frac{1}{2}}^{2} -\frac{2}{3}x + \frac{4}{3}dx = 1 \qquad x^2\Big|_{0}^{\frac{1}{2}} + \left(-\frac{x^2}{3}+\frac{4x}{3}\right)\Bigg|_{\frac{1}{2}}^{2} = 1$$

$$\left(\frac{1}{4}-0\right)+\left(\frac{4}{3}-\frac{7}{12}\right) = 1 \qquad \frac{1}{4}+\frac{3}{4} = 1 \qquad 1 = 1$$

$S_X = [0, 2]$

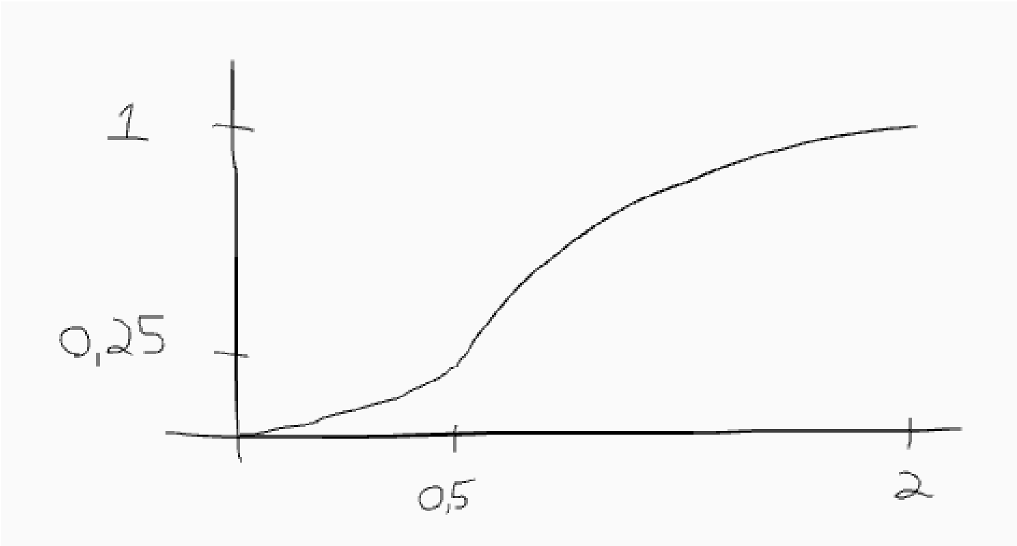
b)

$$F_X(x) \text{ para } \left[0, \frac{1}{2}\right] = \int_{0}^{x} 2tdt \qquad t^2\Big|_{0}^{x} \qquad x^2$$

$$F_X(x) \text{ para } \left[\frac{1}{2}, 2\right] = \int_{\frac{1}{2}}^{x} -\frac{2}{3}t + \frac{4}{3}dt \qquad \frac{1}{3}\int_{\frac{1}{2}}^{x} -2t + 4dt \qquad \frac{1}{3}\left(-t^2+4t\right)\Big|_{\frac{1}{2}}^{x}$$

$$\frac{1}{3}(-x^2+4x-1)$$

$$F_X(x) = \begin{cases} x^2, & \text{se } \leqslant x <\leqslant \frac{1}{2}, \\ \\ \dfrac{1}{3}(-x^2+4x-1), & \text{se } \frac{1}{2} \leqslant x \leqslant 2, \\ \\ 0, & \text{caso contrário.} \end{cases}$$



c)

$$P(0 < x < 1) = P\left(0 < x < \frac{1}{2}\right) + P\left(\frac{1}{2} < x < 1\right)$$

$$\int_0^{\frac{1}{2}} 2x dx + \int_{\frac{1}{2}}^1 \frac{1}{3}(-2x+4)dx \qquad x^2\Big|_0^{\frac{1}{2}} + \frac{1}{3}(-x^2+4x)\Big|_{\frac{1}{2}}^1 \qquad \left(\frac{1}{4}-0\right) + \frac{1}{3}\left(3-\frac{7}{4}\right)$$

$$\frac{1}{4} + \frac{1}{3}\cdot\frac{5}{4} \qquad \frac{3}{12} + \frac{5}{12} \qquad \frac{8}{12} \qquad \frac{3}{4}$$