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Combinatorial (1)

1.1 Permutations

1.1.1 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

1.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

1.1.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

1.2 Partitions and subsets

1.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

1.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$.

1.3 General purpose numbers

1.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$
$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

1.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

1.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

1.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

1.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

1.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

1.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

Data Structures (2)

Bit2dSparse.h

<bits/stdc++.h> a7b778, 19 lines

```
// Time complexity : O(Q log(N)^2)
// Space complexity : O(Q log(N))
namespace Bit2d{
  OrderedSet bit[MAXN];
  void add(int x, int y){
    for(int i = x; i < MAXN; i += i & -i)
      bit[i].insert(mp(y, x));
  }
  void remove(int x, int y){
    for(int i = x; i < MAXN; i += i & -i)
      bit[i].erase(mp(y, x));
  }
  int get(int x, int y){
    int ans = 0;
    for(int i = x; i > 0; i -= i & -i)
```

```
    ans += bit[i].order_of_key(mp(y+1, 0));
    return ans;
  }
};
```

BitRange.h

<bits/stdc++.h> 0e82ae, 36 lines

```
using namespace std;
class BitRange{
private:
  typedef long long t_bit;
  vector<t_bit> bit1, bit2;
  t_bit get(vector<t_bit> &bit, int i){
    t_bit sum = 0;
    for (; i > 0; i -= (i & -i))
      sum += bit[i];
    return sum;
  }
  void add(vector<t_bit> &bit, int i, t_bit value){
    for (; i < (int)bit.size(); i += (i & -i))
      bit[i] += value;
  }
public:
  BitRange(int n){
    bit1.assign(n + 1, 0);
    bit2.assign(n + 1, 0);
  }
  //1-indexed [i, j]
  void add(int i, int j, t_bit v){
    add(bit1, i, v);
    add(bit1, j + 1, -v);
    add(bit2, i, v * (i - 1));
    add(bit2, j + 1, -v * j);
  }
  //1-indexed
  t_bit get(int i){
    return get(bit1, i) * i - get(bit2, i);
  }
  //1-indexed [i,j]
  t_bit get(int i, int j){
    return get(j) - get(i - 1);
  }
};
```

ImplicitTreap.h

<bits/stdc++.h> 16a318, 129 lines

```
using namespace std;
namespace ITreap{
  const int N = 500010;
  typedef long long treap_t;
  treap_t X[N];
  int en = 1, Y[N], sz[N], L[N], R[N], P[N], root;
  const treap_t neutral = 0;
  treap_t op_val[N];
  bool rev[N];
  inline treap_t join(treap_t a, treap_t b, treap_t c){
    return a + b + c;
  }
  void calc(int u) { // update node given children info
    if(L[u]) P[L[u]] = u;
    if(R[u]) P[R[u]] = u;
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
    op_val[u] = join(op_val[L[u]], X[u], op_val[R[u]]);
  }
  void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
    if (rev[u]){
      if(L[u]) rev[L[u]] ^= rev[u];
      if(R[u]) rev[R[u]] ^= rev[u];
      swap(L[u], R[u]);
      rev[u] = false;
    }
  }
  void split(int u, int s, int &l, int &r) { // l gets first s,
       r gets remaining
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split(R[u], s - sz[L[u]] - 1, l, r); R[u
        ] = l; l = u; }
    else { split(L[u], s, l, r); L[u] = r; r = u; }
    P[u] = 0;
    calc(u);
  }
  int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r);
    if(!l || !r) return l + r;
    int u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l;}
    else { L[r] = merge(l, L[r]); u = r;}
    P[u] = 0;
    calc(u);
    return u;
  }
  int new_node(treap_t x){
    P[en] = 0;
    X[en] = x;
    op_val[en] = x;
    rev[en] = false;
    return en++;
  }
  int nth(int u, int idx){
    if(!u)
      return 0;
    unlaze(u);
    if(idx <= sz[L[u]])
      return nth(L[u], idx);
    else if(idx == sz[L[u]] + 1)
      return u;
    else
      return nth(R[u], idx - sz[L[u]] - 1);
  }
//Public
  void init(int n=N-1) { // call before using other funcs
    //init position 0
    sz[0] = 0;
    op_val[0] = neutral;
    //init Treap
    root = 0;
    std::mt19937 rng((int) std::chrono::steady_clock::now().
        time_since_epoch().count());
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i
        ] = R[i] = 0; }
    shuffle(Y + 1, Y + n + 1, rng);
  }
  //0-indexed
  int insert(int idx, int val){
    int a, b;
    split(root, idx, a, b);
    int node = new_node(val);
    root = merge(merge(a, node), b);
    return node;
  }
  //0-indexed
  void erase(int idx){
    int a, b, c, d;
    split(root, idx, a, b);
    split(b, 1, c, d);
    root = merge(a, d);
  }
  //0-indexed
  treap_t nth(int idx){
    int u = nth(root, idx+1);
    return X[u];
  }
  //0-indexed [l, r]
  treap_t query(int l, int r){
    if(l > r) swap(l, r);
    int a, b, c, d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    treap_t ans = op_val[b];
    root = merge(a, merge(b, c));
    return ans;
  }
  //0-indexed [l, r]
  void reverse(int l, int r){
    if (l > r) swap(l, r);
    int a, b, c, d;
    split(root, l, a, d);
    split(d, r - l + 1, b, c);
    if(b)
      rev[b] ^= 1;
    root = merge(a, merge(b, c));
  }
  int getPos(int node){
    int ans = sz[L[node]];
    while(P[node]){
      if(L[P[node]] == node){
        node = P[node];
      }else{
        node = P[node];
        ans += sz[L[node]] + 1;
      }
    }
    return ans;
  }
};
```

LineContainer.h

<bits/stdc++.h> 7fb8cf, 32 lines

```
using ll = long long;
using namespace std;
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b);
  }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
```

```
      isect(x, erase(y));
  }
  ll getMax(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

PolicyBasedTree.h

<bits/stdc++.h>, <ext/pb_ds/tree_policy.hpp>, <ext/pb_ds/assoc_container.hpp> 85999f, 6 lines

```
using namespace __gnu_pbds;
using namespace std;
template <class T> using ordered_set = tree<T, null_type, less<
    T>, rb_tree_tag, tree_order_statistics_node_update>;
template <class K, class V> using ordered_map = tree<K, V, less
    <K>, rb_tree_tag, tree_order_statistics_node_update>;
//order_of_key (k) : Number of items strictly smaller than k .
//find_by_order(k) : K-th element in a set (counting from zero)
    .
```

QueueQuery.h

<bits/stdc++.h> cdb887, 41 lines

```
using namespace std;
class QueueQuery{
private:
  typedef long long t_queue;
  stack<pair<t_queue, t_queue>> s1, s2;
  t_queue cmp(t_queue a, t_queue b){
    return min(a, b);
  }
  void move(){
    if (s2.empty()){
      while (!s1.empty()){
        t_queue element = s1.top().first;
        s1.pop();
        t_queue result = s2.empty() ? element : cmp(element, s2
            .top().second);
        s2.push({element, result});
      }
    }
  }
public:
  void push(t_queue x){
    t_queue result = s1.empty() ? x : cmp(x, s1.top().second);
    s1.push({x, result});
  }
  void pop(){
    move();
    s2.pop();
  }
  t_queue front(){
    move();
    return s2.top().first;
  }
  t_queue query(){
    if (s1.empty() || s2.empty())
      return s1.empty() ? s2.top().second : s1.top().second;
    else
      return cmp(s1.top().second, s2.top().second);
  }
  t_queue size(){
    return s1.size() + s2.size();
  }
};
```

QueryTree.h

<bits/stdc++.h> 7552d0, 54 lines

```
using namespace std;
typedef pair<int, int> Query; // Anything that can be activated
    for a period of time
int n;
int getAnswer();
void rollback(int t);
void insert(Query q);
int getLastVersion();
namespace QueryTree{
  const int MAXN = 200010;
  vector<Query> queries[4*MAXN];
  int T;
  void addQuery(int node, int i, int j, int a, int b, Query &q)
      {
    if ((i > b) or (j < a))
      return;
    if (a <= i and j <= b){
      queries[node].push_back(q);
      return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    addQuery(l, i, m, a, b, q);
    addQuery(r, m + 1, j, a, b, q);
  }
  void dfs(int node, int i, int j, vector<int> &ans){
    int lastTime = getLastVersion();
    for(Query q: queries[node])
      insert(q);
    if( i == j){
      ans[i] = getAnswer();
    }else{
      int m = (i + j) / 2;
      int l = (node << 1);
      int r = l + 1;
      dfs(l, i, m, ans);
      dfs(r, m + 1, j, ans);
    }
    rollback(lastTime);
  }
  // Public:
  void init(int tMax){
    T = tMax;
    for(int i=0; i<=T; i++)
      queries[i].clear();
  }
  void addQuery(int l, int r, Query q){
    addQuery(1, 0, T, l, r, q);
  }
  vector<int> solve(){
    vector<int> ans(T+1);
    dfs(1, 0, T, ans);
    return ans;
  }
};
```

Rmq.h

<bits/stdc++.h> 4ecc87, 32 lines

```
using namespace std;
// Source: https://github.com/brunomaletta/Biblioteca
template<typename T> struct RMQ{
  vector<T> v;
  int n; static const int b = 30;
  vector<int> mask, t;
  int op(int x, int y) { return v[x] < v[y] ? x : y; }
  int msb(int x) { return __builtin_clz(1)-__builtin_clz(x); }
  int small(int r, int sz = b) { return r-msb(mask[r]&((1<<sz)
      -1)); }
  RMQ(const vector<T>& v_) : v(v_), n(v.size()), mask(n), t(n)
      {
    for (int i = 0, at = 0; i < n; mask[i++] = at |= 1) {
      at = (at<<1)&((1<<b)-1);
      while (at and op(i, i-msb(at&-at)) == i) at ^= at&-at;
    }
    for (int i = 0; i < n/b; i++) t[i] = small(b*i+b-1);
    for (int j = 1; (1<<j) <= n/b; j++) for (int i = 0; i+(1<<j
        ) <= n/b; i++)
      t[n/b*j+i] = op(t[n/b*(j-1)+i], t[n/b*(j-1)+i+(1<<(j-1))
          ]);
  }
  int getPos(int l, int r){
    if (r-l+1 <= b) return small(r, r-l+1);
    int ans = op(small(l+b-1), small(r));
    int x = l/b+1, y = r/b-1;
    if (x <= y) {
      int j = msb(y-x+1);
      ans = op(ans, op(t[n/b*j+x], t[n/b*j+y-(1<<j)+1]));
    }
    return ans;
  }
  T queryMin(int l, int r) {
    return v[getPos(l, r)];
  }
};
```

SegmentTree2d.h

<bits/stdc++.h> d52e29, 52 lines

```
using namespace std;
struct SegTree2D{
private:
  int n, m;
  typedef int Node;
  Node neutral = -0x3f3f3f3f;
  vector<vector<Node>> seg;
  Node join(Node a, Node b){
    return max(a, b);
  }
public:
  SegTree2D(int n1, int m1){
    n = n1, m = m1;
    seg.assign(2 * n, vector<Node>(2 * m, 0));
  }
  void update(int x, int y, int val){
    assert(0 <= x && x < n && 0 <= y && y < m);
    x += n, y += m;
    seg[x][y] = val;
    for (int j = y / 2; j > 0; j /= 2)
      seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
    for (x /= 2; x > 0; x /= 2){
      seg[x][y] = join(seg[2 * x][y], seg[2 * x + 1][y]);
      for (int j = y / 2; j > 0; j /= 2){
        seg[x][j] = join(seg[x][2 * j], seg[x][2 * j + 1]);
      }
    }
  }
  vector<int> getCover(int l, int r, int N){
    l = std::max(0, l);
    r = std::min(N, r);
    vector<int> ans;
    for (l += N, r += N; l < r; l /= 2, r /= 2){
      if (l & 1)
        ans.push_back(l++);
      if (r & 1)
        ans.push_back(--r);
```

```
    }
    return ans;
  }
  Node query(int x1, int y1, int x2, int y2){
    auto c1 = getCover(x1, x2 + 1, n);
    auto c2 = getCover(y1, y2 + 1, m);
    Node ans = neutral;
    for (auto i : c1){
      for (auto j : c2){
        ans = join(ans, seg[i][j]);
      }
    }
    return ans;
  }
};
```

SegmentTreeLazy.h

<bits/stdc++.h> f83a6d, 63 lines

```
using namespace std;
namespace SegTreeLazy{
  typedef long long Node;
  Node st[MAXN];
  long long lazy[MAXN];
  int v[MAXN];
  int n;
  Node neutral = 0;
  inline Node join(Node a, Node b){
    return a + b;
  }
  inline void upLazy(int &node, int &i, int &j){
    if (lazy[node] != 0){
      st[node] += lazy[node] * (j - i + 1);
      //st[node] += lazy[node];
      if (i != j){
        lazy[(node << 1)] += lazy[node];
        lazy[(node << 1) + 1] += lazy[node];
      }
      lazy[node] = 0;
    }
  }
  void build(int node, int i, int j){
    if (i == j){
      st[node] = v[i];
      return;
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    build(l, i, m);
    build(r, m + 1, j);
    st[node] = join(st[l], st[r]);
  }
  Node query(int node, int i, int j, int a, int b){
    upLazy(node, i, j);
    if ((i > b) or (j < a))
      return neutral;
    if ((a <= i) and (j <= b)){
      return st[node];
    }
    int m = (i + j) / 2;
    int l = (node << 1);
    int r = l + 1;
    return join(query(l, i, m, a, b), query(r, m + 1, j, a, b))
        ;
  }
  void update(int node, int i, int j, int a, int b, Node value)
      {
    upLazy(node, i, j);
    if ((i > j) or (i > b) or (j < a))
      return;
    if ((a <= i) and (j <= b)){
      lazy[node] = value;
      upLazy(node, i, j);
    }else{
      int m = (i + j) / 2;
      int l = (node << 1);
      int r = l + 1;
      update(l, i, m, a, b, value);
      update(r, m + 1, j, a, b, value);
      st[node] = join(st[l], st[r]);
    }
  }
};
```

SegmentTreePersistent.h

<bits/stdc++.h> 2bf002, 53 lines

```
using namespace std;
namespace PerSegTree{
  const int MAX = 2e5 + 10, UPD = 2e5 + 10, LOG = 20;
  const int MAXS = 4 * MAX + UPD * LOG;
  typedef long long pst_t;
  pst_t seg[MAXS];
  int T[UPD], L[MAXS], R[MAXS], cnt, t;
  int n, *v;
  pst_t neutral = 0;
  pst_t join(pst_t a, pst_t b){
    return a + b;
  }
  pst_t build(int p, int l, int r){
    if (l == r)
      return seg[p] = v[l];
    L[p] = cnt++, R[p] = cnt++;
    int m = (l + r) / 2;
    return seg[p] = join(build(L[p], l, m), build(R[p], m + 1,
        r));
  }
  pst_t query(int a, int b, int p, int l, int r){
    if (b < l or r < a)
      return neutral;
    if (a <= l and r <= b)
      return seg[p];
    int m = (l + r) / 2;
    return join(query(a, b, L[p], l, m), query(a, b, R[p], m +
        1, r));
  }
  pst_t update(int a, int x, int lp, int p, int l, int r){
    if (l == r)
      return seg[p] = x;
    int m = (l + r) / 2;
    if (a <= m)
      return seg[p] = join(update(a, x, L[lp], L[p] = cnt++, l,
          m), seg[R[p] = R[lp]]);
    return seg[p] = join(seg[L[p] = L[lp]], update(a, x, R[lp],
        R[p] = cnt++, m + 1, r));
  }
//Public:
  //O(n)
  void build(int n2, int *v2){
    n = n2, v = v2;
    T[0] = cnt++;
    build(0, 0, n - 1);
  }
  //O(log(n))
  pst_t query(int a, int b, int tt){
    return query(a, b, T[tt], 0, n - 1);
  }
  //O(log(n))
  //update: v[idx] = x;
  int update(int idx, int x, int tt = t){
    update(idx, x, T[tt], T[++t] = cnt++, 0, n - 1);
    return t;
  }
}; // namespace perseg
```

Treap.h

<bits/stdc++.h> a65158, 106 lines

```
using namespace std;
namespace Treap{
  const int N = 500010;
  typedef long long treap_t;
  treap_t X[N];
  int en = 1, Y[N], sz[N], L[N], R[N], root;

  const treap_t neutral = 0;
  treap_t op_val[N];
  inline treap_t join(treap_t a, treap_t b, treap_t c){
    return a + b + c;
  }
  void calc(int u) { // update node given children info
    sz[u] = sz[L[u]] + 1 + sz[R[u]];
    // code here, no recursion
    op_val[u] = join(op_val[L[u]], X[u], op_val[R[u]]);
  }
  void unlaze(int u) {
    if(!u) return;
    // code here, no recursion
  }
  void split(int u, treap_t x, int &l, int &r) { // l gets <= x
      , r gets > x
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(X[u] <= x) { split(R[u], x, l, r); R[u] = l; l = u; }
    else { split(L[u], x, l, r); L[u] = r; r = u; }
    calc(u);
  }
  void split_sz(int u, int s, int &l, int &r) { // l gets first
       s, r gets remaining
    unlaze(u);
    if(!u) return (void) (l = r = 0);
    if(sz[L[u]] < s) { split_sz(R[u], s - sz[L[u]] - 1, l, r);
        R[u] = l; l = u; }
    else { split_sz(L[u], s, l, r); L[u] = r; r = u; }
    calc(u);
  }
  int merge(int l, int r) { // els on l <= els on r
    unlaze(l); unlaze(r);
    if(!l || !r) return l + r;
    int u;
    if(Y[l] > Y[r]) { R[l] = merge(R[l], r); u = l; }
    else { L[r] = merge(l, L[r]); u = r; }
    calc(u);
    return u;
  }
  int new_node(treap_t x){
    X[en] = x;
    op_val[en] = x;
    return en++;
  }
  int nth(int u, int idx){
    if(!u)
      return 0;
    unlaze(u);
    if(idx <= sz[L[u]])
      return nth(L[u], idx);
    else if(idx == sz[L[u]] + 1)
      return u;
    else
```

```
    return nth(R[u], idx - sz[L[u]] - 1);
  }
//Public
  void init(int n=N-1) { // call before using other funcs
    //init position 0
    sz[0] = 0;
    op_val[0] = neutral;
    //init Treap
    root = 0;
    std::mt19937 rng((int) std::chrono::steady_clock::now().
        time_since_epoch().count());
    for(int i = en = 1; i <= n; i++) { Y[i] = i; sz[i] = 1; L[i
        ] = R[i] = 0; }
    shuffle(Y + 1, Y + n + 1, rng);
  }
  void insert(treap_t x){
    int a, b;
    split(root, x, a, b);
    root = merge(merge(a, new_node(x)), b);
  }
  void erase(treap_t x){
    int a, b, c, d;
    split(root, x-1, a, b);
    split(b, x, c, d);
    split_sz(c, 1, b, c);
    root = merge(a, merge(c, d));
  }
  int count(treap_t x){
    int a, b, c, d;
    split(root, x-1, a, b);
    split(b, x, c, d);
    int ans = sz[c];
    root = merge(a, merge(c, d));
    return ans;
  }
  int size(){ return sz[root];}
  //0-indexed
  treap_t nth(int idx){
    int u = nth(root, idx + 1);
    return X[u];
  }
  //Query in k smallest elements
  treap_t query(int k){
    int a, b;
    split_sz(root, k, a, b);
    treap_t ans = op_val[a];
    root = merge(a, b);
    return ans;
  }
};
```

UnionFindWithRollback.h

<bits/stdc++.h> 7db12c, 24 lines

```
using namespace std;
struct RollbackUF {
  vector<int> e;
  vector<tuple<int, int, int, int>> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return st.size(); }
  void rollback(int t) {
    while (st.size() > t){
      auto [a1, v1, a2, v2] = st.back();
      e[a1] = v1; e[a2] = v2;
      st.pop_back();
    }
  }
  bool unite(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.push_back({a, e[a], b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

Dynamic Programming (3)

AlienTrick.h

<bits/stdc++.h> 998fee, 19 lines

```
#define F first
#define S second
using namespace std;
using ll = long long;
using pll = pair<ll, ll>;
pll solveDP(ll C);
ll solveMax(int k){
  ll lo = 0, hi=1e16, ans=1e16;
  while(lo <= hi){
    ll mid = (lo+hi)>>1;
    if(solveDP(mid).S <= k){
      ans = mid;
      hi = mid - 1;
    }else{
      lo = mid + 1;
    }
  }
  return solveDP(ans).F + k*ans;
}
```

DcOptimization.h

<bits/stdc++.h> d21e30, 32 lines

```
using namespace std;
int C(int i, int j);
const int MAXN = 100010;
const int MAXK = 110;
const int INF = 0x3f3f3f3f;
int dp[MAXN][MAXK];
void calculateDP(int l, int r, int k, int opt_l, int opt_r){
  if (l > r)
    return;
  int mid = (l + r) >> 1;
  int ans = -INF, opt = mid;
//  int ans = dp[mid][k-1], opt=mid; //If you accept empty
     subsegment
  for (int i = opt_l; i <= min(opt_r, mid - 1); i++){
    if (ans < dp[i][k - 1] + C(i + 1, mid)){
      opt = i;
      ans = dp[i][k - 1] + C(i + 1, mid);
    }
  }
  dp[mid][k] = ans;
  calculateDP(l, mid - 1, k, opt_l, opt);
  calculateDP(mid + 1, r, k, opt, opt_r);
}
int solve(int n, int k){
  for (int i = 0; i <= n; i++)
    dp[i][0] = -INF;
  for (int j = 0; j <= k; j++)
    dp[0][j] = -INF;
  dp[0][0] = 0;
  for (int j = 1; j <= k; j++)
    calculateDP(1, n, j, 0, n - 1);
  return dp[n][k];
}
```

KnuthOptimization.h

<bits/stdc++.h> edb7ac, 27 lines

```
using namespace std;
typedef long long ll;
const int MAXN = 1009;
const ll INFLL = 0x3f3f3f3f3f3f3f3f;
ll C(int a, int b);
ll dp[MAXN][MAXN];
int opt[MAXN][MAXN];
ll knuth(int n){
  for (int i = 0; i < n; i++){
    dp[i][i] = 0;
    opt[i][i] = i;
  }
  for (int s = 1; s < n; s++){
    for (int i = 0, j; (i + s) < n; i++){
      j = i + s;
      dp[i][j] = INFLL;
      for (int k = opt[i][j - 1]; k < min(j, opt[i + 1][j] + 1)
          ; k++){
        ll cur = dp[i][k] + dp[k + 1][j] + C(i, j);
        if (dp[i][j] > cur){
          dp[i][j] = cur;
          opt[i][j] = k;
        }
      }
    }
  }
  return dp[0][n - 1];
}
```

Geometry (4)

BasicGeometry.h

<bits/stdc++.h> 46fe80, 361 lines

```
using namespace std;
#define POINT_DOUBLE
// Se necessario, apelar para __float128
typedef double ftype;
typedef long double ftLong;
const double EPS = 1e-9;
#define eq(a, b) (abs(a - b) < EPS)
#define lt(a, b) ((a + EPS) < b)
#define gt(a, b) (a > (b + EPS))
#define le(a, b) (a < (b + EPS))
#define ge(a, b) ((a + EPS) > b)
//Begin Point 2D
struct Point2d{
  ftype x, y;
  Point2d() {}
  Point2d(ftype x1, ftype y1) : x(x1), y(y1) {}
  Point2d operator+(const Point2d &t){
    return Point2d(x + t.x, y + t.y);
  }
  Point2d operator-(const Point2d &t){
    return Point2d(x - t.x, y - t.y);
  }
  Point2d operator*(ftype t){
    return Point2d(x * t, y * t);
  }
  Point2d operator/(ftype t){
    return Point2d(x / t, y / t);
  }
  bool operator<(const Point2d &o) const{
    return lt(x, o.x) or (eq(x, o.x) and lt(y, o.y));
```

```
  }
  bool operator==(const Point2d &o) const{
    return eq(x, o.x) and eq(y, o.y);
  }
};
ftLong pw2(ftype a){
  return a * (ftLong)a;
}
//Scalar product
ftLong dot(Point2d a, Point2d b){
  return a.x*(ftLong)b.x + a.y*(ftLong)b.y;
}
ftLong norm(Point2d a){
  return dot(a, a);
}
double len(Point2d a){
  return sqrtl(dot(a, a));
}
double dist(Point2d a, Point2d b){
  return len(a - b);
}
//Vector product
ftLong cross(Point2d a, Point2d b){
  return a.x * (ftLong)b.y - a.y * (ftLong)b.x;
}
//Projection size from A to B
double proj(Point2d a, Point2d b){
  return dot(a, b) / len(b);
}
//The angle between A and B
double angle(Point2d a, Point2d b){
  return acos(dot(a, b) / len(a) / len(b));
}
//Left rotation. Angle in radian
Point2d rotateL(Point2d p, double ang){
  return Point2d(p.x * cos(ang) - p.y * sin(ang), p.x * sin(ang
      ) + p.y * cos(ang));
}
//90 degree left rotation
Point2d perpL(Point2d a){
  return Point2d(-a.y, a.x);
}
//0-> 1o,2o quadrant, 1-> 3o,4o
int half(Point2d &p){
  if (gt(p.y, 0) or (eq(p.y, 0) and ge(p.x, 0)))
    return 0;
  else
    return 1;
}
//angle(a) < angle(b)
bool cmpByAngle(Point2d a, Point2d b){
  int ha = half(a), hb = half(b);
  if (ha != hb){
    return ha < hb;
  }else{
    ftLong c = cross(a, b);
    if(eq(c, 0))
      return lt(norm(a), norm(b));
    else
      return gt(c, 0);
  }
}
inline int sgn(ftLong x){
  return ge(x, 0) ? (eq(x, 0) ? 0 : 1) : -1;
}
//-1: angle(a, b) < angle(b, c)
// 0: angle(a, b) = angle(b, c)
//+1: angle(a, b) > angle(b, c)
int cmpAngleBetweenVectors(Point2d a, Point2d b, Point2d c){
  ftLong dotAB = dot(a, b), dotBC = dot(b, c);
  int sgnAB = sgn(dotAB), sgnBC = sgn(dotBC);
  if(sgnAB == sgnBC){
    //Careful with overflow
    ftLong l = pw2(dotAB)*dot(c, c), r = pw2(dotBC)*dot(a, a);
    if(l == r)
      return 0;
    if(sgnAB == 1)
      return gt(l, r)? -1 : +1;
    return lt(l, r)? -1 : +1;
  }else{
    return (sgnAB > sgnBC)? -1 : +1;
  }
}
//Line parameterized: r1 = a1 + d1*t
//This function can be generalized to 3D
Point2d intersect(Point2d a1, Point2d d1, Point2d a2, Point2d
    d2){
  return a1 + d1 * (cross(a2 - a1, d2) / cross(d1, d2));
}
//Distance between the point(a) and segment(ps1, ps2)
//This function can be generalized to 3D
ftLong distance_point_to_segment(Point2d a, Point2d ps1,
    Point2d ps2) {
  if(ps1 == ps2)
    return dist(ps1, a);
  Point2d d = ps2 - ps1;
  ftLong t = max(ftLong(0), min(ftLong(1), ftLong(dot(a-ps1, d)
      /len(d))));
  Point2d proj = ps1 + Point2d(d.x*t, d.y*t);
  return dist(a, proj);
}
//Distance between the point(a) and line(pl1, pl2)
//This function can be generalized to 3D
double dist(Point2d a, Point2d pl1, Point2d pl2){
  //crs = parallelogram area
  double crs = cross(Point2d(a - pl1), Point2d(pl2 - pl1));
  //h = area/base
  return abs(crs / dist(pl1, pl2));
}
long double area(vector<Point2d> p){
  long double ret = 0;
  for (int i = 2; i < (int)p.size(); i++)
    ret += cross(p[i] - p[0], p[i - 1] - p[0]) / 2.0;
  return abs(ret);
}
long long latticePointsInSeg(Point2d a, Point2d b){
  long long dx = abs(a.x - b.x);
  long long dy = abs(a.y - b.y);
  return gcd(dx, dy) + 1;
}
ftLong signed_area_parallelogram(Point2d p1, Point2d p2,
    Point2d p3){
  return cross(p2 - p1, p3 - p2);
}
long double triangle_area(Point2d p1, Point2d p2, Point2d p3){
  return abs(signed_area_parallelogram(p1, p2, p3)) / 2.0;
}
bool pointInTriangle(Point2d a, Point2d b, Point2d c, Point2d p
    ){
  ftLong s1 = abs(cross(b - a, c - a));
  ftLong s2 = abs(cross(a - p, b - p)) + abs(cross(b - p, c - p
      )) + abs(cross(c - p, a - p));
  return eq(s1, s2);
}
bool clockwise(Point2d p1, Point2d p2, Point2d p3){
  return lt(signed_area_parallelogram(p1, p2, p3), 0);
}
bool counter_clockwise(Point2d p1, Point2d p2, Point2d p3){
  return gt(signed_area_parallelogram(p1, p2, p3), 0);
}
//End Point 2D

//Begin Line
ftLong det(ftype a, ftype b, ftype c, ftype d){
  return a * (ftLong)d - b * (ftLong)c;
}
struct Line{
  ftype a, b, c;
  Line() {}
  Line(ftype a1, ftype b1, ftype c1) : a(a1), b(b1), c(c1){
    normalize();
  }
  Line(Point2d p1, Point2d p2){
    a = p1.y - p2.y;
    b = p2.x - p1.x;
    c = -a * p1.x - b * p1.y;
    normalize();
  }
  void normalize(){
#ifdef POINT_DOUBLE
    ftype z = sqrt(pw2(a) + pw2(b));
#else
    ftype z = __gcd(abs(a), __gcd(abs(b), abs(c)));
#endif
    if(eq(z, 0)) return;
    a /= z;
    b /= z;
    c /= z;
    if (lt(a, 0) or (eq(a, 0) and lt(b, 0))){
      a = -a;
      b = -b;
      c = -c;
    }
  }
};
bool intersect(Line m, Line n, Point2d &res){
  ftype zn = det(m.a, m.b, n.a, n.b);
  if (eq(zn, 0))
    return false;
  res.x = -det(m.c, m.b, n.c, n.b) / zn;
  res.y = -det(m.a, m.c, n.a, n.c) / zn;
  return true;
}
bool parallel(Line m, Line n){
  return eq(det(m.a, m.b, n.a, n.b), 0);
}
bool equivalent(Line m, Line n){
  return eq(det(m.a, m.b, n.a, n.b), 0) &&
         eq(det(m.a, m.c, n.a, n.c), 0) &&
         eq(det(m.b, m.c, n.b, n.c), 0);
}
//Distance from a point(x, y) to a line m
double dist(Line m, ftype x, ftype y){
  return abs(m.a * (ftLong)x + m.b * (ftLong)y + m.c) /
         sqrt(m.a * (ftLong)m.a + m.b * (ftLong)m.b);
}
//End Line

//Begin Segment
struct Segment{
  Point2d a, b;
  Segment() {}
  Segment(Point2d a1, Point2d b1) : a(a1), b(b1) {}
};
bool inter1d(ftype a, ftype b, ftype c, ftype d){
  if (gt(a, b)) swap(a, b);
  if (gt(c, d)) swap(c, d);
```

```
  return le(max(a, c), min(b, d));
}
bool check_intersection(Segment s1, Segment s2){
  Point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
  if (eq(cross(a - c, d - c), 0) && eq(cross(b - c, d - c), 0))
    return inter1d(a.x, b.x, c.x, d.x) && inter1d(a.y, b.y, c.y
        , d.y);
  return sgn(cross(b - a, c - a)) != sgn(cross(b - a, d - a))
      &&
        sgn(cross(d - c, a - c)) != sgn(cross(d - c, b - c));
}
inline bool betw(ftype l, ftype r, ftype x){
  return le(min(l, r), x) and le(x, max(l, r));
}
bool intersect(Segment s1, Segment s2, Segment &ans){
  Point2d a = s1.a, b = s1.b, c = s2.a, d = s2.b;
  if (!inter1d(a.x, b.x, c.x, d.x) || !inter1d(a.y, b.y, c.y, d
      .y))
    return false;
  Line m(a, b);
  Line n(c, d);
  if (parallel(m, n)){
    if (!equivalent(m, n))
      return false;
    if (b < a)
      swap(a, b);
    if (d < c)
      swap(c, d);
    ans = Segment(max(a, c), min(b, d));
    return true;
  }else{
    Point2d p(0, 0);
    intersect(m, n, p);
    ans = Segment(p, p);
    return betw(a.x, b.x, p.x) && betw(a.y, b.y, p.y) &&
           betw(c.x, d.x, p.x) && betw(c.y, d.y, p.y);
  }
}
//End Segment

//Begin Circle
struct Circle{
  ftype x, y, r;
  Circle() {}
  Circle(ftype x1, ftype y1, ftype r1) : x(x1), y(y1), r(r1){};
};
bool pointInCircle(Circle c, Point2d p){
  return ge(c.r, dist(Point2d(c.x, c.y), p));
}
//CircumCircle of a triangle is a circle that passes through
    all the vertices
Circle circumCircle(Point2d a, Point2d b, Point2d c){
  Point2d u((b - a).y, -((b - a).x));
  Point2d v((c - a).y, -((c - a).x));
  Point2d n = (c - b) * 0.5;
  double t = cross(u, n) / cross(v, u);
  Point2d ct = (((a + c) * 0.5) + (v * t));
  double r = dist(ct, a);
  return Circle(ct.x, ct.y, r);
}
//InCircle is the largest circle contained in the triangle
Circle inCircle(Point2d a, Point2d b, Point2d c){
  double m1 = dist(a, b);
  double m2 = dist(a, c);
  double m3 = dist(b, c);
  Point2d ct = ((c * m1) + (b * m2) + a * (m3)) / (m1 + m2 + m3
      );
  double sp = 0.5 * (m1 + m2 + m3);
  double r = sqrt(sp * (sp - m1) * (sp - m2) * (sp - m3)) / sp;
  return Circle(ct.x, ct.y, r);
}
//Minimum enclosing circle, O(n)
Circle minimumCircle(vector<Point2d> p){
  random_shuffle(p.begin(), p.end());
  Circle c = Circle(p[0].x, p[0].y, 0.0);
  for (int i = 0; i < (int)p.size(); i++){
    if (pointInCircle(c, p[i]))
      continue;
    c = Circle(p[i].x, p[i].y, 0.0);
    for (int j = 0; j < i; j++){
      if (pointInCircle(c, p[j]))
        continue;
      c = Circle((p[j].x + p[i].x) * 0.5, (p[j].y + p[i].y) *
          0.5, 0.5 * dist(p[j], p[i]));
      for (int k = 0; k < j; k++){
        if (pointInCircle(c, p[k]))
          continue;
        c = circumCircle(p[j], p[i], p[k]);
      }
    }
  }
  return c;
}
//Return the number of the intersection
int circle_line_intersection(Circle circ, Line line, Point2d &
    p1, Point2d &p2){
  ftLong r = circ.r;
  ftLong a = line.a, b = line.b, c = line.c + line.a * circ.x +
       line.b * circ.y; //take a circle to the (0, 0)
  ftLong x0 = -a * c / (pw2(a) + pw2(b)), y0 = -b * c / (pw2(a)
       + pw2(b));       //(x0, y0) is the shortest distance
      point of the line for (0, 0)
  if (gt(pw2(c), pw2(r) * (pw2(a) + pw2(b)))){
    return 0;
  }
  else if (eq(pw2(c), pw2(r) * (pw2(a) + pw2(b)))){
    p1.x = p2.x = x0 + circ.x;
    p1.y = p2.y = y0 + circ.y;
    return 1;
  }else{
    ftLong d_2 = pw2(r) - pw2(c) / (pw2(a) + pw2(b));
    ftLong mult = sqrt(d_2 / (pw2(a) + pw2(b)));
    p1.x = x0 + b * mult + circ.x;
    p2.x = x0 - b * mult + circ.x;
    p1.y = y0 - a * mult + circ.y;
    p2.y = y0 + a * mult + circ.y;
    return 2;
  }
}
//Return the number of the intersection
int circle_intersection(Circle c1, Circle c2, Point2d &p1,
    Point2d &p2){
  if (eq(c1.x, c2.x) and eq(c1.y, c2.y)){
    if (eq(c1.r, c2.r))
      return -1; //INF
    else
      return 0;
  }else{
    Circle circ(0, 0, c1.r);
    Line line;
    line.a = -2 * (c2.x - c1.x);
    line.b = -2 * (c2.y - c1.y);
    line.c = pw2(c2.x - c1.x) + pw2(c2.y - c1.y) + pw2(c1.r) -
        pw2(c2.r);
    int sz = circle_line_intersection(circ, line, p1, p2);
    p1.x += c1.x;
    p2.x += c1.x;
    p1.y += c1.y;
    p2.y += c1.y;
    return sz;
  }
}
//End Circle
```

ConvexHull.h

"BasicGeometry.h" 253b69, 38 lines

```
using namespace std;
//If accept collinear points then change for <=
bool cw(Point2d a, Point2d b, Point2d c) {
  return lt(cross(b - a, c - b), 0);
}
//If accept collinear points then change for >=
bool ccw(Point2d a, Point2d b, Point2d c) {
  return gt(cross(b - a, c - b), 0);
}
// Returns the points clockwise
vector<Point2d> convex_hull(vector<Point2d> a){
  if (a.size() == 1)
    return a;
  sort(a.begin(), a.end());
  a.erase(unique(a.begin(), a.end()), a.end());
  vector<Point2d> up, down;
  Point2d p1 = a[0], p2 = a.back();
  up.push_back(p1);
  down.push_back(p1);
  for (int i = 1; i < (int)a.size(); i++){
    if ((i == int(a.size() - 1)) || cw(p1, a[i], p2)){
      while (up.size() >= 2 && !cw(up[up.size() - 2], up[up.
          size() - 1], a[i]))
        up.pop_back();
      up.push_back(a[i]);
    }
    if ((i == int(a.size() - 1)) || ccw(p1, a[i], p2)){
      while (down.size() >= 2 && !ccw(down[down.size() - 2],
          down[down.size() - 1], a[i]))
        down.pop_back();
      down.push_back(a[i]);
    }
  }
  a.clear();
  for (int i = 0; i < (int)up.size(); i++)
    a.push_back(up[i]);
  for (int i = down.size() - 2; i > 0; i--)
    a.push_back(down[i]);
  return a;
}
```

ConvexPolygon.h

"ConvexHull.h" 8a1231, 37 lines

```
using namespace std;
//Checks if the point P belongs to the segment AB
bool pointInSegment(Point2d &a, Point2d &b, Point2d &p) {
  if(!eq(cross(a-p, b-p), 0))
    return false;
  return betw(a.x, b.x, p.x) && betw(a.y, b.y, p.y);
}
struct ConvexPolygon{
  vector<Point2d> vp;
  ConvexPolygon(vector<Point2d> aux){
    //The points have to be clockwise
    vp = convex_hull(aux);
  }
  //O(log(N))
  //Accepts points on the edge
  bool pointInPolygon(Point2d point){
    if(vp.size() < 3)
```

```
      return pointInSegment(vp[0], vp[1], point);
    if(!eq(cross(vp[1]-vp[0], point-vp[0]), 0) and sgn(cross(vp
        [1]-vp[0], point-vp[0])) != sgn(cross(vp[1]-vp[0], vp.
        back()-vp[0])) )
      return false;
    if(!eq(cross(vp.back()-vp[0], point-vp[0]), 0) and sgn(
        cross(vp.back()-vp[0], point-vp[0])) != sgn(cross(vp.
        back() - vp[0], vp[1]-vp[0])) )
      return false;
    if(eq(cross(vp[1]-vp[0], point-vp[0]), 0))
      return ge(norm(vp[1]-vp[0]), norm(point-vp[0]));
    int pos = 1, l = 1, r = vp.size() - 2;
    while(l <= r){
      int mid = (l + r)/2;
      if(le(cross(vp[mid] - vp[0], point - vp[0]), 0)){
        pos = mid;
        l = mid+1;
      }else{
        r = mid-1;
      }
    }
    return pointInTriangle(vp[0], vp[pos], vp[pos+1], point);
  }
};
```

GeneralPolygon.h

"BasicGeometry.h" 5cb579, 29 lines

```
const int INSIDE=-1, BOUNDARY=0, OUTSIDE=1;
struct GeneralPolygon{
  vector<Point2d> vp;
  GeneralPolygon(vector<Point2d> aux){
    vp = aux;
  }
  // -1 inside, 0 boundary, 1 outside
  int pointInPolygon(Point2d pt) {
    int n = vp.size(), w = 0;
    for(int i=0; i<n; i++){
      if(pt == vp[i])
        return 0;
      int j = (i+1==n?0:i+1);
      if(vp[i].y == pt.y and vp[j].y == pt.y) {
        if (min(vp[i].x, vp[j].x) <= pt.x and pt.x <= max(vp[i
            ].x, vp[j].x))
          return 0;
      }else{
        bool below = vp[i].y < pt.y;
        if (below != (vp[j].y < pt.y)) {
          auto orientation = cross(pt-vp[i], vp[j]-vp[i]);
          if (orientation == 0) return 0;
          if (below == (orientation > 0))
            w += below ? 1 : -1;
        }
      }
    }
    return (w==0?1:-1);
  }
};
```

NearestPairOfPoints.h

<bits/stdc++.h> fe9435, 64 lines

```
using namespace std;
struct pt {
  long long x, y, id;
  pt(){}
  pt(int _x, int _y, int _id=-1):x(_x), y(_y), id(_id){}
};
namespace NearestPairOfPoints{
  struct cmp_x {
    bool operator()(const pt & a, const pt & b) const {
      return a.x < b.x || (a.x == b.x && a.y < b.y);
    }
  };
  struct cmp_y {
    bool operator()(const pt & a, const pt & b) const {
      return a.y < b.y;
    }
  };
  int n;
  vector<pt> v;
  vector<pt> t;
  double mindist;
  pair<int, int> best_pair;
  void upd_ans(const pt & a, const pt & b) {
    double dist = sqrt((a.x - b.x)*(a.x - b.x) + (a.y - b.y)*(a
        .y - b.y));
    if (dist < mindist) {
      mindist = dist;
      best_pair = {a.id, b.id};
    }
  }
  void rec(int l, int r) {
    if (r - l <= 3) {
      for (int i = l; i < r; ++i) {
        for (int j = i + 1; j < r; ++j) {
          upd_ans(v[i], v[j]);
        }
      }
      sort(v.begin() + l, v.begin() + r, cmp_y());
      return;
    }
    int m = (l + r) >> 1;
    int midx = v[m].x;
    rec(l, m);
    rec(m, r);
    merge(v.begin() + l, v.begin() + m, v.begin() + m, v.begin
        () + r, t.begin(), cmp_y());
    copy(t.begin(), t.begin() + r - l, v.begin() + l);
    int tsz = 0;
    for (int i = l; i < r; ++i) {
      if (abs(v[i].x - midx) < mindist) {
        for (int j = tsz - 1; j >= 0 && v[i].y - t[j].y <
            mindist; --j)
          upd_ans(v[i], t[j]);
        t[tsz++] = v[i];
      }
    }
  }
  pair<int, int> solve(vector<pt> _v){
    v = _v;
    n = v.size();
    t.resize(n);
    sort(v.begin(), v.end(), cmp_x());
    mindist = 1E20;
    rec(0, n);
    return best_pair;
  }
};
```

Point3d.h

<bits/stdc++.h> 9f5f03, 117 lines

```
using namespace std;
#define POINT_DOUBLE
typedef double ftype;
typedef long double ftLong;
const double EPS = 1e-9;
#define eq(a, b) (abs(a-b)<EPS)
#define lt(a, b) ((a+EPS)<b)
#define gt(a, b) (a>(b+EPS))
#define le(a, b) (a<(b+EPS))
#define ge(a, b) ((a+EPS)>b)
//Point3D
struct Point3d{
  ftype x, y, z;
  Point3d() {}
  Point3d(ftype x, ftype y, ftype z) : x(x), y(y), z(z) {}
  Point3d operator+(Point3d t){
    return Point3d(x + t.x, y + t.y, z + t.z);
  }
  Point3d operator-(Point3d t){
    return Point3d(x - t.x, y - t.y, z - t.z);
  }
  Point3d operator*(ftype t){
    return Point3d(x * t, y * t, z * t);
  }
  Point3d operator/(ftype t){
    return Point3d(x / t, y / t, z / t);
  }
};
ftLong dot(Point3d a, Point3d b){
  return a.x * (ftLong)b.x + a.y * (ftLong)b.y + a.z * (ftLong)
      b.z;
}
double len(Point3d a){
  return sqrt(dot(a, a));
}
double dist(Point3d a, Point3d b){
  return len(a-b);
}
double proj(Point3d a, Point3d b){
  return dot(a, b) / len(b);
}
//theta -> XY; phi -> ZY;
Point3d toVetor(double theta, double phi, double r){
  return Point3d(r*cos(theta)*sin(phi), r*sin(theta)*sin(phi),
      r*cos(phi));
}
double getAngleTheta(Point3d p){
  return atan2(p.y, p.x);
}
double getAnglePhi(Point3d p){
  return acos(p.z/len(p));
}
Point3d rotateX(Point3d p, double ang){
  return Point3d(p.x, p.y*cos(ang)-p.z*sin(ang), p.y*sin(ang)+p
      .z*cos(ang));
}
Point3d rotateY(Point3d p, double ang){
  return Point3d(p.x*cos(ang)+p.z*sin(ang), p.y, -p.x*sin(ang)+
      p.z*cos(ang));
}
Point3d rotateZ(Point3d p, double ang){
  return Point3d(p.x*cos(ang)-p.y*sin(ang), p.x*sin(ang)+p.y*
      cos(ang), p.z);
}
//Rotation in relation to the normal axis
Point3d rotateNormal(Point3d v, Point3d n, double ang){
  double theta = getAngleTheta(n);
  double phi = getAnglePhi(n);
  v = rotateZ(v, -theta);
  v = rotateY(v, -phi);
  v = rotateZ(v, ang);
  v = rotateY(v, phi);
  v = rotateZ(v, theta);
  return v;
}
Point3d cross(Point3d a, Point3d b){
```

```
  return Point3d(a.y * b.z - a.z * b.y,
                 a.z * b.x - a.x * b.z,
                 a.x * b.y - a.y * b.x);
}
ftLong triple(Point3d a, Point3d b, Point3d c){
  return dot(a, cross(b, c));
}
Point3d planeIntersect(Point3d a1, Point3d n1, Point3d a2,
    Point3d n2, Point3d a3, Point3d n3){
  Point3d x(n1.x, n2.x, n3.x);
  Point3d y(n1.y, n2.y, n3.y);
  Point3d z(n1.z, n2.z, n3.z);
  Point3d d(dot(a1, n1), dot(a2, n2), dot(a3, n3));
  return Point3d(triple(d, y, z),
                 triple(x, d, z),
                 triple(x, y, d)) / triple(n1, n2, n3);
}
struct Sphere{
  ftype x, y, z, r;
  Sphere(){}
  Sphere(ftype x, ftype y, ftype z,  ftype r):x(x), y(y), z(z),
      r(r){}
};
//Minimum enclosing Sphere, O(n*70000)
//It is also possible to do with ternary search in the 3
    dimensions
Sphere minimumSphere(vector<Point3d> vp){
  Point3d ans(0, 0, 0);
  int n = vp.size();
  for(Point3d p: vp)
    ans = ans + p;
  ans = ans/n;
  double P = 0.1;
  double d = 0, e = 0;
  for(int i = 0; i < 70000; i++){
    int f = 0;
    d = dist(ans, vp[0]);
    for (int j = 1; j < n; j++) {
      e = dist(ans, vp[j]);
      if (d < e) {
        d = e;
        f = j;
      }
    }
    ans = ans + (vp[f]-ans)*P;
    P *= 0.998;
  }
  return Sphere(ans.x, ans.y, ans.z, d);
}
```

Triangle.h

<bits/stdc++.h> 79cef0, 26 lines

```
using namespace std;
typedef long double ld;
const ld PI = acosl(-1);
struct Triangle{
  ld a, b, c;
  Triangle(){}
  Triangle(ld a1, ld b1, ld c1):a(a1), b(b1), c(c1){
    fix();
  }
  ld area(){
    ld s = (a + b + c)/2;
    return sqrtl(s*(s-a)*(s-b)*(s-c));
  }
  void fix(){
    if(a > b) swap(a, b);
    if(a > c) swap(a, c);
    if(b > c) swap(b, c);
  }
  tuple<ld, ld, ld> angle(){
    fix();
    ld h = (2*area())/c;
    ld aa = asin(h/b);
    ld bb = asin(h/a);
    return {aa, bb, PI - aa - bb};
  }
};
```

Graph (5)

2Sat.h

"strongly_connected_component.h" cf8df4, 42 lines

```
using namespace std;
struct SAT{
  typedef pair<int, int> pii;
  vector<pii> edges;
  int n;
  SAT(int size){
    n = 2 * size;
  }
  vector<bool> solve2SAT(){
    vector<bool> vAns(n / 2, false);
    vector<int> comp = SCC::scc(n, edges);
    for (int i = 0; i < n; i += 2){
      if (comp[i] == comp[i + 1])
        return vector<bool>();
      vAns[i / 2] = (comp[i] > comp[i + 1]);
    }
    return vAns;
  }
  int v(int x){
    if (x >= 0)
      return (x << 1);
    x = ~x;
    return (x << 1) ^ 1;
  }
  void add(int a, int b){
    edges.push_back(pii(a, b));
  }
  void addOr(int a, int b){
    add(v(~a), v(b));
    add(v(~b), v(a));
  }
  void addImp(int a, int b){
    addOr(~a, b);
  }
  void addEqual(int a, int b){
    addOr(a, ~b);
    addOr(~a, b);
  }
  void addDiff(int a, int b){
    addEqual(a, ~b);
  }
};
```

Arborescence.h

<bits/stdc++.h>, "../data_structures/union_find_with_rollback.h" 0cb443, 74 lines

```
using ll = long long;
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }
void free(vector<Node*> &v){
  for(auto &x: v)
    delete x;
}
// O(M * log(N))
// return {sum of weights, vector with parents}
pair<ll, vector<int>> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  vector<Node*> vf;
  for (Edge e : g){
    Node* node = new Node{e};
    vf.push_back(node);
    heap[e.b] = merge(heap[e.b], node);
  }
  ll res = 0;
  vector<int> seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1, -1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  for(int s = 0; s < n; ++s) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]){
        free(vf);
        return {-1,{}};
      }
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.unite(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    for(int i = 0; i < qi; ++i) in[uf.find(Q[i].b)] = Q[i];
  }
  for (auto& [u, t, c] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : c) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  for(int i = 0; i < n; ++i) par[i] = in[i].a;
  free(vf);
  return {res, par};
}
```

ArticulationPoint.h

<bits/stdc++.h> 5e7633, 48 lines

```
using namespace std;
const int MAXN = 500010;
//Articulation Point
namespace AP{
  vector<int> adj[MAXN];
  vector<bool> visited, isAP;
  vector<int> tin, low;
  int timer, n;
  void init(int n1){
    n = n1;
    for(int i=0; i<n; i++) adj[i].clear();
  }
  void addEdge(int a, int b){
    adj[a].push_back(b);
    adj[b].push_back(a);
  }
  void dfs(int u, int p = -1) {
    visited[u] = true;
    tin[u] = low[u] = timer++;
    int children=0;
    for (int to : adj[u]) {
      if (to == p) continue;
      if (visited[to]) {
        low[u] = min(low[u], tin[to]);
      } else {
        dfs(to, u);
        low[u] = min(low[u], low[to]);
        if (low[to] >= tin[u] && p!=-1)
          isAP[u] = true;
        ++children;
      }
    }
    if(p == -1 && children > 1)
      isAP[u] = true;
  }
  vector<bool> findArticulationPoint() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    isAP.assign(n, false);
    for (int i = 0; i < n; i++) {
      if (!visited[i])
        dfs(i);
    }
    return isAP;
  }
};
```

Bridge.h

<bits/stdc++.h> 5b478b, 45 lines

```
using namespace std;
const int MAXN = 500010;
typedef pair<int, int> pii;
namespace Bridge{
  vector<int> adj[MAXN];
  vector<bool> visited;
  vector<int> tin, low;
  int timer, n;
  vector<pii> bridges;
  void init(int n1){
    n = n1;
    for(int i=0; i<n; i++) adj[i].clear();
  }
  void addEdge(int a, int b){
    adj[a].push_back(b);
    adj[b].push_back(a);
  }
  void dfs(int u, int p = -1) {
    visited[u] = true;
    tin[u] = low[u] = timer++;
    for (int to : adj[u]) {
      if (to == p) continue;
      if (visited[to]) {
        low[u] = min(low[u], tin[to]);
      } else {
        dfs(to, u);
        low[u] = min(low[u], low[to]);
        if (low[to] > tin[u])
          bridges.push_back({u, to});
      }
    }
  }
  vector<pii> findBridges() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    bridges.clear();
    for (int i = 0; i < n; i++) {
      if (!visited[i])
        dfs(i);
    }
    return bridges;
  }
};
```

CentroidDecomposition.h

<bits/stdc++.h> 8c55e9, 80 lines

```
using namespace std;
typedef long long ll;
// O(N*log(N))
// Centroid Decomposition
const int MAXN = 200010;
namespace CD{
  vector<int> adj[MAXN];
  int dad[MAXN], sub[MAXN];
  bool rem[MAXN];
  int centroidRoot, n;
  void init(int n1){
    n = n1;
    for(int i=0; i<n; i++){
      adj[i].clear();
      rem[i] = false;
    }
  }
  int dfs(int u, int p){
    sub[u] = 1;
    for (int to : adj[u]){
      if (!rem[to] and to != p)
        sub[u] += dfs(to, u);
    }
    return sub[u];
  }
  int centroid(int u, int p, int sz){
    for (auto to : adj[u])
      if (!rem[to] and to != p and sub[to] > sz / 2)
        return centroid(to, u, sz);
    return u;
  }
  void getChildren(int u, int p, int d, vector<int> &v){
    v.push_back(d);
    for(int to: adj[u]){
      if(rem[to] or to == p)
        continue;
      getChildren(to, u, d+1, v);
    }
  }
  ll ans = 0;
  int k;
  int decomp(int u, int p){
    int sz = dfs(u, p);
    int c = centroid(u, p, sz);
    if (p == -1)
      p = c;
    dad[c] = p;
    rem[c] = true;
    // Begin
    vector<int> f(sz+1, 0);
    f[0] = 1;
    for (auto to : adj[c]) if (!rem[to]){
      vector<int> v;
      getChildren(to, c, 1, v);
      for(int d: v){ // Query
        if(d <= k and k-d <= sz)
          ans += f[k-d];
      }
      for(int d: v) // Update
        f[d]++;
    }
    // End
    for (auto to : adj[c]){
      if (!rem[to])
        decomp(to, c);
    }
    return c;
  }
  void addEdge(int a, int b){
    adj[a].push_back(b);
    adj[b].push_back(a);
  }
  // Number of k-size paths: O(N * log(N))
  ll solve(int k1){
    assert(n > 0);
    ans = 0, k = k1;
    centroidRoot = decomp(0, -1);
    return ans;
  }
};
```

Dinic.h

<bits/stdc++.h> 477eed, 112 lines

```
using namespace std;
//O((V^2)*E): for generic graph.
//O(sqrt(V)*E): on unit networks. A unit network is a network
    in which all the edges have unit capacity, and for any
    vertex except s and t either incoming or outgoing edge is
    unique. That's exactly the case with the network we build
    to solve the maximum matching problem with flows.
template <typename flow_t>
struct Dinic{
  struct FlowEdge{
    int from, to, id;
    flow_t cap, flow = 0;
    FlowEdge(int f, int t, flow_t c, int id1) : from(f), to(t),
        cap(c){
      id = id1;
    }
  };
  const flow_t flow_inf = numeric_limits<flow_t>::max();
  vector<FlowEdge> edges;
  vector<vector<int>> adj;
  int n, m = 0;
  int s, t;
```

```
  vector<int> level, ptr;
  queue<int> q;
  bool bfs(){
    while (!q.empty()){
      int u = q.front();
      q.pop();
      for (int id : adj[u]){
        if (edges[id].cap - edges[id].flow < 1)
          continue;
        if (level[edges[id].to] != -1)
          continue;
        level[edges[id].to] = level[u] + 1;
        q.push(edges[id].to);
      }
    }
    return level[t] != -1;
  }
  flow_t dfs(int u, flow_t pushed){
    if (pushed == 0)
      return 0;
    if (u == t)
      return pushed;
    for (int &cid = ptr[u]; cid < (int)adj[u].size(); cid++){
      int id = adj[u][cid];
      int to = edges[id].to;
      if (level[u] + 1 != level[to] || edges[id].cap - edges[id
          ].flow < 1)
        continue;
      flow_t tr = dfs(to, min(pushed, edges[id].cap - edges[id
          ].flow));
      if (tr == 0)
        continue;
      edges[id].flow += tr;
      edges[id ^ 1].flow -= tr;
      return tr;
    }
    return 0;
  }
//Public:
  Dinic(){}
  void init(int _n){
    n = _n;
    adj.resize(n);
    level.resize(n);
    ptr.resize(n);
  }
  void addEdge(int from, int to, flow_t cap, int id=0){
    assert(n>0);
    edges.emplace_back(from, to, cap, id);
    edges.emplace_back(to, from, 0, -id);
    adj[from].push_back(m);
    adj[to].push_back(m + 1);
    m += 2;
  }
  flow_t maxFlow(int s1, int t1){
    s = s1, t = t1;
    flow_t f = 0;
    while (true){
      level.assign(n, -1);
      level[s] = 0;
      q.push(s);
      if (!bfs())
        break;
      ptr.assign(n, 0);
      while (flow_t pushed = dfs(s, flow_inf))
        f += pushed;
    }
    return f;
  }
};
// Returns the minimum cut edge IDs
vector<int> recoverCut(Dinic<int> &d){
  vector<bool> seen(d.n, false);
  queue<int> q;
  q.push(d.s);
  seen[d.s] = true;
  while (!q.empty()){
    int u = q.front();
    q.pop();
    for (int idx : d.adj[u]){
      auto e = d.edges[idx];
      if (e.cap == e.flow)
        continue;
      if (!seen[e.to]){
        q.push(e.to);
        seen[e.to] = true;
      }
    }
  }
  vector<int> ans;
  for(auto e: d.edges){
    if(e.cap > 0 and (e.cap == e.flow) and (seen[e.from] !=
        seen[e.to])){
      if(e.id >= 0) ans.push_back(e.id);
    }
  }
  return ans;
}
```

EdmondBlossoms.h

<bits/stdc++.h> 1f1e8f, 93 lines

```
using namespace std;
const int MAXN = 510;
// Adaptado de: https://github.com/brunomaletta/Biblioteca/blob
    /master/Codigo/Grafos/blossom.cpp
// Edmond's Blossoms algorithm give a maximum matching in
    general graphs (non-bipartite)
// O(N^3)
namespace EdmondBlossoms{
vector<int> adj[MAXN];
int match[MAXN];
int n, pai[MAXN], base[MAXN], vis[MAXN];
queue<int> q;
void init(int n1){
  n = n1;
  for(int i=0; i<n; i++)
    adj[i].clear();
}
void addEdge(int a, int b){
  adj[a].push_back(b);
  adj[b].push_back(a);
}
void contract(int u, int v, bool first = 1) {
  static vector<bool> bloss;
  static int l;
  if (first) {
    bloss = vector<bool>(n, 0);
    vector<bool> teve(n, 0);
    int k = u; l = v;
    while (1) {
      teve[k = base[k]] = 1;
      if (match[k] == -1) break;
      k = pai[match[k]];
    }
    while (!teve[l = base[l]]) l = pai[match[l]];
  }
  while (base[u] != l) {
    bloss[base[u]] = bloss[base[match[u]]] = 1;
    pai[u] = v;
    v = match[u];
    u = pai[match[u]];
  }
  if (!first) return;
  contract(v, u, 0);
  for (int i = 0; i < n; i++) if (bloss[base[i]]) {
    base[i] = l;
    if (!vis[i]) q.push(i);
    vis[i] = 1;
  }
}
int getpath(int s) {
  for (int i = 0; i < n; i++)
    base[i] = i, pai[i] = -1, vis[i] = 0;
  vis[s] = 1; q = queue<int>(); q.push(s);
  while (q.size()) {
    int u = q.front(); q.pop();
    for (int i : adj[u]) {
      if (base[i] == base[u] or match[u] == i) continue;
      if (i == s or (match[i] != -1 and pai[match[i]] != -1))
        contract(u, i);
      else if (pai[i] == -1) {
        pai[i] = u;
        if (match[i] == -1) return i;
        i = match[i];
        vis[i] = 1; q.push(i);
      }
    }
  }
  return -1;
}
typedef pair<int, int> pii;
vector<pii> maximumMatching(){
  vector<pii> ans;
  memset(match, -1, sizeof(match));
  for (int i = 0; i < n; i++) if (match[i] == -1)
    for (int j : adj[i]) if (match[j] == -1) {
      match[i] = j;
      match[j] = i;
      break;
    }
  for (int i = 0; i < n; i++) if (match[i] == -1) {
    int j = getpath(i);
    if (j == -1) continue;
    while (j != -1) {
      int p = pai[j], pp = match[p];
      match[p] = j;
      match[j] = p;
      j = pp;
    }
  }
  for(int i=0; i < n; i++)
    if(i < match[i])
      ans.emplace_back(i, match[i]);
  return ans;
}
};
```

EulerianPath.h

<bits/stdc++.h> a158d0, 45 lines

```
using namespace std;
typedef pair<int, int> pii;
template<bool directed=false> struct EulerianPath{
  vector<vector<pii>> adj;
  vector<int> ans, pos;
  vector<bool> used;
  int n, m;
  EulerianPath(int n1){
```

```
    n = n1; m = 0;
    adj.assign(n, vector<pii>());
  }
  void addEdge(int a, int b) {
    int at = m++;
    adj[a].push_back({b, at});
    if (!directed) adj[b].push_back({a, at});
  }
  void dfs(int u){
    stack<int> st;
    st.push(u);
    while(!st.empty()){
      u = st.top();
      if(pos[u] < adj[u].size()){
        auto [to, id] = adj[u][pos[u]];
        pos[u]++;
        if(!used[id]){
          used[id] = true;
          st.push(to);
        }
      }else{
        ans.push_back(u);
        st.pop();
      }
    }
  }
  // Remember to call the correct src
  // If you want to check if there is an answer remember to
       check if all |components| > 1 of the graph are connected
  vector<int> getPath(int src){
    pos.assign(n, 0);
    used.assign(m, false);
    ans.clear();
    dfs(src);
    reverse(ans.begin(), ans.end());
    return ans;
  }
};
```

FindCycleNegative.h

<bits/stdc++.h> 688fec, 32 lines

```
using namespace std;
typedef long long ll;
typedef tuple<int, int, int> Edge;
vector<int> findNegativeCycle(vector<Edge> edges, int n){
  vector<ll> d(n, 0);
  vector<int> p(n, -1);
  int last = -1;
  for(int i = 0; i < n; ++i){
    last = -1;
    for(auto [u, to, w] : edges){
      if(d[u] + w < d[to]){
        d[to] = d[u] + w;
        p[to] = u;
        last = to;
      }
    }
  }
  if(last == -1){
    return {};
  }else{
    for(int i = 0; i < n; i++)
      last = p[last];
    vector<int> cycle;
    for(int v = last; ; v = p[v]){
      cycle.push_back(v);
      if(v == last && cycle.size() > 1)
        break;
    }
    reverse(cycle.begin(), cycle.end());
    return cycle;
  }
}
```

FlowWithDemand.h

"dinic.h" bb2848, 28 lines

```
using namespace std;
template <typename flow_t>
struct MaxFlowEdgeDemands{
  Dinic<flow_t> mf;
  vector<flow_t> ind, outd;
  flow_t D;
  int n;
  MaxFlowEdgeDemands(int n) : n(n){
    D = 0;
    mf.init(n + 2);
    ind.assign(n, 0);
    outd.assign(n, 0);
  }
  void addEdge(int a, int b, flow_t cap, flow_t demands){
    mf.addEdge(a, b, cap - demands);
    D += demands;
    ind[b] += demands;
    outd[a] += demands;
  }
  bool solve(int s, int t){
    mf.addEdge(t, s, numeric_limits<flow_t>::max());
    for (int i = 0; i < n; i++){
      if (ind[i]) mf.addEdge(n, i, ind[i]);
      if (outd[i]) mf.addEdge(i, n + 1, outd[i]);
    }
    return mf.maxFlow(n, n + 1) == D;
  }
};
```

GraphTheorem.h

<bits/stdc++.h> 0a8d21, 29 lines

```
#define all(x) x.begin(),x.end()
using namespace std;
using ll = long long;
using pii = pair<int, int>;
namespace GraphTheorem{
  // return if a sequence of integers d can be represented as
       the
  // degree sequence of a finite simple graph on n vertices
  bool ErdosGallai(vector<int> d){
    int n = d.size();
    sort(all(d), greater<int>());
    ll sum1 = 0, sum2 = 0;
    int mn = n-1;
    for(int k=1; k<=n; k++){
      sum1 += d[k-1];
      while(k <= mn and k > d[mn])
        sum2 += d[mn--];
      if(mn + 1 < k)
        sum2 -= d[mn++];
      ll a = sum1, b = k*(ll)mn + sum2;
      if(a > b)
        return false;
    }
    return sum1%2 == 0;
  }
  vector<pii> recoverErdosGallai(vector<int> d){
    //Joga todo mundo em um heap e vai removendo o que tem
         maior grau.
  }
};
```

Hld.h

<bits/stdc++.h>, "../data_structures/bit_range.h" 9487f8, 99 lines

```
using namespace std;
#define F first
template <typename T = long long>
class HLD{
private:
  vector<vector<pair<int, T>>> adj;
  vector<int> sz, h, dad, pos;
  vector<T> val, v;
  int t;
  bool edge;
  //Begin Internal Data Structure
  BitRange *bit;
  T neutral = 0;
  inline T join(T a, T b){
    return a+b;
  }
  inline void update(int a, int b, T x){
    bit->add(a+1, b+1, x);
  }
  inline T query(int a, int b){
    return bit->get(a+1, b+1);
  }
  //End Internal Data Structure
  void dfs(int u, int p = -1){
    sz[u] = 1;
    for(auto &viz: adj[u]){
      auto [to, w] = viz;
      if(to == p) continue;
      if(edge) val[to] = w;
      dfs(to, u);
      sz[u] += sz[to];
      if(sz[to] > sz[adj[u][0].F] or adj[u][0].F == p)
        swap(viz, adj[u][0]);
    }
  }
  void build_hld(int u, int p=-1){
    dad[u] = p;
    pos[u] = t++;
    v[pos[u]] = val[u];
    for(auto to: adj[u]) if(to.F != p){
      h[to.F] = (to == adj[u][0]) ? h[u] : to.F;
      build_hld(to.F, u);
    }
  }
  void build(int root, bool is_edge){
    assert(!adj.empty());
    edge = is_edge;
    t = 0;
    h[root] = 0;
    dfs(root);
    build_hld(root);
    //Init Internal Data Structure
    for(int i=0; i<t; i++)
      update(i, i, v[i]);
  }
public:
  ~HLD(){ delete bit; }
  void init(int n){
    dad.resize(n); pos.resize(n); val.resize(n); v.resize(n);
    adj.resize(n); sz.resize(n); h.resize(n);
    bit = new BitRange(n);
  }
  void buildToEdge(int root=0){
    build(root, true);
  }
```

```
  void buildToVertex(vector<T> initVal, int root=0){
    assert(initVal.size() == val.size());
    val = initVal;
    build(root, false);
  }
  void addEdge(int a, int b, T w = 0){
    adj[a].emplace_back(b, w);
    adj[b].emplace_back(a, w);
  }
  T query_path(int a, int b) {
    if (edge and a == b) return neutral;
    if (pos[a] < pos[b]) swap(a, b);
    if (h[a] == h[b]) return query(pos[b]+edge, pos[a]);
    return join(query(pos[h[a]], pos[a]), query_path(dad[h[a]],
          b));
  }
  void update_path(int a, int b, T x) {
    if (edge and a == b) return;
    if (pos[a] < pos[b]) swap(a, b);
    if (h[a] == h[b]) return (void)update(pos[b]+edge, pos[a],
        x);
    update(pos[h[a]], pos[a], x); update_path(dad[h[a]], b, x);
  }
  T query_subtree(int a) {
    if (edge and sz[a] == 1) return neutral;
    return query(pos[a]+edge, pos[a]+sz[a]-1);
  }
  void update_subtree(int a, T x) {
    if (edge and sz[a] == 1) return;
    update(pos[a] + edge, pos[a]+sz[a]-1, x);
  }
  int lca(int a, int b) {
    if (pos[a] < pos[b]) swap(a, b);
    return h[a] == h[b] ? b : lca(dad[h[a]], b);
  }
};
```

Hungarian.h

<bits/stdc++.h> 21d9f6, 63 lines

```
using namespace std;
//input: matrix n x m, n <= m
//return vector p of size n, where p[i] is the match for i
// and minimum cost
// time complexity: O(n^2 * m)
const int ms = 310, INF = 0x3f3f3f3f;
int u[ms], v[ms], p[ms], way[ms], minv[ms];
bool used[ms];
pair<vector<int>, int> solve(const vector<vector<int>> &matrix)
    {
  int n = matrix.size();
  if (n == 0)
    return {vector<int>(), 0};
  int m = matrix[0].size();
  assert(n <= m);
  memset(u, 0, (n + 1) * sizeof(int));
  memset(v, 0, (m + 1) * sizeof(int));
  memset(p, 0, (m + 1) * sizeof(int));
  for (int i = 1; i <= n; i++){
    memset(minv, 0x3f, (m + 1) * sizeof(int));
    memset(way, 0, (m + 1) * sizeof(int));
    for (int j = 0; j <= m; j++)
      used[j] = 0;
    p[0] = i;
    int k0 = 0;
    do{
      used[k0] = 1;
      int i0 = p[k0], delta = INF, k1 = 0;
      for (int j = 1; j <= m; j++){
        if (!used[j]){
          int cur = matrix[i0 - 1][j - 1] - u[i0] - v[j];
          if (cur < minv[j]){
            minv[j] = cur;
            way[j] = k0;
          }
          if (minv[j] < delta){
            delta = minv[j];
            k1 = j;
          }
        }
      }
      for (int j = 0; j <= m; j++){
        if (used[j]){
          u[p[j]] += delta;
          v[j] -= delta;
        }else{
          minv[j] -= delta;
        }
      }
      k0 = k1;
    } while (p[k0]);
    do{
      int k1 = way[k0];
      p[k0] = p[k1];
      k0 = k1;
    } while (k0);
  }
  vector<int> ans(n, -1);
  for (int j = 1; j <= m; j++){
    if (!p[j]) continue;
    ans[p[j] - 1] = j - 1;
  }
  return {ans, -v[0]};
}
```

LctVertex.h

<bits/stdc++.h> 363e20, 113 lines

```
using namespace std;
// Link-Cut Tree - Vertex, undirected version.
// All operations are O(log(n)) amortized.
// Source: https://github.com/brunomaletta/Biblioteca/
typedef long long ll;
typedef pair<int, int> pii;
const int MAXN = 200010;
namespace lct {
  struct node {
    int p, ch[2];
    ll val, sub;
    bool rev;
    int sz;
    ll lazy;
    node() {}
    node(int v) : p(-1), val(v), sub(v), rev(0), sz(1), lazy(0)
        {
      ch[0] = ch[1] = -1;
    }
  };
  node t[MAXN];
  void prop(int x) {
    if (t[x].lazy) {
      t[x].val += t[x].lazy, t[x].sub += t[x].lazy*t[x].sz;
      if (t[x].ch[0]+1) t[t[x].ch[0]].lazy += t[x].lazy;
      if (t[x].ch[1]+1) t[t[x].ch[1]].lazy += t[x].lazy;
    }
    if (t[x].rev) {
      swap(t[x].ch[0], t[x].ch[1]);
      if (t[x].ch[0]+1) t[t[x].ch[0]].rev ^= 1;
      if (t[x].ch[1]+1) t[t[x].ch[1]].rev ^= 1;
    }
    t[x].lazy = 0, t[x].rev = 0;
  }
  void update(int x) {
    t[x].sz = 1, t[x].sub = t[x].val;
    for (int i = 0; i < 2; i++) if (t[x].ch[i]+1) {
      prop(t[x].ch[i]);
      t[x].sz += t[t[x].ch[i]].sz;
      t[x].sub += t[t[x].ch[i]].sub;
    }
  }
  bool is_root(int x) {
    return t[x].p == -1 or (t[t[x].p].ch[0] != x and t[t[x].p].
        ch[1] != x);
  }
  void rotate(int x) {
    int p = t[x].p, pp = t[p].p;
    if (!is_root(p)) t[pp].ch[t[pp].ch[1] == p] = x;
    bool d = t[p].ch[0] == x;
    t[p].ch[!d] = t[x].ch[d], t[x].ch[d] = p;
    if (t[p].ch[!d]+1) t[t[p].ch[!d]].p = p;
    t[x].p = pp, t[p].p = x;
    update(p), update(x);
  }
  int splay(int x) {
    while (!is_root(x)) {
      int p = t[x].p, pp = t[p].p;
      if (!is_root(p)) prop(pp);
      prop(p), prop(x);
      if (!is_root(p)) rotate((t[pp].ch[0] == p)^(t[p].ch[0] ==
          x) ? x : p);
      rotate(x);
    }
    return prop(x), x;
  }
  int access(int v) {
    int last = -1;
    for (int w = v; w+1; update(last = w), splay(v), w = t[v].p
        )
      splay(w), t[w].ch[1] = (last == -1 ? -1 : v);
    return last;
  }
  // Public:
  void makeTree(int v, int w) {
    t[v] = node(w);
  }
  int findRoot(int v) {
    access(v), prop(v);
    while (t[v].ch[0]+1) v = t[v].ch[0], prop(v);
    return splay(v);
  }
  // Checks if v and w are connected
  bool connected(int v, int w) {
    access(v), access(w);
    return v == w ? true : t[v].p != -1;
  }
  // Change v to be root
  void rootify(int v) {
    access(v);
    t[v].rev ^= 1;
  }
  // Sum of the weight in path from v to w
  ll query(int v, int w) {
    rootify(w), access(v);
    return t[v].sub;
  }
  // Sum +x in path from v to w
  void update(int v, int w, int x) {
    rootify(w), access(v);
    t[v].lazy += x;
```

```
  }
  // Add edge (v, w)
  void link(int v, int w) {
    rootify(w);
    t[w].p = v;
  }
  // Remove edge (v, w)
  void cut(int v, int w) {
    rootify(w), access(v);
    t[v].ch[0] = t[t[v].ch[0]].p = -1;
  }
  int lca(int v, int w) {
    access(v);
    return access(w);
  }
}
```

MinCut.h

<bits/stdc++.h> 6f84ae, 50 lines

```
using namespace std;
typedef long long ll;
//This algorithm finds the Global Min-Cut in O(|V|^3)
namespace MinCut{
  const int MAXN = 510;
  bool exist[MAXN], in_a[MAXN];
  ll g[MAXN][MAXN], w[MAXN];
  vector<int> v[MAXN];
  int n;
  void init(int n1){
    n = n1;
    memset(g, 0, sizeof(g));
  }
  void addEdge(int a, int b, int w1){
    if(a == b) return;
    g[a][b] += w1;
    g[b][a] += w1;
  }
  pair<ll, vector<int>> mincut() {
    ll best_cost = 0x3f3f3f3f3f3f3f3fLL;
    vector<int> best_cut;
    for (int i=0; i<n; ++i)
      v[i].assign (1, i);
    memset (exist, true, sizeof(exist));
    for(int ph=0; ph<n-1; ++ph) {
      memset (in_a, false, sizeof in_a);
      memset (w, 0, sizeof w);
      for(int it=0, prev=0; it<n-ph; ++it){
        int sel = -1;
        for(int i=0; i<n; ++i)
          if(exist[i] && !in_a[i] && (sel == -1 || w[i] > w[sel
              ]))
            sel = i;
        if(it == n-ph-1){
          if(w[sel] < best_cost)
            best_cost = w[sel],  best_cut = v[sel];
          v[prev].insert (v[prev].end(), v[sel].begin(), v[sel
              ].end());
          for(int i=0; i<n; ++i)
            g[prev][i] = g[i][prev] += g[sel][i];
          exist[sel] = false;
        }else{
          in_a[sel] = true;
          for(int i=0; i<n; ++i)
            w[i] += g[sel][i];
          prev = sel;
        }
      }
    }
    return {best_cost, best_cut};
  }
};
```

MinimumCostMaximumFlow.h

<bits/stdc++.h> ccd79b, 98 lines

```
using namespace std;
//O(MaxFlow * path) or
//O(N * M * Path) = O(N^2*M^2) or O(N*M^2*log(n)) or O(N^3*M)
//                  SPFA          Dijkstra          Dijkstra
template <class T = int>
class MCMF{
private:
  struct Edge{
    int to;
    T cap, cost;
    Edge(int a, T b, T c) : to(a), cap(b), cost(c) {}
  };
  int n;
  vector<vector<int>> edges;
  vector<Edge> list;
  vector<int> from;
  vector<T> dist, pot;
  vector<bool> visit;
  pair<T, T> augment(int src, int sink){
    pair<T, T> flow = {list[from[sink]].cap, 0};
    for (int v = sink; v != src; v = list[from[v] ^ 1].to){
      flow.first = std::min(flow.first, list[from[v]].cap);
      flow.second += list[from[v]].cost;
    }
    for (int v = sink; v != src; v = list[from[v] ^ 1].to){
      list[from[v]].cap -= flow.first;
      list[from[v] ^ 1].cap += flow.first;
    }
    return flow;
  }
  queue<int> q;
  bool SPFA(int src, int sink){
    T INF = numeric_limits<T>::max();
    dist.assign(n, INF);
    from.assign(n, -1);
    q.push(src);
    dist[src] = 0;
    while (!q.empty()){
      int on = q.front();
      q.pop();
      visit[on] = false;
      for (auto e : edges[on]){
        auto ed = list[e];
        if (ed.cap == 0)
          continue;
        T toDist = dist[on] + ed.cost + pot[on] - pot[ed.to];
        if (toDist < dist[ed.to]){
          dist[ed.to] = toDist;
          from[ed.to] = e;
          if (!visit[ed.to]){
            visit[ed.to] = true;
            q.push(ed.to);
          }
        }
      }
    }
    return dist[sink] < INF;
  }
  void fixPot(){
    T INF = numeric_limits<T>::max();
    for (int i = 0; i < n; i++){
      if (dist[i] < INF)
        pot[i] += dist[i];
    }
  }
public:
  MCMF(int size){
    n = size;
    edges.resize(n);
    pot.assign(n, 0);
    dist.resize(n);
    visit.assign(n, false);
  }
  pair<T, T> solve(int src, int sink){
    pair<T, T> ans(0, 0);
    // Remove negative edges: Johnson's Algorithm
    if (!SPFA(src, sink))
      return ans;
    fixPot();
    // Can use dijkstra to speed up depending on the graph
    while (SPFA(src, sink)){
      auto flow = augment(src, sink);
      // When the priority is the minimum cost and not the flow
      // if(flow.second >= 0)
      //   break;
      ans.first += flow.first;
      ans.second += flow.first * flow.second;
      fixPot();
    }
    return ans;
  }
  void addEdge(int u, int to, T cap, T cost){
    edges[u].push_back(list.size());
    list.push_back(Edge(to, cap, cost));
    edges[to].push_back(list.size());
    list.push_back(Edge(u, 0, -cost));
  }
};
```

StronglyConnectedComponent.h

<bits/stdc++.h> e2d743, 44 lines

```
using namespace std;
typedef pair<int, int> pii;
namespace SCC{
  vector<vector<int>> adj, revAdj;
  vector<bool> visited;
  vector<int> ts, component;
  void dfs1(int u){
    visited[u] = true;
    for(int to : adj[u]){
      if(!visited[to])
        dfs1(to);
    }
    ts.push_back(u);
  }
  void dfs2(int u, int c){
    component[u] = c;
    for(int to : revAdj[u]){
      if(component[to] == -1)
        dfs2(to, c);
    }
  }
  vector<int> scc(int n, vector<pii> &edges){
    adj.assign(n, vector<int>());
    revAdj.assign(n, vector<int>());
    visited.assign(n, false);
    component.assign(n, -1);
    for(auto [a, b] : edges){
      adj[a].push_back(b);
      revAdj[b].push_back(a);
    }
    ts.clear();
    for (int i = 0; i < n; i++){
```

```
      if (!visited[i])
        dfs1(i);
    }
    reverse(ts.begin(), ts.end());
    int comp = 0;
    for (int u : ts){
      if (component[u] == -1)
        dfs2(u, comp++);
    }
    return component;
  }
}
```

TreeId.h

"centroid.h" 286bea, 37 lines

```
#define F first
#define S second
namespace TreeID{
  int id=0;
  map<map<int, int>, int> mpId;
  vector<int> adj[MAXN];
  int treeID(int u, int p){
    map<int, int> mp;
    for(int to: adj[u]){
      if(to != p)
        mp[treeID(to, u)]++;
    }
    if(!mpId.count(mp))
      mpId[mp] = ++id;
    return mpId[mp];
  }
  //Returns a pair of values that represents a tree only. O((N+M)*log(M))
  //0-indexed
  pii getTreeID(vector<pii> &edges, int n){
    for(int i=0; i<n; i++)
      adj[i].clear();
    Centroid::init(n);
    for(pii e: edges){
      adj[e.F].push_back(e.S);
      adj[e.S].push_back(e.F);
      Centroid::addEdge(e.F, e.S);
    }
    pii c = Centroid::findCentroid();
    pii ans(treeID(c.F, -1), treeID(c.S, -1));
    if(ans.F > ans.S)
      swap(ans.F, ans.S);
    return ans;
  }
  bool isomorphic(vector<pii> &tree1, vector<pii> &tree2, int n){
    return getTreeID(tree1, n) == getTreeID(tree2, n);
  }
};
```

Graph (6)

BasicMath.h

<bits/stdc++.h> b11d8a, 34 lines

```
using namespace std;
int fastPow(int base, string bigExp, int mod){
  int ans = 1;
  for(char c: bigExp){
    ans = fastPow(ans, 10, mod);
    ans = (ans*1LL*fastPow(base, c-'0', mod))%mod;
  }
  return ans;
}
//\sum_{i = 0}^{n - 1} floor((a * i + b)/m)
// 0 <= n <= 10^9
// 1 <= m <= 10^9
// 0 <= a, b < m
// O(log(a + b + c + d))
ll floor_sum(ll n, ll m, ll a, ll b) {
  ll ans = 0;
  if (a >= m) {
    ans += (n - 1) * n * (a / m) / 2;
    a %= m;
  }
  if (b >= m) {
    ans += n * (b / m);
    b %= m;
  }
  ll y_max = (a * n + b) / m, x_max = (y_max * m - b);
  if (y_max == 0) return ans;
  ans += (n - (x_max + a - 1) / a) * y_max;
  ans += floor_sum(y_max, a, m, (a - x_max % a) % a);
  return ans;
}
void enumeratingAllSubmasks(int mask){
  for (int s = mask; s; s = (s - 1) & mask)
    cout << s << endl;
}
```

BinomialCoefficients.h

<bits/stdc++.h>, "./basic_math.h", "./modular.h" f706a5, 57 lines

```
using namespace std;
typedef long long ll;

//O(P*log(P))
//C4(n, k, p) = Comb(n, k)%p
vector<int> changeBase(int n, int p){
  vector<int> v;
  while (n > 0){
    v.push_back(n % p);
    n /= p;
  }
  return v;
}
int C4(int n, int k, int p){
  auto vn = changeBase(n, p);
  auto vk = changeBase(k, p);
  int mx = max(vn.size(), vk.size());
  vn.resize(mx, 0);
  vk.resize(mx, 0);
  prevC3(p - 1, p);
  int ans = 1;
  for (int i = 0; i < mx; i++)
    ans = (ans * 1LL * C3(vn[i], vk[i], p)) % p;
  return ans;
}
//O(P^k)
//C5(n, k, p, pk) = Comb(n, k)%(p^k)
int fat_p(ll n, int p, int pk){
  vector<int> fat1(pk, 1);
    int res = 1;
    for(int i=1; i<pk; i++){
    if(i%p == 0)
      fat1[i] = fat1[i-1];
    else
      fat1[i] = (fat1[i-1]*1LL*i)%pk;
  }
  while(n > 1){
    res = (res*1LL*fastPow(fat1[pk-1], n/pk, pk))%pk;
    res = (res*1LL*fat1[n%pk])%pk;
    n /= p;
  }
  return res;
}
ll cnt(ll n, int p){
  ll ans = 0;
  while(n > 1){
    ans += n/p;
    n/=p;
  }
  return ans;
}
int C5(ll n, ll k, int p, int pk){
  ll exp = cnt(n, p) - cnt(n-k, p) - cnt(k, p);
  int d = (fat_p(n-k, p, pk)*1LL*fat_p(k, p, pk))%pk;
  int ans = (fat_p(n, p, pk)*1LL*inv(d, pk))%pk;
  return (ans*1LL*fastPow(p, exp, pk))%pk;
}
```

Catalan.h

<bits/stdc++.h> ecb6b4, 26 lines

```
using namespace std;
const int MOD = 1000000007;
ll C(int n, int k){
  if(k > n)
    return 0;
  return (fat[n]*((ifat[k]*ifat[n-k])%MOD))%MOD;
}
ll catalan(int n){
  return (C(2*n, n) - C(2*n, n-1) + MOD)%MOD;
}
ll f(int x1, int y1, int x2, int y2){
  int y = y2 - y1, x = x2 - x1;
  if(y < 0 or x < 0)
    return 0;
  return C(x + y, x);
}
// o = number of '(', c = number of ')', k = fixed prefix of '(' extra
// Catalan Generalization, open[i] >= close[i] for each 0 <= i < o + c + k
// where open[i] is number of '(' in prefix until i
// and close[i] is number of ')'
ll catalan2(int o, int c, int k){
  int x = o + k - c;
  if(x < 0)
    return 0;
  return (f(k, 0, o+k, c) - f(k, 0, o+k-x-1, c + x + 1) + MOD)%MOD;
}
```

ChineseRemainderTheorem.h

<bits/stdc++.h>, "extended_euclidean.h" 16b826, 26 lines

```
using namespace std;
typedef long long ll;
namespace CRT{
  inline ll normalize(ll x, ll mod){
    x %= mod;
    if (x < 0)
      x += mod;
    return x;
  }
  ll solve(vector<ll> a, vector<ll> m){
    int n = a.size();
    for (int i = 0; i < n; i++)
      normalize(a[i], m[i]);
    ll ans = a[0];
    ll lcm1 = m[0];
    for (int i = 1; i < n; i++){
```

```
      ll x, y;
      ll g = extGcd(lcm1, m[i], x, y);
      if ((a[i] - ans) % g != 0)
        return -1;
      ans = normalize(ans + ((((a[i] - ans) / g) * x) % (m[i] /
            g)) * lcm1, (lcm1 / g) * m[i]);
      lcm1 = (lcm1 / g) * m[i]; //lcm(lcm1, m[i]);
    }
    return ans;
  }
} // namespace CRT
```

DivisionTrick.h

b69a68, 11 lines

```
// O(sqrt(N))
// sum (n/i)
ll divisionTrick(ll n){
  ll ans = 0;
  for(ll l = 1, r; l <= n; l = r + 1) {
    r = n / (n / l);
    // n / i has the same value for l <= i <= r
    ans += (n/l)*(r-l+1);
  }
  return ans;
}
```

EulersTotient.h

<bits/stdc++.h> 246d75, 28 lines

```
using namespace std;
int nthPhi(int n){
  int result = n;
  for (int i = 2; i <= n / i; i++){
    if (n % i == 0){
      while (n % i == 0)
        n /= i;
      result -= result / i;
    }
  }
  if (n > 1)
    result -= result / n;
  return result;
}
vector<int> phiFrom1toN(int n){
  vector<int> vPhi(n + 1);
  vPhi[0] = 0;
  vPhi[1] = 1;
  for (int i = 2; i <= n; i++)
    vPhi[i] = i;
  for (int i = 2; i <= n; i++){
    if (vPhi[i] == i){
      for (int j = i; j <= n; j += i)
        vPhi[j] -= vPhi[j] / i;
    }
  }
  return vPhi;
}
```

ExtendedEuclidean.h

<bits/stdc++.h> d3b52f, 23 lines

```
using namespace std;
typedef long long ll;
ll extGcd(ll a, ll b, ll &x, ll &y){
  if (b == 0){
    x = 1, y = 0;
    return a;
  }else{
    ll g = extGcd(b, a % b, y, x);
    y -= (a / b) * x;
    return g;
  }
}
//a*x + b*y = g
//a*(x-(b/g)*k) + b*(y+(a/g)*k) = g
bool dioEq(ll a, ll b, ll c, ll &x0, ll &y0, ll &g){
  g = extGcd(abs(a), abs(b), x0, y0);
  if (c % g) return false;
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
```

Fft.h

<bits/stdc++.h> c3ee77, 68 lines

```
using namespace std;
struct complex_t {
  double a {0.0}, b {0.0};
  complex_t(){}
  complex_t(double na) : a{na}{}
  complex_t(double na, double nb) : a{na}, b{nb} {}
  const complex_t operator+(const complex_t &c) const {
    return complex_t(a + c.a, b + c.b);
  }
  const complex_t operator-(const complex_t &c) const {
    return complex_t(a - c.a, b - c.b);
  }
  const complex_t operator*(const complex_t &c) const {
    return complex_t(a*c.a - b*c.b, a*c.b + b*c.a);
  }
  const complex_t operator/(const int &c) const {
    return complex_t(a/c, b/c);
  }
};
//using cd = complex<double>;
using cd = complex_t;
const double PI = acos(-1);
void fft(vector<cd> &a, bool invert) {
  int n = a.size();
  for (int i = 1, j = 0; i < n; i++) {
    int bit = n >> 1;
    for (; j & bit; bit >>= 1)
      j ^= bit;
    j ^= bit;
    if (i < j)
      swap(a[i], a[j]);
  }
  for (int len = 2; len <= n; len <<= 1) {
    double ang = 2 * PI / len * (invert ? -1 : 1);
    cd wlen(cos(ang), sin(ang));
    for (int i = 0; i < n; i += len) {
      cd w(1);
      for (int j = 0; j < len / 2; j++) {
        cd u = a[i+j], v = a[i+j+len/2] * w;
        a[i+j] = u + v;
        a[i+j+len/2] = u - v;
        w = w * wlen;
      }
    }
  }
  if (invert){
    for (cd &x : a)
      x = x / n;
  }
}
typedef long long ll;
vector<ll> multiply(vector<int> &a, vector<int> &b) {
  vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
  int n = 1;
  while(n < int(a.size() + b.size()) )
    n <<= 1;
  fa.resize(n);
  fb.resize(n);
  fft(fa, false);
  fft(fb, false);
  for (int i = 0; i < n; i++)
    fa[i] = fa[i]*fb[i];
  fft(fa, true);
  vector<ll> result(n);
  for (int i = 0; i < n; i++)
    result[i] = ll(fa[i].a + 0.5);
  return result;
}
```

FloydCycleFinding.h

<bits/stdc++.h> a950d2, 24 lines

```
using namespace std;
int f(int x);
typedef pair<int, int> pii;
pii floydCycleFinding(int x0){
  int tortoise = f(x0), hare = f(f(x0));
  while(tortoise != hare){
    tortoise = f(tortoise);
    hare = f(f(hare));
  }
  int mu = 0;
  hare = x0;
  while(tortoise != hare){
    tortoise = f(tortoise);
    hare = f(hare);
    mu++;
  }
  int lambda = 1;
  hare = f(tortoise);
  while(tortoise != hare){
    hare = f(hare);
    lambda++;
  }
  return pii(mu, lambda);
}
```

FunctionRootUsingNewton.h

<bits/stdc++.h> 255af8, 50 lines

```
using namespace std;
typedef long double ld;
struct Poly{
  vector<ld> v;
  Poly(vector<ld> &v1):v(v1){}
  //return f(x)
  ld f(ld x){
    ld ans = 0;
    ld e = 1;
    int n = v.size();
    for(int i=0; i<n; i++){
      ans += v[i] * e;
      e *= x;
    }
    return ans;
  }
  //return f'(x)
  ld df(ld x){
    ld ans = 0;
    ld e = 1;
    int n = v.size();
    for(int i=1; i<n; i++){
```

```
      ans += i * v[i] * e;
      e *= x;
    }
    return ans;
  }
  // takes some root of the polynomial
  ld root(ld x0=1){
    const ld eps = 1E-10;
    ld x = x0;
    for (;;) {
      ld nx = x - (f(x)/df(x));
      if (abs(x - nx) < eps)
        break;
      x = nx;
    }
    return x;
  }
  //div f(x) by (x-a)
  void div(ld a){
    int g = (int)v.size() - 1;
    vector<ld> aux(g);
    for(int i=g; i>=1; i--){
      aux[i-1] = v[i];
      v[i-1] += a*aux[i-1];
    }
    v = aux;
  }
};
```

Gauss.h

<bits/stdc++.h> a6d5e8, 43 lines

```
using namespace std;
const int INF = 0x3f3f3f3f;
typedef long double ld;
const ld EPS = 1e-9;
int gauss(vector<vector<ld>> a, vector<ld> &ans) {
  int n = (int) a.size();
  int m = (int) a[0].size() - 1;
  vector<int> where (m, -1);
  for (int col=0, row=0; col<m && row<n; col++) {
    int sel = row;
    for (int i=row; i<n; i++)
      if (abs(a[i][col]) > abs(a[sel][col]))
        sel = i;
    if (abs(a[sel][col]) < EPS)
      continue;
    for (int i=col; i<=m; i++)
      swap(a[sel][i], a[row][i]);
    where[col] = row;
    for (int i=0; i<n; i++){
      if (i != row) {
        ld c = a[i][col] / a[row][col];
        for (int j=col; j<=m; j++)
          a[i][j] -= a[row][j] * c;
      }
    }
    row++;
  }
  ans.assign(m, 0);
  for (int i=0; i<m; i++)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i=0; i<n; i++) {
    ld sum = 0;
    for (int j=0; j<m; j++)
      sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
      return 0;
  }
  for (int i=0; i<m; i++)
    if (where[i] == -1)
      return INF;
  return 1;
}
```

GaussXor.h

<bits/stdc++.h> 3243b0, 40 lines

```
using namespace std;
const int MAXB = 30;
struct GaussXOR {
  int table[MAXB];
  GaussXOR() {
    for(int i = 0; i < MAXB; i++) {
      table[i] = 0;
    }
  }
  int size() {
    int ans = 0;
    for(int i = 0; i < MAXB; i++) {
      if(table[i]) ans++;
    }
    return ans;
  }
  bool isComb(int x) {
    for(int i = MAXB-1; i >= 0; i--) {
      x = std::min(x, x ^ table[i]);
    }
    return x == 0;
  }
  void add(int x) {
    for(int i = MAXB-1; i >= 0; i--) {
      if((table[i] == 0) and ((x>>i) & 1)){
        table[i] = x;
        x = 0;
      } else {
        x = std::min(x, x ^ table[i]);
      }
    }
  }
  int max(){
    int ans = 0;
    for(int i = MAXB-1; i >= 0; i--) {
      ans = std::max(ans, ans ^ table[i]);
    }
    return ans;
  }
};
```

GrayCode.h

6bf231, 9 lines

```
int grayCode(int nth){
  return nth ^ (nth >> 1);
}
int revGrayCode(int g){
  int nth = 0;
  for (; g > 0; g >>= 1)
    nth ^= g;
  return nth;
}
```

Karatsuba.h

<bits/stdc++.h> 73ef6c, 36 lines

```
using namespace std;
// Source: https://github.com/brunomaletta/Biblioteca/blob/
    master/Codigo/Matematica/karatsuba.cpp
//#pragma GCC optimize("Ofast")
//#pragma GCC target ("avx,avx2")
template<typename T> void kar(T* a, T* b, int n, T* r, T* tmp)
    {
  if (n <= 64) {
    for (int i = 0; i < n; i++)
      for (int j = 0; j < n; j++)
        r[i+j] += a[i] * b[j];
    return;
  }
  int mid = n/2;
  T *atmp = tmp, *btmp = tmp+mid, *E = tmp+n;
  memset(E, 0, sizeof(E[0])*n);
  for (int i = 0; i < mid; i++) {
    atmp[i] = a[i] + a[i+mid];
    btmp[i] = b[i] + b[i+mid];
  }
  kar(atmp, btmp, mid, E, tmp+2*n);
  kar(a, b, mid, r, tmp+2*n);
  kar(a+mid, b+mid, mid, r+n, tmp+2*n);
  for (int i = 0; i < mid; i++) {
    T temp = r[i+mid];
    r[i+mid] += E[i] - r[i] - r[i+2*mid];
    r[i+2*mid] += E[i+mid] - temp - r[i+3*mid];
  }
}
// O(n^1.58), Advantages: you can add any module
template<typename T> vector<T> karatsuba(vector<T> a, vector<T>
    b) {
  int n = max(a.size(), b.size());
  while (n&(n-1)) n++;
  a.resize(n), b.resize(n);
  vector<T> ret(2*n), tmp(4*n);
  kar(&a[0], &b[0], n, &ret[0], &tmp[0]);
  return ret;
}
```

Lagrange.h

<bits/stdc++.h> 34df30, 48 lines

```
using namespace std;
typedef long double ld;
struct PointValue{
  ld x, y;
  PointValue(ld x0=0, ld y0=0): x(x0), y(y0){}
};
void mul(vector<ld> &A, int x0){ // multiply A(x) by (x - x0)
  int n = A.size();
  A.push_back(0);
  auto B = A;
  for(int i=n; i>=1; i--){
    A[i] = A[i-1];
  }
  A[0] = 0;
  for(int i=0; i<n+1; i++)
    A[i] -= B[i]*x0;
}
void div(vector<ld> &A, int x0){ // multiply A(x) by (x - x0)
  int g = (int)A.size() - 1;
  vector<ld> aux(g);
  for(int i=g; i>=1; i--){
    aux[i-1] = A[i];
    A[i-1] += x0*aux[i-1];
  }
  A = aux;
}
// Change Polynomial Representation from Point-Value to
    Coefficient
// O(n^2)
vector<ld> LagrangeInterpolation(vector<PointValue> vp){
  vector<ld> A(1, 1);
  int n = vp.size();
```

```
  for(int i=0; i<n; i++)
    mul(A, vp[i].x);
  vector<ld> ans(n, 0);
  for(int i=0; i<n; i++){
    ld x = vp[i].x, y = vp[i].y;
    div(A, x);
    ld d = 1;
    for(int j=0; j<n; j++){
      if(j != i)
        d *= (x - vp[j].x);
    }
    for(int j=0; j<n; j++)
      ans[j] += A[j]*(y/d);
    mul(A, vp[i].x);
  }
  return ans;
}
```

Ntt.h

<bits/stdc++.h> b371b9, 133 lines

```
using namespace std;
typedef long long ll;
const int MOD = 998244353;
inline int modMul(int a, int b) {
  return (int) ((a*(ll)b) % MOD);
}
namespace ntt {
  int base = 1;
  vector<int> roots = {0, 1};
  vector<int> rev = {0, 1};
  int max_base = -1;
  int root = -1;
  inline int power(int a, long long b) {
    int res = 1;
    while (b > 0) {
      if (b & 1)
        res = modMul(res, a);
      a = modMul(a, a);
      b >>= 1;
    }
    return res;
  }
  inline int inv(int a) {
    a %= MOD;
    if (a < 0) a += MOD;
    int b = MOD, u = 0, v = 1;
    while(a){
      int t = b / a;
      b -= t * a; swap(a, b);
      u -= t * v; swap(u, v);
    }
    assert(b == 1);
    if (u < 0) u += MOD;
    return u;
  }
  void init() {
    int tmp = MOD - 1;
    max_base = 0;
    while (tmp % 2 == 0) {
      tmp /= 2;
      max_base++;
    }
    root = 2;
    while (true) {
      if (power(root, 1 << max_base) == 1) {
        if (power(root, 1 << (max_base - 1)) != 1) {
          break;
        }
      }
      root++;
    }
  }
  void ensure_base(int nbase) {
    if (max_base == -1)
      init();
    if (nbase <= base)
      return;
    assert(nbase <= max_base);
    rev.resize(1 << nbase);
    for (int i = 0; i < (1 << nbase); i++)
      rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (nbase - 1));
    roots.resize(1 << nbase);
    while (base < nbase) {
      int z = power(root, 1 << (max_base - 1 - base));
      for (int i = 1 << (base - 1); i < (1 << base); i++) {
        roots[i << 1] = roots[i];
        roots[(i << 1) + 1] = modMul(roots[i], z);
      }
      base++;
    }
  }
  void fft(vector<int> &a) {
    int n = (int) a.size();
    assert((n & (n - 1)) == 0);
    int zeros = __builtin_ctz(n);
    ensure_base(zeros);
    int shift = base - zeros;
    for (int i = 0; i < n; i++) {
      if (i < (rev[i] >> shift)) {
        swap(a[i], a[rev[i] >> shift]);
      }
    }
    for (int k = 1; k < n; k <<= 1) {
      for (int i = 0; i < n; i += 2 * k) {
        for (int j = 0; j < k; j++) {
          int x = a[i + j];
          int y = modMul(a[i + j + k], roots[j + k]);
          a[i + j] = x + y - MOD;
          if (a[i + j] < 0) a[i + j] += MOD;
          a[i + j + k] = x - y + MOD;
          if (a[i + j + k] >= MOD) a[i + j + k] -= MOD;
        }
      }
    }
  }
  vector<int> multiply(vector<int> a, vector<int> b, int eq =
      0) {
    int need = (int) (a.size() + b.size() - 1);
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    a.resize(sz);
    b.resize(sz);
    fft(a);
    if (eq)
      b = a;
    else
      fft(b);
    int inv_sz = inv(sz);
    for (int i = 0; i < sz; i++)
      a[i] = modMul(modMul(a[i], b[i]), inv_sz);
    reverse(a.begin() + 1, a.end());
    fft(a);
    a.resize(need);
    return a;
  }
  vector<int> pow(vector<int> a, ll e){
    int need = (int) ( (a.size()-1)*e + 1);
    int nbase = 0;
    while ((1 << nbase) < need) nbase++;
    ensure_base(nbase);
    int sz = 1 << nbase;
    a.resize(sz);
    fft(a);
    int inv_sz = ntt::inv(sz);
    for (int i = 0; i < sz; i++)
      a[i] = modMul(power(a[i], e), inv_sz);
    reverse(a.begin() + 1, a.end());
    fft(a);
    a.resize(need);
    return a;
  }
};
```

Prime.h

<bits/stdc++.h>, "basic_math.h" 2a1280, 55 lines

```
using namespace std;
typedef unsigned long long ull;
ull modMul(ull a, ull b, ull mod){
  return (a * (__uint128_t)b) % mod;
}
bool checkComposite(ull n, ull a, ull d, int s){
  ull x = fastPow(a, d, n);
  if (x == 1 or x == n - 1)
    return false;
  for (int r = 1; r < s; r++){
    x = modMul(x, x, n);
    if (x == n - 1LL)
      return false;
  }
  return true;
};
bool millerRabin(ull n){
  if (n < 2)
    return false;
  int r = 0;
  ull d = n - 1LL;
  while ((d & 1LL) == 0){
    d >>= 1;
    r++;
  }
  for (ull a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}){
    if (n == a)
      return true;
    if (checkComposite(n, a, d, r))
      return false;
  }
  return true;
}
ull pollard(ull n){
  auto f = [n](ull x) { return modMul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 0, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1){
    if (x == y)
      x = ++i, y = f(x);
    if ((q = modMul(prd, max(x, y) - min(x, y), n)))
      prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n){
  if (n == 1)
    return {};
  if (millerRabin(n))
    return {n};
```

```
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), r.begin(), r.end());
  return l;
}
```

SieveAndPrimes.h

<bits/stdc++.h> fbecb5, 27 lines

```
using namespace std;
typedef long long ll;
int mobius[1000010];
void sieveMobius(ll l) {
  sieve(l);
  mobius[1] = 1;
  for(int i=2; i<=l; i++)
    mobius[i] = 0;
  for(ll p: primes){
    if(p > l) break;
    for(ll j = p; j <= l; j += p){
      if(mobius[j] != -1){
        mobius[j]++;
        if(j%(p*p) == 0)
          mobius[j] = -1;
      }
    }
  }
  for(int i=2; i<=l; i++){
    if(mobius[i] == -1)
      mobius[i] = 0;
    else if(mobius[i]%2 == 0)
      mobius[i] = 1;
    else
      mobius[i] = -1;
  }
}
```

SimpsonIntegration.h

<bits/stdc++.h> f21f13, 13 lines

```
using namespace std;
double f(double x);
const int N = 1000000;
double simpson_integration(double a, double b){
  double h = (b - a) / N;
  double s = f(a) + f(b); // a = x_0 and b = x_2n
  for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's formula
    double x = a + h * i;
    s += f(x) * ((i & 1) ? 4 : 2);
  }
  s *= h / 3;
  return s;
}
```

XorAndOrConvolution.h

<bits/stdc++.h> 8dfd2b, 63 lines

```
using namespace std;
typedef long long ll;
void xorFWHT(vector<ll> &P, bool inverse=false){
  int n = P.size();
  for(int len = 1; 2 * len <= n; len <<= 1){
    for(int i = 0; i < n; i += 2 * len){
      for(int j = 0; j < len; j++){
        ll u = P[i + j];
        ll v = P[i + len + j];
        P[i + j] = u + v;
        P[i + len + j] = u - v;
      }
    }
  }
  if(inverse){
    for (int i = 0; i < n; i++){
      P[i] /= n;
    }
  }
}
void orFWHT(vector<ll> &P, bool inverse=false){
  int n = P.size();
  for(int len = 1; 2 * len <= n; len <<= 1){
    for(int i = 0; i < n; i += 2 * len){
      for(int j = 0; j < len; j++){
        if(inverse)
          P[i + len + j] -= P[i + j];
        else
          P[i + len + j] += P[i + j];
      }
    }
  }
}
void andFWHT(vector<ll> &P, bool inverse=false){
  int n = P.size();
  for(int len = 1; 2 * len <= n; len <<= 1){
    for(int i = 0; i < n; i += 2 * len){
      for(int j = 0; j < len; j++){
        ll u = P[i + j];
        ll v = P[i + len + j];
        if(inverse){
          P[i + j] = v - u;
          P[i + len + j] = u;
        }else{
          P[i + j] = v;
          P[i + len + j] = u + v;
        }
      }
    }
  }
}
vector<ll> convolution(vector<ll> a, vector<ll> b){
  int mx = max(a.size(), b.size());
  int n = 1;
  while(n < mx)
    n <<= 1;
  a.resize(n, 0); b.resize(n, 0);
  xorFWHT(a); xorFWHT(b);
  for(int i=0; i<n; i++)
    a[i] *= b[i];
  xorFWHT(a, true);
  return a;
}
```

Strings (7)

AhoCorasick.h

<bits/stdc++.h> 372026, 126 lines

```
#define F first
#define S second
using namespace std;
const int K = 26;
inline int getID(char c){
  return c-'a';
}
namespace Aho{
  struct Vertex {
    int next[K], go[K];
    int leaf = -1; // CAUTION with repeated strings!
    int p = -1, sz, match=-1;
    char pch;
    int suff_link = -1;
    int end_link = -1;
    Vertex(int p1=-1, char ch1='$', int sz1=0) : p(p1), pch(ch1){
      fill(begin(next), end(next), -1);
      fill(begin(go), end(go), -1);
      sz = sz1;
    }
  };
  vector<Vertex> trie;
  void init(){
    trie.clear();
    trie.emplace_back();
  }
  int add_string(string const& s, int id=1) {
    int v = 0;
    for (char ch : s) {
      int c = getID(ch);
      if (trie[v].next[c] == -1) {
        trie[v].next[c] = trie.size();
        trie.emplace_back(v, ch, trie[v].sz+1);
      }
      v = trie[v].next[c];
    }
    trie[v].leaf = id;
    return v;
  }
  int go(int v, char ch);
  int get_suff_link(int v) {
    if (trie[v].suff_link == -1) {
      if (v == 0 || trie[v].p == 0)
        trie[v].suff_link = 0;
      else
        trie[v].suff_link = go(get_suff_link(trie[v].p), trie[v].pch);
    }
    return trie[v].suff_link;
  }
  int get_end_link(int v) {
    if (trie[v].end_link == -1) {
      if (v == 0 || trie[v].p == 0){
        trie[v].end_link = 0;
      }else{
        int suff_link = get_suff_link(v);
        if(trie[suff_link].leaf != -1)
          trie[v].end_link = suff_link;
        else
          trie[v].end_link = get_end_link(suff_link);
      }
    }
    return trie[v].end_link;
  }
  int go(int v, char ch) {
    int c = getID(ch);
    if (trie[v].go[c] == -1) {
      if (trie[v].next[c] != -1)
        trie[v].go[c] = trie[v].next[c];
      else
        trie[v].go[c] = (v == 0) ? 0 : go(get_suff_link(v), ch);
    }
    return trie[v].go[c];
  }
};
//Aplication:
typedef pair<int, int> pii;
void addMatch(vector<pii> &ans, int v, int i){
```

```
  // This runs at most sqrt(N) times:1+2+3+4+..+sqrt(N)=N
  while(v != 0){
    // The string id is Aho::trie[v].leaf
    ans.emplace_back(i - Aho::trie[v].sz + 1, i);
    v = Aho::get_end_link(v);
  }
}
//Get match positions: O(answer) = O(N * sqrt(N))
vector<pii> whatMatch(string t){
  int state = 0;
  int i=0;
  vector<pii> ans;
  for(char c : t){
    state = Aho::go(state, c);
    if(Aho::trie[state].leaf != -1)
      addMatch(ans, state, i);
    else
      addMatch(ans, Aho::get_end_link(state), i);
    i++;
  }
  sort(ans.begin(), ans.end());
  return ans;
}
int countMatch(int v){
  if(Aho::trie[v].match == -1) {
    if (v == 0 || Aho::trie[v].p == 0){
      if(Aho::trie[v].leaf != -1)
        Aho::trie[v].match = 1;
      else
        Aho::trie[v].match = 0;
    }else{
      if(Aho::trie[v].leaf != -1)
        Aho::trie[v].match = 1 + countMatch(Aho::get_end_link(v
            ));
      else
        Aho::trie[v].match = countMatch(Aho::get_end_link(v));
    }
  }
  return Aho::trie[v].match;
}
//Get match amount: O(t)
long long matchAmount(string t){
  int state = 0;
  long long ans = 0;
  for(char c : t){
    state = Aho::go(state, c);
    ans += countMatch(state);
  }
  return ans;
}
```

Eertree.h

<bits/stdc++.h> 4c95c7, 87 lines

```
using namespace std;
const int MAXN = 100010;
typedef long long ll;
namespace eertree{
  struct Node {
    int i, j;
    int sz, suf;
    int to[26]; //Can change to vector<pii>
  };
  Node tree[MAXN];
  int f[MAXN], cnt[MAXN], p[MAXN];
  int currNode, n, len;
  char s[MAXN];
  int newNode(int l, int r){
    Node &no = tree[++n];
    f[n] = p[n] = 0;
    no.i = l, no.j = r;
    no.sz = r-l+1;
    memset(no.to, 0, sizeof(no.to));
    return n;
  }
  void init(){
    n = len = 0;
    newNode(0, -2);
    tree[1].suf = 1;
    newNode(0, -1);
    tree[2].suf = 1;
    currNode = 1;
  }
  int getId(char c){
    return c-'a';
  }
  // O(1) amortized
  void add(char c){
    int tmp = currNode, idx = len++, idC = getId(c);
    s[idx] = c;
    while (true) {
      int sz = tree[tmp].sz;
      if (idx - sz >= 1 and s[idx] == s[idx-sz-1])
        break;
      tmp = tree[tmp].suf;
    }
    if(tree[tmp].to[idC] != 0) {
      currNode = tree[tmp].to[idC];
    }else{
      currNode = newNode(idx - (tree[tmp].sz + 2) + 1, idx);
      tree[tmp].to[idC] = currNode;
      tmp = tree[tmp].suf;
      if (tree[currNode].sz == 1) {
        tree[currNode].suf = 2;
      }else{
        while (true) {
          int sz = tree[tmp].sz;
          if (idx-sz >= 1 and s[idx] == s[idx-sz-1])
            break;
          tmp = tree[tmp].suf;
        }
        tree[currNode].suf = tree[tmp].to[idC];
      }
      p[currNode] = p[tree[currNode].suf] + 1;
    }
    f[currNode]++;
  }
  //Returns the total of distinct palindrome substrings
  int size(){
    return n - 2;
  }
  //Returns the number of the suffix that is palindrome. Online
      .
  int countSuffix(){
    return p[currNode];
  }
  // Calculates the number of equal palindromes and saves in
      cnt
  // Returns the total of palindrome substrings
  ll precompute(){
    ll ans = 0;
    for(int i=0; i<=n; i++) cnt[i] = f[i];
    for(int i=n; i>=3; i--){
      ans += cnt[i];
      cnt[tree[i].suf] += cnt[i];
    }
    return ans;
  }
  // Call precompute before
  int count(int id){
    return cnt[id];
  }
};
```

Hashing.h

<bits/stdc++.h> b37aec, 34 lines

```
using namespace std;
struct StringHashing{
  const uint64_t MOD = (1LL<<61) - 1;
  const int base = 31;
  uint64_t modMul(uint64_t a, uint64_t b){
    uint64_t l1 = (uint32_t)a, h1 = a>>32, l2 = (uint32_t)b, h2
        = b>>32;
    uint64_t l = l1*l2, m = l1*h2 + l2*h1, h = h1*h2;
    uint64_t ret = (l&MOD) + (l>>61) + (h << 3) + (m >> 29) +
        ((m << 35) >> 3) + 1;
    ret = (ret & MOD) + (ret>>61);
    ret = (ret & MOD) + (ret>>61);
    return ret-1;
  }
  int getInt(char c){
    return c-'a'+1;
  }
  vector<uint64_t> hs, p;
//Public:
  StringHashing(string s){
    int n = s.size();
    hs.resize(n); p.resize(n);
    p[0] = 1;
    hs[0] = getInt(s[0]);
    for(int i=1; i<n; i++){
      p[i] = modMul(p[i-1], base);
      hs[i] = (modMul(hs[i-1], base) + getInt(s[i]))%MOD;
    }
  }
  uint64_t getValue(int l, int r){
    if(l > r) return -1;
    uint64_t res = hs[r];
    if(l > 0) res = (res + MOD - modMul(p[r-l+1], hs[l-1]))%MOD
        ;
    return res;
  }
};
```

Kmp.h

<bits/stdc++.h> da8117, 37 lines

```
using namespace std;
// "abcabcd" is [0,0,0,1,2,3,0]
// "aabaaab" is [0,1,0,1,2,2,3]
vector<int> kmp(string s) {
  int n = (int)s.length();
  // pi[i] is the length of the longest proper prefix of the
      substring
  // s[0..i] which is also a suffix of this substring.
  vector<int> pi(n);
  for (int i = 1; i < n; i++) {
    int j = pi[i-1];
    while (j > 0 and s[i] != s[j])
      j = pi[j-1];
    if (s[i] == s[j])
      j++;
    pi[i] = j;
  }
  return pi;
}
int K = 26;
inline int getID(char c){
```

```
  return c-'a';
}
vector<vector<int>> computeAutomaton(string s) {
  s += '#';
  int n = s.size();
  vector<int> pi = kmp(s);
  vector<vector<int>> aut(n, vector<int>(26));
  for(int i = 0; i < n; i++){
    for(int c = 0; c < K; c++){
      if(i > 0 and c != getID(s[i]))
        aut[i][c] = aut[pi[i-1]][c];
      else
        aut[i][c] = i + (c == getID(s[i]));
    }
  }
  return aut;
}
```

Manacher.h

<bits/stdc++.h> 005335, 32 lines

```
using namespace std;
// source: https://github.com/brunomaletta/Biblioteca/blob/
    master/Codigo/Strings/manacher.cpp
// ret[2*i] = larger size palindrome centered on i
// ret[2*i+1] = larger size palindrome centered on i and i + 1
vector<int> manacher(const string &s) {
  int l = 0, r = -1, n = s.size();
  vector<int> d1(n), d2(n);
  for (int i = 0; i < n; i++) {
    int k = i > r ? 1 : min(d1[l+r-i], r-i);
    while (i+k < n && i-k >= 0 && s[i+k] == s[i-k]) k++;
    d1[i] = k--;
    if (i+k > r) l = i-k, r = i+k;
  }
  l = 0, r = -1;
  for (int i = 0; i < n; i++) {
    int k = i > r ? 0 : min(d2[l+r-i+1], r-i+1); k++;
    while (i+k <= n && i-k >= 0 && s[i+k-1] == s[i-k]) k++;
    d2[i] = --k;
    if (i+k-1 > r) l = i-k, r = i+k-1;
  }
  vector<int> ret(2*n-1);
  for (int i = 0; i < n; i++) ret[2*i] = 2*d1[i]-1;
  for (int i = 0; i < n-1; i++) ret[2*i+1] = 2*d2[i+1];
  return ret;
}
struct Palindrome {
  vector<int> man;
  Palindrome(const string &s) : man(manacher(s)) {}
  bool isPalindrome(int i, int j) {
    return man[i+j] >= j-i+1;
  }
};
```

MinCyclicString.h

<bits/stdc++.h> 468079, 20 lines

```
using namespace std;
string min_cyclic_string(string s){
  s += s;
  int n = s.size();
  int i = 0, ans = 0;
  while (i < n / 2){
    ans = i;
    int j = i + 1, k = i;
    while (j < n && s[k] <= s[j]){
      if (s[k] < s[j])
        k = i;
      else
        k++;
      j++;
    }
    while (i <= k)
      i += j - k;
  }
  return s.substr(ans, n / 2);
}
```

SuffixArray.h

<bits/stdc++.h> c98212, 90 lines

```
#define all(x) x.begin(),x.end()
using namespace std;
typedef pair<int, int> pii;
vector<int> sort_cyclic_shifts(vector<int> &v) {
  int n = v.size();
  const int alphabet = n+1;
  vector<int> p(n), c(n), cnt(alphabet, 0);
  for(int i = 0; i < n; i++)
    cnt[v[i]]++;
  for(int i = 1; i < alphabet; i++)
    cnt[i] += cnt[i-1];
  for(int i = 0; i < n; i++)
    p[--cnt[v[i]]] = i;
  c[p[0]] = 0;
  int classes = 1;
  for(int i = 1; i < n; i++) {
    if(v[p[i]] != v[p[i-1]])
      classes++;
    c[p[i]] = classes - 1;
  }
  vector<int> pn(n), cn(n);
  for(int h = 0; (1 << h) < n; ++h) {
    //Ordenando pelo second no RadixSort
    int h2 = (1 << h);
    for(int i = 0; i < n; i++){
      pn[i] = p[i] - h2;
      if(pn[i] < 0) pn[i] += n;
    }
    fill(cnt.begin(), cnt.begin() + classes, 0);
    for(int i = 0; i < n; i++)
      cnt[c[p[i]]]++;
    for(int i = 1; i < classes; i++)
      cnt[i] += cnt[i-1];
    for(int i = n-1; i >= 0; i--)
      p[--cnt[c[pn[i]]]] = pn[i];
    cn[p[0]] = 0;
    classes = 1;
    for(int i = 1; i < n; i++){
      pii cur(c[p[i]], c[(p[i] + h2) % n]);
      pii prev(c[p[i-1]], c[(p[i-1] + h2) % n]);
      if(cur != prev)
        ++classes;
      cn[p[i]] = classes - 1;
    }
    c.swap(cn);
  }
  return p;
}
// O(N*log(N))
vector<int> sa_construction(vector<int> v) {
  auto aux = v;
  sort(all(aux));
  for(int &x: v)
    x = (lower_bound(all(aux), x) - aux.begin()) + 1;
  v.push_back(0);
  vector<int> suffix = sort_cyclic_shifts(v);
  suffix.erase(suffix.begin());
  return suffix;
}
// Kasai's algorithm: O(N)
vector<int> lcp_construction(vector<int> const& v, vector<int>
    const& suf) {
  int n = v.size();
  vector<int> rank(n, 0);
  for(int i = 0; i < n; i++)
    rank[suf[i]] = i;
  int k = 0;
  vector<int> lcp(n-1, 0);
  for(int i = 0; i < n; i++){
    if (rank[i] == n - 1) {
      k = 0; continue;
    }
    int j = suf[rank[i] + 1];
    while (i + k < n && j + k < n && v[i+k] == v[j+k])
      k++;
    lcp[rank[i]] = k;
    if (k) k--;
  }
  return lcp;
}
// (ss[i] = k) --> {s[i..k], s[i..k+1], ..., s[i..n-1]}
vector<int> getDistinctSubstrings(vector<int> &v){
  int n = v.size();
  auto suf = sa_construction(v);
  auto lcp = lcp_construction(v, suf);
  vector<int> ss(n);
  ss[suf[0]] = suf[0] + 0;
  for(int i=1; i<n; i++)
    ss[suf[i]] = suf[i] + lcp[i-1];
  return ss;
}
```

SuffixTree.h

<bits/stdc++.h> b403fd, 106 lines

```
typedef long long ll;
using namespace std;
namespace SuffixTree {
const int NS = 60; //Number of strings
const int MAXN = 100010; //Number of letters
int cn, cd, ns, en = 1, lst;
string S[NS]; int lastS = -1;
/* sufn[si][i] no do sufixo S[si][i...] */
vector<int> sufn[NS];
struct Node {
  int l, r, si=0;
  int p, suf=0;
  map<char, int> adj;
  Node() : l(0), r(-1){ suf = p = 0; }
  Node(int l1, int r1, int s1, int p1) : l(l1), r(r1), si(s1),
      p(p1) {}
  inline int len() { return r - l + 1; }
  inline int operator[](int i) { return S[si][l + i]; }
  inline int& operator()(char c) { return adj[c]; }
};
Node t[2*MAXN];
inline int new_node(int l, int r, int s, int p) {
  t[en] = Node(l, r, s, p);
  return en++;
}
void init(){
  t[0] = Node();
  cn=0, cd=0, ns=0, en = 1, lst=0;
  lastS = -1;
}
//The strings are inserted independently
void add_string(string s, char id='$') {
  assert(id < 'A');
```

```
  s += id;
  S[++lastS] = s;
  sufn[lastS].resize(s.size() + 1);
  cn = cd = 0;
  int i = 0; const int n = s.size();
  for(int j = 0; j < n; j++){
    for(; i <= j; i++) {
      if(cd == t[cn].len() && t[cn](s[j]))
        cn = t[cn](s[j]), cd = 0;
      if(cd < t[cn].len() && t[cn][cd] == s[j]) {
        cd++;
        if(j < (int)s.size() - 1) break;
        else {
          if(i) t[lst].suf = cn;
          for(; i <= j; i++) {
            sufn[lastS][i] = cn;
            cn = t[cn].suf;
          }
        }
      } else if(cd == t[cn].len()) {
        sufn[lastS][i] = en;
        if(i) t[lst].suf = en;
        lst = en;
        t[cn](s[j]) = new_node(j, n - 1, lastS, cn);
        cn = t[cn].suf;
        cd = t[cn].len();
      } else {
        int mid = new_node(t[cn].l, t[cn].l + cd - 1, t[cn].si,
              t[cn].p);
        t[t[cn].p](t[cn][0]) = mid;
        if(ns) t[ns].suf = mid;
        if(i) t[lst].suf = en;
        lst = en;
        sufn[lastS][i] = en;
        t[mid](s[j]) = new_node(j, n - 1, lastS, mid);
        t[mid](t[cn][cd]) = cn;
        t[cn].p = mid; t[cn].l += cd;
        cn = t[mid].p;
        int g = cn? j - cd : i + 1;
        cn = t[cn].suf;
        while(g < j && g + t[t[cn](S[lastS][g])].len() <= j)
          cn = t[cn](S[lastS][g]), g += t[cn].len();
        if(g == j)
          ns = 0, t[mid].suf = cn, cd = t[cn].len();
        else
          ns = mid, cn = t[cn](S[lastS][g]), cd = j - g;
      }
    }
  }
}
bool match(string &s, int i=0, int no=0, int iEdge=0){
  if(i == (int)s.size())
    return true;
  if(iEdge == t[no].len()){ //I arrived at the Node
    if(t[no].adj.count(s[i]))
      return match(s, i+1, t[no].adj[s[i]], 1);
    else
      return false;
  }
  if(t[no][iEdge] == s[i])
    return match(s, i+1, no, iEdge+1);
  return false;
}
typedef tuple<int, int, int> tp;
// O(n), substring <i, l, r> = s[i..l], s[i..l+1], ..., s[i..r]
void getDistinctSubstrings(vector<tp> &v, int no=0, int d=0){
  d += t[no].len() - t[no].adj.empty();
  int l = t[no].l, r = t[no].r - t[no].adj.empty();
  if(l <= r){
    v.emplace_back(r - d + 1, l, r);
  }
  for(auto [x, to]: t[no].adj)
    getDistinctSubstrings(v, to, d);
}
};
```

Trie.h

<bits/stdc++.h> bb31a2, 45 lines

```
using namespace std;
const int K = 26;
inline int getId(char c){
  return c - 'a';
}
struct Vertex {
  int next[K];
  int leaf;
  int count;
  Vertex() {
    fill(begin(next), end(next), -1);
    leaf = 0;
    count = 0;
  }
};
struct Trie{
  vector<Vertex> trie;
  Trie(){
    trie.emplace_back();
  }
  void add(string const& s) {
    int v = 0;
    trie[v].count++;
    for(char ch: s) {
      int c = getId(ch);
      if (trie[v].next[c] == -1) {
        trie[v].next[c] = trie.size();
        trie.emplace_back();
      }
      v = trie[v].next[c];
      trie[v].count++;
    }
    trie[v].leaf++;
  }
  int countStr(string const& s) {
    int v = 0;
    for (char ch : s) {
      int c = getId(ch);
      if (trie[v].next[c] == -1)
        return 0;
      v = trie[v].next[c];
    }
    return trie[v].leaf;
  }
};
```

Zfunction.h

<bits/stdc++.h> af7999, 18 lines

```
using namespace std;
// z[i] is the length of the longest common prefix between s
     [0..(n-1)] and the suffix of s[i..(n-1)].
// z[0] is generally not well defined.
// "aaabaab" - [0,2,1,0,2,1,0]
// "abacaba" - [0,0,1,0,3,0,1]
vector<int> z_function(string s) {
  int n = (int) s.length();
  vector<int> z(n);
  for (int i = 1, l = 0, r = 0; i < n; i++){
    if (i <= r)
      z[i] = min (r - i + 1, z[i - l]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]])
      z[i]++;
    if (i + z[i] - 1 > r)
      l = i, r = i + z[i] - 1;
  }
  return z;
}
```

Various (8)

CountingInversions.h

<bits/stdc++.h> 8f7598, 29 lines

```
using namespace std;
typedef long long ll;
const int INF = 0x3f3f3f3f;
// Counting Inversions: O(N*log(N))
ll ci(vector<int> &v){
  int n = v.size();
  ll inv = 0LL;
  if(n <= 1)
    return 0;
  vector<int> u1, u2;
  for(int i=0; i < n/2; i++)
    u1.push_back(v[i]);
  for(int i=n/2; i < n; i++)
    u2.push_back(v[i]);
  inv += ci(u1);
  inv += ci(u2);
  u1.push_back(INF);
  u2.push_back(INF);
  int ini1=0, ini2=0;
  for(int i=0; i < n; i++){
    if(u1[ini1] <= u2[ini2]){
      v[i] = u1[ini1++];
    }else{
      v[i] = u2[ini2++];
      inv += u1.size() - ini1 - 1;
    }
  }
  return inv;
}
```

Fastio.h

<bits/stdc++.h> a4a1a2, 21 lines

```
int readInt () {
  bool minus = false;
  int result = 0;
  char ch;
  ch = getchar();
  while (true) {
    if (ch == '-') break;
    if (ch >= '0' && ch <= '9') break;
    ch = getchar();
  }
  if (ch == '-') minus = true; else result = ch-'0';
  while (true) {
    ch = getchar();
    if (ch < '0' || ch > '9') break;
    result = result*10 + (ch - '0');
  }
  if (minus)
    return -result;
  else
    return result;
}
```

Histogram.h

<bits/stdc++.h> c50cd1, 18 lines

```
using namespace std;
typedef long long ll;
// Largest Rectangular Area in a Histogram
ll histogram(vector<int> v){
  int n = v.size();
  v.push_back(0);
  ll ans = 0;
  stack<int> st;
  for(int i = 0; i<=n; i++){
    while(st.size() && v[st.top()] >= v[i]){
      int idx = st.top(); st.pop();
      int L = st.size() ? st.top() : -1;
      ans = max(ans, (i-L-1) * (ll)v[idx]);
    }
    st.push(i);
  }
  return ans;
}
```

IdentifyPattern.h

<bits/stdc++.h> 9b5e96, 26 lines

```
using namespace std;
typedef pair<int, int> pii;
// Return the pattern of vector in O(N): pair<cycle start,
     cycle size>
pii identifyPattern(vector<int> v){
  int n = v.size();
  reverse(v.begin(), v.end());
  vector<int> pi(n);
  for (int i = 1; i < n; i++) {
    int j = pi[i-1];
    while (j > 0 and v[i] != v[j])
      j = pi[j-1];
    if (v[i] == v[j])
      j++;
    pi[i] = j;
  }
  tuple<int, int, int> ans(n, 1, n-1);
  for(int i=1; i<=n; i++){
    int p = i - pi[i-1];
    if(p == 0)
      continue;
    int idx = n-i;
    ans = min(ans, {idx+p, p, idx});
  }
  auto [sum, p, idx] = ans;
  return pii(idx, p);
}
```

Kadane.h

<bits/stdc++.h> 0ac31b, 28 lines

```
using namespace std;
typedef long long ll;
// Largest Sum Contiguous Subarray: O(N)
ll kadane(vector<ll> &v);
// Largest Sum Submatrix: O(N^3)
ll kadane2d(vector<vector<int>> &mat){
  if(mat.size() == 0) return 0;
  int n = mat.size(), m = mat[0].size();
  ll ans = 0;
  vector<ll> v(m);
  for(int a=0; a<n; a++){
    fill(v.begin(), v.end(), 0);
    for(int b=a; b<n; b++){
      for(int k=0; k<m; k++)
        v[k] += mat[b][k];
      ans = max(ans, kadane(v));
    }
  }
  return ans;
}
ll circularKadane(vector<ll> v){
  ll ans1 = kadane(v);
  ll sum = 0;
  for(int i=0; i < (int)v.size(); i++){
    sum += v[i]; v[i] = -v[i];
  }
  return max(ans1, sum + kadane(v));
}
```

Lis.h

<bits/stdc++.h> 7f1cad, 23 lines

```
using namespace std;
vector<int> lis(vector<int> &v){
  vector<int> st, ans;
  vector<int> pos(v.size()+1), dad(v.size()+1);
  for(int i=0; i < (int)v.size(); i++){
    auto it = lower_bound(st.begin(), st.end(), v[i]); // Do
        not accept repeated values
    //auto it = upper_bound(st.begin(), st.end(), v[i]); //
        Accept repeated values
    int p = it-st.begin();
    if(it==st.end())
      st.push_back(v[i]);
    else
      *it = v[i];
    pos[p] = i;
    dad[i] = (p==0)? -1 : pos[p-1];
  }
  int p = pos[st.size() - 1];
  while(p >= 0){
    ans.push_back(v[p]);
    p=dad[p];
  }
  reverse(ans.begin(), ans.end());
  return ans;
}
```

MoAlgorithm.h

<bits/stdc++.h> 9c2afa, 29 lines

```
using namespace std;
const int BLOCK_SIZE = 700;
void remove(int idx);
void add(int idx);
void clearAnswer();
int getAnswer();
struct Query{
  int l, r, idx;
  bool operator<(Query other) const{
    if (l / BLOCK_SIZE != other.l / BLOCK_SIZE)
      return l < other.l;
    return (l / BLOCK_SIZE & 1) ? (r < other.r) : (r > other.r)
        ;
  }
};
vector<int> mo_s_algorithm(vector<Query> queries){
  vector<int> answers(queries.size());
  sort(queries.begin(), queries.end());
  clearAnswer();
  int L = 0, R = 0;
  add(0);
  for(Query q : queries){
    while(q.l < L) add(--L);
    while(R < q.r) add(++R);
    while(L < q.l) remove(L++);
    while(q.r < R) remove(R--);
    answers[q.idx] = getAnswer();
  }
  return answers;
}
```

ParallelBinarySearch.h

<bits/stdc++.h> 709720, 29 lines

```
using namespace std;
const int MAXN = 100010;
int ans[MAXN];
bool test(int x);
void add(int k);
void remove(int k);
void solve(int i, int j, vector<int> &v){
  if(v.empty())
    return;
  if(i == j){
    for(int x: v)
      ans[x] = i;
    return;
  }
  int mid = (i+j)/2;
  for(int k=i; k<=mid; k++)
    add(k);
  vector<int> left, right;
  for(int x: v){
    if(test(x))
      left.push_back(x);
    else
      right.push_back(x);
  }
  solve(mid+1, j, right);
  for(int k=mid; k>=i; k--)
    remove(k); // Or roolback();
  solve(i, mid, left);
}
```

Polyominoes.h

<bits/stdc++.h> a7d4a0, 67 lines

```
#define F first
#define S second
using namespace std;
const int MAXP = 10;
typedef pair<int, int> pii;
//This implementation considers the rotations as distinct
//              0, 10, 10+9, 10+9+8...
int pos[11] = {0, 10, 19, 27, 34, 40, 45, 49, 52, 54, 55};
struct Polyominoes{
  pii v[MAXP];
  int64_t id;
  int n;
  Polyominoes(){
    n = 1;
    v[0] = {0, 0};
    normalize();
  }
  pii& operator[](int i){
    return v[i];
  }
  bool add(int a, int b){
    for(int i=0; i<n; i++)
      if(v[i].F == a and v[i].S == b)
        return false;
    v[n++] = pii(a, b);
    normalize();
    return true;
```

```
  }
  void normalize(){
    int mnx=100, mny=100;
    for(int i=0; i<n; i++)
      mnx = min(mnx, v[i].F), mny = min(mny, v[i].S);
    id = 0;
    for(int i=0; i<n; i++){
      v[i].F -= mnx, v[i].S -= mny;
      id |= (1LL<<(pos[v[i].F] + v[i].S));
    }
  }
};
vector<Polyominoes> polyominoes[MAXP+1];
int dx[] = {0, 0, -1, 1};
int dy[] = {-1, 1, 0, 0};
void buildPolyominoes(int mxN=10){
  for(int i=0; i<=mxN; i++)
    polyominoes[i].clear();
  Polyominoes init;
  queue<Polyominoes> q;
  unordered_set<int64_t> used;
  q.push(init);
  used.insert(init.id);
  while(!q.empty()){
    Polyominoes u = q.front(); q.pop();
    polyominoes[u.n].push_back(u);
    if(u.n == mxN)
      continue;
    for(int i=0; i<u.n; i++){
      for(int j=0; j<4; j++){
        Polyominoes to = u;
        bool ok = to.add(to[i].F + dx[j], to[i].S + dy[j]);
        if(ok and !used.count(to.id)){
          q.push(to);
          used.insert(to.id);
        }
      }
    }
  }
}
```

Pragma.h

7dc8f7, 3 lines

```
#pragma GCC optimize("O3", "unroll-loops")
#pragma GCC target("avx2")
#pragma GCC target("popcnt")
```

RandomFunction.h

<bits/stdc++.h> a3b3ce, 8 lines

```
using namespace std;
mt19937 rng((int) std::chrono::steady_clock::now().
    time_since_epoch().count());
inline int rand(int l, int r){
  return uniform_int_distribution<int>(l, r)(rng);
}
void randomShuffle(vector<int> v){
  shuffle(v.begin(), v.end(), rng);
}
```

SchedulingJobs.h

<bits/stdc++.h> 0bbaec, 16 lines

```
using namespace std;
typedef long long ll;
struct Job {
  int t, c, idx;
  Job(int t1=0, int c1=0, int i=0):t(t1), c(c1), idx(i){}
};
//Penalty functions fi(t) = c[i]*t
bool cmp1(Job a, Job b){
  return a.c*(ll)b.t > b.c*(ll)a.t;
}
//Penalty functions fi(t) = c[i]*e^(alfa*t)
const double alfa = 2;
const double EPS = 1e-9;
bool cmp2(Job a, Job b){
  return (1 - exp(alfa*a.t))/a.c > (1 - exp(alfa*b.t))/b.c +
      EPS;
}
```

Simplex.h

<bits/stdc++.h> 7eb83b, 78 lines

```
using namespace std;
// Caution: long double can give TLE
typedef double ld;
typedef vector<ld> vd;
typedef vector<vd> vvd;
typedef vector<int> vi;
const ld EPS = 1e-9;
/*
 * Algorithm : Simplex ( Linear Programming )
 * Author : Simon Lo
*/
struct LPSolver {
  int m, n;
  vi B, N;
  vvd D;
  LPSolver(const vvd &A, const vd &b, const vd &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, vd(n +
        2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i
        ][j] = A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D
        [i][n + 1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }
  void Pivot(int r, int s) {
    ld inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv
        ;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }
  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || (D[x][j] == D[x][s]
            && N[j] < N[s])) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[
            r][s] ||
            ((D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s])
                && B[i] < B[r])) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }
  ld Solve(vd &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1])
        r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
          numeric_limits<ld>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || (D[i][j] == D[i
              ][s] && N[j] < N[s])) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<ld>::infinity();
    x = vd(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n
        + 1];
    return D[m][n + 1];
  }
  void clear() {
    for (int i = 0; i < m; i++){
      D[i].clear();
    }
    D.clear();
    B.clear();
    N.clear();
  }
};
```

SpragueGrundy.h

<bits/stdc++.h> 878745, 23 lines

```
using namespace std;
const int MAXN = 1010;
int version;
int used[MAXN];
int mex() {
  for(int i=0; ; ++i)
    if(used[i] != version)
      return i;
}
int g[MAXN];
// Can remove 1, 2 and 3
void grundy(){
  //Base case depends on the problem
  g[0] = 0; g[1] = 1; g[2] = 2;
  //Inductive case
  for(int i=3; i<MAXN; i++){
    version++;
    used[g[i-1]] = version;
    used[g[i-2]] = version;
    used[g[i-3]] = version;
    g[i] = mex();
  }
}
```