2D Localization in the Far-Field with Elevated URA

Music Algorithm

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March 4, 2025

URA Model

The URA creates a three-dimensional field, as its antennas are distributed in the x and z dimensions. This configuration allows for the estimation of arrival angles in both the horizontal plane (φ) and the vertical plane (θ), providing spatial diversity. However, when the URA is at the same plane as the user, the variation in distances between the antennas and the source is small, especially in the z dimension. This can limit the accuracy of elevation angle estimation and introduce ambiguities in the localization of multiple users.

URA Model

By elevating the URA, we significantly increase the variation in distances between each antenna and the user, making the phase patterns more distinct. This improves the spatial separation of sources, enhances angular resolution, and makes the system more robust against interference and ambiguities. Thus, elevating the URA strengthens spatial diversity, leading to better performance in position estimation and interference mitigation.

URA Model

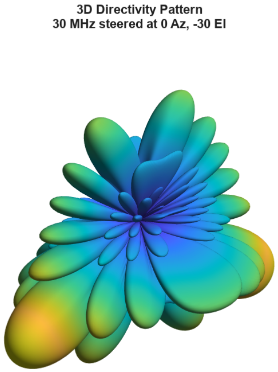

Figura: Beamforming elevated URA

URA Model

The user is located in the xy-plane, with their positions expressed as:

$$\mathbf{u} = \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix}, \quad \mathbf{p}_m = \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}, \quad m = 1, \ldots, M$$

where $\mathbf{u}$ represents the location of the user, and $\mathbf{p}_m$ represents the location of the m-th antenna. For antenna configurations with elevation, we consider z_{elev} as the average height of the array.

$$\mathbf{a}(\mathbf{u}) = \left[1,\, e^{-\frac{j2\pi}{\lambda}(d_2 - d_1)},\, \ldots,\, e^{-\frac{j2\pi}{\lambda}(d_m - d_1)}\right]^\top \quad \mathbf{a} \in \mathbb{C}^m$$

URA Model

Assuming the user is in the xy-plane and the array is in the xz-plane, and considering the elevation of the URA in meters (z_{elev}), we have:

$$\mathbf{u} = \begin{bmatrix} x_u \\ y_u \\ 0 \end{bmatrix}, \quad \mathbf{p}_m = \begin{bmatrix} x_m \\ 0 \\ z_m \end{bmatrix}, \quad m = 1, \ldots, M$$

So that each antenna of the URA is located at:

$$\mathbf{p}_m = (x_m, z_m) = (m \cdot \delta_x, n \cdot \delta_z + z_{\text{elev}}) \tag{1}$$

URA Model

The reference antenna position at the bottom-left corner a_{11} is $(0, 0, z_{\text{elev}})$.

$$d_{m,n} = (d_2 - d_1) = \sqrt{(x_u - x_m)^2 + (y_u - y_m)^2 + (z_u - z_m)^2}$$

Given a uniform spacing for δ_x and δ_z, we obtain the steering vector for the URA using the Euclidean distance:

$$\mathbf{a}(\mathbf{u}) = [a_{11}, a_{12}, \cdots, a_{mn}]$$

Where, given the pre-established conditions in this presentation, we have:

$$a_{mn} = e^{-j\frac{2\pi}{\lambda}\sqrt{[x_u - (m\cdot\delta_x)]^2 + y_u^2 + [z_u - (n\cdot\delta_z + z_{\text{elev}})]^2}}$$

URA Model - Far-Field Approximation

In the far-field, we assume that the user is far from the array, meaning:

$$y_u \gg x_m, z_n$$

When y_u is very large, we can use the Taylor approximation for square roots:

$$\sqrt{y_u^2 + \Delta} \approx y_u + \frac{\Delta}{2y_u}, \quad \text{if } \Delta \ll y_u^2$$

Thus, the distance can be approximated as:

$$d_{mn} = y_u - x_m \sin\theta \cos\phi - z_n \sin\phi$$

And the steering coefficient becomes:

$$a_{mn} = e^{j\frac{2\pi}{\lambda}(x_m \sin\theta \cos\phi - z_n \sin\phi)}$$

It is noteworthy that this approximation has an error e.

URA Model

The Fraunhofer distance (d_F) provides the minimum distance at which it is possible to approximate the curvature of a wave as planar. Thus, the steering vector model has limitations for distances shorter than d_{FA}.

$$d_{FA} = \frac{2D^2}{\lambda}$$

Modeling the channel

The average propagation can be described by a parametric channel gain model, where g is the large-scale fading coefficient, $g = \frac{1}{\mathsf{PL}}$, adjustable for different scenarios. Defining $\Upsilon = (\frac{\lambda}{4\pi})^{\alpha}$, observe which Υ is correlated with the system frequency:

$$g_{ij} = \Upsilon \left(\frac{1}{d_{ij}} \right)^{\alpha}$$

Path loss can be decomposed into three terms, describing the phenomena of path loss, shadowing, and fading. Each term introduces different variations in the signal amplitude due to the area over which each term is averaged. The area mean is the long-term path loss term, characterizing a slow variation, and measures the average attenuation of the signal amplitude relative to distance, being deterministic.

Modeling the channel

At a given instant l, we can model the signals incident on the array:

$$\underbrace{\begin{bmatrix} y_{11}(l) \\ y_{12}(l) \\ \vdots \\ y_{mn}(l) \end{bmatrix}}_{\mathbf{y}\in\mathbb{C}^{m\cdot n}} = [\mathbf{g}^\top]^{\frac{1}{2}} \underbrace{\begin{bmatrix} a_{11}(\mathbf{u}_1) & a_{11}(\mathbf{u}_2) & \dots & a_{11}(\mathbf{u}_k) \\ a_{12}(\mathbf{u_2}) & a_{12}(\mathbf{u_2}) & \dots & a_{12}(\mathbf{u}_k) \\ \vdots & \vdots & \ddots & \vdots \\ a_{mn}(\mathbf{u_3}) & a_{mn}(\mathbf{u_3}) & \dots & a_{mn}(\mathbf{u}_k) \end{bmatrix}}_{\mathbf{A}\in\mathbb{C}^{(m\cdot n)\times k}} \underbrace{\begin{bmatrix} s_1(l) \\ s_2(l) \\ \vdots \\ s_k(l) \end{bmatrix}}_{\mathbf{s}\in\mathbb{C}^k} + \underbrace{\begin{bmatrix} n_1(l) \\ n_2(l) \\ \vdots \\ n_m(l) \end{bmatrix}}_{\mathbf{n}\in\mathbb{C}^m}.$$

where $\mathbf{g}^\top = \Big[g_1, g_2, \cdots, g_k\Big] \in \mathbb{R}^{1\times k}$. $y_{mn} \in \mathbb{C}$ is the linear combination of k transmitted signals, with the effects of the channel, additive noise at the receivers, and phase shifts due to the array response:

$$y_{mn}(l) = \sum_{k=1}^{K} \sqrt{g_k} s_k(l) e^{-j\gamma_2 e^{-j\frac{2\pi}{\lambda}\sqrt{[x_u-(m\cdot\delta_x)]^2+y_u^2+[z_u-(n\cdot\delta_z+z_{\text{elev}})]^2}}} + n(l)$$

where $n(l) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$.

Modeling the channel

We construct the matrix $\mathbf{Y} \in \mathbb{C}^{m \times a}$ (a is the number of samples) with the sample functions of the process, where each coefficient $y_{mn}(l)$ is the combination of signals received by an antenna in the array at a given instant l.

$$\mathbf{Y} = \sqrt{\boldsymbol{G}}\mathbf{A} \cdot \mathbf{S} + \mathbf{N}, \quad \mathbf{N} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2 \mathbf{I}_M)$$

where $\boldsymbol{G}^{\frac{1}{2}} = \text{diag}(\sqrt{g_1}, \sqrt{g}_2, \cdots, \sqrt{g_k})$, $\mathbf{A}$ is the steering vector, $\mathbf{S}$ is the matrix of transmitted signals, and $\mathbf{N}$ is the noise matrix. In this way, we characterize the process based on its statistical averages.

MUSIC

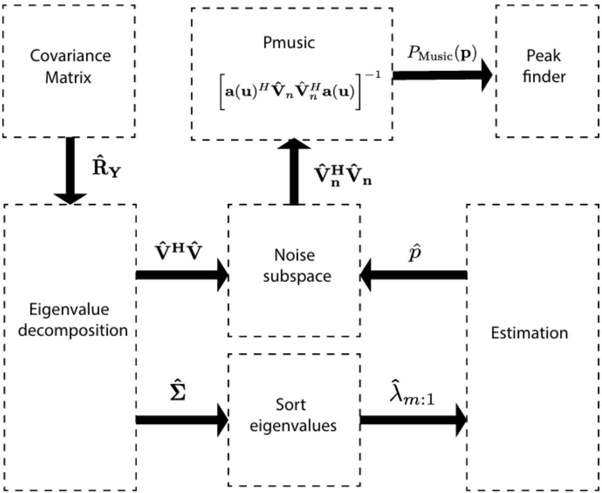

Figura: diagram MUSIC

Cuadro: Values used on simulation parameters (MUSIC)

Parameters	**Values**
Snapshots	$N = 1$ realization
Number of users	$K = 1$ User
Number of antennas	$M = 8 \times 8$
ULA spacing	$\Delta = 0,5\lambda$
Carrier Frequency	$f_c = 15{,}0$Ghz
Transmitted signal power	$P = 100$mW
Noise power	$\sigma^2 = 10$mW
Path-loss Expoent	$\alpha = 2$

Results

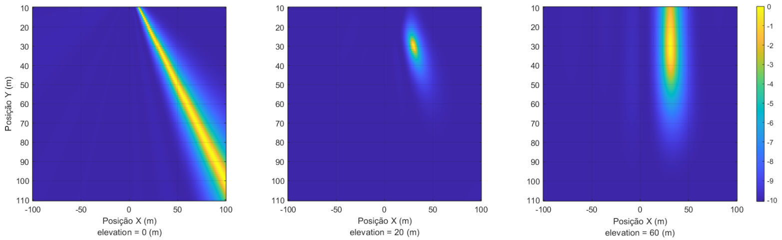

Figura: Pseudo-spectrum for different elevations

Results

By keeping the array at the same level as the user, we observe low phase variation in the y-dimension, which results in a high correlation between the signals received by the antennas, making it impossible to determine the user's distance. Elevating the array in space directly increases phase variation, reducing the correlation between signals and improving spatial diversity in the y and z dimensions. The figure shows that this elevation converges to an optimal point.

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