Comparative Analysis of Spatial and Spectral Methods in GNN for Power Flow in Electrical Power Systems

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Outline

Current Section

Purpose:

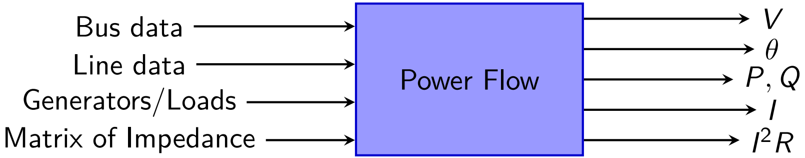
- Determine steady-state voltages, power flows, and losses in electrical systems.

Key Components:

- Buses: nodes, loads, generators, transmission lines:

Applications

- Voltage stability, contingency analysis, support operational planning, and economic dispatch.



The power flow problem is defined by a set of nonlinear equations derived from Kirchhoff's laws

$$P_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}), \tag{1}$$

$$Q_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}), \tag{2}$$

$$\Delta P_i = P_i^{\text{specified}} - \boxed{V_i} \sum_{j=1}^{N} \boxed{V_j} (G_{ij} \cos \boxed{\theta_{ij}} + B_{ij} \sin \boxed{\theta_{ij}}) \tag{3}$$

$$\Delta Q_i = Q_i^{\text{specified}} - \boxed{V_i} \sum_{j=1}^{N} \boxed{V_j} (G_{ij} \sin \boxed{\theta_{ij}} - B_{ij} \cos \boxed{\theta_{ij}}) \tag{4}$$

Limitations of Traditional Methods (e.g., Newton-Raphson):

- High computational cost due to solving large nonlinear equation systems. Inefficient for large-scale networks, especially under real-time constraints.

Approximate Methods (e.g., DC Flow)

- Fast computational times. Lower accuracy is due to neglect of reactive power and assuming small voltage angle differences.

GNN: Combines the structural properties of power grids with machine learning, including physical constraints, offering efficient modeling and acceptable accuracy in suitable time frames. [4, 5, 6, 8]

Current Section

Background

Graph Neural Networks

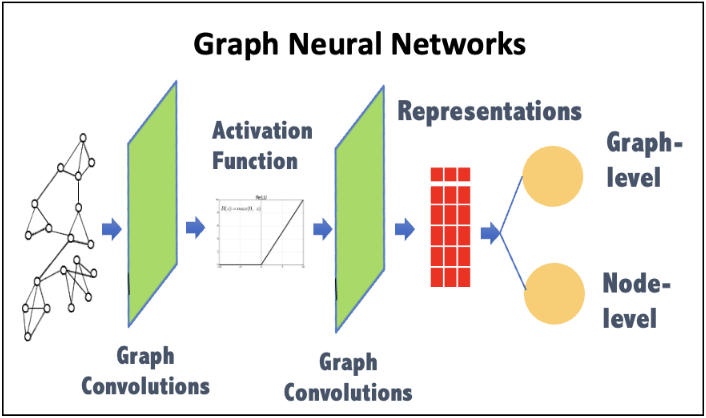


Figure: Source: https://web.stanford.edu/class/cs224w/

Background
Taxonomies of GNNs

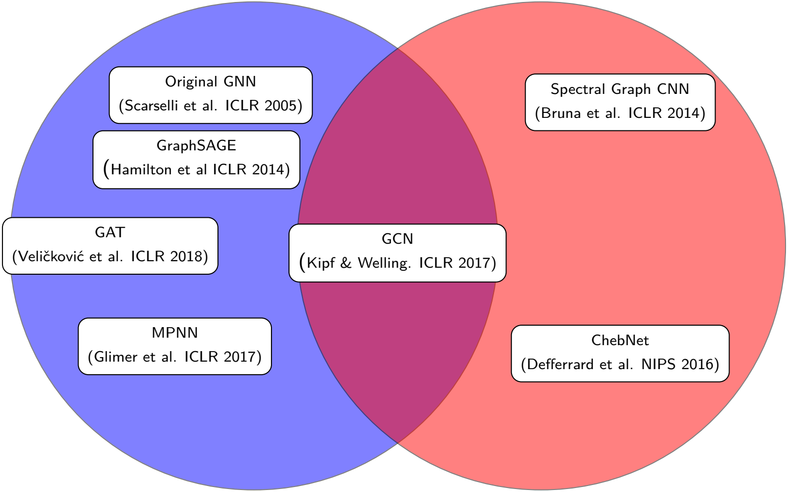


Figure: Based in https://ai.tencent.com/ailab/ml/KDD-Deep-Graph-Learning.html

In the context of GNN, a graph is represented as $G = (V, E)$, where V is a set of nodes and E is the edges connecting the nodes.

Adjacency matrix

$$A_{ij} = \begin{cases} 1 & \text{if } e_{ij} \in E \\ 0 & \text{if } e_{ij} \notin E \end{cases}$$

Degree matrix

$$D_{ij} = \begin{cases} \deg(v_i) & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Laplacian

$$L = D - A.$$

Normalized Laplacian

$$\tilde{L} = I - D^{-1/2} A D^{-1/2}.$$

The eigenvalues and eigenvectors of the $\tilde{L}$ are used to perform spectral graph convolutions. The convolution operation is expressed as:

$$g \star x = F^{-1}(F(g) \cdot F(x)) = U(U^T g \cdot U^T x), \tag{5}$$

where g is the filter in the spectral domain. The ChebNet method [1] approximates the convolution operation using Chebyshev polynomials.

$$g \star x \approx \sum_{k=0}^{K} \theta_k T_k(\tilde{L}) x, \tag{6}$$

where $\tilde{L} = \frac{2}{\lambda_{\max}} L - I$ is the rescaled Laplacian, $\lambda_{\max}$ is the largest eigenvalue of L, θ_k are the Chebyshev coefficients, and $T_k(\tilde{L})$ are defined as:

$$T_0(\tilde{L}) = I, \quad T_1(\tilde{L}) = \tilde{L}, \quad T_k(\tilde{L}) = 2\tilde{L} T_{k-1}(\tilde{L}) - T_{k-2}(\tilde{L}). \tag{7}$$

This method reduces computational complexity by avoiding the need for eigenvector computation.

Graph Convolutional Network (GCN) based in [3], simplify the expansion of Chebyshev polynomials, assuming mainly that $K = 1$ and the $\lambda_{max} = 2$. The convolution operation is now:

$$H^{(l+1)} = \sigma(\tilde{D}^{-1/2}\tilde{A}\tilde{D}^{-1/2}H^{(l)}W^{(l)}), \tag{8}$$

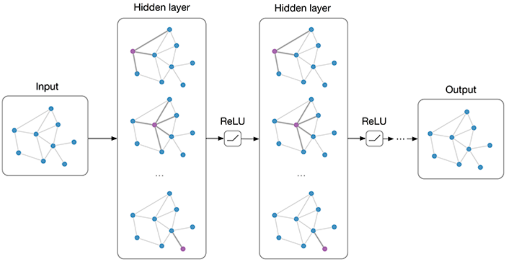


Figure: Graph Convolutional Network Multilayer

Source from: https://tkipf.github.io/graph-convolutional-networks/

Spatial Approaches operate directly on the graph structure; in this way, the convolution operation is based on the local neighborhood.

GraphSAGE based in [2], is an inductive framework that generates node embeddings by sampling and aggregating features from a node's local neighborhood. The layer-wise propagation rule is defined as:

$$h_v^{(l+1)} = \sigma\left(W^{(l)} \cdot \text{AGGREGATE}^{(l)}\left(\{h_u^{(l)}, \forall u \in \mathcal{N}(v)\}\right)\right), \qquad (9)$$

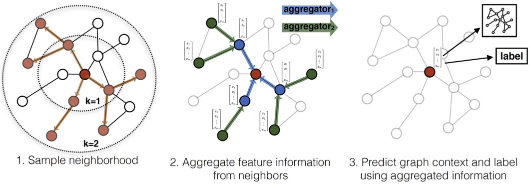
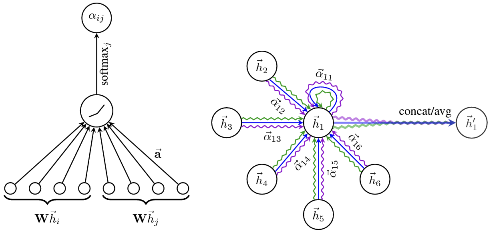


Figure: Source from: [2]

GAT [7] incorporate the attention mechanism assigning different weights to different neighbors. The attention coefficient α_{ij} is computed as:

$$\alpha_{ij} = \frac{\exp\left(\text{LeakyReLU}\left(a^T[Wh_i \| Wh_j]\right)\right)}{\sum_{k\in\mathcal{N}(i)} \exp\left(\text{LeakyReLU}\left(a^T[Wh_i \| Wh_k]\right)\right)}, \tag{10}$$

The node representation is then updated as:



$$h_i^{(l+1)} = \sigma\Big(\sum_{j\in\mathcal{N}(i)} \alpha_{ij} W h_j\Big)$$

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Methodology

The computational frameworks used are PyTorch Geometric (PyG) and PandaPower, mainly to apply GNN algorithms and create synthetic data to support the benchmark test cases.

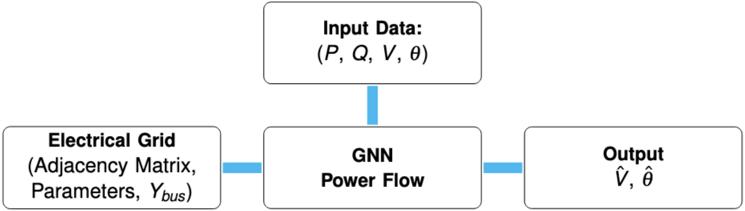

Figure: Flowchart Experiment of GNN Power Flow.

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Experiments

The test cases utilized from the pandapower library are as follows:

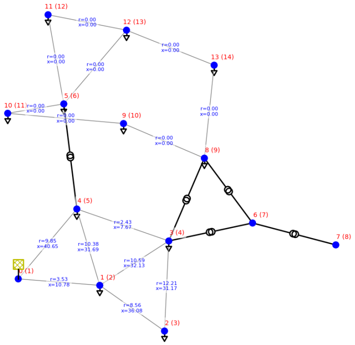

Figure: Test Case 14-Buses

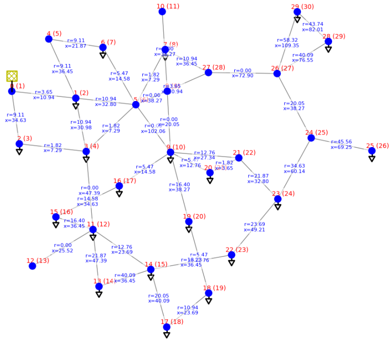

Figure: Test Case 30-Buses

In the experiments, node data is used.

Experiments

Dataset:

- Two test cases: 14 buses and 30 buses are used.
- Both datasets containing 2.000 independent observations

Objective:

Predict V and θ node level with features P, Q

Loss Function:

$$\mathcal{L}(\hat{V}, \hat{\theta}, V, \theta) = \frac{1}{N} \sum_{i=1}^{N} \left((\hat{V}_i - V_i)^2 + (\hat{\theta}_i - \theta_i)^2 \right) \tag{12}$$

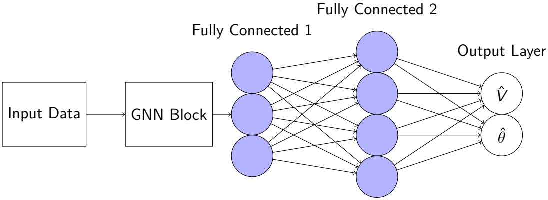

Figure: Network architecture used for all GNN blocks in the experiment.

Results

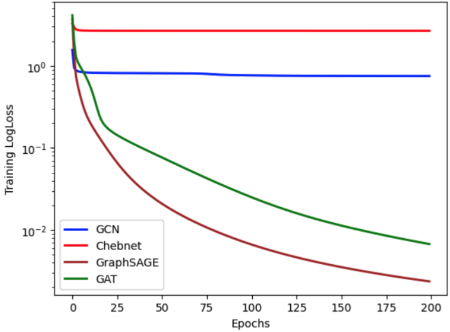

Figure: Training LogLoss Test Case 14-Bus

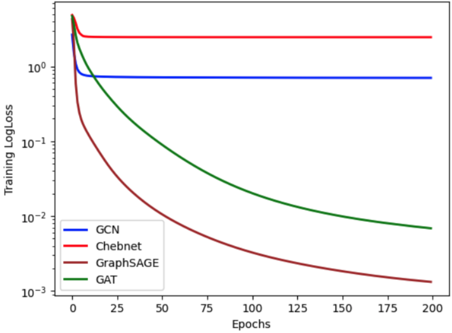

Figure: Training LogLoss Test Case 30-Bus

Results

Model	# Parameters	Taxonomy	Train Time	MAPE	R^2	RMSE	MAE
GCN	2.308	Hybrid	12 min	3,2%	0.99	0.82	0.31
ChebNet	6.120	Spectral	23 min	4,85%	0.96	1.66	0.68
GraphSAGE	2.768	Spatial	10 min	0,79%	0.99	0.11	0.05
GAT	2.768	Spatial	16 min	0,80%	0.99	0.12	0.06

Table: Performance on test case 14 - PandaPower

Model	# Parameters	Taxonomy	Train Time	MAPE	R^2	RMSE	MAE
GCN	4,900	Hybrid	13 min	3.25%	0.99	0.86	0.38
ChebNet	23,048	Spectral	27 min	4.04%	0.96	1.58	0.66
GraphSAGE	5,872	Spatial	12 min	0.53%	0.99	0.084	0.04
GAT	5,872	Spatial	17 min	0.60%	0.99	0.1	0.05

Table: Performance on test case 30 - PandaPower

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Conclusion

- **Observed Superior Performance of Spatial Methods**: Based on the results, GraphSAGE and GAT demonstrate better performance compared to spectral methods, with lower prediction errors and reduced computational demands
- **Effective Use of Local Information**: GraphSAGE and GAT effectively leverage local node-specific information, a crucial advantage in power systems where local interactions heavily influence problem dynamics.
- **Limitations of Spectral Methods**: ChebNet and GCN exhibit higher prediction errors and increased training times, likely due to their reliance on global graph structures, which can dilute critical local information.
- **Future Work**: Further studies on real-world, enhancing scalability, and ensuring the adaptability of GNN-based models for real-time applications.

Q&A

Current Section

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Thank you!!