Macroeconomics 2 Presentation Part III equations

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1 A Behavioral Model

Let's ignore the first two equations, since they are the same as (28) and (29), that will be explained later.

1.1 Introduction

Equation 1

% equation 1%

$$\mathrm{x_t} = \mathrm{M} \cdot \mathbb{E}_\mathrm{t}\left[\mathrm{x_{t+1}} - \sigma(\mathrm{i_t} - \mathbb{E}_\mathrm{t}\left[\pi_\mathrm{t+1}\right] - \mathrm{r^n_t})\right] \tag{1}$$

Equation 2

% equation 2%

$$\pi_\mathrm{t} = \beta \cdot \mathrm{M^f}\mathbb{E}_\mathrm{t}\left[\pi_\mathrm{t+1}\right] + \kappa \cdot \mathrm{x_t} \tag{2}$$

1.2 Basic Setup and the Household's Problem

Equation 3

% equation 3%

$$U = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right] \tag{3}$$

Equation (3) is just the flow utility of the Household, with :

- β the discount factor
- c_t the consumption of the houshold at time t
- N_t the work of the household at time t
- γ determines the concavity of the utility function with respect to the consumption, i.e. the importance of consumption in the utility function
- ϕ determines the concavity of the utility function with respect to work, i.e. the importance of work in the utility function

%There is some computation between (3) and (4) !!!%

Equation 4

% equation 4%

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{4}$$

Equation (4) is the law of motion of the real financial wealth of the household, where :

- k_t is the real financial welath of the household at time t
- r_t is the real interest rate
- w_t is the real wage
- y_t is the agent's real income, defined as $y_t = w_t \cdot N_t + y_t^f$, with y_t^f the profit income (or the income from firms) at time t

Equation 5

% equation 5%

$$\mathbf{X}_{t+1} = \mathbf{G}^{\mathbf{X}} \left(\mathbf{X}_t, \boldsymbol{\epsilon}_{t+1} \right) \tag{5}$$

Equation (5) describes the evolution of macroeconomic variables, where :

- $\mathbf{X}_t$ is the state vector, including several macroeconomic variables of time t, like ζ_t the aggregate TFP, and the announced actions in monetary and fiscal policy
- $\mathbf{G}^X$ the equilibrium transition function, i.e. the function that gives the macroeconomic variables at time $t+1$ from the macroeconomic variables at the previous period
- ϵ_t is the innovation in the economy at time t, with $\mathbb{E}_t \left[\epsilon_{t+1} \right] = 0$, that depends on the equilibrium policies of the agent and of the government

Equation 6

% equation 6%

$$k_{t+1} = G^k(c_t, N_t, k_t, \mathbf{X}_t) := (1 + \bar{r} + \hat{r}(\mathbf{X}_t))(k_t + \bar{y} + \hat{y}(N_t, \mathbf{X}_t) - c_t) \tag{6}$$

Equation (6) is the application of the consideration of a set of macroeconomic variables on the law of motion of real financial wealth k_t, where :

- $\bar{r}$ is the steady state value of the real interest rate, that does not depend on time
- $\hat{r}(\mathbf{X}_t)$ is the value of the deviation from the steady state of the real interest rate, that depends on the state vector $\mathbf{X}_t$ at time t
- $\bar{y}$ is the steady state value of the agent's real income, that does not depend on time
- $\hat{y}(N_t, \mathbf{X}_t)$ is the deviation from the steady state of the agent's real income, that depends on the number of hours worked at time t and on the state vector at time t
- c_t is the aggregate consumption level at time t of the agent

Equation 7

% equation 7%

$$\mathbf{X}_{t+1} = \boldsymbol{\Gamma}\mathbf{X}_t + \mathbf{epsilon}_{t+1} \tag{7}$$

Equation (7) describes the linear version of the equilibrium transition function, it is the linearization of the law of motion, where :

- Γ is a squared matrix that multiplies the state vector
- $\mathbf{X}_t$ is the state vector at time t
- ϵ_t is the innovation shock

Equation 8 (Assumption 1)

% equation 8%

$$\mathbf{X}_{t+1} = \bar{m} \cdot \mathbf{G}^{\mathbf{X}}(\mathbf{X}_t, \boldsymbol{\epsilon}_{t+1}) \tag{8}$$

Equation (8) describes the Cognitive Discounting of the State Vector, i.e. the perception by behavioral agents of the law of motion of the macroeconomic variables, where :

- $\bar{m} \in [0n1]$ is the coginitive discount factor measuring the attention to the future

Equation 9

% equation 9%

$$\mathbf{X}_{t+1} = \bar{m}(\boldsymbol{\Gamma}\mathbf{X}_t + \boldsymbol{\epsilon}_{t+1}) \tag{9}$$

Equation (9) is just the linearized version of the perception by behavioral agents of the law of motion of the state vector.

Equation 10

$$\mathbb{E}_t^{BR}\left[\mathbf{X}_{t+k}\right] = \bar{m}^k \mathbb{E}_t\left[\mathbf{X}_{t+k}\right] \tag{10}$$

Equation (10) defines the expectation of behavioral agents in function of the rational perception of the law of motion of the state vector, where :

- $k \geq 0$ a time period in discrete context
- $\mathbb{E}_t^{BR}\left[\mathbf{X}_{t+k}\right]$ is the expected value of the state vector at time $t + k$ by behavioral agents (or subjective/behavioral expectation operator)

- $\bar{\mathrm{m}}^{\mathrm{k}}$ is the cognitive discounting effect at period $\mathrm{t+k}$
- $\mathbb{E}_{\mathrm{t}}\left[\mathbf{X}_{\mathrm{t+k}}\right]$ is the rational expectation of the state vector at time $\mathrm{t+k}$

Equation 11 (Lemma 1)

$$\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right]=\bar{\mathrm{m}}^{\mathrm{k}}\mathbb{E}_{\mathrm{t}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right] \tag{11}$$

Equation (11) defines in the general case the behavioral expectation operator, for any function of the state vector, where :

- $\mathrm{k} \geq 0$ a time period in discrete context
- $\mathrm{z}(\cdot)$ is a function, such that $\mathrm{z}(0)=0$
- $\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right]$ is the expected value of the image of the state vector by the function $\mathrm{z}(\cdot)$ at time $\mathrm{t+k}$ by behavioral agents
- $\bar{\mathrm{m}}^{\mathrm{k}}$ is the cognitive discounting effect at period $\mathrm{t+k}$
- $\mathbb{E}_{\mathrm{t}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right]$ is the rational expectation of the image of the state vector by the function $\mathrm{z}(\cdot)$ at time $\mathrm{t+k}$

Equation 12

$$\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\bar{\mathrm{r}}+\hat{\mathrm{r}}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right]=\bar{\mathrm{r}}+\bar{\mathrm{m}}^{\mathrm{k}}\mathbb{E}_{\mathrm{t}}\left[\hat{\mathrm{r}}(\mathbf{X}_{\mathrm{t+k}})\right] \tag{12}$$

Equation (12) is an example of the Lemma 1 applied to the interest rate, where :

- $\mathrm{k} \geq 0$ a time period in discrete context
- $\bar{\mathrm{r}}$ the steady state level of the real interest rate, that does not depend on time,
- $\hat{\mathrm{r}}(\mathbf{X}_{\mathrm{t+k}})$ is the equilibrium transition function defining the value of the deviation from the steady state of the real interest rate in function of the state vector at time $\mathrm{t+k}$
- $\bar{\mathrm{r}}+\hat{\mathrm{r}}(\mathbf{X}_{\mathrm{t}})=\mathrm{r}_{\mathrm{t}}(\mathbf{X}_{\mathrm{t}})$ is the value of the real interest rate at time t
- $\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\bar{\mathrm{r}}+\hat{\mathrm{r}}(\mathbf{X}_{\mathrm{t+k}})\right]$ is the expected value of the real interest at time $\mathrm{t+k}$ by behavioral agents
- $\mathbb{E}_{\mathrm{t}}\left[\hat{\mathrm{r}}(\mathbf{X}_{\mathrm{t+k}})\right]$ is the rational expectation of value of the deviation of the real itnerest rate from the steady state at time $\mathrm{t+k}$

1.3 The Firm's problem

Equation 13

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon}\, di \right)^{\frac{1}{1-\varepsilon}} \tag{13}$$

Equation (13) describes the aggregate price level, where :

- P_t is the aggregate price level of the economy at time t
- $i \in [0,1]$ is the firm index
- ε is the elasticity of subsititution between goods

Equation 14

$$\nu^0(q_{i\tau}, \mu_\tau, c_\tau) := (e^{q_{i\tau}} - (1-\tau_f)e^{-\mu_\tau})\, e^{-\varepsilon q_{i\tau}} c_\tau \tag{14}$$

Equation (14) describes the profit of the firm before the lump sum tax of the government, where :

- ν is the real profit of the firm
- $q_{i\tau} = \ln\left(\frac{P_{i\tau}}{P_\tau}\right) = p_{i\tau} - p_\tau$ is the real log price at time τ
- $\tau_f = \frac{1}{\varepsilon}$ it the corrective wage subsidy from the government, funded by the lump sum tax
- $\mu_\tau = \zeta_t - \ln(\omega_t)$ is the labor wedge, which is zero at efficiency
- ε is the elasticity of subsititution between goods
- c_τ is the aggregate level of consumption

Equation 15

$$\nu\,(q_{it}, \mathbf{X}_\tau) := \nu^0\,(q_{it} - \Pi(\mathbf{X}_\tau), \mu(\mathbf{X}_\tau), c(\mathbf{X}_\tau)) \tag{15}$$

Equation 15 describes the flow profit of the firm in function of the real log price and of the extended macro state vector, where :

- $q_{it} = \ln\left(\frac{P_{it}}{P_t}\right) = p_{it} - p_t$ is the real log price
- $\mathbf{X}_\tau = (\mathbf{X}_\tau^{\mathcal{M}}, \Pi_\tau)$ is the extended macro state vector, with $\mathbf{X}^{\mathcal{M}_\tau}$ the vector of macro variables, including ζ_τ and possible announcements about future policy

- $\Pi\left(\mathbf{X}_\tau\right) := \mathrm{p}_\tau - \mathrm{p}_\mathrm{t} = \pi_{\mathrm{t}+1} + ... + \pi_\tau$ is the inflation between times t and τ
- $\mathrm{q}_{\mathrm{it}} - \Pi(\mathbf{X}_\tau) = \mathrm{q}_{\mathrm{i}\tau}$ is the real price of the firm if they didn't change its price between t and τ
- $\mu\left(\mathbf{X}_\tau\right)$ is the labor wedge in function of the extended state vector at time t
- $\mathrm{c}(\mathbf{X}_\tau)$ is the aggregate consumption level in function of the extended state vector at time t

Equation 16

Equation (16) . . . , where :

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Equation

$$\text{EQ} \tag{16}$$

Equation () . . . , where :

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Equation

$$\text{EQ} \tag{17}$$

Equation () . . . , where :

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$$\text{EQ} \tag{18}$$

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