Macroeconomics 2 Presentation equations summary

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1 A Behavioral Model

Let's ignore the first two equations, since they are the same as (28) and (29), that will be explained later.

1.1 Introduction

Equation 1

$$\mathrm{x_t} = \mathrm{M} \cdot \mathbb{E}_\mathrm{t}\left[\mathrm{x_{t+1}} - \sigma(\mathrm{i_t} - \mathbb{E}_\mathrm{t}\left[\pi_\mathrm{t+1}\right] - \mathrm{r^n_t})\right] \tag{1}$$

Equation 2

$$\pi_\mathrm{t} = \beta \cdot \mathrm{M^f}\mathbb{E}_\mathrm{t}\left[\pi_\mathrm{t+1}\right] + \kappa \cdot \mathrm{x_t} \tag{2}$$

1.2 Basic Setup and the Household's Problem

Equation 3

$$U = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \right] \tag{3}$$

Equation (3) is just the flow utility of the Household, with :

- β the discount factor
- c_t the consumption of the houshold at time t
- N_t the work of the household at time t
- γ determines the concavity of the utility function with respect to the consumption, i.e. the importance of consumption in the utility function
- ϕ determines the concavity of the utility function with respect to work, i.e. the importance of work in the utility function

%There is some computation between (3) and (4) !!!%

Equation 4

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \tag{4}$$

Equation (4) is the law of motion of the real financial wealth of the household, where :

- k_t is the real financial welath of the household at time t
- r_t is the real interest rate
- w_t is the real wage
- y_t is the agent's real income, defined as $y_t = w_t \cdot N_t + y_t^f$, with y_t^f the profit income (or the income from firms) at time t

Equation 5

$$\mathbf{X}_{t+1} = \mathbf{G}^{\mathbf{X}}\left(\mathbf{X}_t, \boldsymbol{\epsilon}_{t+1}\right) \tag{5}$$

Equation (5) describes the evolution of macroeconomic variables, where :

- $\mathbf{X}_t$ is the state vector, including several macroeconomic variables of time t, like ζ_t the aggregate TFP, and the announced actions in monetary and fiscal policy
- $\mathbf{G}^X$ the equilibrium transition function, i.e. the function that gives the macroeconomic variables at time $t + 1$ from the macroeconomic variables at the previous period
- ϵ_t is the innovation in the economy at time t, with $\mathbb{E}_t\left[\epsilon_{t+1}\right] = 0$, that depends on the equilibrium policies of the agent and of the government

Equation 6

$$k_{t+1} = G^k(c_t, N_t, k_t, \mathbf{X}_t) := (1 + \bar{r} + \hat{r}(\mathbf{X}_t))(k_t + \bar{y} + \hat{y}(N_t, \mathbf{X}_t) - c_t) \tag{6}$$

Equation (6) is the application of the consideration of a set of macroeconomic variables on the law of motion of real financial wealth k_t, where :

- $\bar{r}$ is the steady state value of the real interest rate, that does not depend on time
- $\hat{r}(\mathbf{X}_t)$ is the value of the deviation from the steady state of the real interest rate, that depends on the state vector $\mathbf{X}_t$ at time t
- $\bar{y}$ is the steady state value of the agent's real income, that does not depend on time
- $\hat{y}(N_t, \mathbf{X}_t)$ is the deviation from the steady state of the agent's real income, that depends on the number of hours worked at time t and on the state vector at time t
- c_t is the aggregate consumption level at time t of the agent

Equation 7

$$\mathbf{X}_{t+1} = \mathbf{\Gamma}\mathbf{X}_t + \mathbf{epsilon}_{t+1} \tag{7}$$

Equation (7) describes the linear version of the equilibrium transition function, it is the linearization of the law of motion, where :

- Γ is a squared matrix that multiplies the state vector
- $\mathbf{X}_t$ is the state vector at time t
- ϵ_t is the innovation shock

Equation 8 (Assumption 1)

$$\mathbf{X}_{\mathrm{t+1}} = \bar{\mathrm{m}} \cdot \mathbf{G}^{\mathbf{X}}(\mathbf{X}_{\mathrm{t}}, \boldsymbol{\epsilon}_{\mathrm{t+1}}) \tag{8}$$

Equation (8) describes the Cognitive Discounting of the State Vector, i.e. the perception by behavioral agents of the law of motion of the macroeconomic variables, where :

- $\bar{\mathrm{m}} \in [0, 1]$ is the cognitive discount factor measuring the attention to the future

Equation 9

$$\mathbf{X}_{\mathrm{t+1}} = \bar{\mathrm{m}}(\boldsymbol{\Gamma}\mathbf{X}_{\mathrm{t}} + \boldsymbol{\epsilon}_{\mathrm{t+1}}) \tag{9}$$

Equation (9) is just the linearized version of the perception by behavioral agents of the law of motion of the state vector.

Equation 10

$$\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\mathbf{X}_{\mathrm{t+k}}\right] = \bar{\mathrm{m}}^{\mathrm{k}}\mathbb{E}_{\mathrm{t}}\left[\mathbf{X}_{\mathrm{t+k}}\right] \tag{10}$$

Equation (10) defines the expectation of behavioral agents in function of the rational perception of the law of motion of the state vector, where :

- $\mathrm{k} \geq 0$ a time period in discrete context
- $\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\mathbf{X}_{\mathrm{t+k}}\right]$ is the expected value of the state vector at time $\mathrm{t+k}$ by behavioral agents (or subjective/behavioral expectation operator)
- $\bar{\mathrm{m}}^{\mathrm{k}}$ is the cognitive discounting effect at period $\mathrm{t+k}$
- $\mathbb{E}_{\mathrm{t}}\left[\mathbf{X}_{\mathrm{t+k}}\right]$ is the rational expectation of the state vector at time $\mathrm{t+k}$

Equation 11 (Lemma 1)

$$\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right] = \bar{\mathrm{m}}^{\mathrm{k}}\mathbb{E}_{\mathrm{t}}\left[\mathrm{z}\left(\mathbf{X}_{\mathrm{t+k}}\right)\right] \tag{11}$$

Equation (11) defines in the general case the behavioral expectation operator, for any function of the state vector, where :

- $\mathrm{k} \geq 0$ a time period in discrete context
- $\mathrm{z}(\cdot)$ is a function, such that $\mathrm{z}(0) = 0$

- $\mathbb{E}_t^{BR}\left[z\left(\mathbf{X}_{t+k}\right)\right]$ is the expected value of the image of the state vector by the function $z(\cdot)$ at time $t+k$ by behavioral agents
- $\bar{m}^k$ is the cognitive discounting effect at period $t+k$
- $\mathbb{E}_t\left[z\left(\mathbf{X}_{t+k}\right)\right]$ is the rational expectation of the image of the state vector by the function $z(\cdot)$ at time $t+k$

Equation 12

$$\mathbb{E}_t^{BR}\left[\bar{r}+\hat{r}\left(\mathbf{X}_{t+k}\right)\right]=\bar{r}+\bar{m}^k\mathbb{E}_t\left[\hat{r}(\mathbf{X}_{t+k})\right] \tag{12}$$

Equation (12) is an example of the Lemma 1 applied to the interest rate, where :

- $k \geq 0$ a time period in discrete context
- $\bar{r}$ the steady state level of the real interest rate, that does not depend on time,
- $\hat{r}(\mathbf{X}_{t+k})$ is the equilibrium transition function defining the value of the deviation from the steady state of the real interest rate in function of the state vector at time $t+k$
- $\bar{r}+\hat{r}(\mathbf{X}_t)=r_t(\mathbf{X}_t)$ is the value of the real interest rate at time t
- $\mathbb{E}_t^{BR}\left[\bar{r}+\hat{r}(\mathbf{X}_{t+k})\right]$ is the expected value of the real interest at time $t+k$ by behavioral agents
- $\mathbb{E}_t\left[\hat{r}(\mathbf{X}_{t+k})\right]$ is the rational expectation of value of the deviation of the real itnerest rate from the steady state at time $t+k$

1.3 The Firm's problem

Equation 13

$$P_t=\left(\int_0^1 P_{it}^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}} \tag{13}$$

Equation (13) describes the aggregate price level, where :

- P_t is the aggregate price level of the economy at time t
- $i \in [0,1]$ is the firm index
- ε is the elasticity of subsititution between goods

Equation 14

$$\nu^0(\mathrm{q}_{\mathrm{i}\tau}, \mu_\tau, \mathrm{c}_\tau) := \left(\mathrm{e}^{\mathrm{q}_{\mathrm{i}\tau}} - (1-\tau_{\mathrm{f}})\mathrm{e}^{-\mu_\tau}\right) \mathrm{e}^{-\varepsilon \mathrm{q}_{\mathrm{i}\tau}} \mathrm{c}_\tau \tag{14}$$

Equation (14) describes the profit of the firm before the lump sum tax of the government, where :

- ν is the real profit of the firm
- $\mathrm{q}_{\mathrm{i}\tau} = \ln\left(\frac{\mathrm{P}_{\mathrm{i}\tau}}{\mathrm{P}_\tau}\right) = \mathrm{p}_{\mathrm{i}\tau} - \mathrm{p}_\tau$ is the real log price at time τ
- $\tau_{\mathrm{f}} = \frac{1}{\varepsilon}$ it the corrective wage subsidy from the government, funded by the lump sum tax
- $\mu_\tau = \zeta_{\mathrm{t}} - \ln(\omega_{\mathrm{t}})$ is the labor wedge, which is zero at efficiency
- ε is the elasticity of subsititution between goods
- c_τ is the aggregate level of consumption

Equation 15

$$\nu\left(\mathrm{q}_{\mathrm{it}}, \mathbf{X}_\tau\right) := \nu^0\left(\mathrm{q}_{\mathrm{it}} - \Pi(\mathbf{X}_\tau), \mu(\mathbf{X}_\tau), \mathrm{c}(\mathbf{X}_\tau)\right) \tag{15}$$

Equation 15 describes the flow profit of the firm in function of the real log price and of the extended macro state vector, where :

- $\mathrm{q}_{\mathrm{it}} = \ln\left(\frac{\mathrm{P}_{\mathrm{it}}}{\mathrm{P}_{\mathrm{t}}}\right) = \mathrm{p}_{\mathrm{it}} - \mathrm{p}_{\mathrm{t}}$ is the real log price
- $\mathbf{X}_\tau = (\mathbf{X}_\tau^{\mathcal{M}}, \Pi_\tau)$ is the extended macro state vector, with $\mathbf{X}^{\mathcal{M}_\tau}$ the vector of macro variables, including ζ_τ and possible announcements about future policy
- $\Pi\left(\mathbf{X}_\tau\right) := \mathrm{p}_\tau - \mathrm{p}_{\mathrm{t}} = \pi_{\mathrm{t}+1} + ... + \pi_\tau$ is the inflation between times t and τ
- $\mathrm{q}_{\mathrm{it}} - \Pi(\mathbf{X}_\tau) = \mathrm{q}_{\mathrm{i}\tau}$ is the real price of the firm if they didn't change its price between t and τ
- $\mu\left(\mathbf{X}_\tau\right)$ is the labor wedge in function of the extended state vector at time t
- $\mathrm{c}(\mathbf{X}_\tau)$ is the aggregate consumption level in function of the extended state vector at time t

Equation 16

$$\max_{\mathrm{q_{it}}} \mathbb{E}_{\mathrm{t}}\left[\sum_{\tau=\mathrm{t}}^{\infty}(\beta\theta)^{\tau-\mathrm{t}}\,\frac{\mathrm{c}\left(\mathbf{X}_{\tau}^{-\gamma}\right)}{\mathrm{c}\left(\mathbf{X}_{\mathrm{t}}^{-\gamma}\right)}\nu\left(\mathrm{q}_{\mathrm{it},\mathbf{X}_{\tau}}\right)\right] \tag{16}$$

Equation (16) describes the maximisation program of the firm given that they have a Calvo-like probability of θ of being able to change their price at each period, where :

- t is the initial period
- τ is the time period index
- $\mathrm{q_{it}}$ is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{\mathrm{c}\left(\mathbf{X}_{\tau}^{-\gamma}\right)}{\mathrm{c}\left(\mathbf{X}_{\mathrm{t}}^{-\gamma}\right)}$ is the adjustment in the stochastic discount factor between times t and τ

Equation 17

$$\max_{\mathrm{q_{it}}} \mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}\left[\sum_{\tau=\mathrm{t}}^{\infty}(\beta\theta)^{\tau-\mathrm{t}}\,\frac{\mathrm{c}\left(\mathbf{X}_{\tau}^{-\gamma}\right)}{\mathrm{c}\left(\mathbf{X}_{\mathrm{t}}^{-\gamma}\right)}\nu\left(\mathrm{q}_{\mathrm{it},\mathbf{X}_{\tau}}\right)\right] \tag{17}$$

Equation 17 describes the maximisation program of the behavioral firm, i.e. such that it is maximisation the behavioral expectation operator of the flow profit, where :

- $\mathbb{E}_{\mathrm{t}}^{\mathrm{BR}}$ is the behavioral/subjective expected value operator
- t is the initial period
- τ is the time period index
- $\mathrm{q_{it}}$ is the real log price of the firm at time t
- β is the discount factor
- θ is the Calvo like probability that the firm can change its price at any period
- $\frac{\mathrm{c}\left(\mathbf{X}_{\tau}^{-\gamma}\right)}{\mathrm{c}\left(\mathbf{X}_{\mathrm{t}}^{-\gamma}\right)}$ is the adjustment in the stochastic discount factor between times t and τ

1.4 Model solution

Equation 18

$$\hat{c}_t = \mathbb{E}_t \left[\hat{c}_{t+1} - \frac{1}{\gamma R} \hat{r}_t \right] \tag{18}$$

Equation (18) is the linearized version of the Euler equation obtained from the presented model. It is also called the investment-savings (IS) curve, where :

- $\hat{c}_t$ is the value of the deviation from the steady state of the aggregate consumption at time t
- $\mathbb{E}_t [\hat{c}_{t+1}]$ is the rational expectation of the value of the deviation from the steady state of the aggregate consumption at time $t + 1$
- γ is the factor of the importance of consumption
- $R := 1 + \bar{r}$ is defined from the real intereste rate at the steady state (cf. page 7 of the article)

Equation 19

$$\hat{c}_t = M \cdot \mathbb{E}_t [\hat{c}_{t+1} - \sigma \hat{r}_t] \tag{19}$$

Equation (19) is the application of Lemma 1 (equation (11)) on the previous Euler equation, i.e. a cognitively discounted aggregate Euler equation, where :

- M is the macro parameter of attention, such that $M = \bar{m}$ here
- $\sigma = \frac{1}{\gamma R}$

Equation 20

$$N_t^{\phi} = \omega_t c_t^{\gamma} \tag{20}$$

Equation (20) the result of the static First Order Condition for labor supply, where :

- N_t is the quantity of labor provided at time t
- ω_t is the real wage at time t
- c_t is the aggregate quantity of consumption at time t
- γ is the consumption importance in the utility

Equation 21

$$\hat{\mathrm{c}}_{\mathrm{t}}^{\mathrm{n}} = \frac{1+\phi}{\gamma+\phi}\zeta_{\mathrm{t}} \tag{21}$$

Equation (21) . . . , where :

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Equation 22

$$\hat{\mathrm{c}}_{\mathrm{t}}^{\mathrm{n}} = \mathrm{M} \cdot \mathbb{E}_{\mathrm{t}}\left[\hat{\mathrm{c}}_{\mathrm{t+1}}^{\mathrm{n}}\right] - \sigma \hat{\mathrm{r}}_{\mathrm{t}}^{\mathrm{n}} \tag{22}$$

Equation (22) . . . , where :

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Equation 23

$$\mathrm{r}_{\mathrm{t}}^{\mathrm{n0}} = \bar{\mathrm{r}} + \frac{1+\phi}{\sigma(\gamma+\phi)}\left(\mathrm{M} \cdot \mathbb{E}_{\mathrm{t}}\left[\zeta_{\mathrm{t+1}}\right] - \zeta_{\mathrm{t}}\right) \tag{23}$$

Equation (23) . . . , where :

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Equation 24

$$\mathrm{x}_{\mathrm{t}} = \mathrm{M} \cdot \mathbb{E}_{\mathrm{t}}\left[\mathrm{x}_{\mathrm{t+1}}\right] - \sigma(\hat{\mathrm{r}}_{\mathrm{t}} - \hat{\mathrm{r}}_{\mathrm{t}}^{\mathrm{n}}) \tag{24}$$

Equation (24) . . . , where :

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Equation 25

$$\mathrm{x}_{\mathrm{t}} = \mathrm{M} \cdot \mathbb{E}_{\mathrm{t}}\left[\mathrm{x}_{\mathrm{t+1}}\right] - \sigma(\mathrm{i}_{\mathrm{t}} - \mathbb{E}_{\mathrm{t}}\left[\pi_{\mathrm{t+1}}\right] - \mathrm{r}_{\mathrm{t}}^{\mathrm{n}}) \tag{25}$$

Equation () . . . , where :

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Equation 26

$$\mathrm{x_t} = -\sigma \sum_{\mathrm{k} \geq 0} \mathrm{M} \cdot \mathbb{E}_\mathrm{t} \left[\hat{\mathrm{r}}_\mathrm{t+k} - \hat{\mathrm{r}}^\mathrm{n}_\mathrm{t+k} \right] \tag{26}$$

Equation (26) . . . , where :

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Equation 27

$$\mathrm{p}^*_\mathrm{t} = \mathrm{p_t} + (1 - \beta\theta) \sum_{\mathrm{k}=0}^{\infty} (\beta\theta\bar{\mathrm{m}})^\mathrm{k} \cdot \mathbb{E}_\mathrm{t} \left[\pi\mathrm{t} + 1 + ... + \pi_\mathrm{t+k} - \mu_\mathrm{t+k} \right] \tag{27}$$

Equation (27) . . . , where :

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1.5 A Behavioral New Keynesian Model

Equation 28 - Proposition 2, first equation

$$\mathrm{x_t} = \mathrm{M} \cdot \mathbb{E}_\mathrm{t} \left[\mathrm{x_{t+1}} \right] - \sigma(\mathrm{i_t} - \mathbb{E}_\mathrm{t} \left[\pi_\mathrm{t+1} \right] - \mathrm{r}^\mathrm{n}_\mathrm{t}) \tag{28}$$

Equation (28) . . . , where :

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Equation 29 - Proposition 2, second equation

$$\pi_\mathrm{t} = \beta \cdot \mathrm{M}^\mathrm{f} \mathbb{E}_\mathrm{t} \left[\pi_\mathrm{t+1} \right] + \kappa \cdot \mathrm{x_t} \tag{29}$$

Equation (29) . . . , where :

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Equation 30

$$\begin{cases} \mathrm{M} = \bar{\mathrm{m}} \\ \sigma = \frac{1}{\gamma \mathrm{R}} \\ \mathrm{M}^{\mathrm{f}} = \bar{\mathrm{m}} \left(\theta + \frac{1-\beta\theta}{1-\beta\theta\bar{\mathrm{m}}} (1-\theta) \right) \end{cases} \tag{30}$$

Equation (30) . . . , where :

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