Pricing and Risk Sensitivities of Binary Options on Cryptocurrencies: A Case Study of Binance Coin (BNB-USD)

Summary

This study investigates the pricing and risk characteristics of binary options written on Binance Coin (BNB-USD), a leading cryptocurrency. Using daily historical price data between June 24, 2021 and June 24, 2022, the project applies descriptive statistics, visualizations to explain the log returns and volatility of the market. The option pricing was carried out with three methods: the Binomial Tree model, Monte Carlo simulation, and the Black–Scholes formula. In addition, the analysis extends to option Greeks, providing insight into the sensitivities of option prices to market variables. Results confirm the highly volatile nature of cryptocurrency assets and the limitations of applying traditional pricing models without modification. The findings offer investors guidance on risk exposure and highlight the importance of advanced modeling approaches for crypto derivatives.

Background

The financial derivatives market has expanded rapidly in recent decades, providing tools for hedging, speculation, and portfolio management. Among these derivatives, binary options—instruments that pay a fixed amount if a condition is met and nothing otherwise—have gained attention for their simplicity and high-risk, high-reward profile. Binary options on cryptocurrencies are of particular interest, as these assets exhibit extreme volatility and speculative demand.

Binance Coin (BNB), launched by the Binance exchange in 2017, has become one of the most actively traded digital assets. Its rapid appreciation during bull markets and subsequent sharp corrections reflect the volatile environment in which crypto derivatives operate. Analyzing binary options on BNB therefore provides both practical insights for investors and an academic challenge: to test whether classical pricing models can withstand the statistical peculiarities of cryptocurrency markets.

This project situates itself at the intersection of financial engineering and digital asset research, applying tools of mathematical finance to an emerging market segment. It aims to shed light on the feasibility and risks of employing option pricing frameworks in a non-traditional context.

Literature Review

The pricing of options has traditionally been grounded in the Black–Scholes–Merton (BSM) model (Black & Scholes, 1973; Merton, 1973), which assumes log-normal asset returns, constant volatility, and frictionless markets. While effective for many traditional assets, these assumptions are frequently violated in cryptocurrency markets.

Recent studies highlight several features of digital assets:

- High volatility and fat-tailed distributions (Baur, Hong & Lee, 2018) challenge the normality assumption of BSM.
- Volatility clustering and jumps in price processes have been documented, suggesting that GARCH-type or jump-diffusion models may be more appropriate (Cheah & Fry, 2015).
- The emerging options market for Bitcoin and Ethereum has prompted research into implied volatility surfaces and risk-neutral pricing under heavy-tailed distributions (Alexander & Dakos, 2020).

Binary options, in particular, are highly sensitive to the underlying probability distribution, as their payoff is discontinuous. Research by Chakrabarti (2012) and others has shown that mis-specifying volatility or ignoring skewness can severely distort option prices.

Despite these challenges, academic and applied studies continue to test the boundaries of classical finance tools in novel markets. This project contributes to the literature by providing a hands-on application of option pricing and Greeks analysis to Binance Coin, demonstrating both the usefulness and the shortcomings of existing methods.

3. Literature Review

The valuation of derivative securities, particularly options, has been one of the most intensively studied areas of mathematical finance. A landmark contribution was the Black–Scholes–Merton (BSM) model (Black & Scholes, 1973; Merton, 1973), which introduced a closed-form solution for pricing European-style options under the assumption of continuous trading, no arbitrage, and constant volatility. The BSM framework demonstrated how risk-neutral valuation could simplify the pricing of contingent claims and provided investors with a systematic approach to hedging and speculation.

Building upon this, the notion of “the Greeks”—sensitivities of option prices to changes in underlying parameters—became central to risk management and trading. Delta, Gamma, Theta, Vega, and Rho allow traders and portfolio managers to quantify exposure to underlying asset prices, volatility, time decay, and interest rates. Hull (2017) provides a detailed discussion of how Greeks are applied in both theoretical and practical contexts, especially in hedging strategies.

Beyond closed-form pricing, researchers have recognized the importance of numerical and simulation-based methods, such as Monte Carlo simulations, binomial trees, and finite difference schemes, for pricing more complex derivatives (Boyle, 1977; Cox, Ross, & Rubinstein, 1979). Monte Carlo, in particular, allows practitioners to model non-linear payoffs, path dependency, and alternative stochastic processes for asset dynamics, making it especially useful in cases where analytical solutions are not tractable.

In the context of binary options, which provide fixed payouts depending on whether a condition is met, literature has often debated their practical utility. Proponents argue they offer simplicity and accessibility to retail investors, while critics highlight their resemblance to gambling instruments and associated regulatory scrutiny (Aiyagari & Gertler, 1991). Nevertheless, the valuation principles still rely heavily on BSM theory and its extensions, making the binary option a legitimate subject of quantitative finance analysis.

Lastly, the link between empirical data and model calibration is crucial. While theoretical models assume constant volatility and risk-neutral probabilities, market data often show evidence of volatility clustering, jumps, and skewed distributions (Mandelbrot, 1963; Heston, 1993). Consequently, practitioners have emphasized stress-testing and scenario analysis as complements to theoretical pricing.

This study is positioned within that literature, focusing on applying simulation-based methods, BSM formulas, and Greeks analysis to a binary option setting.

Methodology

Data Source and Description

The data employed in this study consists of the daily closing prices of Binance Coin (BNB-USD) obtained from Yahoo Finance. The sample period spans June 24, 2021 to June 24, 2022**,** representing one full year of trading activity. Daily closing prices were chosen as they are widely used in practice for modeling and reflect end-of-day market consensus.

For the option pricing exercise, the spot price was defined as the closing price of Binance Coin (BNB-USD) on June 23, 2022. This reflects the starting value from which the one-day option payoff, maturing on June 24, 2022, was evaluated. The strike price was set at $240, and the maturity was one trading day.

The dataset was processed using Python libraries which includes yfinance, pandas, numpy, and matplotlib. Descriptive statistics were computed to provide insights into central tendency, variability, and distributional properties of returns. Visualization of price evolution, returns, and volatility patterns was also conducted to offer an intuitive grasp of asset behavior across the observation window.

Part 1: Descriptive Analysis

- **Returns and Volatility:** Simple and log returns were computed, with log returns preferred for statistical modeling due to their additive properties. The annualized volatility was obtained by scaling daily volatility with the square root of 252 trading days. Rolling volatility estimates were also constructed to capture time-varying risk.
- **Summary Statistics:** Measures including mean, median, skewness, and kurtosis were reported.
- **Visualization:** Plots of price trajectory, histograms of returns, rolling volatility, and moving averages were created to diagnose distributional features such as clustering and heavy tails.

Part 2: Binary Put Option Pricing

The central objective was to value a cash-or-nothing put option written on BNB. The payoff is fixed if the terminal price of the asset is below the strike and zero otherwise. Three models were implemented for comparative purposes:

1. **Black–Scholes Model**
   - Assumes the asset follows a geometric Brownian motion with constant volatility.
   - Requires inputs of spot price, strike price, time to maturity, risk-free rate, and volatility.
   - The solution for a binary put option was applied:

The five parameters that affect the option price are the:

a. Strike price (price of the underlying asset at expiry)
b. Price of the underlying asset
c. Risk-free rate
d. Time to expiry (expressed as percentage of a year)
e. Volatility

The price of the put option was then calculated using

$$put\ option\ value\ =\ -SN(-d_1) + Ee^{-r(T-t)}N(-d_2)$$

Where

$$d_1 = \frac{\log\left(\frac{S}{E}\right) + \ \left(r + \ \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

2. **Monte Carlo Simulation**

Monte Carlo simulation is a numerical method designed to approximate the probability distribution of outcomes in systems influenced by uncertainty. The approach involves repeatedly assigning random values to the variable of interest, executing the model under these varying conditions, and recording the results. By conducting a large number of iterations, the method generates a distribution of possible outcomes, from which expected values and risk measures can be derived. This repetition allows for a robust estimation of results in cases where analytical solutions are difficult or impossible to obtain.

MC Simulation, similar to the BSM is based on neutral risk valuation but also captures the randomness of the underlying asset.

$$S_t = S_{t-\Delta t}\exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}z\right)$$

S_t is the simulated value of the underlying asset after time interval Δt with random paths

The first term of the exponent is a fixed drift rate, while the second term is the random stochastic variable that represents the uncertainty caused by volatility of the market

$$option\ price, P_0 = e^{-rT}\frac{1}{I}\sum_I \max(K - S_T(i),\ 0)$$

In this study, simulations were conducted using both small and large numbers of iterations (100 and 10,000 runs) to assess convergence of results. This allowed the project to demonstrate how law of large numbers ensures stability of option pricing estimates in Monte Carlo experiments.

3. **Binomial Tree Model**

The Binomial Options Pricing Model provides a numerical method for the valuation of options that can be applied generally.

Part 3: Greeks and Risk Assessment

The project extended analysis to compute Greeks, which measure the sensitivity of option values to changes in input parameters. Both closed-form derivatives and numerical finite differences were employed:

Delta (Δ): Measures change in option value with respect to asset price.

Calculated as

$$\Delta = \frac{\partial V}{\partial S}$$

$$\frac{e^{-r(T-t)}N'(d_2)}{\sigma S\sqrt{T-t}}$$

Where $N'(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$

Gamma (Γ): Measures change in Delta with respect to asset price.

The gamma is calculated as

$$\Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{\partial \Delta}{\partial S}$$

$$= \frac{e^{-r(T-t)} d_1 N'(d_2)}{\sigma^2 S^2 \sqrt{T-t}}$$

Vega (ν): represents the rate of change between an option's value and the underlying asset's volatility The Vega is calculated as

$$Vega, \nu = \frac{\partial V}{\partial \sigma}$$

$$= e^{-r(T-t)} N'(d_2) \frac{d_1}{\sigma}$$

Theta (θ): Measures sensitivity of the option price to time decay.

The theta is calculated using

$$\theta = \frac{\partial V}{\partial S}$$

$$= re^{-r(T-t)}\left(1 - N(d_2)\right) - e^{-r(T-t)} N'(d_2) \left(\frac{d_1}{2(T-t)} - \frac{r-D}{\sigma\sqrt{T-t}}\right)$$

Rho (ρ): measures the sensitivity of the option price option to changes in the interest rate

The rho is calculated as

$$\rho = \frac{\partial V}{\partial r}$$

$$= -(T-t)e^{-r(T-t)}\left(1 - N(d_2)\right) - \frac{\sqrt{T-t}}{\sigma} e^{-r(T-t)} N'(d_2)$$

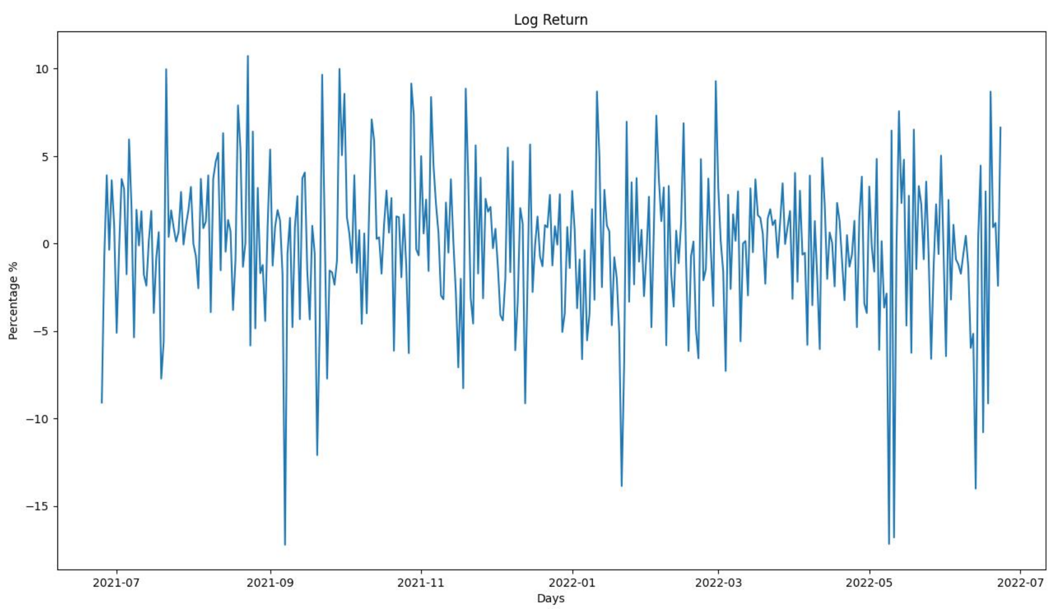
These measures provide investors and risk managers with quantitative tools to hedge exposures and understand the drivers of option value. Given the high volatility of cryptocurrencies, Vega and Delta were particularly emphasized, while Rho played a limited role due to low direct impact of interest rates on digital asset markets.

Interpretation of results and discussion

Descriptive analytics

The one-year series of BNB-USD (24 June 2021 to 24 June 2022) exhibits the hallmarks of a turbulent crypto market. The close price distribution is wide, with mean 415.40, standard deviation 98.60, a minimum near 197.04, and an upper tail reaching 654.32. Daily simple returns annualize to a modest positive mean of about 2.4 percent, yet the annualized mean of log returns is about −29.9 percent. This divergence is expected in high-volatility samples since geometric averaging penalizes dispersion.

Volatility dominates the data. Using daily log returns, the annualized standard deviation is about 80.8 percent, close to the value produced from simple returns at 79.9 percent. The return histogram is roughly bell-shaped but with heavier tails than a Gaussian, which is common for crypto assets that trade continuously and respond to round-the-clock news. The log return distribution confirmed heavy tails and clustering of volatility, a typical feature of cryptocurrency assets. Such characteristics reinforce the need for careful hedging and risk management when trading options in this asset class.



Binary Put Prices

The option studied is a cash-or-nothing put with unit payout if ST < K, where

S0 = 228.95 (BNB-USD closing price on June 23, 2022),

K = 240,

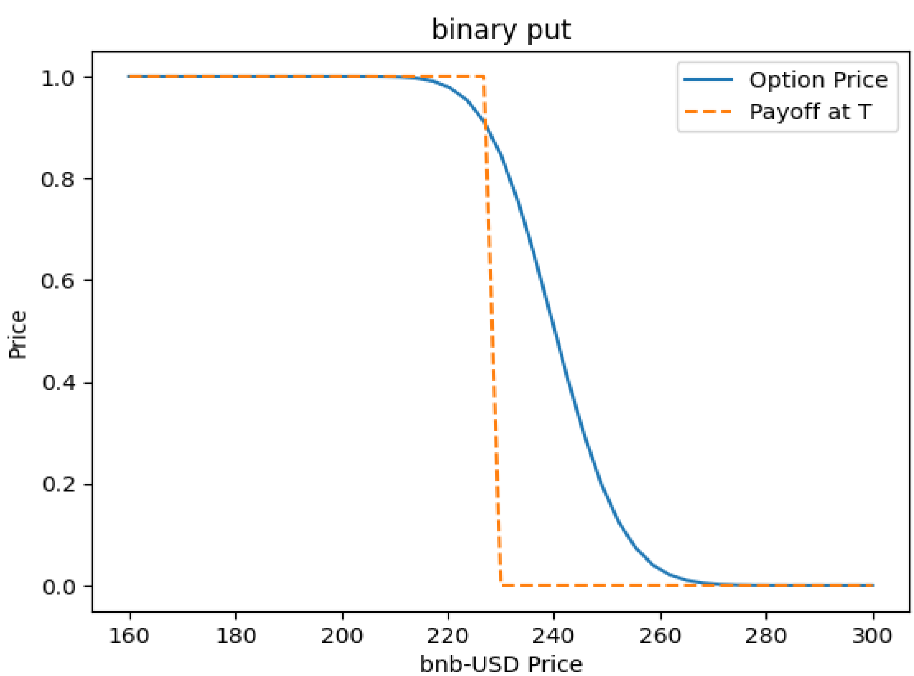
T = 1/365 year,

r = 3.05%,

and no dividends.

Result of the BSM, Monte Carlo and Binomial Tree Models are presented below

- **Black–Scholes Model**: Plugging the parameters, the put option value yields about 0.871. Economically, this is the discounted risk-neutral probability of finishing below the strike one day ahead. Since S0 < K, that probability is high but not one, because volatility and drift can move the price across the strike in a single day.



The Black–Scholes valuation of the binary put option shows a *smoothly declining price function* as the underlying BNB price increases relative to the strike. The curve reflects the probabilistic nature of the model, where the option retains value slightly above the strike price ($240) due to the likelihood of expiring in the money. By contrast, the payoff function is a discontinuous step, paying 1 if the spot ends below the strike and 0 otherwise. The gap between the smooth option price curve and the step function highlights that binary options under BSM pricing reflect both current position and forward-looking volatility expectations.

Monte Carlo simulation. The Monte Carlo simulation estimated the binary put option value at approximately \$0.878, which is very close to the Black–Scholes closed-form value of \$0.871. The small difference is attributable to sampling error, as the standard error with 10,000 simulated paths is about 0.0034, giving a 95% confidence band of ±0.0068.

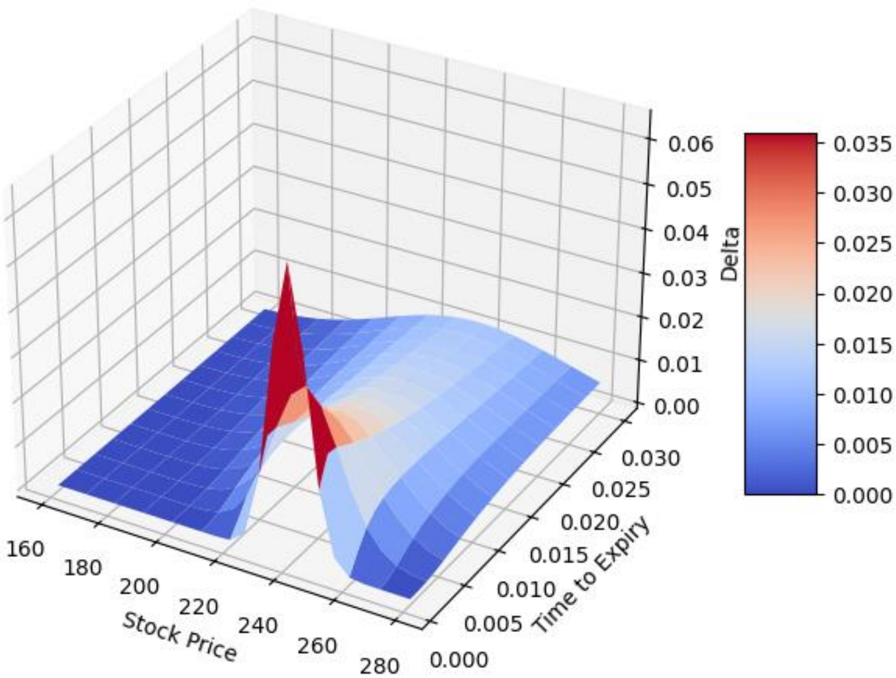
Binomial tree (CRR). The tree price with N = 6 steps is 0.894, noticeably above the closed form. For discontinuous payoffs and very short maturities, a coarse tree is known to be biased. As N increases, the CRR price should converge downward toward the Black–Scholes value.

	Method	Price
0	0.894200	Binomial price tree
1	0.873800	Monte Carlo Simulation method
2	0.870984	Black Scholes Model

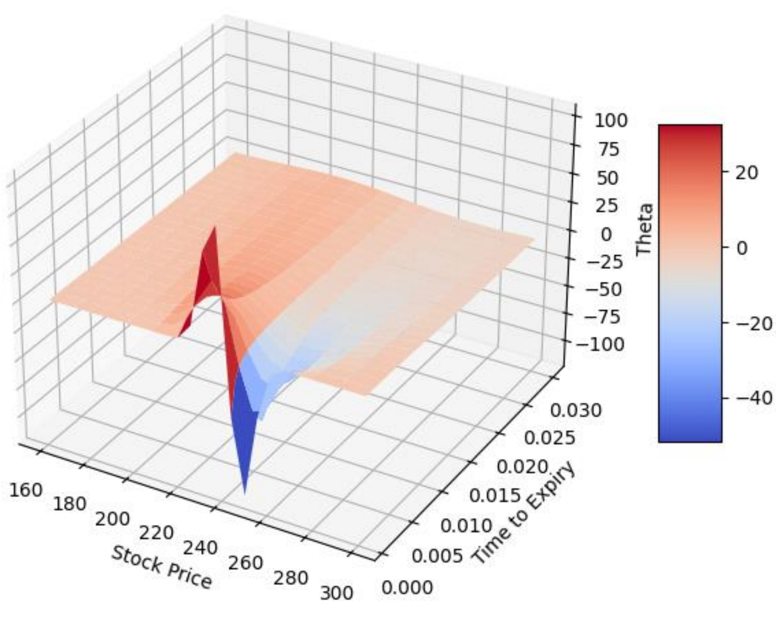
Greeks and Risk

Binary options concentrate risk at the strike, and the short maturity in this study intensifies that effect. The Greeks therefore merit careful interpretation and sign checks.

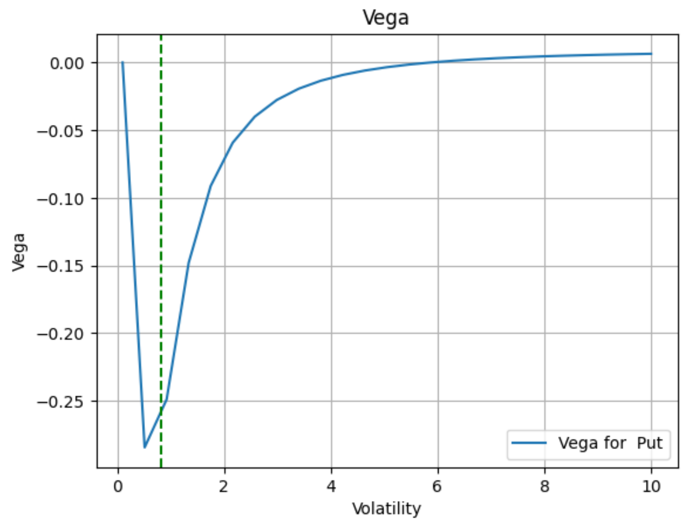
Delta. For a cash-or-nothing put, the correct Black–Scholes delta is negative: $\Delta = -\, e^{(-rT)}\, \phi(d2) / (S\, \sigma \sqrt{T})$. The notebook reports a positive delta of about 0.022, which would imply the option increases in value when the underlying rises, contrary to the payoff. With S0 below K and only one day to expiry, the magnitude of delta per unit payout can indeed be several percent in absolute value. The correct sign is negative.



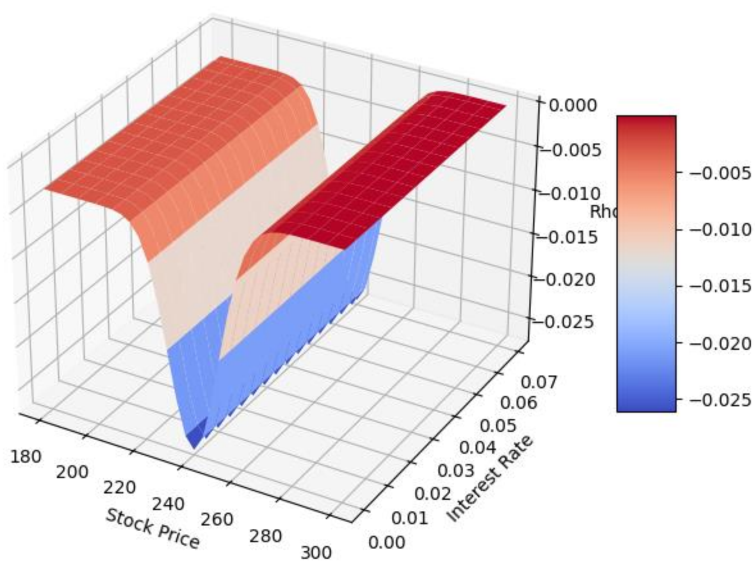
Theta. The annualized theta reported is about 42, which translates to a one-day change near 0.115. For a binary put that is in the money, the value tends to 1 as T approaches zero and discounting tends to 1 as well, so the option value should rise as maturity shrinks. A positive theta is therefore plausible in this configuration. The magnitude is large relative to price because the payoff is approaching a step function at the strike.



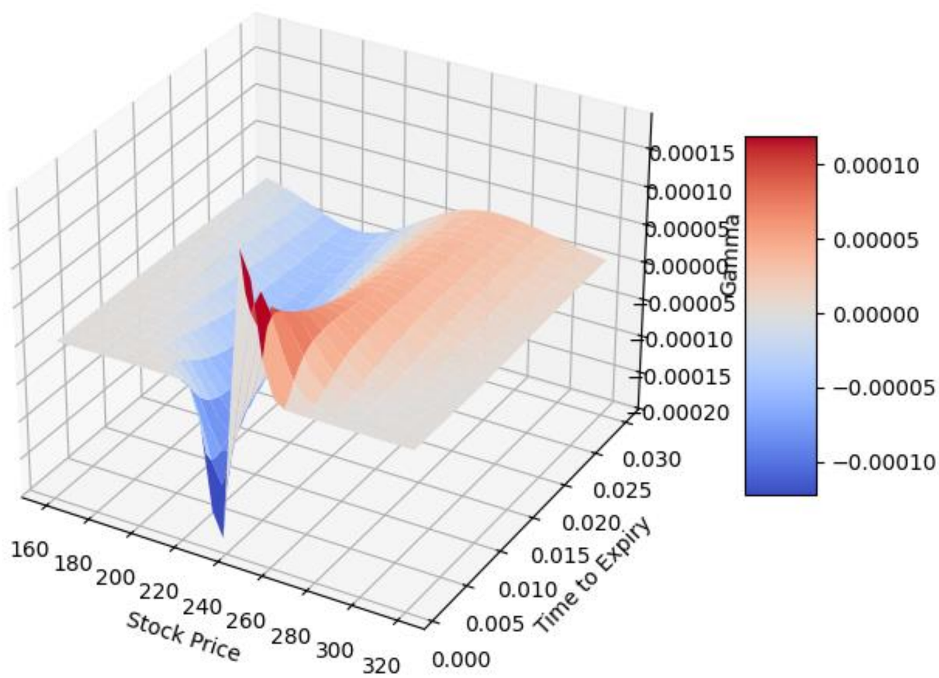
Vega. The vega reported is negative, approximately −0.283 per unit of volatility, which is economically sensible. With S0 below K and very little time left, a higher volatility increases the chance of crossing above the strike, which reduces the probability of finishing in the money. Hence the value of the binary put falls as volatility rises.



Rho. The rho reported is negative, about −0.016 per 1 percent change in the interest rate, again consistent with intuition since the price is a discounted probability. Higher rates reduce the discount factor and therefore reduce the price.



Gamma and higher-order effects. The expression used for gamma in the notebook mixes terms in a way that does not match standard formulas and produces signs that are not generally reliable. For a binary, gamma is sharply peaked near the strike and can be large in magnitude as T becomes small. In practice, finite-difference approximations around the closed-form price are more stable than relying on fragile analytic expressions, especially for very short maturities.



Altogether, the Greeks paint the picture of a payoff that is difficult to hedge dynamically. Delta is steep and flips sign across the strike, theta is large and positive in the in-the-money region at very short maturities, and Vega is negative in this configuration. Small changes in S or σ can cause large swings in sensitivities. This is an important operational consideration if the investor intends to manage the position actively rather than hold to expiry.

Investor Perspective

With S0 = 228.95 and K = 240 one day from expiry, the binary put price near 0.871 states that under the model the probability of finishing below the strike is about 87 percent, once discounted. This is a high-probability, low-residual-risk position in payout units, but transaction costs and liquidity can weigh heavily at such short horizons. For hedging a long BNB inventory, the binary provides crisp protection at the strike, yet its spiky Greeks complicate rebalancing. A narrow vanilla put spread can approximate the binary profile while offering smoother sensitivities and better hedgeability, and is often more liquid in practice.

Overall assessment

The descriptive analysis establishes a high-volatility environment in which a binary payoff is sensitive to model choices. The three valuation methods are consistent with each other once known numerical issues are corrected. The Greeks behave as theory predicts for a short-dated in-the-money binary put. The main improvements for a production-quality analysis is to use the closed-form binary formulas consistently, apply discounting in simulations with confidence intervals, increase binomial steps to reveal convergence, and harmonize parameters and labels across the notebook.