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Page No.	
Date	

Homework 6

Prob 1:

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Soln:
1.a] P(toothache) : Probability of toothache is true.

$$\sum_{\langle cavity\rangle\langle catch\rangle} P(\text{toothache}, \text{cavity}, \text{catch})$$

P(toothache| cavity, catch) + P(toothache | ¬cavity, catch) + P(toothache| cavity, ¬catch) + P(toothache| ¬cavity, ¬catch)

= 0.108 + 0.016 + 0.012 + 0.064
= 0.2

1.b] $P(\text{cavity}) = \sum_{\langle toothache\rangle\langle catch\rangle} P(\text{cavity}, \text{toothache}, \text{catch})$

= P(cavity| toothache, catch) + P(cavity | ¬toothache, catch) + P(cavity| toothache, ¬catch) + P(cavity | ¬toothache, ¬catch)

= 0.108 + 0.072 + 0.012 + 0.008
= 0.2

1.c] $P(\text{toothache} \mid \text{cavity}) = \dfrac{P(\text{toothache}, \text{cavity})}{P(\text{cavity})}$

$$= \frac{\sum_{\langle catch\rangle} P(\text{toothache}, \text{cavity}, \text{catch})}{\sum_{\langle toothache\rangle\langle catch\rangle} P(\text{cavity}, \text{toothache}, \text{catch})}$$

Page No.
Date

$= \frac{P(toothache, cavity, catch) + P(toothache, cavity, \neg catch)}{P(cavity)}$

$= \frac{0.108 + 0.012}{0.2}$

$= \frac{0.12}{0.2}$

$= 0.6.$

1.d) $P(cavity \mid toothache \vee catch)$: Probability Value for cavity is true, given that either toothache or catch is true

$= \frac{P(cavity, toothache \vee catch)}{P(toothache \vee catch)}$

$= P(toothache \vee catch) = P(toothache) + P(catch) - P(toothache \wedge catch)$

$= 0.2 + (0.108 + 0.016 + 0.072 + 0.144) - \sum P(toothache, catch, cavity / \neg cavity)$

$= 0.2 + 0.34 - (0.108 + 0.016)$

$= 0.416,$

$P(toothache \vee catch) = 0.416$

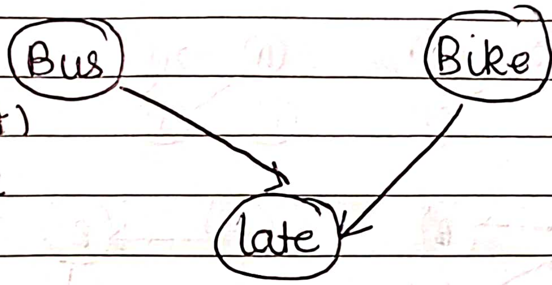
$P(cavity \mid toothache \vee catch) = \frac{P(cavity, toothache \vee catch)}{P(toothache \vee catch)}$

$= \frac{0.108 + 0.012 + 0.072}{0.416}$

$= \boxed{0.4615}$

Problem2: I don't have car. I come to work either by bike or by bus. if I take the bus, there is 10% chance that I'm late. If I take bike there is 2% chance that I'm late. I take bike 4 days out of 5. today I late.

→



$P(Bus = t) = 0.2$

$P(Bike = t) = 0.8$

$P(\text{late} \mid Bus) = 0.1$, $P(\text{late} \mid Bike) = 0.02$

$P(Bike) = \frac{4}{5} = 0.8$

$P(Bus) = \frac{1}{5} = 0.2$

Given late is true

$$P(Bus \mid Late) = \frac{P(Bus) \cdot P(\text{late} \mid Bus)}{P(\text{late})}$$

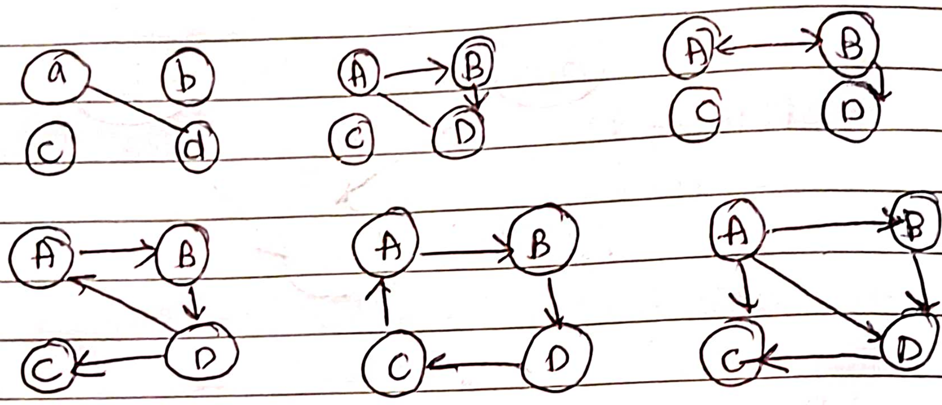
$$= \frac{0.2 \times 0.1}{P(\text{late} \mid Bus) \cdot P(Bus) + P(\text{late} \mid Bike) \cdot P(Bike)}$$

$$= \frac{0.2 * 0.1}{(0.1 * 0.2) + (0.02 * 0.8)}$$

$$= \frac{0.02}{0.036} = \frac{20}{36} = 0.555$$

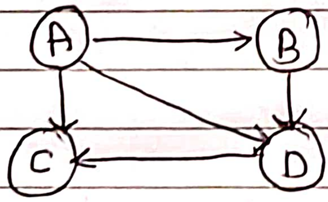
Problem 3 3.a] circle letter/s that corresponds to all valid Bayesian networks in following figure. if there is not any valid Bayesian network, circle None.

Bayesian Network None



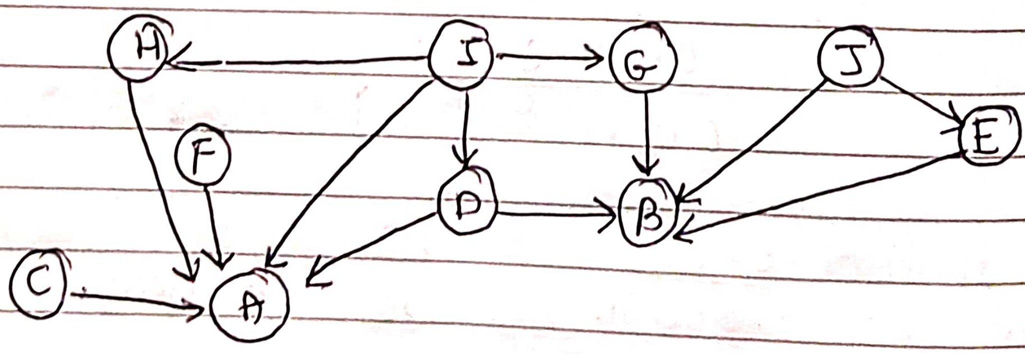
Bayesian Network

→ The only valid Bayesian network here is (f)



as it is non cyclic and has directions to nodes

3.b] for following Bayesian Network, write down the joint distribution expression of all variables/ nodes in terms of product of conditional probability of all variables

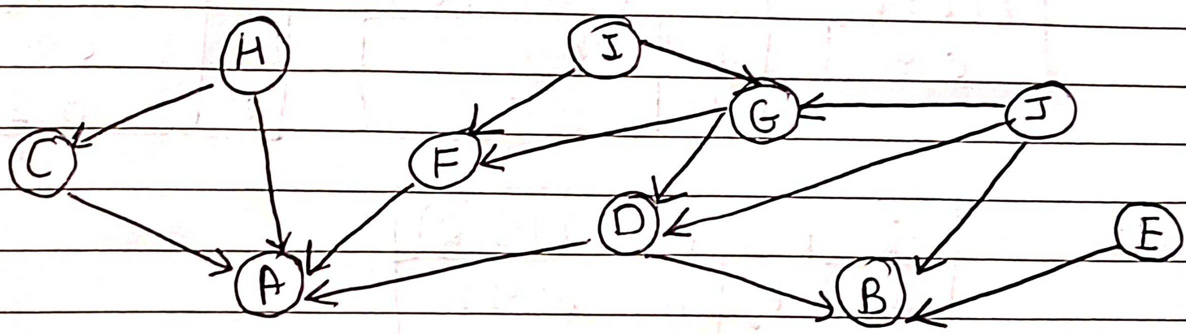


Page No.	
Date	

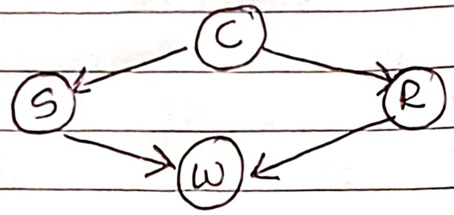
The factored form of the joint distribution is
P(C). P(A|C, H, E, I, D). P(F). P(H|I). P(I).
P(D|I). P(G|I). P(J). P(E|J)

3.C Draw the Bayesian Network that Corresponds to following factored joint Probability distribution given: P(A|C, D, F, H), P(B|D, E, J) P(C|H),
P(D|G, J) P(E) P(F|G, I) P(G|I, J)
P(H) P(I) P(J)

→



Problem 4] Below is Bayesian network for WetGrass Problem. Some prior Probabities and conditional Probability tables are given, all variables are Boolean Variables that can take Values true (t) or false (f)



$P(C=t)$
0.5

C	$P(S=t \mid C)$
t	0.1
f	0.5

C	$P(R=t \mid C)$
t	0.8
f	0.2

S	R	$P(W=t \mid S,R)$
t	t	0.99
t	f	0.90
f	t	0.90
f	f	0.00

a] Calculate value for joint Probability $P(C=f, R=f, S=t, W=t)$

→ $P(C=f, R=f, S=t, W=t)$

$= P(C=f) * P(R=f \mid C=f) * P(S=t \mid C=f) * P(W=t \mid R=f \mid S=t)$

(Based on $P(X_1 \ldots X_n) = \prod_{i=1} P(X_i \mid \text{Parent}(X_i))$)

$= 0.5 * 0.8 * 0.5 * 0.90$

$= \boxed{0.18}$

b] you absreve that $W = t$ & $S = b$. perform inferrence to obtain the posteriori Probability that weather is cloudy, that is $P(C = t \mid W = t, S = b)$ show your work

→ $P(C = t \mid W = t, S = b)$

Posterior Probality that the weather is cloudy based on observation that the grass is wet & sprinkler is not on

$$\frac{P(C = t, W = t, S = b)}{P(W = t, S = b)}$$

$$P(C = t \mid W = t, S = b) = \frac{\sum_{R = t, b} P(C = t, W = t, S = b)}{\sum_{\substack{R = t, b \\ C = t, b}} P(W = t, S = b)}$$

$$= \frac{P(C = t, R = t, W = t, S = b) + P(C = t, R = b, W = t, S = b)}{P(R = t, C = t, W = t, S = b) + P(R = b, C = t, W = t, S = b) + P(R = t, C = b, W = t, S = b) + P(R = b, C = b, W = t, S = b))}$$

$$= \frac{(0.5 * 0.8 * 0.9 * 0.9) + (0.5 * 0.2 * 0 * 0.9)}{(0.5 * 0.8 * 0.9 * 0.9) + (0.5 * 0.2 * 0 * 0.9) + (0.5 * 0.2 * 0.9 * 0.5) + (0.5 * 0.8 * 0 * 0.5)}$$

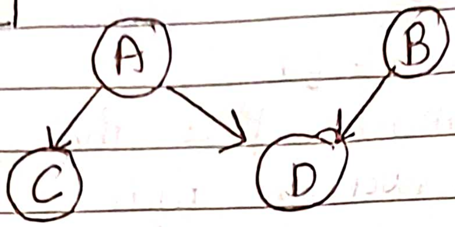
$$= \frac{0.324 + 0}{0.324 + 0.045}$$

$$= \frac{0.324}{0.369} = \boxed{0.878}$$

Problem 5 Consider the following Bayesian Network containing four Boolean Random Variables

P(A = t)
0.1

P(B = t)
0.5

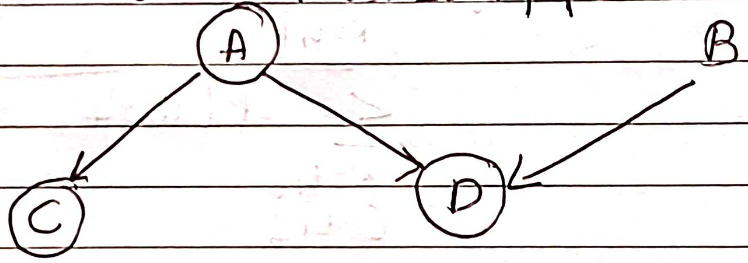


P(C = t | A = t)	0.7
P(C = t | A = f	0.2

P(D = t | A = t, B = t	
	0.9
	0.6
	0.7
	0.3

5.a compute joint Probability $p(A=f, B=t, C=f, D=t)$

→



$P(A=f, B=t, C=f, D=t)$

$= P(A=f) \cdot P(B=t) \cdot P(C=f \mid A=f) \cdot P(D=t \mid A=f, B=t)$

$= 0.9 \times 0.5 \times 0.8 \times 0.6$

$\boxed{= 0.216}$

5.b Compute the Posterior Probability $P(A=t \mid B=t, C=t, D=t)$

→ $P(A=t \mid B=t, C=t, D=t)$

$$= \frac{P(A=t, B=t, C=t, D=t)}{P(B=t, C=t, D=t)}$$

$$= \frac{P(A=t) \cdot P(B=t) \cdot P(C=t \mid A=t) \cdot P(D=t \mid A=t, B=t)}{\sum_{A=t,f} P(B=t, C=t, D=t)}$$

Page No.
Date

$$\sum_{A=t,f} P(B=t, C=t, D=t)$$

$P(A=t, B=t, C=t, D=t) + P(A=f, B=t, C=t, D=t)$

$P(A=t, B=t, C=t, D=t) = P(A=t) . P(B=t) . P(C=t|A=t) . P(D=t| A=t, B=t)$

$= 0.1 * 0.5 * 0.7 * 0.9$

$= 0.0315$

$P(A=f, B=t, C=t, D=t) = P(A=f) . P(B=t) . P(C=f|A=f) . P(D=t|A=t, B=t)$

$= 0.1 * 0.5 * 0.7 * 0.9$

$= 0.0315$

$P(A=f, B=t, C=t, D=t) = P(A=f) . P(B=t) . P(C=t|A=f) . P(D=t| A=f, B=t$

$= 0.9 * 0.5 * 0.2 * 0.6$

$= 0.054$

$\therefore P(A=t | B=t, C=t, D=t) = \frac{0.0315}{0.0315+0.054}$

$$\boxed{P(A=t| B=t, C=t, D=t) = 0.368}$$