TABLE 1. General trend in relative humidity at 22 representative weather stations

Locality	N. Latitude	E. Longitude	Observational period (n)	Mean relative humidity a per cent	b per cent per year	Correlation between r and t
Abashiri	44° 01′	144° 17′	1891-1959 (69)	78·8	− 0·025	− 0·26
Asahikawa	43° 46′	142° 22′	1917-1959 (43)	80·3	− 0·119	− 0·72
Obihiro	42° 55′	143° 13′	1916-1959 (44)	77·2	− 0·024	− 0·19
Suttsu	42° 47′	140° 14′	1888-1959 (72)	77·5	− 0·019	− 0·21
Mizusawa	39° 08′	141° 08′	1902-1958 (57)	80·2	+ 0·029	+ 0·46
Ishinomaki	38° 26′	141° 18′	1888-1959 (72)	79·9	− 0·041	− 0·43
Yamagata	38° 15′	140° 21′	1911-1959 (49)	79·0	− 0·081	− 0·70
Kanazawa	36° 33′	136° 39′	1909-1959 (51)	76·6	+ 0·011	+ 0·04
Maebashi	36° 24′	139° 04′	1897-1959 (63)	70·3	− 0·072	− 0·75
Kumagaya	36° 09′	139° 23′	1897-1959 (63)	74·1	− 0·043	− 0·50
Takayama	36° 00′	137° 15′	1904-1959 (56)	80·1	− 0·047	− 0·68
Tokyo	35° 41′	139° 46′	1923-1959 (37)	71·9	− 0·087	− 0·52
Yokohama	35° 26′	139° 39′	1928-1959 (32)	74·6	− 0·085	− 0·40
Hikone	35° 16′	136° 15′	1894-1959 (66)	78·9	+ 0·005	+ 0·08
Nagoya	35° 10′	136° 58′	1923-1959 (37)	76·0	− 0·078	− 0·47
Kyoto	35° 01′	135° 44′	1914-1959 (46)	75·4	− 0·116	− 0·76
Hamada	34° 54′	132° 04′	1893-1959 (67)	73·2	− 0·031	− 0·32
Osaka	34° 39′	135° 32′	1934-1959 (26)	72·1	− 0·148	− 0·78
Tadotsu	34° 16′	133° 45′	1893-1959 (67)	75·4	0·000	0·00
Izuhara	34° 12′	129° 18′	1896-1959 (64)	73·2	− 0·047	− 0·53
Fukuoka	33° 35′	130° 23′	1839-1959 (21)	76·2	− 0·137	− 0·44
Kumamoto	32° 49′	130° 43′	1902-1959 (58)	76·8	− 0·053	− 0·59
					(Mean − 0·055)	

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A Markov chain model for daily rainfall occurrence at Tel Aviv

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Summary

A Markov chain probability model is found to fit Tel Aviv data of daily rainfall occurrence. This accounts for the form of the distributions of dry and of wet spells and of weather ' cycles ' which have been presented in earlier papers. Further aspects of rainfall occurrence patterns may be derived as well, and are found to fit the data. In particular, the distribution of the number of rainy days per week, month or other period is obtained. Numbers of rainy days in different months are apparently independent.

1. Introduction

A Markov chain model is shown in this paper to fit daily rainfall occurrence data for Tel Aviv, and various derived properties of the model are found to fit equally well. Several authors have found wet and dry spells to have geometric or related distributions (Belasco 1948; Williams 1952; Longley 1953; Ramabhadran 1954; Neumann 1955). Such distributions arise under the Markov chain model, but may also arise otherwise. The existence of an underlying Markov chain has been suggested before (Brooks and Carruthers 1953), but does not seem to have been investigated hitherto. The model gives a basic probable representation for the spells distributions, and goes further in making it possible to derive several other properties of rainfall occurrence patterns.

The importance of this model is in giving a good fit to various aspects of rainfall occurrence patterns whilst being very simple and requiring only two parameters. Of course simplicity is achieved at the expense of ignoring possible small deviations, but such deviations do not show significantly on the data for 27 seasons analysed here, and must therefore be, at most, very minor.

The fit of the model to Tel Aviv rainfall, which is connected with the passage of depressions and frontal systems, stands in contrast to data from England where persistence increases for the first few days of spell length (Lawrence 1954; 1957).

2. The Markov chain model and derived properties

The definition of the Markov chain model in terms of rainfall occurrence is formulated in this section, and several properties are stated – without proof. For their derivation the reader is referred to texts on Markov chains (e.g., Feller 1957) and to earlier papers of the present authors (Gabriel and Neumann (1957) who derive Eq. (9) below; Gabriel (1959) derives Eqs. (10), (11) and (12) and discusses the other formulae briefly).

It is assumed that the probability of rainfall on any day depends only on whether the previous day was wet or dry, i.e., whether rainfall did or did not occur. Given the event on the previous day, then, the probability of rainfall is assumed independent of events of further preceding days. Such a probability model is referred to as a Markov chain whose parameters are the two conditional probabilities :

$$p_1 = \mathrm{P}_r\,\{\text{wet day} \mid \text{previous day wet}\} \qquad (1)$$

$$p_0 = \mathrm{P}_r\,\{\text{wet day} \mid \text{previous day dry}\} \qquad (2)$$

This model is formulated entirely in terms of occurrence and non-occurrence of rainfall on any day; no account being taken of amounts of precipitation or any other meteorological observations. It is not suggested in terms of a physical explanation of rainfall occurrence but merely as a statistical description of the observations.

Given the Markov chain and estimates of its two basic probabilities, one can derive various properties of rainfall occurrence patterns. Thus the probabilities of rainfall i days after a wet or a dry day are :

$$P + (1 - P)\,d^i \qquad (3)$$

or

$$P + Pd^i \qquad (4)$$

respectively, where

$$d = p_1 - p_0 \qquad (5)$$

and

$$P = p_0/(1 - d) \qquad (6)$$

the latter being the absolute probability of a day being wet.

A wet spell of length k is defined as a sequence of k-wet days preceded and followed by dry days. Dry spells are defined correspondingly. Weather cycles are defined as combinations of a wet spell and an adjacent dry spell. A wet spell of length k is thus equivalent to a recurrence time of $k + 1$ days for dry weather, and correspondingly for dry spells and recurrence of wet days. The distributions of spells by length are obtained from those of recurrence times and are found to be geometric, with the probability of a wet spell of length k being :

$$(1 - p_1)\,{p_1}^{k-1} \qquad (7)$$

and the probability of a dry spell of length m being :

$$p_0\,(1 - p_0)^{m-1} \qquad (8)$$

The lengths of successive spells are readily seen to be independent and the probability of a weather cycle of n days is :

$$p_0\,(1 - p_1)\,\frac{(1 - p_0)^{n-1} - {p_1}^{n-1}}{1 - p_0 - p_1} \qquad (9)$$

The probability of exactly s wet days among n days following a wet day is :

$$P_r\,\{s \mid n, 1\} = {p_1}^s\,(1 - p_0)^{n-s} \sum_{c=1}^{c_1} \binom{s}{a} \binom{n - s - 1}{b - 1} \left(\frac{1 - p_1}{1 - p_0}\right)^b \left(\frac{p_0}{p_1}\right)^a . \qquad (10)$$

where

$$c_1 = \begin{cases} n + \frac{1}{2} - |2s - n + \frac{1}{2}| & \text{if } s < n \\ 0 \text{ (and the sum is understood to involve only this term)} & \text{if } s = n, \end{cases}$$

and a and b are the least integers not smaller than $\frac{1}{2}(c - 1)$ and $\frac{1}{2}c$, respectively. Similarly, following a dry day, the probability is :

$$P_r\{S \mid n, 0\} = p_1^s (1 - p_0)^{n-s} \sum_{c=1}^{c_0} \binom{s-1}{b-1} \binom{n-s}{a} \left(\frac{1-p_1}{1-p_0}\right)^a \left(\frac{p_0}{p_1}\right)^b . \qquad (11)$$

where

$$c_0 = \begin{cases} n + \frac{1}{2} - |2s - n - \frac{1}{2}| & \text{if } s > 0 \\ 0 \text{ (and the sum is understood to involve only this term).} & \text{if } s = 0 \end{cases}$$

and a, b are defined as above. The probability of s wet among any n days is then

$$P_r\{s \mid n\} = P\, P_r\{s \mid n, 1\} + (1 - P)\, P_r\{s \mid n, 0\} \qquad (12)$$

where P is defined as in Eq. (6) above.

For large n the distribution of the number of wet days tends to normality with mean and variance

$$E(s) = nP \qquad (13)$$

and

$$Var(s) = nP(1 - P)\frac{1 + d}{1 - d} \qquad (14)$$

The correlation between numbers of wet days in successive periods of n days results entirely from the dependence of the event on the first day of the second period on that on the last day of the first period. Clearly the correlation will be small for large n. Thus successive long periods will have virtually independent numbers of wet days.

3. Fit of the model

All the above properties of rainfall occurrence are derived from the probability model and require only the two probabilities p_1 and p_0 for their calculation. No further coefficients of periodicity, persistence or other parameters are required. The remarkable thing is the adequate representation this simple model gives of actual observations on rainfall at Tel Aviv. The fit of the model has been tested on data of daily rainfall in Tel Aviv (Nahmani Street) for the 27 rainy seasons 1923/24-1949/50. Days were classified as wet or dry according to whether there had or had not been recorded at least 0·1 mm of precipitation in the 24 hr from 8 a.m. to 8 a.m. the following day.

TABLE 1. Wet days and all days classified by rainfall occurrence on preceding day and by month. Estimates of conditional probabilities of rainfall occurrence

	Preceding day	Actual day Wet	Total	Estimate of probability p_1	p_0
November	Wet	117	195	0·600	
	Dry	80	615		0·130
December	Wet	213	326	0·653	
	Dry	117	511		0·229
January	Wet	260	386	0·674	
	Dry	132	451		0·293
February	Wet	214	326	0·656	
	Dry	101	437		0·231
March	Wet	114	215	0·530	
	Dry	100	622		0·161
April	Wet	48	104	0·462	
	Dry	51	706		0·072