

WILEY SERIES IN PROBABILITY AND STATISTICS

FIFTH EDITION

Time Series Analysis

Forecasting and Control

George E. P. Box • Gwilym M. Jenkins

Gregory C. Reinsel • Greta M. Ljung

WILEY

TIME SERIES ANALYSIS

WILEY SERIES IN PROBABILITY AND STATISTICS

Established by WALTER A. SHEWHART and SAMUEL S. WILKS

Editors: *David J. Balding, Noel A. C. Cressie, Garrett M. Fitzmaurice, Geof H. Givens, Harvey Goldstein, Geert Molenberghs, David W. Scott, Adrian F. M. Smith, Ruey S. Tsay, Sanford Weisberg*

Editors Emeriti: *J. Stuart Hunter, Iain M. Johnstone, Joseph B. Kadane, Jozef L. Teugels*

A complete list of the titles in this series appears at the end of this volume.

TIME SERIES ANALYSIS

Forecasting and Control

Fifth Edition

**GEORGE E. P. BOX
GWILYM M. JENKINS
GREGORY C. REINSEL
GRETA M. LJUNG**

WILEY

Copyright 2016 by John Wiley & Sons, Inc. All rights reserved

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permission>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Box, George E. P.

Time series analysis : forecasting and control. – Fifth edition / George E.P. Box, Gwilym M. Jenkins, Gregory C. Reinsel, Greta M. Ljung.

pages cm

Includes bibliographical references and index.

ISBN 978-1-118-67502-1 (cloth : alk. paper) 1. Time-series analysis. 2. Prediction theory. 3. Transfer functions. 4. Feedback control systems—Mathematical models. I. Jenkins, Gwilym M. II. Reinsel, Gregory C. III. Ljung, Greta M., 1941- IV. Title.

QA280.B67 2016

519.5'5—dc23

2015015492

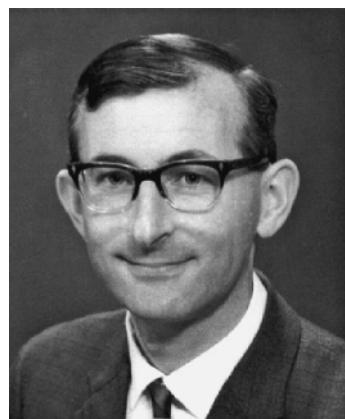
Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

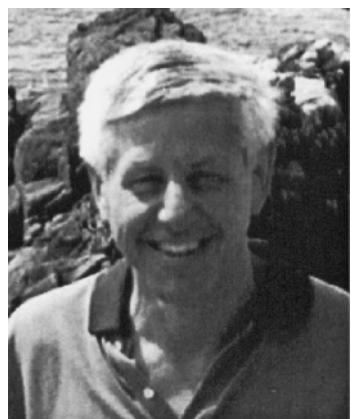
To the memory of



George E. P. Box



Gwilym M. Jenkins



Gregory C. Reinsel

CONTENTS

PREFACE TO THE FIFTH EDITION	xix
PREFACE TO THE FOURTH EDITION	xxiii
PREFACE TO THE THIRD EDITION	xxv
1 Introduction	1
1.1 Five Important Practical Problems, 2	
1.1.1 Forecasting Time Series, 2	
1.1.2 Estimation of Transfer Functions, 3	
1.1.3 Analysis of Effects of Unusual Intervention Events to a System, 4	
1.1.4 Analysis of Multivariate Time Series, 4	
1.1.5 Discrete Control Systems, 5	
1.2 Stochastic and Deterministic Dynamic Mathematical Models, 6	
1.2.1 Stationary and Nonstationary Stochastic Models for Forecasting and Control, 7	
1.2.2 Transfer Function Models, 11	
1.2.3 Models for Discrete Control Systems, 13	
1.3 Basic Ideas in Model Building, 14	
1.3.1 Parsimony, 14	
1.3.2 Iterative Stages in the Selection of a Model, 15	
Appendix A1.1 Use of the R Software, 17	
Exercises, 18	

PART ONE STOCHASTIC MODELS AND THEIR FORECASTING	19
2 Autocorrelation Function and Spectrum of Stationary Processes	21
2.1 Autocorrelation Properties of Stationary Models, 21	
2.1.1 Time Series and Stochastic Processes, 21	
2.1.2 Stationary Stochastic Processes, 24	
2.1.3 Positive Definiteness and the Autocovariance Matrix, 26	
2.1.4 Autocovariance and Autocorrelation Functions, 29	
2.1.5 Estimation of Autocovariance and Autocorrelation Functions, 30	
2.1.6 Standard Errors of Autocorrelation Estimates, 31	
2.2 Spectral Properties of Stationary Models, 34	
2.2.1 Periodogram of a Time Series, 34	
2.2.2 Analysis of Variance, 35	
2.2.3 Spectrum and Spectral Density Function, 36	
2.2.4 Simple Examples of Autocorrelation and Spectral Density Functions, 40	
2.2.5 Advantages and Disadvantages of the Autocorrelation and Spectral Density Functions, 43	
Appendix A2.1 Link Between the Sample Spectrum and Autocovariance Function Estimate, 43	
Exercises, 44	
3 Linear Stationary Models	47
3.1 General Linear Process, 47	
3.1.1 Two Equivalent Forms for the Linear Process, 47	
3.1.2 Autocovariance Generating Function of a Linear Process, 50	
3.1.3 Stationarity and Invertibility Conditions for a Linear Process, 51	
3.1.4 Autoregressive and Moving Average Processes, 52	
3.2 Autoregressive Processes, 54	
3.2.1 Stationarity Conditions for Autoregressive Processes, 54	
3.2.2 Autocorrelation Function and Spectrum of Autoregressive Processes, 56	
3.2.3 The First-Order Autoregressive Process, 58	
3.2.4 Second-Order Autoregressive Process, 59	
3.2.5 Partial Autocorrelation Function, 64	
3.2.6 Estimation of the Partial Autocorrelation Function, 66	
3.2.7 Standard Errors of Partial Autocorrelation Estimates, 66	
3.2.8 Calculations in R, 67	
3.3 Moving Average Processes, 68	
3.3.1 Invertibility Conditions for Moving Average Processes, 68	
3.3.2 Autocorrelation Function and Spectrum of Moving Average Processes, 69	
3.3.3 First-Order Moving Average Process, 70	
3.3.4 Second-Order Moving Average Process, 71	
3.3.5 Duality Between Autoregressive and Moving Average Processes, 75	
3.4 Mixed Autoregressive–Moving Average Processes, 75	

3.4.1	Stationarity and Invertibility Properties,	75
3.4.2	Autocorrelation Function and Spectrum of Mixed Processes,	77
3.4.3	First Order Autoregressive First-Order Moving Average Process,	78
3.4.4	Summary,	81
Appendix A3.1	Autocovariances, Autocovariance Generating Function, and Stationarity Conditions for a General Linear Process,	82
Appendix A3.2	Recursive Method for Calculating Estimates of Autoregressive Parameters,	84
	Exercises,	86
4	Linear Nonstationary Models	88
4.1	Autoregressive Integrated Moving Average Processes,	88
4.1.1	Nonstationary First-Order Autoregressive Process,	88
4.1.2	General Model for a Nonstationary Process Exhibiting Homogeneity,	90
4.1.3	General Form of the ARIMA Model,	94
4.2	Three Explicit Forms for the ARIMA Model,	97
4.2.1	Difference Equation Form of the Model,	97
4.2.2	Random Shock Form of the Model,	98
4.2.3	Inverted Form of the Model,	103
4.3	Integrated Moving Average Processes,	106
4.3.1	Integrated Moving Average Process of Order (0, 1, 1),	107
4.3.2	Integrated Moving Average Process of Order (0, 2, 2),	110
4.3.3	General Integrated Moving Average Process of Order (0, d , q),	114
Appendix A4.1	Linear Difference Equations,	116
Appendix A4.2	IMA(0, 1, 1) Process with Deterministic Drift,	121
Appendix A4.3	ARIMA Processes with Added Noise,	122
A4.3.1	Sum of Two Independent Moving Average Processes,	122
A4.3.2	Effect of Added Noise on the General Model,	123
A4.3.3	Example for an IMA(0, 1, 1) Process with Added White Noise,	124
A4.3.4	Relation between the IMA(0, 1, 1) Process and a Random Walk,	125
A4.3.5	Autocovariance Function of the General Model with Added Correlated Noise,	125
	Exercises,	126
5	Forecasting	129
5.1	Minimum Mean Square Error Forecasts and Their Properties,	129
5.1.1	Derivation of the Minimum Mean Square Error Forecasts,	131
5.1.2	Three Basic Forms for the Forecast,	132
5.2	Calculating Forecasts and Probability Limits,	135
5.2.1	Calculation of ψ Weights,	135
5.2.2	Use of the ψ Weights in Updating the Forecasts,	136
5.2.3	Calculation of the Probability Limits at Different Lead Times,	137
5.2.4	Calculation of Forecasts Using R,	138
5.3	Forecast Function and Forecast Weights,	139

5.3.1	Eventual Forecast Function Determined by the Autoregressive Operator, 140
5.3.2	Role of the Moving Average Operator in Fixing the Initial Values, 140
5.3.3	Lead l Forecast Weights, 142
5.4	Examples of Forecast Functions and Their Updating, 144
5.4.1	Forecasting an IMA(0, 1, 1) Process, 144
5.4.2	Forecasting an IMA(0, 2, 2) Process, 147
5.4.3	Forecasting a General IMA(0, d , q) Process, 149
5.4.4	Forecasting Autoregressive Processes, 150
5.4.5	Forecasting a (1, 0, 1) Process, 153
5.4.6	Forecasting a (1, 1, 1) Process, 154
5.5	Use of State-Space Model Formulation for Exact Forecasting, 155
5.5.1	State-Space Model Representation for the ARIMA Process, 155
5.5.2	Kalman Filtering Relations for Use in Prediction, 157
5.5.3	Smoothing Relations in the State Variable Model, 160
5.6	Summary, 162
Appendix A5.1	Correlation Between Forecast Errors, 164
A5.1.1	Autocorrelation Function of Forecast Errors at Different Origins, 164
A5.1.2	Correlation Between Forecast Errors at the Same Origin with Different Lead Times, 165
Appendix A5.2	Forecast Weights for any Lead Time, 166
Appendix A5.3	Forecasting in Terms of the General Integrated Form, 168
A5.3.1	General Method of Obtaining the Integrated Form, 168
A5.3.2	Updating the General Integrated Form, 170
A5.3.3	Comparison with the Discounted Least-Squares Method, 171
	Exercises, 174

PART TWO STOCHASTIC MODEL BUILDING 177

6	Model Identification 179
6.1	Objectives of Identification, 179
6.1.1	Stages in the Identification Procedure, 180
6.2	Identification Techniques, 180
6.2.1	Use of the Autocorrelation and Partial Autocorrelation Functions in Identification, 180
6.2.2	Standard Errors for Estimated Autocorrelations and Partial Autocorrelations, 183
6.2.3	Identification of Models for Some Actual Time Series, 185
6.2.4	Some Additional Model Identification Tools, 190
6.3	Initial Estimates for the Parameters, 194
6.3.1	Uniqueness of Estimates Obtained from the Autocovariance Function, 194
6.3.2	Initial Estimates for Moving Average Processes, 194
6.3.3	Initial Estimates for Autoregressive Processes, 196

6.3.4	Initial Estimates for Mixed Autoregressive–Moving Average Processes, 197	
6.3.5	Initial Estimate of Error Variance, 198	
6.3.6	Approximate Standard Error for \bar{w} , 199	
6.3.7	Choice Between Stationary and Nonstationary Models in Doubtful Cases, 200	
6.4	Model Multiplicity, 202	
6.4.1	Multiplicity of Autoregressive–Moving Average Models, 202	
6.4.2	Multiple Moment Solutions for Moving Average Parameters, 204	
6.4.3	Use of the Backward Process to Determine Starting Values, 205	
Appendix A6.1	Expected Behavior of the Estimated Autocorrelation Function for a Nonstationary Process, 206	
	Exercises, 207	
7	Parameter Estimation	209
7.1	Study of the Likelihood and Sum-of-Squares Functions, 209	
7.1.1	Likelihood Function, 209	
7.1.2	Conditional Likelihood for an ARIMA Process, 210	
7.1.3	Choice of Starting Values for Conditional Calculation, 211	
7.1.4	Unconditional Likelihood, Sum-of-Squares Function, and Least-Squares Estimates, 213	
7.1.5	General Procedure for Calculating the Unconditional Sum of Squares, 216	
7.1.6	Graphical Study of the Sum-of-Squares Function, 218	
7.1.7	Examination of the Likelihood Function and Confidence Regions, 220	
7.2	Nonlinear Estimation, 226	
7.2.1	General Method of Approach, 226	
7.2.2	Numerical Estimates of the Derivatives, 227	
7.2.3	Direct Evaluation of the Derivatives, 228	
7.2.4	General Least-Squares Algorithm for the Conditional Model, 229	
7.2.5	ARIMA Models Fitted to Series A–F, 231	
7.2.6	Large-Sample Information Matrices and Covariance Estimates, 233	
7.3	Some Estimation Results for Specific Models, 236	
7.3.1	Autoregressive Processes, 236	
7.3.2	Moving Average Processes, 238	
7.3.3	Mixed Processes, 238	
7.3.4	Separation of Linear and Nonlinear Components in Estimation, 239	
7.3.5	Parameter Redundancy, 240	
7.4	Likelihood Function Based on the State-Space Model, 242	
7.5	Estimation Using Bayes' Theorem, 245	
7.5.1	Bayes' Theorem, 245	
7.5.2	Bayesian Estimation of Parameters, 246	
7.5.3	Autoregressive Processes, 247	
7.5.4	Moving Average Processes, 249	
7.5.5	Mixed Processes, 250	
Appendix A7.1	Review of Normal Distribution Theory, 251	

A7.1.1	Partitioning of a Positive-Definite Quadratic Form,	251
A7.1.2	Two Useful Integrals,	252
A7.1.3	Normal Distribution,	253
A7.1.4	Student's <i>t</i> Distribution,	255
Appendix A7.2	Review of Linear Least-Squares Theory,	256
A7.2.1	Normal Equations and Least Squares,	256
A7.2.2	Estimation of Error Variance,	257
A7.2.3	Covariance Matrix of Least-Squares Estimates,	257
A7.2.4	Confidence Regions,	257
A7.2.5	Correlated Errors,	258
Appendix A7.3	Exact Likelihood Function for Moving Average and Mixed Processes,	259
Appendix A7.4	Exact Likelihood Function for an Autoregressive Process,	266
Appendix A7.5	Asymptotic Distribution of Estimators for Autoregressive Models,	274
Appendix A7.6	Examples of the Effect of Parameter Estimation Errors on Variances of Forecast Errors and Probability Limits for Forecasts,	277
Appendix A7.7	Special Note on Estimation of Moving Average Parameters,	280
	Exercises,	280

8 Model Diagnostic Checking 284

8.1	Checking the Stochastic Model,	284
8.1.1	General Philosophy,	284
8.1.2	Overfitting,	285
8.2	Diagnostic Checks Applied to Residuals,	287
8.2.1	Autocorrelation Check,	287
8.2.2	Portmanteau Lack-of-Fit Test,	289
8.2.3	Model Inadequacy Arising from Changes in Parameter Values,	294
8.2.4	Score Tests for Model Checking,	295
8.2.5	Cumulative Periodogram Check,	297
8.3	Use of Residuals to Modify the Model,	301
8.3.1	Nature of the Correlations in the Residuals When an Incorrect Model Is Used,	301
8.3.2	Use of Residuals to Modify the Model,	302
	Exercises,	303

9 Analysis of Seasonal Time Series 305

9.1	Parsimonious Models for Seasonal Time Series,	305
9.1.1	Fitting Versus Forecasting,	306
9.1.2	Seasonal Models Involving Adaptive Sines and Cosines,	307
9.1.3	General Multiplicative Seasonal Model,	308
9.2	Representation of the Airline Data by a Multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ Model,	310
9.2.1	Multiplicative $(0, 1, 1) \times (0, 1, 1)_{12}$ Model,	310
9.2.2	Forecasting,	311
9.2.3	Model Identification,	318
9.2.4	Parameter Estimation,	320

9.2.5	Diagnostic Checking, 324	
9.3	Some Aspects of More General Seasonal ARIMA Models, 325	
9.3.1	Multiplicative and Nonmultiplicative Models, 325	
9.3.2	Model Identification, 327	
9.3.3	Parameter Estimation, 328	
9.3.4	Eventual Forecast Functions for Various Seasonal Models, 329	
9.3.5	Choice of Transformation, 331	
9.4	Structural Component Models and Deterministic Seasonal Components, 331	
9.4.1	Structural Component Time Series Models, 332	
9.4.2	Deterministic Seasonal and Trend Components and Common Factors, 335	
9.4.3	Estimation of Unobserved Components in Structural Models, 336	
9.5	Regression Models with Time Series Error Terms, 339	
9.5.1	Model Building, Estimation, and Forecasting Procedures for Regression Models, 340	
9.5.2	Restricted Maximum Likelihood Estimation for Regression Models, 344	
Appendix A9.1	Autocovariances for Some Seasonal Models, 345	
	Exercises, 349	
10	Additional Topics and Extensions	352
10.1	Tests for Unit Roots in ARIMA Models, 353	
10.1.1	Tests for Unit Roots in AR Models, 353	
10.1.2	Extensions of Unit Root Testing to Mixed ARIMA Models, 358	
10.2	Conditional Heteroscedastic Models, 361	
10.2.1	The ARCH Model, 362	
10.2.2	The GARCH Model, 366	
10.2.3	Model Building and Parameter Estimation, 367	
10.2.4	An Illustrative Example: Weekly S&P 500 Log Returns, 370	
10.2.5	Extensions of the ARCH and GARCH Models, 372	
10.2.6	Stochastic Volatility Models, 377	
10.3	Nonlinear Time Series Models, 377	
10.3.1	Classes of Nonlinear Models, 378	
10.3.2	Detection of Nonlinearity, 381	
10.3.3	An Empirical Example, 382	
10.4	Long Memory Time Series Processes, 385	
10.4.1	Fractionally Integrated Processes, 385	
10.4.2	Estimation of Parameters, 389	
	Exercises, 392	
PART THREE	TRANSFER FUNCTION AND MULTIVARIATE MODEL BUILDING	395
11	Transfer Function Models	397
11.1	Linear Transfer Function Models, 397	

11.1.1	Discrete Transfer Function,	398
11.1.2	Continuous Dynamic Models Represented by Differential Equations,	400
11.2	Discrete Dynamic Models Represented by Difference Equations,	404
11.2.1	General Form of the Difference Equation,	404
11.2.2	Nature of the Transfer Function,	406
11.2.3	First- and Second-Order Discrete Transfer Function Models,	407
11.2.4	Recursive Computation of Output for Any Input,	412
11.2.5	Transfer Function Models with Added Noise,	413
11.3	Relation Between Discrete and Continuous Models,	414
11.3.1	Response to a Pulsed Input,	415
11.3.2	Relationships for First- and Second-Order Coincident Systems,	417
11.3.3	Approximating General Continuous Models by Discrete Models,	419
Appendix A11.1	Continuous Models with Pulsed Inputs,	420
Appendix A11.2	Nonlinear Transfer Functions and Linearization,	424
	Exercises,	426

12 Identification, Fitting, and Checking of Transfer Function Models 428

12.1	Cross-Correlation Function,	429
12.1.1	Properties of the Cross-Covariance and Cross-Correlation Functions,	429
12.1.2	Estimation of the Cross-Covariance and Cross-Correlation Functions,	431
12.1.3	Approximate Standard Errors of Cross-Correlation Estimates,	433
12.2	Identification of Transfer Function Models,	435
12.2.1	Identification of Transfer Function Models by Prewhitening the Input,	437
12.2.2	Example of the Identification of a Transfer Function Model,	438
12.2.3	Identification of the Noise Model,	442
12.2.4	Some General Considerations in Identifying Transfer Function Models,	444
12.3	Fitting and Checking Transfer Function Models,	446
12.3.1	Conditional Sum-of-Squares Function,	446
12.3.2	Nonlinear Estimation,	447
12.3.3	Use of Residuals for Diagnostic Checking,	449
12.3.4	Specific Checks Applied to the Residuals,	450
12.4	Some Examples of Fitting and Checking Transfer Function Models,	453
12.4.1	Fitting and Checking of the Gas Furnace Model,	453
12.4.2	Simulated Example with Two Inputs,	458
12.5	Forecasting with Transfer Function Models Using Leading Indicators,	461
12.5.1	Minimum Mean Square Error Forecast,	461
12.5.2	Forecast of CO ₂ Output from Gas Furnace,	465
12.5.3	Forecast of Nonstationary Sales Data Using a Leading Indicator,	468

12.6	Some Aspects of the Design of Experiments to Estimate Transfer Functions,	469
Appendix A12.1	Use of Cross-Spectral Analysis for Transfer Function Model Identification,	471
A12.1.1	Identification of Single-Input Transfer Function Models,	471
A12.1.2	Identification of Multiple-Input Transfer Function Models,	472
Appendix A12.2	Choice of Input to Provide Optimal Parameter Estimates,	473
A12.2.1	Design of Optimal Inputs for a Simple System,	473
A12.2.2	Numerical Example,	476
	Exercises,	477
13	Intervention Analysis, Outlier Detection, and Missing Values	481
13.1	Intervention Analysis Methods,	481
13.1.1	Models for Intervention Analysis,	481
13.1.2	Example of Intervention Analysis,	484
13.1.3	Nature of the MLE for a Simple Level Change Parameter Model,	485
13.2	Outlier Analysis for Time Series,	488
13.2.1	Models for Additive and Innovational Outliers,	488
13.2.2	Estimation of Outlier Effect for Known Timing of the Outlier,	489
13.2.3	Iterative Procedure for Outlier Detection,	491
13.2.4	Examples of Analysis of Outliers,	492
13.3	Estimation for ARMA Models with Missing Values,	495
13.3.1	State-Space Model and Kalman Filter with Missing Values,	496
13.3.2	Estimation of Missing Values of an ARMA Process,	498
	Exercises,	502
14	Multivariate Time Series Analysis	505
14.1	Stationary Multivariate Time Series,	506
14.1.1	Cross-Covariance and Cross-Correlation Matrices,	506
14.1.2	Covariance Stationarity,	507
14.1.3	Vector White Noise Process,	507
14.1.4	Moving Average Representation of a Stationary Vector Process,	508
14.2	Vector Autoregressive Models,	509
14.2.1	VAR(p) Model,	509
14.2.2	Moment Equations and Yule–Walker Estimates,	510
14.2.3	Special Case: VAR(1) Model,	511
14.2.4	Numerical Example,	513
14.2.5	Initial Model Building and Least-Squares Estimation for VAR Models,	515
14.2.6	Parameter Estimation and Model Checking,	518
14.2.7	An Empirical Example,	519
14.3	Vector Moving Average Models,	524
14.3.1	Vector MA(q) Model,	524
14.3.2	Special Case: Vector MA(1) Model,	525
14.3.3	Numerical Example,	525

14.3.4	Model Building for Vector MA Models,	526
14.4	Vector Autoregressive–Moving Average Models,	527
14.4.1	Stationarity and Invertibility Conditions,	527
14.4.2	Covariance Matrix Properties of VARMA Processes,	528
14.4.3	Nonuniqueness and Parameter Identifiability for VARMA Models,	528
14.4.4	Model Specification for VARMA Processes,	529
14.4.5	Estimation and Model Checking for VARMA Models,	532
14.4.6	Relation of VARMA Models to Transfer Function and ARMAX Models,	533
14.5	Forecasting for Vector Autoregressive–Moving Average Processes,	534
14.5.1	Calculation of Forecasts from ARMA Difference Equation,	534
14.5.2	Forecasts from Infinite VMA Form and Properties of Forecast Errors,	536
14.6	State-Space Form of the VARMA Model,	536
14.7	Further Discussion of VARMA Model Specification,	539
14.7.1	Kronecker Structure for VARMA Models,	539
14.7.2	An Empirical Example,	543
14.7.3	Partial Canonical Correlation Analysis for Reduced-Rank Structure,	545
14.8	Nonstationarity and Cointegration,	546
14.8.1	Vector ARIMA Models,	546
14.8.2	Cointegration in Nonstationary Vector Processes,	547
14.8.3	Estimation and Inferences for Cointegrated VAR Models,	549
Appendix A14.1	Spectral Characteristics and Linear Filtering Relations for Stationary Multivariate Processes,	552
A14.1.1	Spectral Characteristics for Stationary Multivariate Processes,	552
A14.1.2	Linear Filtering Relations for Stationary Multivariate Processes,	553
	Exercises,	554

PART FOUR DESIGN OF DISCRETE CONTROL SCHEMES 559

15	Aspects of Process Control	561
15.1	Process Monitoring and Process Adjustment,	562
15.1.1	Process Monitoring,	562
15.1.2	Process Adjustment,	564
15.2	Process Adjustment Using Feedback Control,	566
15.2.1	Feedback Adjustment Chart,	567
15.2.2	Modeling the Feedback Loop,	569
15.2.3	Simple Models for Disturbances and Dynamics,	570
15.2.4	General Minimum Mean Square Error Feedback Control Schemes,	573
15.2.5	Manual Adjustment for Discrete Proportional–Integral Schemes,	575

15.2.6	Complementary Roles of Monitoring and Adjustment, 578
15.3	Excessive Adjustment Sometimes Required by MMSE Control, 580
15.3.1	Constrained Control, 581
15.4	Minimum Cost Control with Fixed Costs of Adjustment and Monitoring, 582
15.4.1	Bounded Adjustment Scheme for Fixed Adjustment Cost, 583
15.4.2	Indirect Approach for Obtaining a Bounded Adjustment Scheme, 584
15.4.3	Inclusion of the Cost of Monitoring, 585
15.5	Feedforward Control, 588
15.5.1	Feedforward Control to Minimize Mean Square Error at the Output, 588
15.5.2	An Example: Control of the Specific Gravity of an Intermediate Product, 591
15.5.3	Feedforward Control with Multiple Inputs, 593
15.5.4	Feedforward–Feedback Control, 594
15.5.5	Advantages and Disadvantages of Feedforward and Feedback Control, 596
15.5.6	Remarks on Fitting Transfer Function–Noise Models Using Operating Data, 597
15.6	Monitoring Values of Parameters of Forecasting and Feedback Adjustment Schemes, 599
Appendix A15.1	Feedback Control Schemes Where the Adjustment Variance Is Restricted, 600
A15.1.1	Derivation of Optimal Adjustment, 601
A15.1.2	Case Where δ Is Negligible, 603
Appendix A15.2	Choice of the Sampling Interval, 609
A15.2.1	Illustration of the Effect of Reducing Sampling Frequency, 610
A15.2.2	Sampling an IMA(0, 1, 1) Process, 610
	Exercises, 613

PART FIVE CHARTS AND TABLES	617
COLLECTION OF TABLES AND CHARTS	619
COLLECTION OF TIME SERIES USED FOR EXAMPLES IN THE TEXT AND IN EXERCISES	625
REFERENCES	642
INDEX	659

PREFACE TO THE FIFTH EDITION

This book describes statistical models and methods for analyzing discrete time series and presents important applications of the methodology. The models considered include the class of autoregressive integrated moving average (ARIMA) models and various extensions of these models. The properties of the models are examined and statistical methods for model specification, parameter estimation, and model checking are presented. Applications to forecasting nonseasonal as well as seasonal time series are described. Extensions of the methodology to transfer function modeling of dynamic relationships between two or more time series, modeling the effects of intervention events, multivariate time series modeling, and process control are discussed. Topics such as state-space and structural modeling, nonlinear models, long-memory models, and conditionally heteroscedastic models are also covered. The goal has been to provide a text that is practical and of value to both academicians and practitioners.

The first edition of this book appeared in 1970 and around that time there was a great upsurge in research on time series analysis and forecasting. This generated a large influx of new ideas, modifications, and improvements by many authors. For example, several new research directions began to emerge in econometrics around that time, leading to what is now known as time series econometrics. Many of these developments were reflected in the fourth edition of this book and have been further elaborated upon in this new edition.

The main goals of preparing a new edition have been to expand and update earlier material, incorporate new literature, enhance and update numerical illustrations through the use of R, and increase the number of exercises in the book. Some of the chapters in the previous edition have been reorganized. For example, Chapter 14 on multivariate time series analysis has been reorganized and expanded, placing more emphasis on vector autoregressive (VAR) models. The VAR models are by far the most widely used multivariate time series models in applied work. This edition provides an expanded treatment of these models that includes software demonstrations.

Chapter 10 has also been expanded and updated. This chapter covers selected topics in time series analysis that either extend or supplement material discussed in earlier chapters.

This includes unit roots testing, modeling of conditional heteroscedasticity, nonlinear models, and long memory models. A section of unit root testing that appeared in Chapter 7 of the previous edition has been expanded and moved to Section 10.1 in this edition. Section 10.2 deals with autoregressive conditionally heteroscedastic models, such as the ARCH and GARCH models. These models focus on the variability in a time series and are useful for modeling the volatility or variability in economic and financial series, in particular. The treatment of the ARCH and GARCH models has been expanded and several extensions have been added.

Elsewhere in the text, the exposition has been enhanced by revising, modifying, and omitting text as appropriate. Several tables have either been edited or replaced by graphs to make the presentation more effective. The number of exercises has been increased throughout the text and they now appear at the end of each chapter.

A further enhancement to this edition is the use of the statistical software R for model building and forecasting. The R package is available as a free download from the R Project for Statistical Computing at www.r-project.org. A brief description of the software is given in Appendix A1.1 of Chapter 1. Graphs generated using R now appear in many of the chapters along with R code that will help the reader reconstruct the graphs. The software is also used for numerical illustration in many of the examples in the text.

The fourth edition of this book was published by Wiley in 2008. Plans for a new edition began during the fall of 2012. I was deeply honored when George Box asked me to help him with this update. George was my Ph.D. advisor at the University of Wisconsin-Madison and remained a dear friend to me over the years as he did to all his students. Sadly, he was rather ill when the plans for this new edition were finalized towards the end of 2012. He did not have a chance to see the project completed as he passed away in March of 2013. I am deeply grateful for the opportunity to work with him and for the confidence he showed in assigning me this task. The book is dedicated to his memory and to the memory of his distinguished co-authors Gwilym Jenkins and Gregory Reinsel. Their contributions were many and they are all missed.

I also want to express my gratitude to several friends and colleagues in the time series community who have read the manuscript and provided helpful comments and suggestions. These include Ruey Tsay, William Wei, Sung Ahn, and Raja Velu who have read Chapter 14 on multivariate time series analysis, and David Dickey, Johannes Ledolter, Timo Teräsvirta, and Niels Haldrup who have read Chapter 10 on special topics. Their constructive comments and suggestions are much appreciated. Assistance and support from Paul Lindholm in Finland is also gratefully acknowledged. The use of R in this edition includes packages developed for existing books on time series analysis such as Cryer and Chan (2010), Shumway and Stoffer (2011), and Tsay (2014). We commend these authors for making their code and datasets available for public use through the R Project.

Research for the original version of this book was supported by the Air Force Office of Scientific Research and by the British Science Research Council. Research incorporated in the third edition was partially supported by the Alfred P. Sloan Foundation and by the National Aeronautics and Space Administration. Permission to reprint selected tables from *Biometrika Tables for Statisticians*, Vol. 1, edited by E. S. Pearson and H. O. Hartley is also acknowledged. On behalf of my co-authors, I would like to thank George Tiao, David Mayne, David Pierce, Granville Tunnicliffe Wilson, Donald Watts, John Hampton, Elaine Hodkinson, Patricia Blant, Dean Wichern, David Bacon, Paul Newbold, Hiro Kanemasu, Larry Haugh, John MacGregor, Bovas Abraham, Johannes Ledolter, Gina Chen, Raja Velu, Sung Ahn, Michael Wincek, Carole Leigh, Mary Esser, Sandy Reinsel, and

Meg Jenkins, for their help, in many different ways, in preparing the earlier editions. A very special thanks is extended to Claire Box for her long-time help and support.

The guidance and editorial support of Jon Gurstelle and Sari Friedman at Wiley is gratefully acknowledged. We also thank Stephen Quigley for his help in setting up the project, and Katrina Maceda and Shikha Pahuja for their help with the production.

Finally, I want to express my gratitude to my husband Bert Beander for his encouragement and support during the preparation of this revision.

GRETA M. LJUNG

*Lexington, MA
May 2015*

PREFACE TO THE FOURTH EDITION

It may be of interest to briefly recount how this book came to be written. Gwilym Jenkins and I first became friends in the late 1950s. We were intrigued by an idea that a chemical reactor could be designed that optimized itself automatically and could follow a moving maximum. We both believed that many advances in statistical theory came about as a result of interaction with researchers who were working on real scientific problems. Helping to design and build such a reactor would present an opportunity to further demonstrate this concept.

When Gwilym Jenkins came to visit Madison for a year, we discussed the idea with the famous chemical engineer Olaf Hougen, then in his eighties. He was enthusiastic and suggested that we form a small team in a joint project to build such a system. The National Science Foundation later supported this project. It took 3 years, but suffice it to say, that after many experiments, several setbacks, and some successes the reactor was built and it worked.

As expected, this investigation taught us a lot. In particular, we acquired proficiency in the manipulation of difference equations that were needed to characterize the dynamics of the system. It also gave us a better understanding of nonstationary time series required for realistic modeling of system noise. This was a happy time. We were doing what we most enjoyed doing: interacting with experimenters in the evolution of ideas and the solution of real problems, with real apparatus and real data.

Later there was fallout in other contexts, for example, advances in time series analysis, in forecasting for business and economics, and also developments in statistical process control (SPC) using some notions learned from the engineers.

Originally Gwilym came for a year. After that I spent each summer with him in England at his home in Lancaster. For the rest of the year, we corresponded using small reel-to-reel tape recorders. We wrote a number of technical reports and published some papers but eventually realized we needed a book. The first two editions of this book were written during a period in which Gwilym was, with extraordinary courage, fighting a debilitating illness to which he succumbed sometime after the book had been completed.

Later Gregory Reinsel, who had profound knowledge of the subject, helped to complete the third edition. Also in this fourth edition, produced after his untimely death, the new material is almost entirely his. In addition to a complete revision and updating, this fourth edition resulted in two new chapters: Chapter 10 on nonlinear and long memory models and Chapter 12 on multivariate time series.

This book should be regarded as a tribute to Gwilym and Gregory.

I was especially blessed to work with two such gifted colleagues.

GEORGE E. P. BOX

*Madison, Wisconsin
March 2008*

PREFACE TO THE THIRD EDITION

This book is concerned with the building of stochastic (statistical) models for time series and their use in important areas of application. This includes the topics of forecasting, model specification, estimation, and checking, transfer function modeling of dynamic relationships, modeling the effects of intervention events, and process control. Coincident with the first publication of *Time Series Analysis: Forecasting and Control*, there was a great upsurge in research in these topics. Thus, while the fundamental principles of the kind of time series analysis presented in that edition have remained the same, there has been a great influx of new ideas, modifications, and improvements provided by many authors.

The earlier editions of this book were written during a period in which Gwilym Jenkins was, with extraordinary courage, fighting a slowly debilitating illness. In the present revision, dedicated to his memory, we have preserved the general structure of the original book while revising, modifying, and omitting text where appropriate. In particular, Chapter 7 on estimation of ARMA models has been considerably modified. In addition, we have introduced entirely new sections on some important topics that have evolved since the first edition. These include presentations on various more recently developed methods for model specification, such as canonical correlation analysis and the use of model selection criteria, results on testing for unit root nonstationarity in ARIMA processes, the state-space representation of ARMA models and its use for likelihood estimation and forecasting, score tests for model checking, structural components, and deterministic components in time series models and their estimation based on regression-time series model methods. A new chapter (12) has been developed on the important topic of *intervention and outlier* analysis, reflecting the substantial interest and research in this topic since the earlier editions.

Over the last few years, the new emphasis on industrial quality improvement has strongly focused attention on the role of control both in process *monitoring* and in process *adjustment*. The control section of this book has, therefore, been completely rewritten to serve as an introduction to these important topics and to provide a better understanding of their relationship.

The objective of this book is to provide practical techniques that will be available to most of the wide audience who could benefit from their use. While we have tried to remove the inadequacies of earlier editions, we have not attempted to produce here a rigorous mathematical treatment of the subject.

We wish to acknowledge our indebtedness to Meg (Margaret) Jenkins and to our wives, Claire and Sandy, for their continuing support and assistance throughout the long period of preparation of this revision.

Research on which the original book was based was supported by the Air Force Office of Scientific Research and by the British Science Research Council. Research incorporated in the third edition was partially supported by the Alfred P. Sloan Foundation and by the National Aeronautics and Space Administration. We are grateful to Professor E. S. Pearson and the Biometrika Trustees for permission to reprint condensed and adapted forms of Tables 1, 8, and 12 of *Biometrika Tables for Statisticians*, Vol. 1, edited by E. S. Pearson and H. O. Hartley, to Dr. Casimer Stralkowski for permission to reproduce and adapt three figures from his doctoral thesis, and to George Tiao, David Mayne, Emanuel Parzen, David Pierce, Granville Wilson, Donald Watts, John Hampton, Elaine Hodkinson, Patricia Blant, Dean Wichern, David Bacon, Paul Newbold, Hiro Kanemasu, Larry Haugh, John MacGregor, Bovas Abraham, Gina Chen, Johannes Ledolter, Greta Ljung, Carole Leigh, Mary Esser, and Meg Jenkins for their help, in many different ways, in preparing the earlier editions.

GEORGE BOX AND GREGORY REINSEL

1

INTRODUCTION

A *time series* is a sequence of observations taken sequentially in time. Many sets of data appear as time series: a monthly sequence of the quantity of goods shipped from a factory, a weekly series of the number of road accidents, daily rainfall amounts, hourly observations made on the yield of a chemical process, and so on. Examples of time series abound in such fields as economics, business, engineering, the natural sciences (especially geophysics and meteorology), and the social sciences. Examples of data of the kind that we will be concerned with are displayed as time series plots in Figures 2.1 and 4.1. An intrinsic feature of a time series is that, typically, adjacent observations are *dependent*. The nature of this dependence among observations of a time series is of considerable practical interest. *Time series analysis* is concerned with techniques for the analysis of this dependence. This requires the development of stochastic and dynamic models for time series data and the use of such models in important areas of application.

In the subsequent chapters of this book, we present methods for building, identifying, fitting, and checking models for time series and dynamic systems. The methods discussed are appropriate for discrete (sampled-data) systems, where observation of the system occurs at equally spaced intervals of time.

We illustrate the use of these time series and dynamic models in five important areas of application:

1. The *forecasting* of future values of a time series from current and past values.
2. The determination of the *transfer function* of a system subject to inertia—the determination of a dynamic input–output model that can show the effect on the output of a system of any given series of inputs.
3. The use of indicator input variables in transfer function models to represent and assess the effects of unusual *intervention* events on the behavior of a time series.

4. The examination of interrelationships among several related time series variables of interest and determination of appropriate *multivariate* dynamic models to represent these joint relationships among the variables over time.
5. The design of simple *control schemes* by means of which potential deviations of the system output from a desired target may, so far as possible, be compensated by adjustment of the input series values.

1.1 FIVE IMPORTANT PRACTICAL PROBLEMS

1.1.1 Forecasting Time Series

The use at time t of available observations from a time series to forecast its value at some future time $t + l$ can provide a basis for (1) economic and business planning, (2) production planning, (3) inventory and production control, and (4) control and optimization of industrial processes. As originally described by Holt et al. (1963), Brown (1962), and the Imperial Chemical Industries (ICI) monograph on short term forecasting (Coutie, 1964), forecasts are usually needed over a period known as the *lead time*, which varies with each problem. For example, the lead time in the inventory control problem was defined by Harrison (1965) as a period that begins when an order to replenish stock is placed with the factory and lasts until the order is delivered into stock.

We will assume that observations are available at *discrete*, equispaced intervals of time. For example, in a sales forecasting problem, the sales z_t in the current month t and the sales $z_{t-1}, z_{t-2}, z_{t-3}, \dots$ in previous months might be used to forecast sales for lead times $l = 1, 2, 3, \dots, 12$ months ahead. Denote by $\hat{z}_t(l)$ the forecast made at *origin* t of the sales z_{t+l} at some future time $t + l$, that is, at *lead time* l . The function $\hat{z}_t(l)$, which provides the forecasts at origin t for all future lead times, based on the available information from the current and previous values $z_t, z_{t-1}, z_{t-2}, z_{t-3}, \dots$ through time t , will be called the *forecast function* at origin t . Our objective is to obtain a forecast function such that the mean square of the deviations $z_{t+l} - \hat{z}_t(l)$ between the actual and forecasted values is as small as possible for each lead time l .

In addition to calculating the best forecasts, it is also necessary to specify their accuracy, so that, for example, the risks associated with decisions based upon the forecasts may be calculated. The accuracy of the forecasts may be expressed by calculating *probability limits* on either side of each forecast. These limits may be calculated for any convenient set of probabilities, for example, 50 and 95%. They are such that the realized value of the time series, when it eventually occurs, will be included within these limits with the stated probability. To illustrate, Figure 1.1 shows the last 20 values of a time series culminating at time t . Also shown are forecasts made from origin t for lead times $l = 1, 2, \dots, 13$, together with the 50% probability limits.

Methods for obtaining forecasts and estimating probability limits are discussed in detail in Chapter 5. These forecasting methods are developed based on the assumption that the time series z_t follows a *stochastic* model of known form. Consequently, in Chapters 3 and 4 a useful class of such time series models that might be appropriate to represent the behavior of a series z_t , called autoregressive integrated moving average (ARIMA) models, are introduced and many of their properties are studied. Subsequently, in Chapters 6, 7, and 8 the practical matter of how these models may be developed for actual time series data is explored, and the methods are described through the three-stage procedure of tentative