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FAST-TRACK PAPER

Applicability of time-to-failure analysis to accelerated strain before earthquakes and volcanic eruptions

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SUMMARY

We examine quantitatively the ranges of applicability of the equation $\Omega = A + B[1 - t/t_f]^m$ for predicting 'system-sized' failure times t_f in the Earth. In applications Ω is a proxy measure for strain or crack length, and A, B and the index m are model parameters determined by curve fitting. We consider constitutive rules derived from (a) Charles' law for subcritical crack growth; (b) Voight's equation; and (c) a simple percolation model, and show in each case that this equation holds only when $m < 0$. When $m > 0$, the general solution takes the form $\Omega = A + B[1 + t/T]^m$, where T is a positive time constant, and no failure time can be defined. Reported values for volcanic precursors based on rate data are found to be within the range of applicability of time-to-failure analysis ($m < 0$). The same applies to seismic moment release before earthquakes, at the expense of poor retrospective predictability of the time of the *a posteriori*-defined main shock. In contrast, reported values based on increasing cumulative Benioff strain occur in the region where a system-sized failure time cannot be defined ($m > 0$; commonly $m \approx 0.3$). We conclude on physical grounds that cumulative seismic moment is preferred as the most direct measure of seismic strain. If cumulative Benioff strain is to be retained on empirical grounds, then it is important that these data either be re-examined with the independent constraint $m < 0$, or that for the case $0 < m + 1 < 1$, a specific correction for the time-integration of cumulative data be applied, of the form $\Sigma\Omega = At + B'\{1 - [1 - t/t_f]^{m+1}\}$.

Key words: earthquake prediction, percolation, rock fracture, subcritical crack growth, volcanic activity.

INTRODUCTION

The rate dependence for material failure can often be described by the general non-linear equation

$$d^2\Omega/dt^2 - a(d\Omega/dt)^\alpha = 0\,, \tag{1}$$

where Ω is related to precursory strain, a is a constant and α is an exponent that measures the degree of non-linearity (Voight 1988, 1989). This equation has been applied in retrospective mode both to predict the time of volcanic eruptions (e.g. Voight 1988; Kilburn & Voight 1998) and to the prediction of earthquake failure times, the latter usually based on the observation of accelerating Benioff strain (e.g. Bufe & Varnes 1993; Bowman *et al.* 1998).

Eq. (1) has been justified in part by comparison with equations for quasi-static, subcritical crack growth (e.g. Varnes 1989; Kilburn & Voight 1998). In this case the fundamental non-linear equation takes the form of Charles' law:

$$\frac{dx}{dt} = V_0\left(\frac{K}{K_0}\right)^p, \tag{2}$$

where x is the crack length at time t, V_0 is the starting velocity at time $t_0 = 0$, K is the stress intensity, a measure of the degree of concentration of stress at the crack tip, K_0 is the stress intensity at time t_0 and the exponent p is known as the stress corrosion index.

In this paper we examine explicitly the relationship between the non-linear exponents p and α, and suggest a more general solution to eq. (1) than is currently applied, and where critically the form of the equation depends on the value of these exponents. We then develop a simple percolation model for time-dependent failure, which has the same form, with a critical exponent v that can similarly be related to p, and is restricted to the range $v > 0$, where a finite failure time can be derived. The range of applicability for time-to-failure analysis in the three different theories is found to be the same. We then compare the values of the different exponents reported from field observations of volcanoes and earthquakes. First we summarize the background theory and derive explicit relationships between the relevant power-law exponents.

BACKGROUND THEORY

In this section we examine three independent hypotheses for predicting failure times in the Earth: (a) Charles' law for subcritical crack growth; (b) Voight's (1988) equation; and (c) a simple percolation model.

(a) Subcritical crack growth

The stress intensity K takes the general form

$$K = Y\sigma x^{1/2}, \tag{3}$$

where Y is a dimensionless constant that depends on the loading configuration and the mode of failure, σ is the stress applied at a remote boundary, and x is the crack length (e.g. Atkinson 1987). From (2) and (3), assuming a constant remotely applied stress, we find after integration

$$x = \frac{x_0}{\left[1 - \left(\frac{p-2}{2}\right)\frac{V_0 t}{x_0}\right]^{(2/p-2)}}, \quad p \neq 2, \tag{4}$$

or

$$x = x_0[1 + t/(m\tau)]^m, \quad p \neq 2, \tag{5}$$

where

$$\left.\begin{aligned} m &= 2/(2-p) \\ \tau &= x_0/V_0 \end{aligned}\right\}, \quad p \neq 2. \tag{6}$$

The time constant τ measures the ratio of the initial crack length to the starting velocity; both are positive, so $\tau > 0$. For the case $p > 2$, or $m < 0$, this reduces to an equation of the form

$$x = x_0[1 - t/t_f]^m, \quad p > 2, \quad m < 0. \tag{7a}$$

where t_f is the failure time, defined for the singular point $x(t) \to \infty$, or

$$t_f = -m\tau > 0, \quad m < 0 \tag{7b}$$

(e.g. Das & Scholz 1981). For a time-varying stress, a similar equation holds to a good approximation if the earthquake stress drop is small compared to the ambient tectonic stress (Main 1988). The failure time defined in (7) applies to events that are 'system-sized', that is, that are effectively infinite, representing an instability in the crack growth. However, the term 'failure' here implies only the final stages of accelerating crack growth, when dynamic effects not considered in the model dominate. As a consequence, the quasi-static singularity in (7a) represents an upper bound to the dynamic failure time at the inertial limit.

For the case $m > 0$, we find instead an equation of the form

$$\left.\begin{aligned} x &= x_0[1 + t/T]^m \\ T &= m\tau > 0 \end{aligned}\right\}, \quad p < 2, \quad m > 0. \tag{8}$$

Note that the sign of the exponent m determines the form of the equation, in particular whether or not a negative sign appears within the square brackets in the equations for crack growth (7 and 8). A finite failure time can be defined only for $m < 0$. In the case $m > 0$, $x \to \infty$ when $t_f \to \infty$ so the true failure time is infinite. If we consider the limit of long times, $t/T \gg 1$, then (8) reduces to a simple power law with no singularity: $x \sim t^m$.

To solve for the case $p = 2$, we put this condition directly into eq. (2), the resulting differential equation being linear, with the solution

$$x = x_0\, e^{t/\tau}, \quad p = 2. \tag{9}$$

In this case $x \to \infty$ only when $t = t_f \to \infty$, so again the failure time is infinite, even though there is a characteristic time τ associated with the process. Curves for $x(t)$ for the cases $m = -0.5$, 0, 0.5 and 2 are shown in Fig. 1, plotted against normalized time $t' = t/t_f (m < 0)$, or $t' = t/\tau (m > 0)$, as appropriate.

The velocity of crack growth can be calculated by differentiating the expressions for crack length in (7) and (8), resulting in the form

$$\frac{dx}{dt} = \begin{cases} V_0/[1 - t/t_f]^n, & n > 1 \\ V_0/[1 + t/T]^n, & n < 1 \end{cases} \tag{10}$$

where

$$n = 1 - m = p/(p-2). \tag{11}$$

Eq. (11) corrects an algebraic error in the derivation of Varnes (1989). For $n > 1$ or $p > 2$, a failure time identical to that given in eq. (7) can be calculated from $dx/dt \to \infty$, with the same range of applicability. Again t_f is undefined for $m > 0$ ($n < 1, p < 2$).

If we differentiate (10) again, two different regimes can be defined for the case of infinite failure time: $m > 0, p < 2$. Accelerating crack growth ($d^2x/dt^2 > 0$) occurs for $1 < m < \infty$ ($0 < p < 2$) and decelerating crack growth for $0 < m < 1$ ($-\infty < p < 0$). In both cases crack growth is stable. Decelerating crack growth is unlikely for the tensile failure in the double torsion loading geometry described by Atkinson (1987), but may occur for the case of a dilute population of cracks in the early stages of damage, e.g. Costin (1987; Fig. 5.10b).

(b) Voight's equation

Eq. (2) is posed in terms of a velocity or first derivative. Voight (1988) suggested the more general form (1) from the acceleration or second derivative. Motivated by the above, we consider solutions of the general form

$$\Omega = A + B[1 + t/(m\tau)]^m, \quad m \neq 0, \tag{12}$$

where the constant A emerges due to the second derivative, and the starting value for the parameter Ω is $\Omega_0 = \Omega(0) = A + B$. After differentiation this reduces to a similar form to (10):

$$\left.\begin{aligned} d\Omega/dt &= \dot{\Omega}_0[1 + t/(m\tau)]^{m-1} \\ \dot{\Omega}_0 &= B/\tau \end{aligned}\right\}, \quad m \neq 0, \tag{13}$$

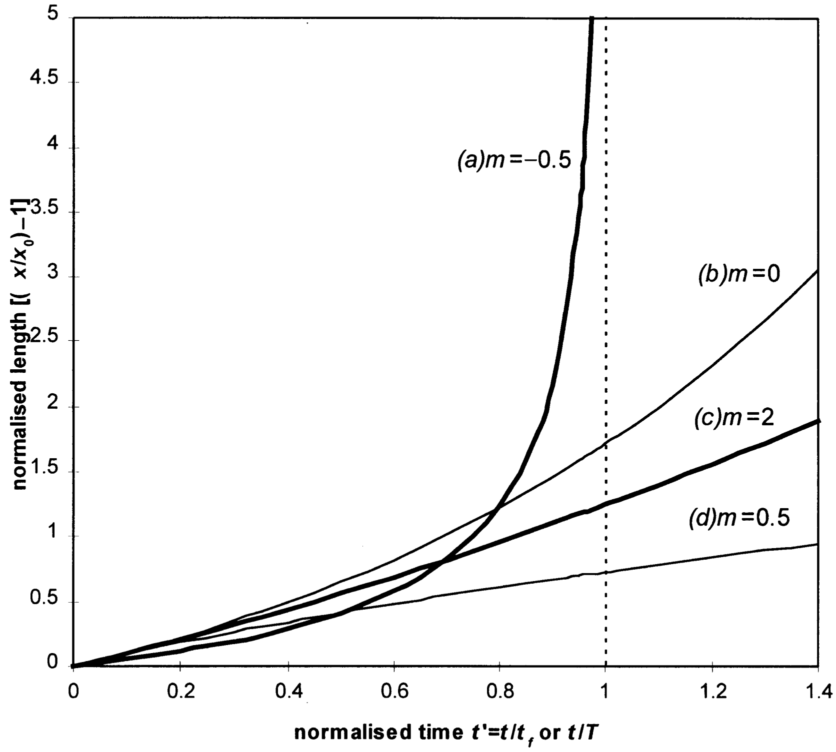

Figure 1. Plot of the normalized crack length $[(x/x_0)-1]$ as a function of the normalized time t' for the cases (*a*) $m=-0.5$, $t_f=1$, $t'=t/t_f$; (*b*) $m=0$, $\tau=1$, $t'=t/\tau$; (c) $m=2$, $\tau=1$, $t'=t/\tau$; (d) $m=0.5$, $\tau=1$, $t'=t/\tau$. A finite failure time (indicated by the dashed vertical line) is only defined for $m<0$.

where $\dot{\Omega}_0>0$ is the initial strain rate, whence $B>0$. If we differentiate (13), we obtain the form of eq. (1) with

$$\left.\begin{aligned} a &= \left(\frac{m-1}{m\tau}\right)\left(\frac{B}{\tau}\right)^{1-\alpha} \\ \alpha &= \frac{m-2}{m-1} \end{aligned}\right\}, \quad m\neq 0, \tag{14}$$

The acceleration at time $t=0$ is

$$\ddot{\Omega}_0=\left(\frac{m-1}{m}\right)\frac{B}{\tau^2}, \quad m\neq 0. \tag{15}$$

Accelerating strain $\ddot{\Omega}_0>0$ occurs when either $m<0$ or $m>1$; decelerating strain occurs when $0<m<1$. These ranges are identical to those derived above for accelerating or decelerating subcritical crack growth. A finite failure time is predicted equivalently when Ω_0, $\dot{\Omega}_0$, $\ddot{\Omega}_0\rightarrow\infty$, of the form

$$\left.\begin{aligned} \Omega &= A+B[1-t/t_f]^m \\ t_f &= -m\tau \end{aligned}\right\}, \quad m<0. \tag{16}$$

Note again that this equation is a special form of the more general form (13). Critically, the presence of a negative or positive sign in the term in square brackets depends on the value of *m*.

(c) Percolation theory

In this section we show that the approach to the percolation threshold or critical point can also be described by a temporal power-law increase of similar form to (7) or (16). First, we note that the correlation length (which is related to the size of the largest connected cluster of broken elements) increases non-linearly as the probability p of local failure approaches the percolation threshold p_c,

$$\xi\propto|p-p_c|^{-\nu}, \quad \nu>0 \tag{17}$$

(e.g. Stauffer & Aharony 1994). At the percolation threshold, the largest connected cluster of broken elements spans the two sides of the model space, producing a 'system-sized' event: $\xi\rightarrow\infty$. The percolation model is stochastic, and ignores any deterministic interactions that may occur before or after an individual element breaks. The simple site percolation problem produces $\nu=4/3$ in two dimensions, and $\nu=0.9$ in three (Stauffer & Aharony 1994, p. 60). When deterministic local interactions are taken into account, the condition $\nu>0$ still holds for a variety of critical point systems (e.g. Stanley 1971, Table 3.4). That is, the stochastic percolation model produces the same qualitative behaviour as the deterministic–stochastic statistical mechanics models for behaviour near the critical point, although the parameters may be different.

We now develop a simple 2-D model for the evolution of the correlation length as a function of time as the percolation threshold is approached. The percolation model assumes that each element is broken randomly, irrespective of its neighbours. We reproduce this by assuming a random distribution of local breaking strains ε_F and neglecting elastic interactions. We subject the model to a constant externally applied strain rate $d\varepsilon/dt=\dot{\varepsilon}_0$ and consider all elements with $\varepsilon^{ij}\geq\varepsilon_F^{ij}$ to have broken (stippled area in Fig. 2). The number of new elements that break in a unit time interval Δt is then $\Delta n=Np'(\varepsilon_F)\Delta\varepsilon_F=Np'(t)\Delta t$, where N is the total number of model elements and

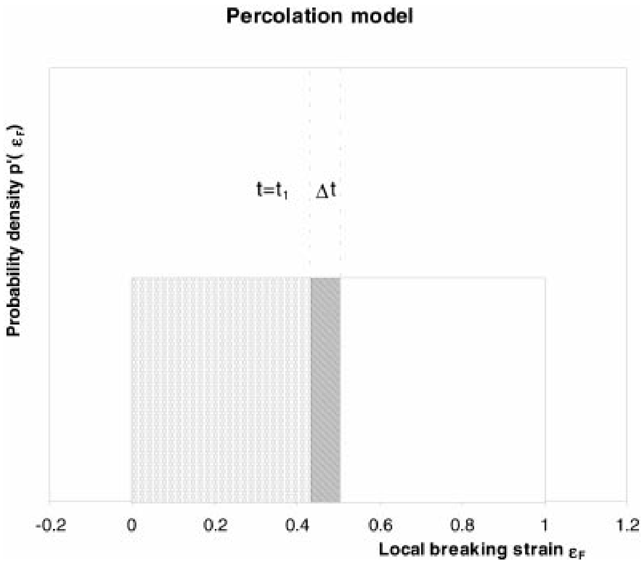

Figure 2. A simple percolation model for failure. The probability of failure of an element is random in space, implying a random probability distribution of breaking strains $p'(\varepsilon_F) = \text{const.}$ The stippled area under the curve corresponds to the total integrated probability $p(t_1)$, the shaded area to the number that break in the time increment Δt, and the white area to the remaining intact elements.

p' here is a probability density (shaded area in Fig. 2). The probability density (rate) of failure is then

$$p'(t) = p'(\varepsilon_F)\Delta\varepsilon_F/\Delta t = p'(\varepsilon_F)\dot{\varepsilon}_0 \,. \tag{18}$$

For a random distribution of breaking strengths $p'(\varepsilon_F) = \text{const.} = c$ and for a linear loading rate $\dot{\varepsilon} = \text{const.}$ From the normalization criterion $\int_0^{\varepsilon_F^{\max}} p'dp = 1$, we have $c = 1/\varepsilon_F^{\max}$. For the normalized units shown in Fig. 2, $c = 1$. The cumulative probability of failure of an element at time t, in normalized units, is then

$$p = \int_0^t p'(t)dt = \dot{\varepsilon}_0 t \,. \tag{19}$$

Since p is linear in time, eq. (17) reduces to

$$\xi(t) \propto |t - t_c|^{-\nu} \,, \quad \nu > 0 \,. \tag{20}$$

Other distributions of local breaking strain or loading rate, as long as $\dot{\varepsilon}_0 > 0$, will change the magnitude but not the sign of ν. Thus, the size of the largest crack $x \approx \xi$ (which can now have a rough rather than a smooth periphery) increases with the same form as (7), where $m = -\nu$, and with the same range of applicability ($m < 0$, $\nu > 0$).

COMPARISON WITH DATA

Applications of eq. (1) to earthquake or volcanic data so far have used only the form

$$\Omega = A + B'[t_f - t]_m \,, \quad m \neq 0 \,, \tag{21}$$

where $B' = B/t_f^m$ (e.g. Bufe & Varnes 1993; Bowman *et al.* 1998), and without explicitly considering the appropriate ranges of applicability of the more general solution to (1) proposed here (eq. 13). We now examine the values reported in the literature for proxy measurements of strain before earthquakes and volcanic eruptions.

Proxy measures of strain

The choice of parameters is based to some extent on the availability of data, since in general we do not have good direct measures of strain as would be the case in a laboratory experiment. In fact, direct geodetic measurements of strain are often available only for specific transects, and generally show no detectable precursory strain increase (e.g. Argus & Lyzenga 1994). In future, more detailed measures of strain will become available through satellite interferometry, but for the present, the main measures of strain used in time-to-failure analysis are indirect. The main proxy measures for strain are inferred from the radiated seismic energy, the seismic moment or the seismic event rate.

The Benioff 'strain' is based on essentially dimensional arguments, assuming a locally elastic earthquake mechanism that neglects the contribution of energy lost to permanent deformation or heat. In this case, the radiated seismic energy scales with strain squared, $E = \mu\varepsilon^2/2$, where μ is the shear modulus of the brittle crust. The Benioff 'strain' is then defined to be $\varepsilon \propto E^{0.5}$. However, this strain is not directly related to a measurable tectonic strain, because it measures only the energy radiated in elastic waves, normally only a small and variable proportion of the total energy released during seismic slip. The physical meaning of a *cumulative* Benioff 'strain' is then also not clear.

In a more complete derivation, Kostrov (1974) showed that the strain tensor due to seismic deformation takes the form $\varepsilon_{ij} = \Sigma_{k=1}^{N} M_{ij}^k/(2\mu V)$, where V is the volume of seismic deformation and M_{ij}^k is the seismic moment tensor of the kth individual event. Here the strain tensor is a measurable variable, defining the deformation that has occurred between the boundaries of the volume V, for example the sample ends in a laboratory test or the rigid interiors of two plates. It is therefore a measure of regional rather than local strain. Since it is directly related to an observable strain, this moment-based definition of seismic strain release is in common use, for example in seismotectonics and seismic hazard analysis, where often there is a good match between the seismic and the independently measured tectonic strain tensor (e.g. Jackson & McKenzie 1988; Amelung & King 1997; Ward 1998).

Although not a direct measure of strain, the seismic event rate has been found to have the same form as (2), with virtually identical values for the relevant exponents (Meredith & Atkinson 1983). The event rate is then proportional to the crack velocity or strain rate.

Volcanic data

When we examine the literature for volcanic precursors, we find that when (1) is applied to a range of pre-eruptive processes, α has been found to lie between 1 and 2 (typically closer to 2), irrespective of the process that Ω describes (Voight 1988; Cornelius & Voight 1995; Kilburn & Voight 1998). This implies that the failure times for these cases do have a physical meaning, and that eq. (16) can be applied to the data without considering the possibility of a positive sign in the brackets in eq. (13) for the case $m > 0$. In a recent example, this equation gives a good retrospective prediction of the eruption time of the Soufriere Hills volcano, Montserrat, by plotting the inverse seismic event rate $(dN/dt)^{-1}$ as a function of time. This method explicitly uses the rate criterion $\dot{\Omega} \to \infty$, where $\Omega \equiv N$.

Earthquake data

We have shown, for three different models, that when $m \geq 0$ there is no singularity in Ω, and hence no well-defined failure time. However, a significant number of reported values for m for earthquake data fall within this range. Bufe & Varnes (1993) tabulated 18 determinations of the exponent m for the case of accelerating cumulative seismic moment release M, cumulative Benioff strain $E^{0.5}$ or the cumulative numbers of seismic events N. In some t_f was fixed, since the time of the main shock was known, and in others t_f was allowed to be a free parameter in the curve fit, although all were analysed in retrospective mode. For the Benioff strain, 11 out of 15 reported determinations of m are positive, and hence fall outside the range of applicability of time-to-failure analysis, implying that the quoted 'failure times' do not have their usual physical significance. The problem can be traced to allowing t_f and m to be independent variables in the line fit, when we can see from the more general solution (13) above that they are not.

For the case of cumulative seismic moment release, allowing t_f to be a free parameter, m is always negative, and hence the applied theory is valid. However, despite the better theoretical link to a measurable tectonic strain, and the physical validity of the exponents found, this case leads to a poorer agreement with the time of the *a posteriori*-defined main shock (Bufe & Varnes 1993). This occurs because the acceleration of seismic moment is much more non-linear (as would be expected for negative values of m) and hence has less predictive accuracy. We conclude that strain based on the cumulative seismic moment is preferable for assessing the physical basis and the magnitude of the relevant critical exponents for time-to-failure analysis, because (a) the results are both directly comparable to a measured regional tectonic strain, and (b) the reported exponents found are within the correct range.

If the Benioff strain is retained as a proxy measure (albeit untestable against the independently measured regional strain at present) then the earthquake data for Benioff strain should be re-examined with the more general solution, that is, allowing positive or negative values of the exponent m to determine the form of the equation. A simple possible alternative is provided below for the Benioff strain.

DISCUSSION

An important distinction between earthquake and volcanic data is that the former is usually analysed in terms of rate (time-differentiated) data, whereas the latter is usually analysed in terms of cumulative (time-integrated) data, usually in the form of a *cumulative* Benioff strain. Integrating the general solution (13) we obtain

$$\sum \Omega = \int_0^t \Omega dt$$

$$= At + \left(\frac{m\tau B}{m+1}\right)\{[1 + t/(m\tau)]^{m+1} - 1\}\,, \quad m \neq 0\,. \tag{22}$$

When $m < 0$, this reduces to the form:

$$\sum \Omega = At + B'[1 - (1 - t/t_f)^{m+1}]\,, \quad m < 0\,, \tag{23}$$

where here $B' = Bt_f/(m+1)$. At $t = 0$, $\Sigma\Omega(0) = 0$. For $m < 0$, the singularity that was present in the Benioff strain disappears in the integrated Benioff strain, because the area under the curve remains finite. In this case, at $t = t_f$, we find $\Sigma\Omega(t_f) = At_f + B'$. This defines a long-term seismic strain rate

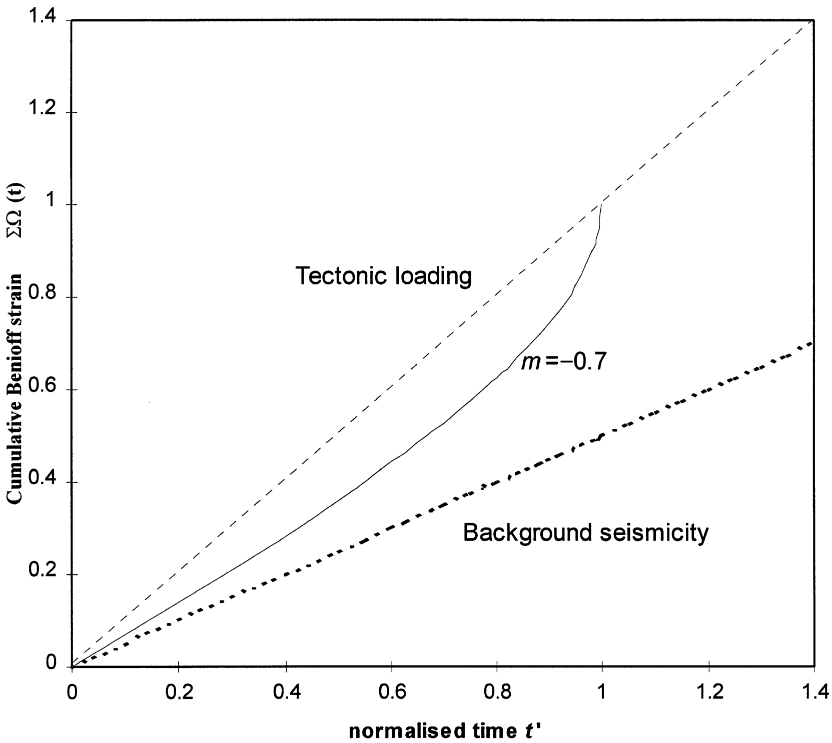

Figure 3. Plot of the cumulative Benioff strain from eq. (23), shown as a solid line, for the case $m + 1 = 0.3$. The background component and the total long-term seismic strain release rate are shown as straight dashed lines.

$\Sigma\Omega(t_f)/t_f = A + B'/t_f = \eta\dot{\varepsilon}_T$, where η is the ratio of seismic to tectonic strain. The presence of a 'background' rate A allows the possibility of a stationary random component to be included in the model. This is important because even completely random data have a positive regression coefficient when analysed on a cumulative frequency basis. The parameter A here serves to measure the magnitude of this component. More complex models are required if the background rate is non-stationary (e.g. Gross & Rundle 1998).

Fig. 3 shows an example of eq. (23) for $\eta = 1$; $A = B' = 0.5$ and $m = -0.7$ ($m + 1 = 0.3$). The significance of this choice is that the non-linear exponent is often found to be 0.3 in such cumulative data. The presence of a linear background rate in the equation means that we do not have to treat a linear or a non-linear fit to the data as separate possibilities, for example as in Bowman *et al.* (1998); we just allow the relative significance of both possibilities to be determined by the data. We also note from this plot that the failure time is defined when the curve recovers the long-term seismic strain deficit, more in common with the first-order slip-predictable model of Shimazaki & Nakata (1980) than their time-predictable version.

CONCLUSIONS

Time-to-failure analysis has specific ranges of applicability ($m < 0$) that are identical for three different theoretical models: subcritical crack growth, Voight's equation, and a simple percolation model. These are in good agreement with published critical point exponents for a variety of physical systems ($v = -m > 0$). When applied to published volcanic data, we also find $m < 0$ in all cases. For earthquake data the cumulative seismic moment also has this necessary physical property, at the expense of poor predictability of the failure time. Seismic moment is also directly comparable to independent measures of strain in common use elsewhere in seismotectonics (Kostrov 1974). On both counts, seismic moment should be preferred as a measure of strain in time-to-failure analysis.

When $m > 0$, it is not sufficient to use the same equation with different parameters; rather, the equation takes a different form. As a consequence, earthquake data for Benioff strain with $m > 0$, quite common in the literature, should be re-examined. A simple correction involving adding a background strain rate will give a solution that preserves both the commonly used form of the equation and the measured values of the exponents. It also allows naturally for a random background component. The question of how Benioff strain relates to an observable strain has yet to be resolved.

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