2025 Energy Production Optimization

Unit Commitment Problem – Final Report

Xander Pauwels, Xuanbo Zhao, Noam Diop, Gabriela Rueda

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Abstract

This paper tackles the Unit Commitment Problem (UCP), which aims to minimize energy production costs while meeting hourly demand. We compare two integer linear programming formulations that handle minimum uptime and downtime: one using only status variables Takriti et al. (2000) and another using additional start-up/shut-down variables Rajan and Takriti (2005). We then expand the model to include non-controllable renewable sources. Our findings show that the first formulation yields lower computational times, making it the most efficient alternative.

1 Introduction

The 2025 Energy Production Optimization case addresses the Unit Commitment (UC) problem for a fleet of power generators. The objective is to minimize total production cost while satisfying hourly demand and respecting generator constraints. It is important to take into account that each generator is characterized by different technical features, which include: minimum and maximum output levels, up time and downtime requirements, start-up, no-load, and variable production costs.

We begin by formulating the UC as an Integer Linear Program (ILP) using only binary on/off variables, following Takriti et al. (2000). We then present an alternative formulation by Rajan and Takriti (2005), which introduces start-up and shutdown variables that better capture transitions in generator status. Beyond these formulations, we extend the model to include renewable energy, analyzing the system's performance under varying levels of renewable energy penetration and curtailment penalties.

From a societal perspective, the relevance of this paper lies in the shift to low-carbon energy, reducing emissions and lowering costs. As the integration of renewable energy increases, planning tools are essential for resilient energy systems. While from the academic point of view, the UC problem remains a key topic in operations research. Our comparison of two established formulations provides insight into their relative performances. Furthermore, by extending the framework to include renewable integration and uncertainty, we contribute to the literature on stochastic and mixed-integer optimization methods.

Our main contributions are threefold. First, we start by comparing two established ILP formulations of the UC problem and demonstrate that the first formulation, which uses only binary on/off variables, achieves optimal solutions with lower computational time. Second, we extend these models to include renewable energy and investigate the effects of different curtailment policies. Finally, in the face of renewable energy uncertainty, we develop a two-stage stochastic unit commitment model. Stage 1 fixes non-anticipative decisions (e.g., unit commitment and startup) without future renewable data, while Stage 2 adjusts generation and curtailment once the scenario is revealed. Although scenario probabilities can be region-specific, we simulate uncertainty using random probabilities, improving decision-making over deterministic methods.

The remainder of the paper is structured as follows. Section 2 provides description of the Unit Commitment problem. In Section 3, we detail the methodological approach. Section 4 reports the results of our numerical experiments, comparing computational performance and solution quality. Finally, Section 5 summarizes the main findings, discusses their implications, and outlines directions for future research.

2 Problem description

The problem involves minimizing the energy production cost under the following requirements:

The Unit Commitment problem involves several key constraints and assumptions. Firstly, the hourly demand must be met, meaning that the total output of all generators at each time period should be sufficient to meet the demand. Additionally, each generator's output must stay within its minimum and maximum operational limits. Furthermore, generators that are turned on must remain on for a specified number of time periods, while those that are turned off must remain off for a minimum number of time periods.

In terms of assumptions, the problem is simplified by the following factors. The technical requirements, such as the capacity limits and operational constraints, are considered to be constant over time. Moreover, the future demand is known at all periods, meaning that decisions can be made based on the full knowledge of future requirements. The model assumes that, without loss of generality, $L, l \leq T - 1$, where T represents the total number of time periods. Finally, the relationship between the variable cost and production level is assumed to be linear, simplifying the cost structure for decision-making purposes.

3 Methodology

In this section, we will outline the methodological approach. We present both the binary on/off and the alternative start-up/shutdown formulations of the UCP. Additionally, we describe how renewable energy sources and uncertainty are integrated.

3.1 Formulations 1 and 2

We begin with the first formulation, which uses only binary on/off variables to represent the operational status of each unit. Let i index the generating units and $t = 1, 2, \ldots, T$ index the time periods.

Each unit is characterized by a set of parameters, including:

q_i and Q_i: Minimum and maximum power output of unit i,

c_i^{SU}, c_i^{NL}, c_i^{var}: Start-up, no-load, and variable production costs of unit i,

l_i, L_i: Minimum downtime and minimum uptime requirements,

d_t: Electricity demand at time t,

λ: Penalty factor for excess (curtailed) energy,

$u_{i,0} \in \{0, 1\}$: Initial on/off status of unit i,

U_1^i: Number of consecutive periods unit i has been on at $t = 0$ (if $u_{i,0} = 1$),

U_0^i: Number of consecutive periods unit i has been off at $t = 0$ (if $u_{i,0} = 0$).

The formulation introduces the following decision variables:

$u_{i,t} \in \{0, 1\}$: On/off status of unit i at time t,

$p_{i,t} \geq 0$: Power output of unit i at time t,

$s_t \geq 0$: Excess energy at time t,

$v_{i,t} \in \{0, 1\}$: Start-up indicator for unit i at time t.

This setup lays the foundation for the mathematical formulation of the UCP, which we describe next.

To model the UCP, we formulate a mixed-integer linear program. The objective is to minimize the total cost, which consists of no-load costs (incurred whenever a generator is on), variable production costs (proportional to output), start-up costs (incurred when a unit turns on), and a penalty for excess generation.

The constraints in our formulation serve several key functions. Constraint (2) ensures that each generator's production $p_{i,t}$ remains within its minimum (q_i) and maximum (Q_i) output limits when the generator is on (i.e., when $u_{i,t} = 1$); if $u_{i,t} = 0$, then the production is forced to be 0. Constraint (3) enforces the demand balance, requiring that the total generation plus any curtailed energy s_t exactly meets the demand d_t in each period. Constraints (4) and (5) impose the minimum uptime and downtime requirements, respectively; for example, if a unit starts at time t (as indicated by $u_{i,t} - u_{i,t-1} = 1$), then constraint (4) forces the unit to remain on for at least $L_i - 1$ subsequent periods, while constraint (5) ensures that once a unit is turned off, it remains off for at least $l_i - 1$ periods. Constraint (6) defines the start-up variable $v_{i,t}$ by capturing the increase in the on/off status from $t - 1$ to t. Additionally, Constraints (7) and (8) incorporate the initial conditions by forcing a unit to remain on or off for the required number of periods if its initial on (or off) duration is less than its minimum uptime (or downtime). Finally, Constraint (9) specifies the domains of the decision variables, ensuring that binary variables are restricted to $\{0, 1\}$ and that production and curtailed energy are nonnegative.

$$\min \quad \sum_{t=1}^{T} \sum_{i \in I} \left(c_i^{NL} u_{i,t} + c_i^{var} p_{i,t} + c_i^{SU} v_{i,t} \right) + \lambda \sum_{t=1}^{T} s_t \tag{1}$$

$$\text{s.t.} \quad q_i u_{i,t} \leq p_{i,t} \leq Q_i u_{i,t}, \quad \forall i \in I, \quad \forall t = 1, \ldots, T, \tag{2}$$

$$\sum_{i \in I} p_{i,t} = d_t + s_t, \quad \forall t = 1, \ldots, T, \tag{3}$$

$$u_{i,t} - u_{i,t-1} \leq u_{i,t+s}, \quad \forall i \in I, \quad \forall t = 1, \ldots, T-1, \quad \forall s = 1, \ldots, \min(L_i - 1, T - t), \tag{4}$$

$$u_{i,t-1} - u_{i,t} \leq 1 - u_{i,t+s}, \quad \forall i \in I, \quad \forall t = 1, \ldots, T-1, \quad \forall s = 1, \ldots, \min(l_i - 1, T - t), \tag{5}$$

$$v_{i,t} \geq u_{i,t} - u_{i,t-1}, \quad \forall i \in I, \quad \forall t = 1, \ldots, T, \tag{6}$$

$$u_{i,t} = 1, \quad \text{if } u_{i,0} = 1 \text{ and } U_1^i < L_i, \quad \forall t = 1, \ldots, \min(T, L_i - U_1^i), \tag{7}$$

$$u_{i,t} = 0, \quad \text{if } u_{i,0} = 0 \text{ and } U_0^i < l_i, \quad \forall t = 1, \ldots, \min(T, l_i - U_0^i), \tag{8}$$

$$u_{i,t} \in \{0, 1\}, \quad v_{i,t} \in \{0, 1\}, \quad p_{i,t} \geq 0, \quad s_t \geq 0, \quad \forall i \in I, \quad \forall t = 1, \ldots, T. \tag{9}$$

In Formulation 2, we extend the binary model by explicitly modeling start-up and shutdown decisions using two new binary variables, $v_{i,t}$ and $w_{i,t}$. The constraint (13) ensures the proper update of a generator's status over time, meaning that a unit is on at time t if it was on in the previous period and is not subsequently shut down, or if it is turned on via $v_{i,t}$. Furthermore, the constraint (14) enforces the minimum uptime requirement by preventing the unit from shutting down before it has been on for the minimum number of periods once started, while the constraint (15) ensures the minimum downtime requirement is met by restricting the unit from restarting immediately after a shutdown. Together, these constraints not only capture the logical transitions in unit status but also accurately impose the operational restrictions of minimum up- and downtime, thereby improving the model's representation of generator behavior.

$$\min \quad \sum_{t=1}^{T} \sum_{i \in I} \left(c_i^{NL} u_{i,t} + c_i^{var} p_{i,t} + c_i^{SU} v_{i,t} \right) + \lambda \sum_{t=1}^{T} s_t \tag{10}$$

$$\text{s.t.} \quad q_i u_{i,t} \leq p_{i,t} \leq Q_i u_{i,t}, \quad \forall i \in I, \forall t = 1, \ldots, T, \tag{11}$$

$$\sum_{i \in I} p_{i,t} = d_t + s_t, \quad \forall t = 1, \ldots, T, \tag{12}$$

$$u_{i,t} = u_{i,t-1} + v_{i,t} - w_{i,t}, \quad \forall i \in I, \forall t = 1, \ldots, T, \tag{13}$$

$$v_{i,t} + \sum_{\tau=t+1}^{\min(t+L_i-1,T)} w_{i,\tau} \leq 1, \quad \forall i \in I, \forall t = 1, \ldots, T, \tag{14}$$

$$w_{i,t} + \sum_{\tau=t+1}^{\min(t+l_i-1,T)} v_{i,\tau} \leq 1, \quad \forall i \in I, \forall t = 1, \ldots, T, \tag{15}$$

$$u_{i,t} = 1, \quad \text{if } u_{i,0} = 1 \text{ and } U_1^i < L_i, \quad \forall t = 1, \ldots, \min(T, L_i - U_1^i), \tag{16}$$

$$u_{i,t} = 0, \quad \text{if } u_{i,0} = 0 \text{ and } U_0^i < l_i, \quad \forall t = 1, \ldots, \min(T, l_i - U_0^i), \tag{17}$$

$$u_{i,t} \in \{0,1\}, \quad v_{i,t} \in \{0,1\}, \quad w_{i,t} \in \{0,1\}, \quad p_{i,t} \geq 0, \quad s_t \geq 0, \quad \forall i \in I, \forall t = 1, \ldots, T. \tag{18}$$

3.2 Renewable Energy Included

To extend our unit commitment model to account for renewable energy sources, we introduce a renewable production term into the demand balance constraint. In our model, the renewable production for each time period is represented as $\alpha \cdot p_{RE,t}$, where $p_{RE,t}$ is derived from one of five renewable production scenarios, and the penetration factor $\alpha \in [0,1]$ indicates the fraction of the forecasted renewable output that is available. The modified demand balance constraint becomes:

$$\sum_{i \in I} p_{i,t} + \alpha p_t^{RE} = d_t + s_t, \quad \forall t = 1, \ldots, T$$

In this formulation, s_t captures any energy curtailment and incurs a penalty cost λ. By varying α and λ (with $\lambda \in \{1, 10, 100\}$) across the five renewable scenarios, we investigate the sensitivity of the optimal unit commitment decisions to both renewable penetration and the economic implications of renewable curtailment.

3.3 Dealing with Uncertainty

Next, in order to account for the unpredictable nature of renewable energy production, we introduce a two-stage decision process:

Stage 1: Non-Anticipative Decisions
In this stage, decisions must be made without knowing the future renewable production. These decisions include the unit commitment variables (e.g., $u_{g,t}$ for unit on/off status and $v_{g,t}$ for startup decisions). Since the renewable scenario is not known at $t = 1$, these decisions must be fixed and applied consistently across all possible scenarios.

Stage 2: Scenario-Dependent Decisions
Once the renewable production is revealed in later time periods, decisions that can be adjusted are made in Stage 2. These include the generation output $p_{g,t}^{(s)}$ and load shedding (or curtailment) $s_t^{(s)}$ for each scenario s. In this stage, the model adapts to the realized scenario to ensure that demand is met while minimizing the operating cost.

Now, we define our mathematical notation: Let S denote the set of scenarios, with each scenario $s \in S$ having probability π_s, which will be randomized in this case since we don't have any information about different scenarios, and let the renewable production in period t under scenario s be $pRE_t^{(s)}$. Then:

1. Demand Balance Constraint (for each period t and scenario s):

$$\sum_{g \in G} p_{g,t}^{(s)} + \alpha\, pRE_t^{(s)} = demand_t + s_t^{(s)}, \quad \forall t,\ \forall s \in S.$$

2. Capacity Constraint (for each unit g, period t, and scenario s):

$$\text{min\_capacity}_g \cdot u_{g,t} \leq p_{g,t}^{(s)} \leq \text{max\_capacity}_g \cdot u_{g,t}, \quad \forall g,\ \forall t,\ \forall s \in S.$$

   Here, the Stage 1 decision variables $u_{g,t}$ and $v_{g,t}$ remain unchanged across all scenarios.

3. Objective Function:

$$\min \quad \underbrace{\sum_{g \in G} \sum_{t=1}^{T} \Big(\text{no\_load\_cost}_g \cdot u_{g,t} + \text{startup\_cost}_g \cdot v_{g,t} \Big)}_{\text{Stage 1 Fixed Costs}}$$
$$+ \underbrace{\sum_{s \in S} \pi_s \left[\sum_{g \in G} \sum_{t=1}^{T} \text{variable\_cost}_g \cdot p_{g,t}^{(s)} + \lambda \sum_{t=1}^{T} s_t^{(s)} \right]}_{\text{Stage 2 Expected Costs}}.$$

A possible extension of this model is to adjust the probabilities assigned to different scenarios based on geographic context. For instance, scenarios with minimal sunlight are less likely in desert regions, while tropical areas may experience greater weather variability. Similarly, areas with high wind patterns would assign higher probabilities to scenarios with high wind energy output. Geographic location, thus, plays a crucial role in determining not only how much renewable energy a country utilizes but also which specific sources are used.

Therefore, the probabilities attributed to the different scenarios should be region dependent. In our case we don't have any information about the exact location of the countries we are interested in so we make the probability random. However, in real-world application, these probabilities could be modeled as functions of regional parameters. We can formalize this relationship as :

$$\pi_s = f(\text{location}, \text{season}, \text{weather patterns}, s)$$

where π_s is the probability of scenario s, and f refers to geographic and meteorological data. To further analyze the impact of renewable uncertainty on our decisions, we will randomize the scenario probabilities π_s by setting a fixed random seed (e.g., 42) and generating multiple distinct probability vectors. For each randomized set, we will solve the two-stage stochastic model to obtain the corresponding optimal objective value and record the generation usage of each conventional unit.

4 Numerical results

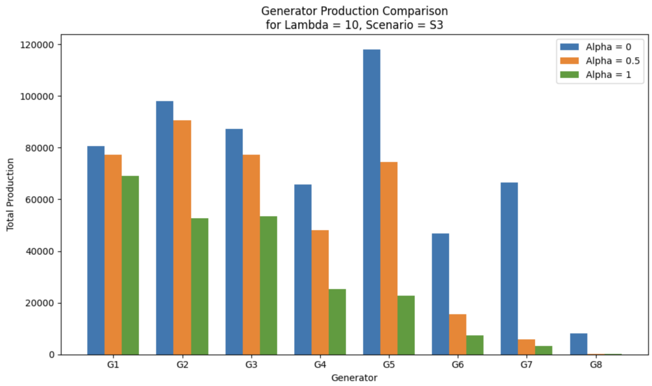
In this section, we present the numerical results and compare the computation times of the different formulations. Both models are implemented in Python using Pyomo and solved with the Gurobi solver. To ensure the correctness of our implementations, we begin by testing with the small examples provided. Provided that the optimal costs match, it confirms the accuracy of our models. After that, we run the implementations with the realistic demand data. Table 1 shows the optimal total costs and computation times generated by each formulation where we set $\alpha = 0$ and $\lambda = 10$

Table 1: Comparison of the two formulations (Gaps = 0%)

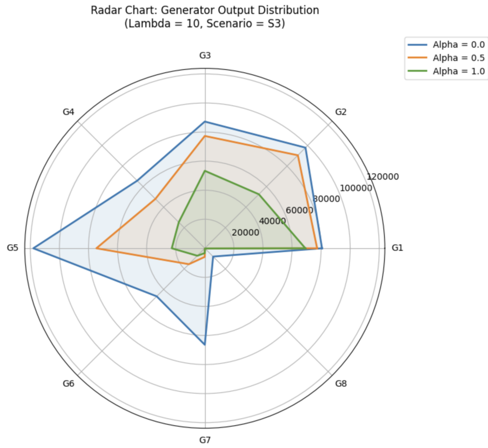
Formulation	**Optimal Total Cost**	**Computation Time**
Formula 1 (Takriti et al., 2000)	18,109,828.92	0.44 seconds
Formula 2 (Rajan and Takriti, 2005)	18,109,828.92	0.68 seconds

We can see that Formulation 1 performs better, as it has a lower computation time. This is due to tighter LP relaxations. Formulation 2 have more constraints and variables than the first model. Again, we observe that the objective value is the same, meaning that both formulations are equivalent.

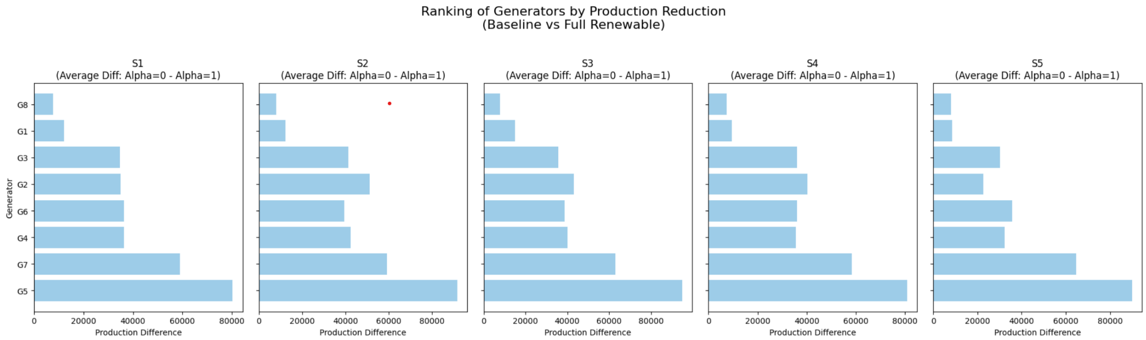
Building on this foundation, we extended our analysis by incorporating renewable energy into the models. Figure 1a and 1b illustrate how the cumulative generation from each traditional unit changes with varying α (for the selected penalty value of $\lambda = 10$). The results indicate that as renewable penetration increases, the overall contribution from conventional generators decreases. In particular, looking at figure 1c, units with high startup and variable costs (e.g., Unit G2 and Unit G4) are curtailed earlier than lower-cost baseload units such as G1. This behavior aligns with the intuition that renewables, operating at nearly zero marginal cost, displace more expensive and flexible units while leaving economically efficient baseload units in service.



(a) Generator Production Comparison



(b) Generator Output Distribution



(c) Ranking of Generators by Production Reduction

Figure 1: Combined Graphs Layout

Integrating renewable energy influences unit commitment decisions in two significant ways. First, renewable generation's nearly zero marginal cost means that higher renewable penetration leads to lower overall production costs. Second, the model tends to curtail expensive, flexible peaking units while favoring the continued operation of lower-cost baseload units. This selective displacement arises from the differences in cost structures and the operational constraints of conventional generators.

Next, in order to address the inherent uncertainty at the beginning of $t = 1$ regarding which renewable energy production scenario will occur, we implement a two-stage stochastic programming approach. In the first stage, non-anticipative decisions such as the unit commitment variables $u_{g,t}$ and startup decisions $v_{g,t}$ are made without prior knowledge of the future renewable output. In the second stage, after the scenario is revealed, scenario-dependent variables $p_{g,t}^{(s)}$ (generation output) and $s_t^{(s)}$ (curtailment) are adjusted to meet the demand, while minimizing the operating cost.

In this study, we randomly generated 20 different sets of scenario probability vectors, $\pi = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$, using a fixed random seed of 42. For each set, we solve the two-stage stochastic model with the parameters $\alpha = 1$ and $\lambda_{\text{penalty}} = 10$ and record the corresponding optimal value. Table 2 summarizes the results, listing the optimal objective value for each iteration along with the respective scenario probabilities.

From the table, we observe that:

The optimal objective value varies slightly across the 20 different iterations, reflecting the sensitivity of the model to the scenario probability distribution.

The different distributions in π_1 through π_5 illustrate the random nature of scenario probabilities and how they influence the overall optimal cost.

These results demonstrate the model's capability to handle uncertainty in renewable production by fixing the Stage 1 decisions and adapting the Stage 2 decisions based on the realized scenario.

Table 2: Simulations with corresponding objective values & probability of each scenario

Iteration	**OptimalValue**	π_1	π_2	π_3	π_4	π_5
1	6711169.84	0.191390711	0.198553275	0.052841963	0.285991219	0.271222831
2	6750445.003	0.181577350	0.086204402	0.141788299	0.339111930	0.251318019
3	6375282.390	0.124551068	0.340643722	0.352787065	0.093743765	0.088274380
4	6683292.913	0.118923499	0.024659203	0.223172848	0.278523932	0.354720517
5	6413833.496	0.041149114	0.246363672	0.205931216	0.149753784	0.356802214
6	6343749.041	0.106815026	0.463051584	0.209067391	0.153286597	0.067779402
7	6653951.797	0.187821769	0.153284496	0.190989177	0.181343889	0.286560669
8	6341575.810	0.009918411	0.354208015	0.223543125	0.177406162	0.234924287
9	6535687.645	0.211589751	0.219385763	0.312974147	0.250121139	0.005929200
10	6397615.363	0.104065046	0.391023011	0.219258414	0.141858099	0.143795430
11	6650496.536	0.214026055	0.240466722	0.154153959	0.354899425	0.036453839
12	6623446.594	0.253656352	0.218524211	0.215421239	0.282228430	0.030169768
13	6685425.702	0.133039479	0.136795722	0.182402192	0.339700593	0.208062014
14	6537661.484	0.165937060	0.064049313	0.318132574	0.082307522	0.369573531
15	6415152.675	0.091830087	0.257001795	0.299518403	0.069345277	0.282304437
16	6615111.806	0.265394603	0.179352160	0.277158697	0.134816462	0.143278078
17	6521335.866	0.305139725	0.318202707	0.212466610	0.002981093	0.161209865
18	6441583.070	0.049972066	0.284178227	0.347857106	0.253684174	0.064308427
19	6600691.825	0.139020670	0.245155163	0.200403798	0.274001844	0.141418525
20	6727089.439	0.238446709	0.086304022	0.195043421	0.213098222	0.267107626

5 Conclusion

In this paper we address the problem of energy production planning by formulation and solving the unit commitment problem. The aim is to minimize the total production cost while ensuring that demand is satisfied and operational constraints like minimum uptime, minimum downtime and initial generator conditions are met. We developed two mathematical formulations following Rajan and Takriti (2005) and Takriti et al. (2000). We used the programming language Python and the solver Gurobi. Our numerical results show that, although both models provide the same optimal total outputs,that formulation 1 provides a lower computation time and therefore performs better.

Furthermore, we integrated a renewable energy source in our model, resulting in a decrease in overall production costs and the curtailment of expensive, flexible units while favoring the continued operation of low-cost base load units.

We performed simulation to test our two-stage stochastic unit commitment model, which separates non-anticipative decisions from scenario-dependent adjustments. Compared to previous approaches, our model advances decision-making by better handling renewable production uncertainty and providing more robust and adaptable operational strategies.

Future work may improve the model by adjusting scenario probabilities based on geographic and climatic factors for more realistic renewable potential estimates. In addition, incorporating CO_2 emission constraints can further restrict fossil-fuel generation and promote renewables. Finally, exploring potential nonlinearities in variable costs—where marginal effects cause cost changes as production levels vary—would add depth to the economic modeling. These enhancements would lead to a more accurate, robust, and adaptable unit commitment framework under uncertainty.

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