Image Denoising using Decomposition methods: Comparative Analysis

SIGNAL AND IMAGE PROCESSING

TEAM 7

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Decomposition Methods used:

Singular Value Decomposition (SVD)

Principle Component Analysis (PCA)

Discrete Cosine Transform (DCT)

Wavelet Transform

Non-Negative Matrix Factorisation (NMF)

Image used

3 Different types of noise in 5 levels
(10%, 20%, 30%, 40%, 50%)

- Gaussian Noise
- Salt & Pepper
- Speckle

Singular Value Decomposition (SVD) Denoising

- SVD is used to decompose an image into three matrices: the left singular matrix, the right singular matrix, and the diagonal matrix of singular values :

$$A = U*\sum*V^T$$

(where A is the image matrix)

- To denoise the image:

$$A' = U*\Sigma'*V^T$$

(Σ' is the truncated version of the Σ matrix after applying threshold)

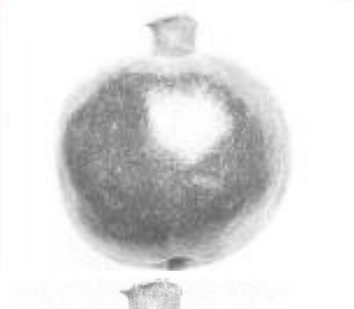
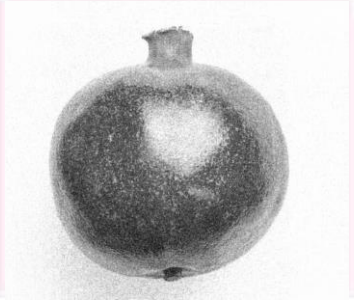
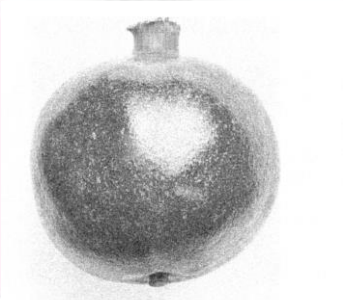
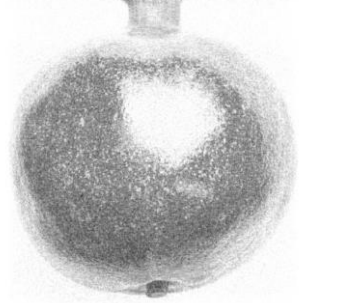
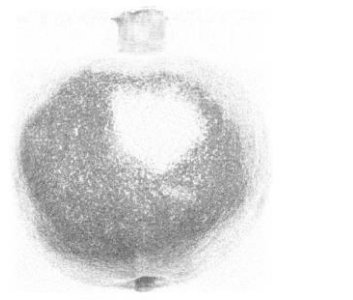
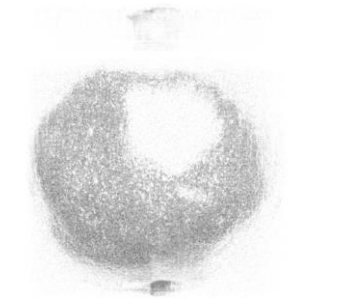
- The thresholding step(where the threshold is t) is used to suppress singular values that are likely to be due to noise, while preserving those that are likely to be part of the underlying signal and so the truncated S' is calculated as:

$$\Sigma' = \text{diag}([\sigma_1, \sigma_1, \sigma_1 \ldots . \sigma_1, 0 \ldots \ldots ,0])$$

Original Image

Gaussian Noise

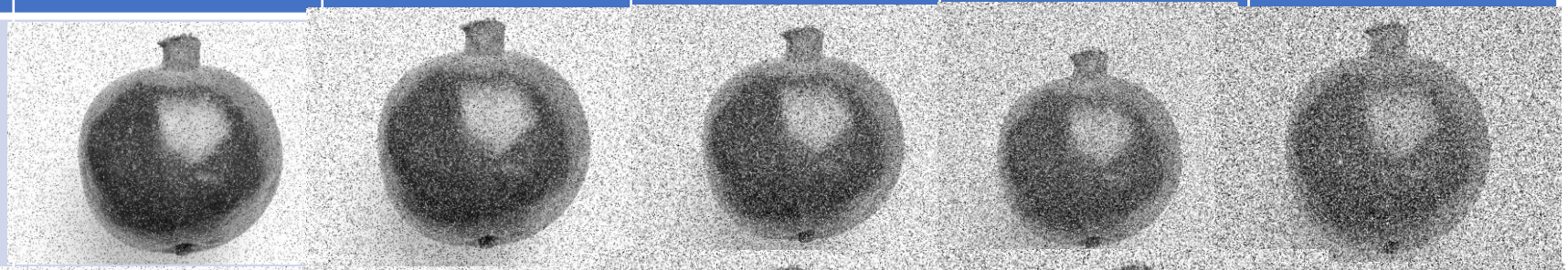
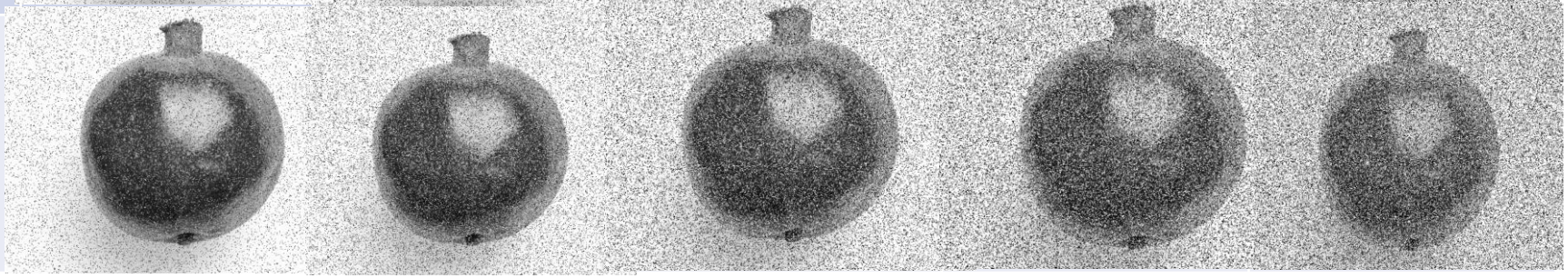
Black and White Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Salt And Pepper Noise

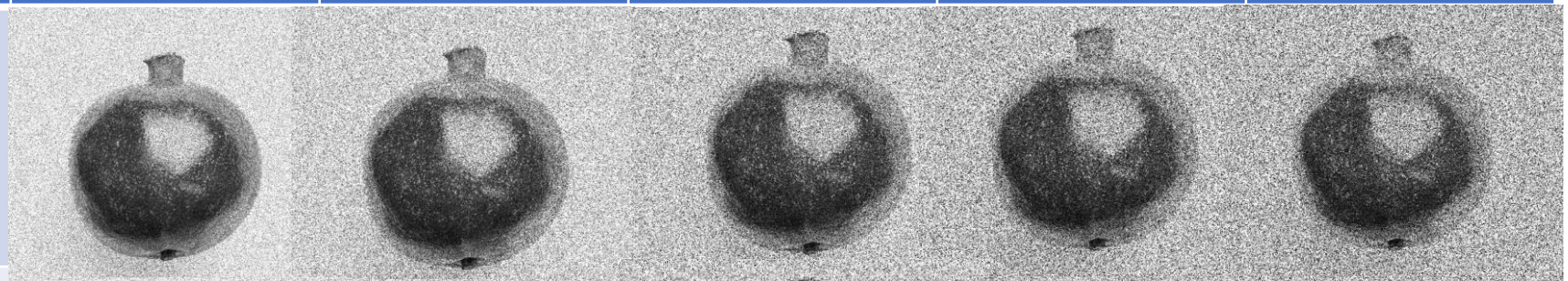
Black and White Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Speckle

Black and White Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Principal Component Analysis (PCA) Denoising

- PCA has wide application in statistical pattern recognition and signal processing

- In PCA-based denoising, the original image is transformed into PCA domain and thereby preserves the important principal components only and removes the noise components

Given an image $X = \begin{bmatrix} -x_1- \\ -x_2- \\ -x_3- \\ -x_4- \\ -x_5- \end{bmatrix}_{n \times m}$

1. Compute Row-wise mean

$$\underset{n \times 1}{\bar{X}} = \frac{1}{n} \sum_{j=1}^{n} x_j$$

2. Build average matrix:

$$[--\bar{x}--] \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{bmatrix}$$

3. Subtract mean from data matrix:

$$\underset{n \times m}{B} = X - \bar{X}$$

Based on eigenvalues and eigenvectors:

4. Compute the covariance matrix of the centered data:

$$C = B^T B$$
$$n \, x \, n$$

5. Compute eigenvalues and eigenvectors of covarience matrix

$$C * U_i = \lambda * U_i$$
$$n \, x \, n \quad n \, x \, 1$$

6. Compute the projection of the noisy image onto the subspace spanned by the largest k eigenvectors(principle components):

$$x = U_k^T * B$$
$$K \, x \, n$$

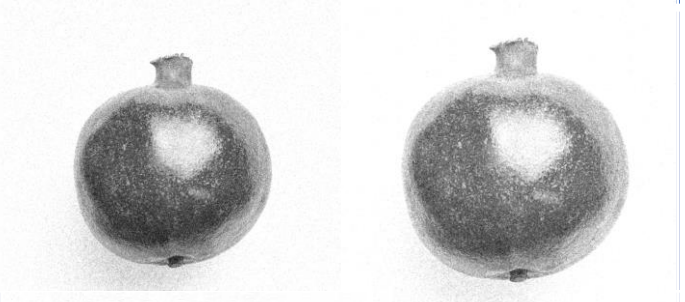
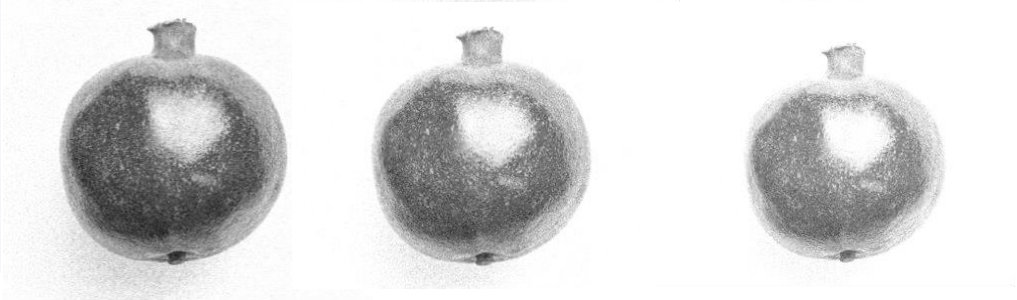
7. Reconstruct the denoised image by projecting the compressed image back onto the original space:

$$X = U_k * x + \overline{X}$$
$$N \, x \, k K \, x \, m$$

Original Image

Grey-scale Image

Gaussian Noise

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Salt And Pepper Noise

Grey-scale Image

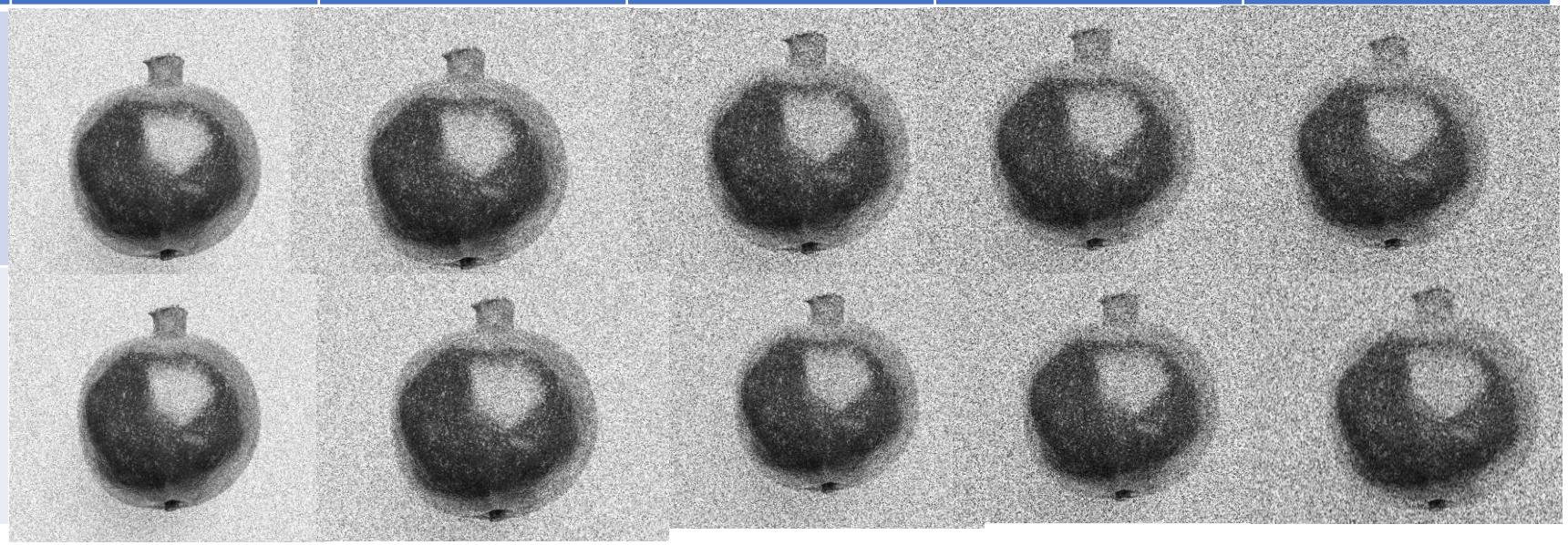
Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Speckle

Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					



Discrete Cosine Transform (DCT) Denoising

- DCT (Discrete Cosine Transform) denoising is a technique used to remove noise from a signal by applying a DCT transform to the signal, thresholding the DCT coefficients, and then reconstructing the signal using the threshold coefficients.

- The thresholding step is used to suppress coefficients that are likely to be due to noise, while preserving those that are likely to be part of the underlying signal.

Equation

The mathematical representation of DCT of an image is given by ,

$$F(u,v) = \mathrm{C}(u)\,\mathrm{C}(v) \sum_{0}^{M-1} x \sum_{0}^{N-1} y\ \mathrm{f}(x,y)\cos\big((2x+1)u\pi/2N\big)\cos\big((2y+1)v\pi/2M\big)$$

where F(u,v) is the DCT coefficients of the image, f(x,y) is the original image, C(u) and C(v) are normalization constants, and M and N are the number of rows and columns in the image.

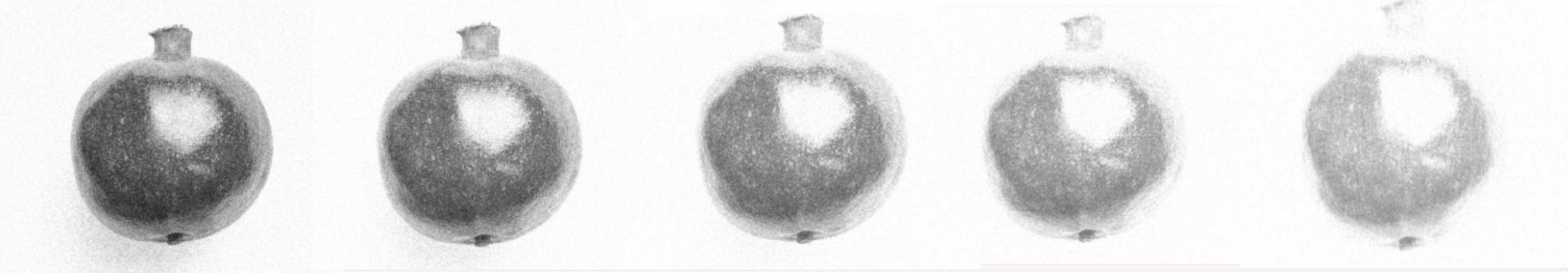
The inverse DCT (IDCT) is then applied to the threshold DCT coefficients to obtain the denoised image. The mathematical representation of IDFC is given by,

$$f(x,y) = \sum_{0}^{M-1} u \sum_{0}^{N-1} v \; \mathrm{C}(u)\,\mathrm{C}(v)\,\mathrm{F}(u,v)\cos\left((2x+1)u\pi/2N\right)\cos\left((2y+1)v\pi/2M\right)$$

Original Image

Grey-scale Image

Gaussian Noise

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Salt And Pepper Noise

Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Speckle

Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Wavelet transform

Wavelet based denosing represents an image as linear combination of wavelet coefficients which preserves the important features of image.

The main three process:

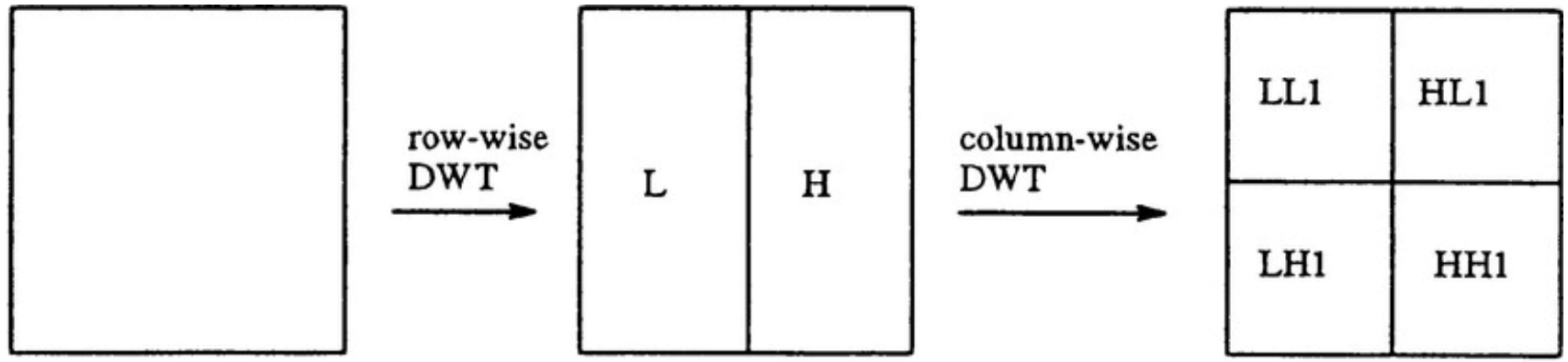
- STEP 1: Transforming the image into a set of wavelet coefficients
  – to represent image characetersitcs in different frequency bands
- STEP 2 : Detect Noise and Remove them from wavelet coefficients using thresholding , -wavelet shrinkage and wavelet domain filtering
- SETP 3: Apply inverse wavelet transform to the image
  - reconstructed the image from the modified wavelet coefficients

RESULT : image with reduced noise

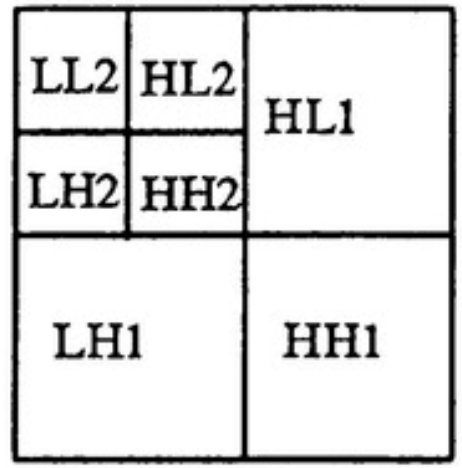
Wavelet based denoised scheme

Decompositon

Reconstruction



(a) First level of decomposition



(b) Second level of decomposition

Original Image

Grey-scale Image

Gaussian Noise

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Salt And Pepper Noise

Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Speckle

Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Non-Negative Matrix Factorization

Non-Negative Matrix Factorization (NMF) is a technique that can be used for image denoising by decomposing the image matrix into two non-negative matrices.

One represents the **basis** of the image and the other represents **coefficient.**

By decomposing the image matrix in this way, the noise can be separated from the useful information in the images.

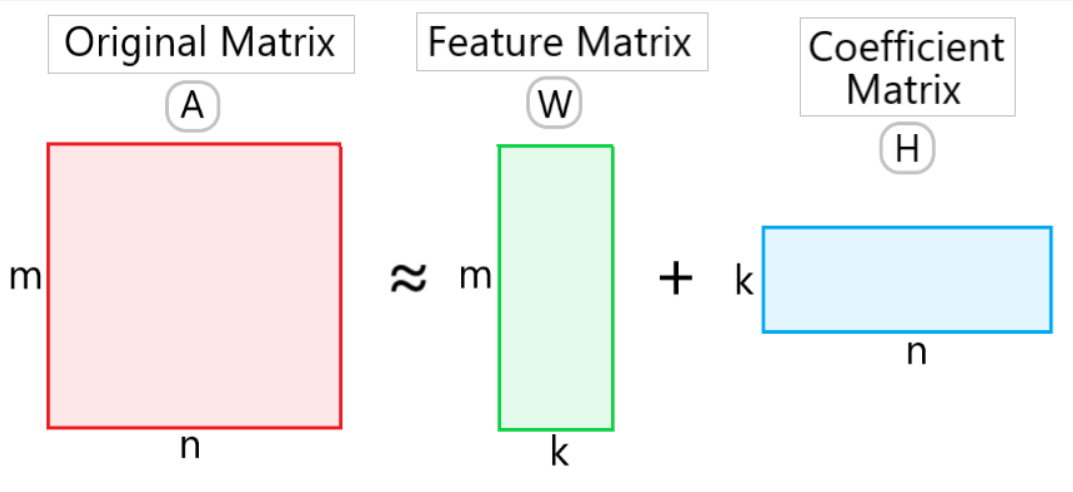
Mathematical equation

Given a non-negative matrix A of size $m \times n$, NMF finds two non-negative matrices W of size $m \times k$ and H of size $k \times n$ such that:

$$A \approx W * H$$

Where k is the rank of the approximation. ***k ≤ min(m,n)***

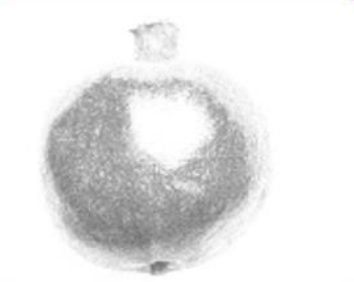
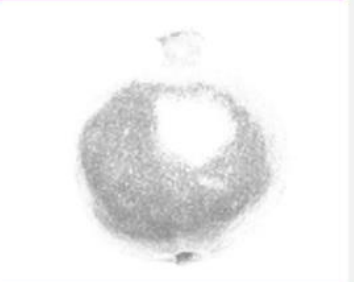
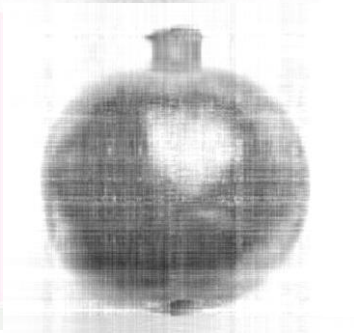
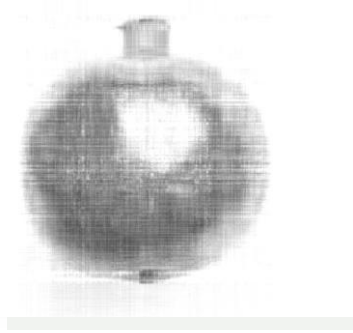
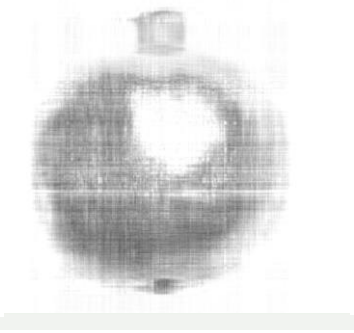
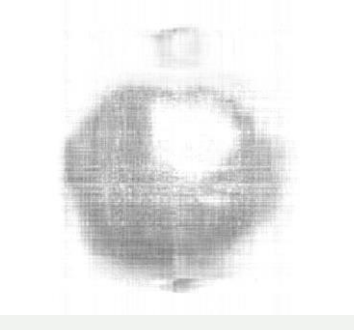
By minimizing the approximation error, NMF finds the two non-negative matrices W and H that best approximate the original matrix A.



Original Image

Grey-scale Image

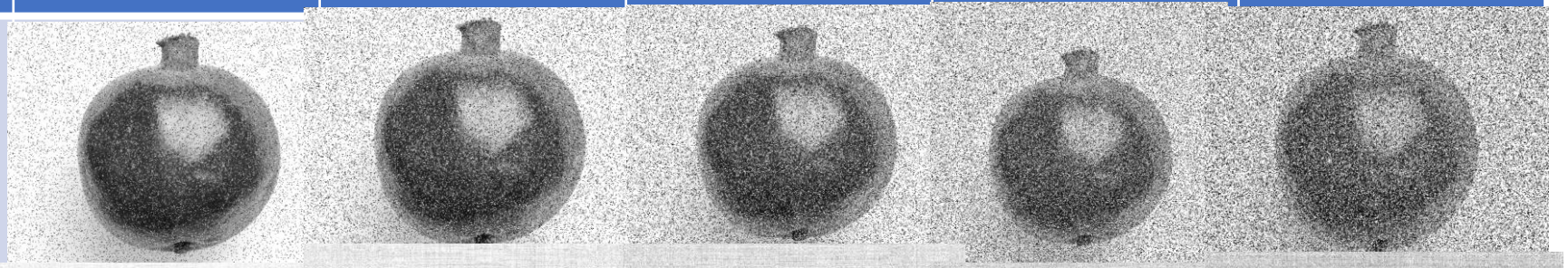
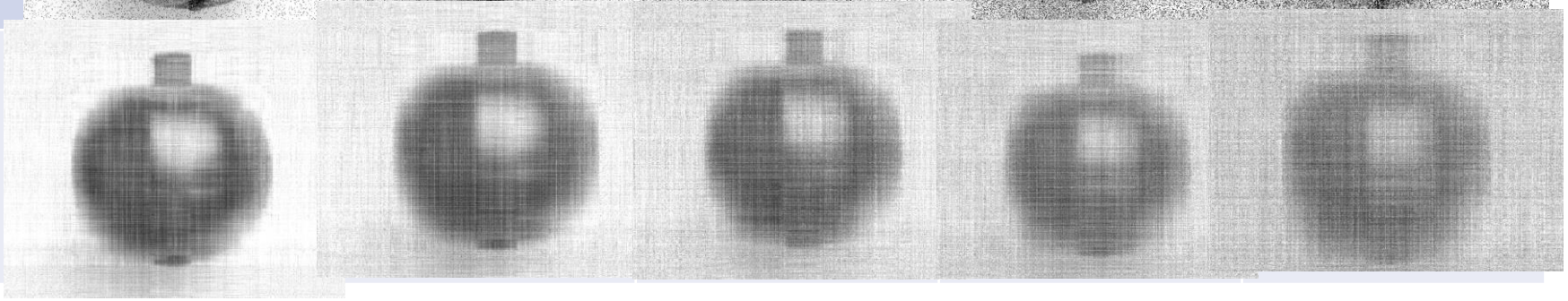
Gaussian Noise

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

Salt And Pepper Noise

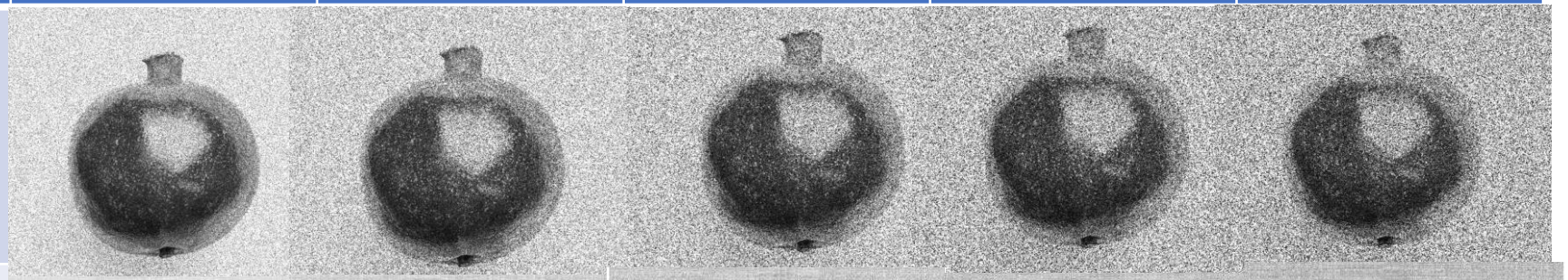
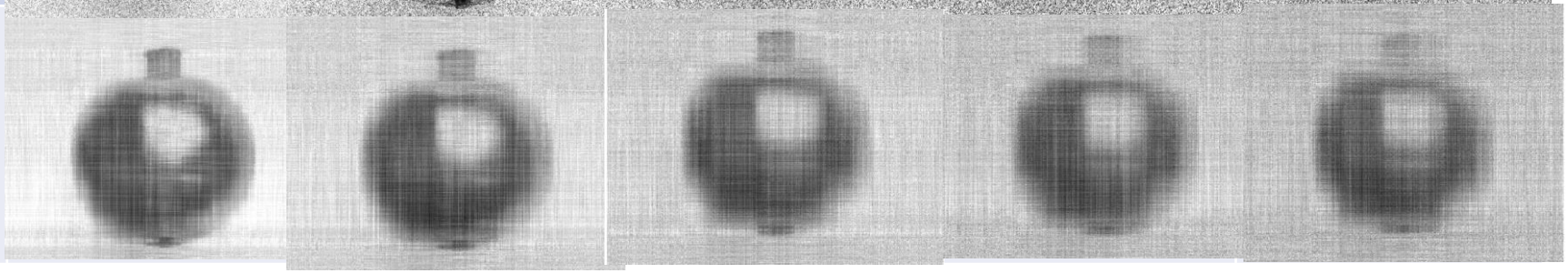
Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					

Original Image

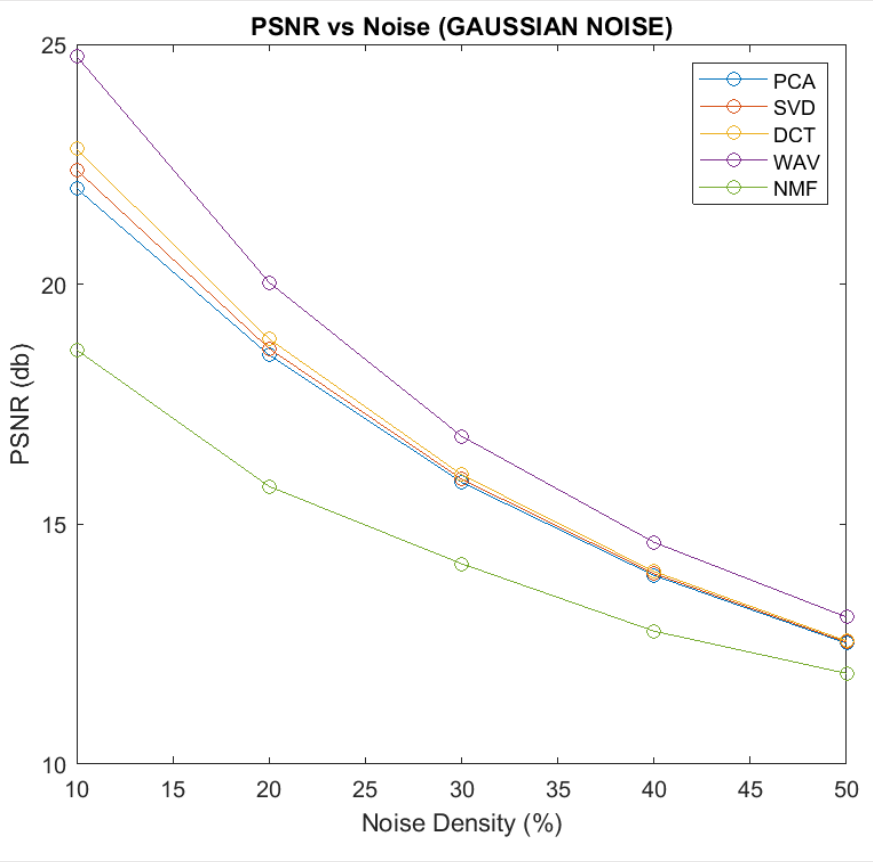
Speckle

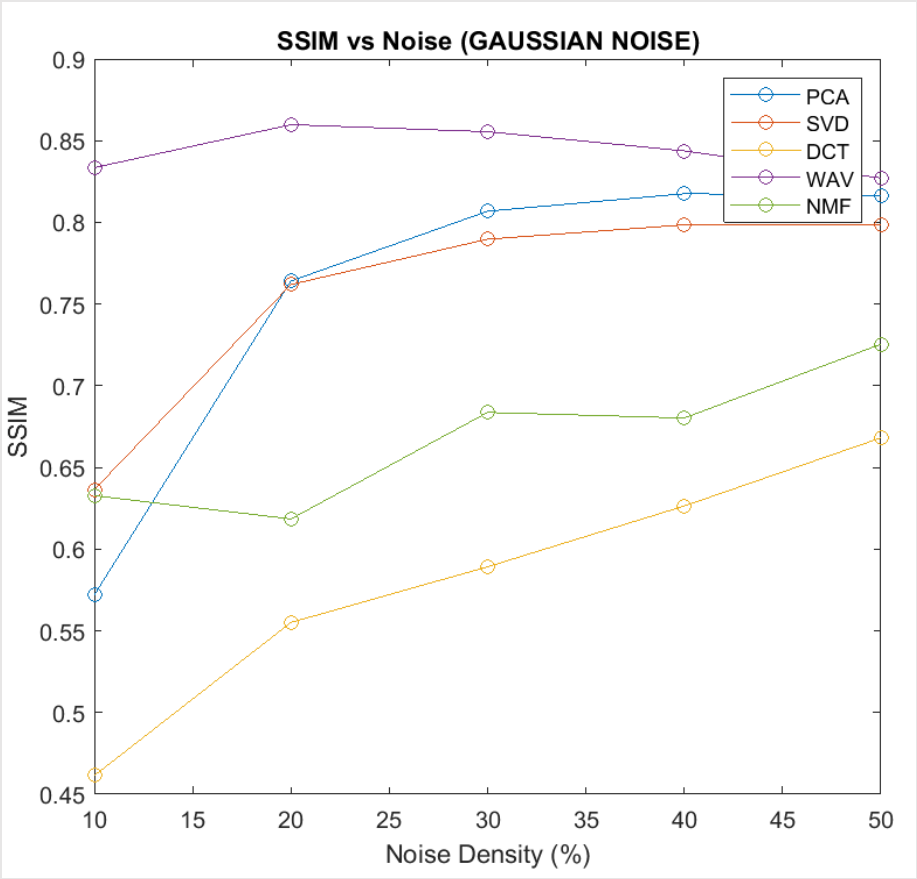
Grey-scale Image

Noise Levels	10%	20%	30%	40%	50%
Noisy					
Denoised					


Results

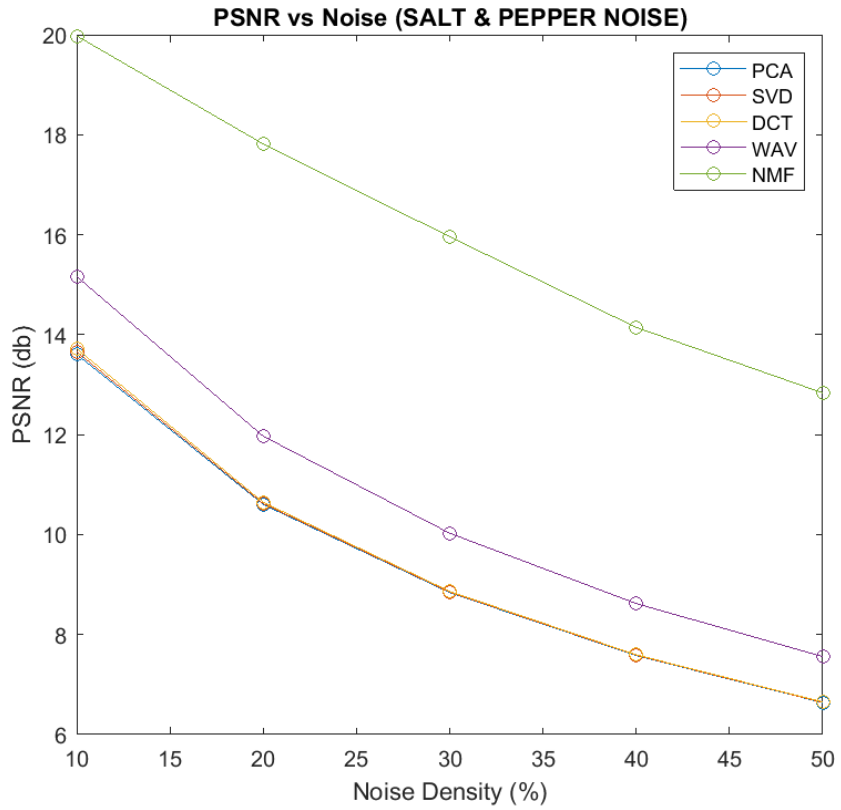
PSNR, SSIM - Pomegranate

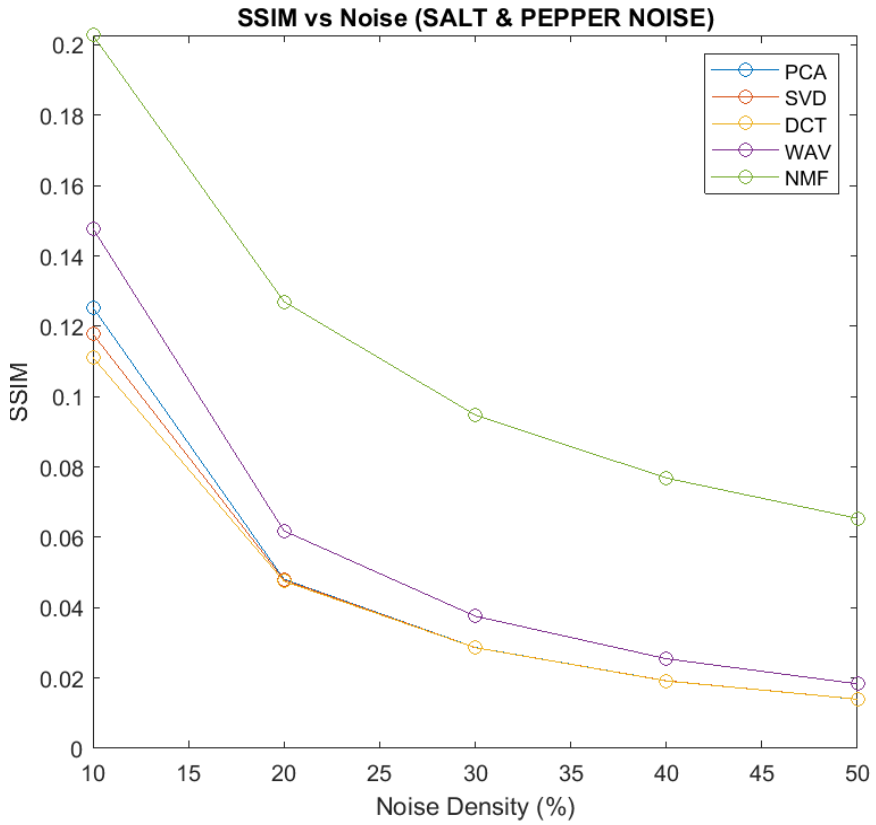




GAUSSIAN NOISE

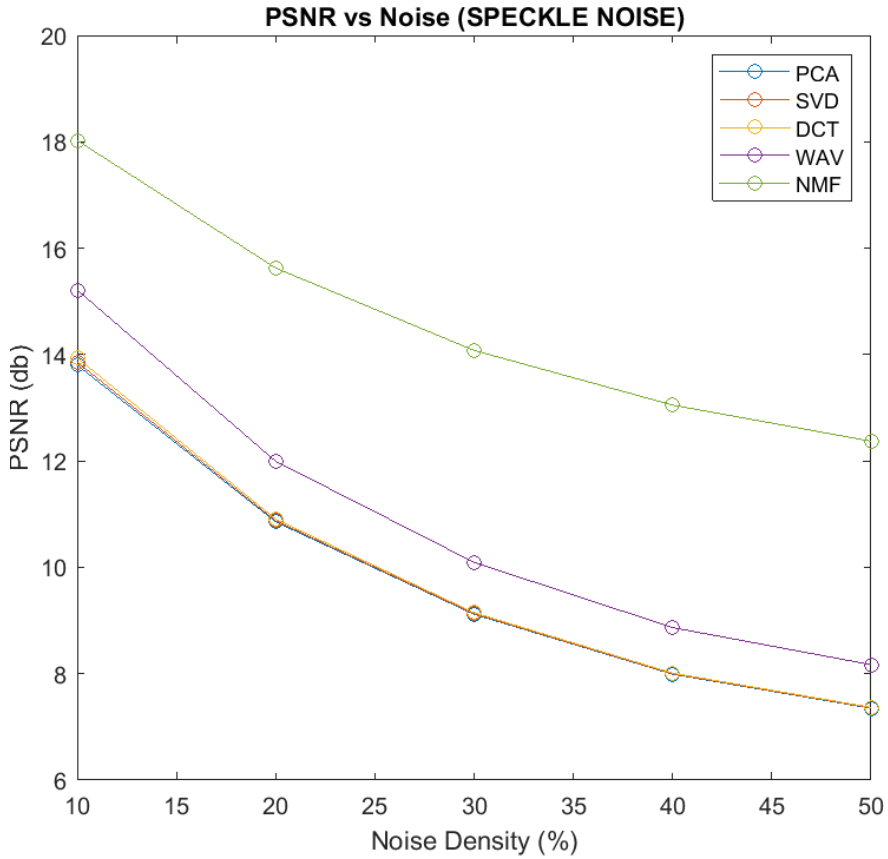
METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	21.9965	18.5247	15.8735	13.9318	12.5189
	SVD	22.3833	18.6599	15.9400	13.9691	12.5400
	DCT	22.8177	18.8594	16.0331	14.0158	12.5674
	WAVELET	24.7507	20.0377	16.8348	14.6229	13.0634
	NMF	18.6328	15.7875	14.1767	12.7723	11.8909
SSIM	PCA	0.5720	0.7641	0.8068	0.8175	0.8165
	SVD	0.6365	0.7620	0.7897	0.7985	0.7894
	DCT	0.4614	0.5551	0.5892	0.6264	0.6681
	WAVELET	0.8335	0.8598	0.8555	0.8437	0.8271
	NMF	0.6327	0.6185	0.6837	0.6803	0.7255

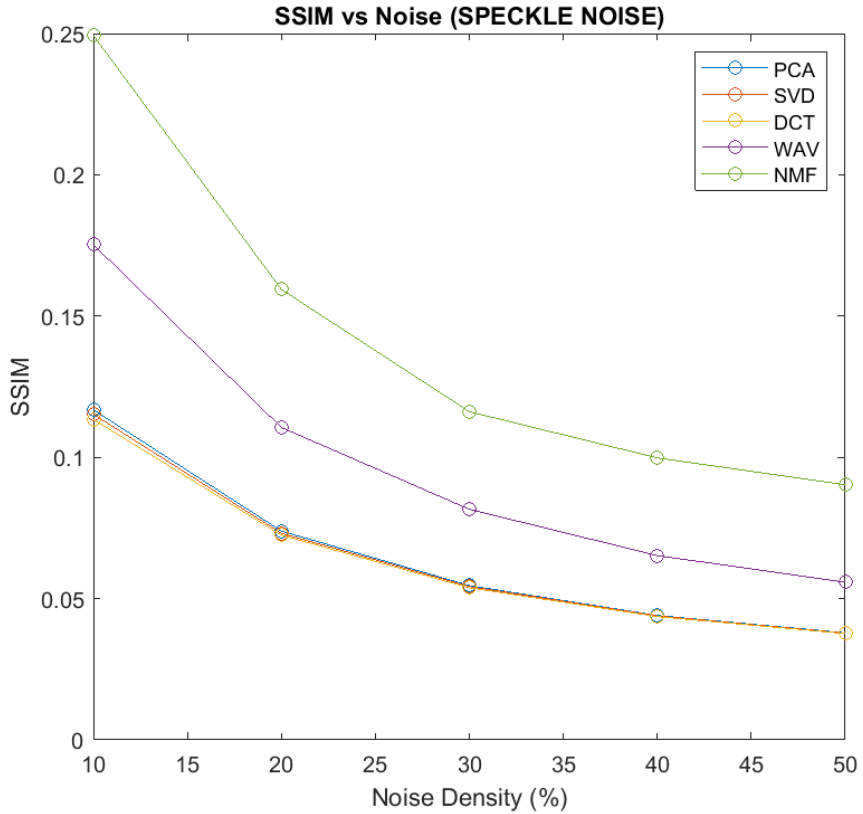




SALT & PEPPER NOISE

METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	13.6068	10.6028	8.8402	7.5796	6.6387
	SVD	13.6550	10.6200	8.8496	7.5856	6.6434
	DCT	13.7193	10.6463	8.8643	7.5655	6.6505
	WAVELET	15.1626	11.9686	10.0270	8.6208	7.5595
	NMF	19.9769	17.8147	15.9585	14.1441	12.8347
SSIM	PCA	0.1253	0.0481	0.0287	0.0481	0.0140
	SVD	0.1178	0.0479	0.0286	0.0192	0.0140
	DCT	0.1112	0.0475	0.0286	0.0192	0.0140
	WAVELET	0.1478	0.0618	0.0376	0.0255	0.0183
	NMF	0.2027	0.1270	0.0948	0.0769	0.0654





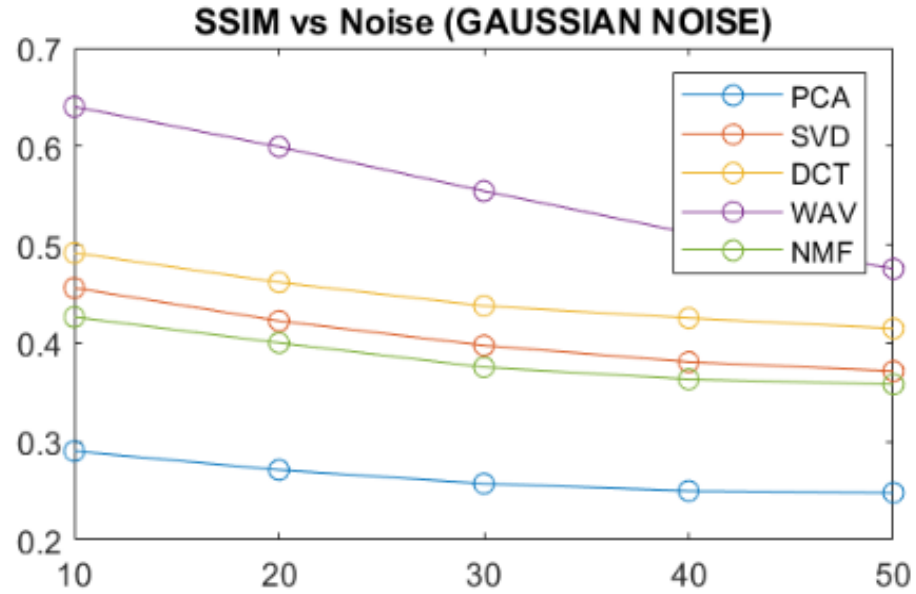
SPECKLE NOISE

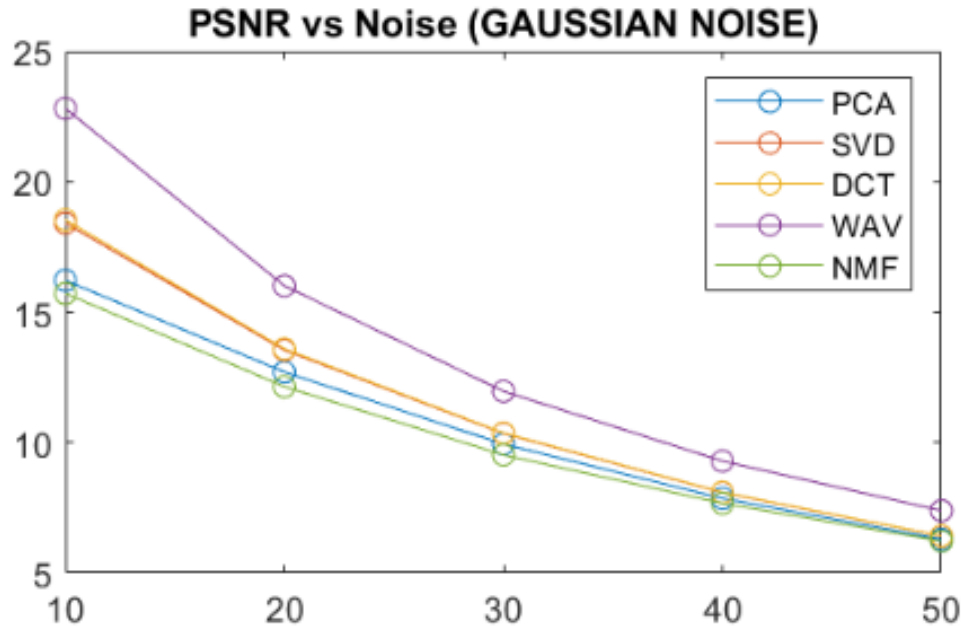
METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	13.8017	10.8543	9.1166	7.9911	7.3508
	SVD	13.8571	10.8753	9.1278	7.9990	7.3571
	DCT	13.9357	11.9938	9.1466	8.0116	7.3673
	WAVELET	15.2148	19.4703	10.0931	8.8673	8.1699
	NMF	18.0263	15.6529	14.0781	13.0508	12.3712
SSIM	PCA	0.1168	0.0739	0.0546	0.0440	0.0379
	SVD	0.1152	0.0731	0.0543	0.0439	0.0378
	DCT	0.1134	0.0725	0.0538	0.0436	0.0376
	WAVELET	0.1753	0.1106	0.0816	0.0652	0.05588
	NMF	0.2493	0.1594	0.1162	0.0998	0.0903

PSNR, SSIM - Lena

GAUSSIAN NOISE

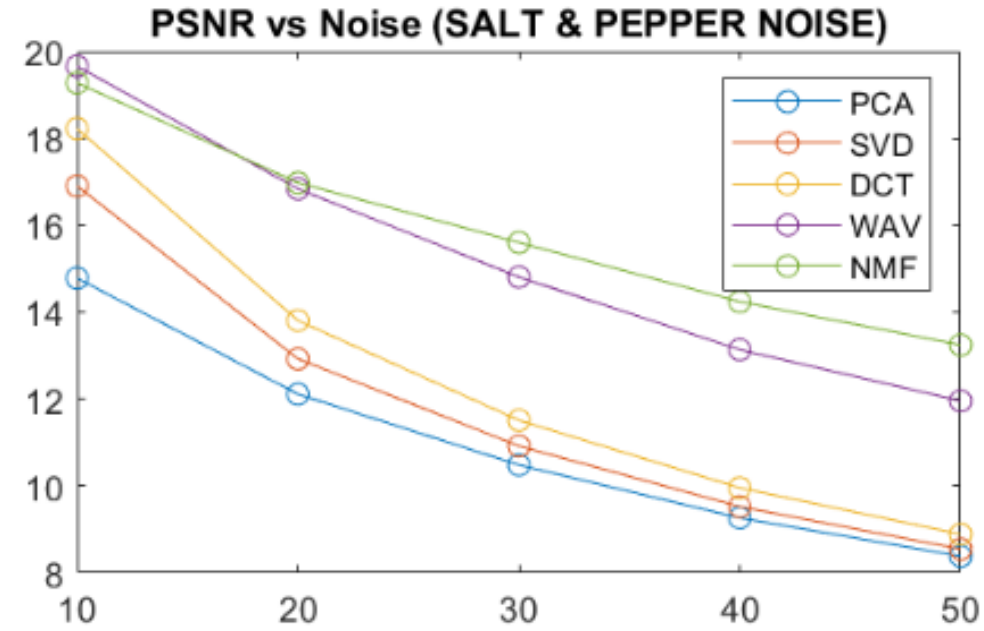
METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	16.2155	12.7046	9.9429	7.8622	6.2946
	SVD	18.4119	13.5474	10.3482	8.0837	6.4121
	DCT	18.5348	13.5941	10.3699	8.0983	6.4239
	WAVELET	22.8078	16.0089	11.9670	9.2912	7.3901
	NMF	15.7177	12.1451	9.5272	7.6730	6.2264
SSIM	PCA	0.2.903	0.2712	0.2573	0.3500	0.2481
	SVD	0.4561	0.4226	0.3977	0.3809	0.3717
	DCT	0.4918	0.4618	0.4379	0.4255	0.4149
	WAVELET	0.6402	0.5994	0.5545	0.5119	0.4756
	NMF	0.4265	0.4003	0.3756	0.3634	0.3581

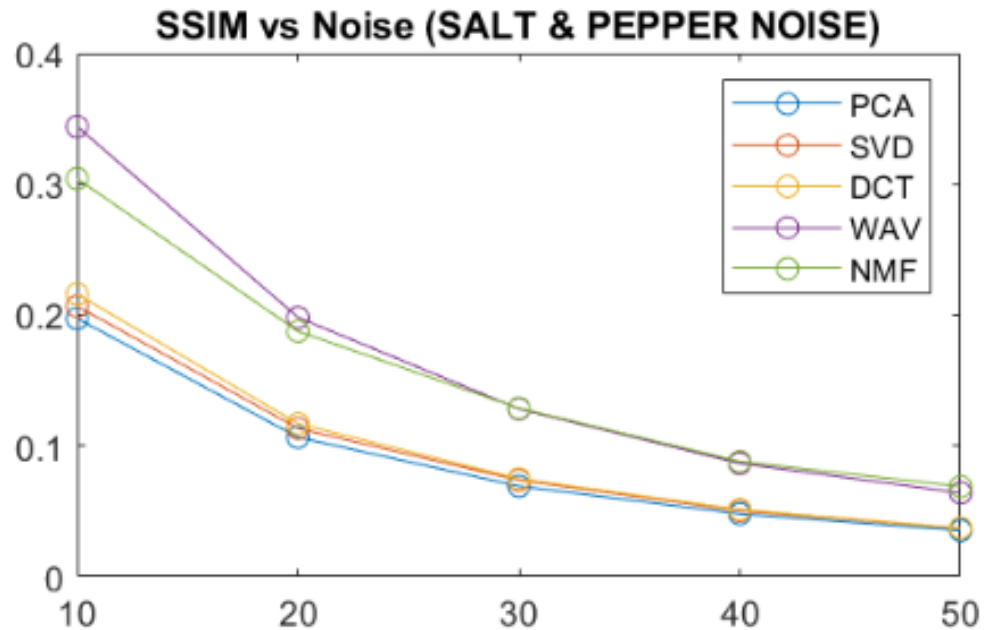




SALT & PEPPER NOISE

METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	14.7820	12.1122	10.4847	9.2667	8.3846
	SVD	16.9024	12.9245	10.9207	9.5262	8.5498
	DCT	18.2205	13.8064	11.5122	9.9579	8.8914
	WAVELET	19.6572	16.8346	14.8706	13.1355	11.9577
	NMF	19.2710	16.9759	15.6010	14.2403	13.2453
SSIM	PCA	0.1974	0.1066	0.0693	0.0481	0.0354
	SVD	0.2068	0.1134	0.0738	0.0504	0.0371
	DCT	0.2160	0.1170	0.0748	0.0513	0.0372
	WAVELET	0.3441	0.1982	0.1283	0.0867	0.0641
	NMF	0.3043	0.1876	0.1288	0.0880	0.0690





SPECKLE NOISE

METRICS	METHOD	10%	20%	30%	40%	50%
PSNR	PCA	16.8775	14.4576	12.9814	11.9479	11.2760
	SVD	20.7919	16.2719	14.0884	12.7261	11.8875
	DCT	22.0099	17.6688	15.1359	13.5730	12.5911
	WAVELET	21.7217	19.4703	17.6459	16.2348	15.3613
	NMF	20.1368	19.5562	18.7499	17.9583	17.5122
SSIM	PCA	0.2763	0.1928	0.1518	0.1256	0.1138
	SVD	0.3378	0.2091	0.1620	0.1328	0.1198
	DCT	0.3636	0.2199	0.1565	0.1284	0.1145
	WAVELET	0.5435	0.3903	0.3078	0.2611	0.2321
	NMF	0.3862	0.3050	0.2486	0.2059	0.1939