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IMAGE DENOISING USING DECOMPOSITION METHODS

Submitted by:

Dinesh Kumar M R (CB.EN.U4AIE20011)

Jangala Gouthami (CB.EN.U4AIE20023)

K S Paval (CB.EN.U4AIE20047)

Shreya Sanghamitra (CB.EN.U4AIE20066)

N T Shrish Surya (CB.EN.U4AIE20067)

Under the supervision of
Dr.Sowmya V

DEPT. OF COMPUTATIONAL ENGINEERING AND NETWORKING
AMRITA SCHOOL OF ENGINEERING

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Abstract

Image denoising is the process of removing noise from an image while preserving the underlying signal. The decomposition method of image denoising involves decomposing the image into various components such as: B. Separate the low and high frequency components and apply different denoising techniques to each component. Each presented technique has its own strengths and weaknesses. In this project, we present a comprehensive comparative analysis of these techniques for a wide range of noise densities. All decomposition techniques are implemented in MATLAB, evaluated, and compared to standard benchmark image data and qualitative measures, namely peak signal-to-noise ratio (PSNR) and structural similarity index (SSIM). Therefore, this project presents a comprehensive comparative analysis of various modern noise reduction techniques.

1 Introduction

Image noise refers to random variations in brightness and color information that are not usually part of the original image. These fluctuations can be caused by various factors such as: B. Sensor noise in digital cameras, electromagnetic interference, or quantization errors in image compression. Noise affects the overall image quality and makes it difficult to process and analyze. To visually distinguish whether an image contains impulse noise, an observer should look for unusually bright or dark pixels. These bright and dark pixels correspond to the maximum (255) and minimum (0) values a pixel can capture (for 8-bit images, 0 and 255 are the minimum and maximum possible values). The process of removing this noise using various image processing techniques is commonly called denoising.

Various prior art denoising methods have been proposed over the years to restore images corrupted by Gaussian, salt and pepper, and speckle noise. The main goal of this project is to provide the reader with knowledge of the various decomposition methods presented and the various categories of knowledge to which these methods can be assigned. The main contributions of the paper are as follows.

- We have summarized the working techniques of most of the decomposition methods
- For better understanding by the reader, visual representation of selected benchmark images decomposed using various techniques are also given.

2 Literature Review

Image denoising is a well-studied problem in the field of image processing and computer vision. The goal of image denoising is to remove noise from an image while preserving important details and features. There are many different techniques for denoising images, each with its own strengths and weaknesses but we are only focusing on the decomposition methods like:

1. Principal Component Analysis (PCA) based denoising is to represent an image as a linear combination of a set of basis images, which are called principal components.
2. Singular Value Decomposition (SVD) based denoising is to represent an image as a linear combination of a set of basis images, which are called singular vectors.
3. Discrete Cosine Transform (DCT) based denoising is to represent an image as a linear combination of a set of basis images, which are called cosine functions.
4. Wavelet Transform based denoising is to decompose the image into different frequency bands using a wavelet transform, and then apply denoising techniques to the different bands.
5. Non-negative Matrix Factorization (NMF) based denoising is to represent an image as a linear combination of a set of basis images, which are called non-negative factors.

3 Objective

Mathematically, the problem of image denoising can be modeled as follows:

$$y = x + n \tag{1}$$

where y is the observed noisy image, x is the unknown clean image, and n represents additive noise with standard deviation σ_n which can be estimated in applications by the proposed methods.The goal of noise reduction is to reduce noise in natural images while minimizing the loss of original features and improving the signal-to-noise ratio (SNR). The main challenges of image denoising are:

- Flat areas should be smoothed,
- edges should be protected from blurring,
- textures should be preserved, and
- new artifacts should not be created.

4 Theoretical Background

The proposed work in this project focuses on denoising images using decomposed methods. The goal of denoising is to improve the overall visual quality of the image and enhance the performance of image processing and analysis tasks that follow.

Decomposition-based methods involve breaking down the image into a set of basic components, such as the methods we proposed PCA, SVD, DCT, Wavelet transform and NMF. Overall, the choice of denoising method depends on the type of noise present in the image and the desired trade-off between noise reduction and preservation of image details.

5 Evaluation parameters

5.1 Peak signal-to-noise ratio (PSNR)

PSNR is a measure of the quality of the reconstructed image compared to the original image, especially in terms of signal-to-noise ratio (SNR). It is most commonly used in the field of image and video compression. PSNR is defined as the ratio of the maximum possible power of the signal to the power of the noise. Usually expressed in decibels (dB), the higher the PSNR value, the better the image quality. PSNR is calculated as:

$$PSNR = 10 \log_{10}\left(\frac{MAX^2}{MSE}\right) \tag{2}$$

Where, MAX: maximum possible pixel value (usually 255 for 8-bit images) and MSE: mean squared error between the original and reconstructed images.

5.2 Structural similarity index (SSIM)

SSIM is a method of measuring similarity between two images. This is a full reference quality assessment method, meaning that the original image is required as a reference for comparison. SSIM was designed to address some of the limitations of other image quality metrics, such as: B. Maximum signal-to-noise ratio (PSNR) considering both structural and luminance information in the image.

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1) + (2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{3}$$

Where, $\mu_x(\sigma_x)$ and $\mu_y(\sigma_y)$ are the mean (standard deviation) in the x and y directions, respectively. While the constant C_1 and C_2 are selected such that the maximum (minimum) value of SSIM is 1 (0).

6 Methodology

6.1 Principal Component Analysis (PCA)

The basic idea is to represent an image as a set of principal components. A principal component is a linear combination of the original pixels that capture the main features of the image. Noise is represented by small coefficients of the less important principal components. You can remove or reduce this to improve the image quality. PCA can be used as a preprocessing step before applying other image restoration techniques such as filtering and inpainting.

Let x be a zero-mean random variable. Suppose we want the direction w such that the projection of x along this direction has maximum variance:

$$\max_w (w'x) \quad \text{st.} \quad w = 1. \tag{4}$$

We have

$$\begin{aligned}(w'x) &= w'xx'w \qquad (5)\\ &= w'\Sigma w. \qquad (6)\end{aligned}$$

The Lagrangian is

$$L = w'\Sigma w + \lambda(w'w - 1). \tag{7}$$

The stationary condition is

$$\begin{aligned}\frac{\partial L}{\partial w} &= 2\Sigma w - 2\lambda w = 0, \qquad (8)\\ \Sigma w &= \lambda w. \qquad (9)\end{aligned}$$

Thus w is an eigenvector of Σ. Since

$$w'\Sigma w = w'(\lambda w) = \lambda, \tag{10}$$

the direction with maximum variance is the largest eigenvector. This procedure can be iterated to get the second largest variance projection (orthogonal to the first one), and so on.

6.2 Singular Value Decomposition (SVD)

The singular value decomposition (SVD) is a matrix factorization. If A is an $n \times m$ matrix, then we may write A as a product of three factors:

$$A = U\Sigma V^* \quad , \tag{11}$$

where U is an orthogonal $n \times n$ matrix, V is an orthogonal $m \times m$ matrix, V^* is the transpose of V, and Σ is an $n \times m$ matrix that has all zeros except for its diagonal entries, which are non-negative real numbers.

However, (11) can be alternatively expressed in summation form as:

$$A = \sum_{i=1}^{N} s_i u_i v_i \tag{12}$$

where if A is an image, it can be expressed as a weighted sum of component images:

$$u_i v_i \tag{13}$$

where u_i's are the columns of the output basis, v_i's are the rows of the input basis, and s_i's are called the singular values which form the diagonal of the S matrix. The singular values can be thought of as a weighting factor of the component images.

The basic idea is to use SVD to decompose a noisy image into a series of simpler images or "components". The noise is then removed by discarding the low-energy components and recombining the remaining components to produce a denoised image. This process is also called "image compression" or "dimensionality reduction".

6.3 Discrete Cosine Transform (DCT)

DCT (Discrete Cosine Transform) can be used to denoise an image by transforming the image from the spatial domain to the frequency domain. In this case the noise usually appears as high frequency components. By removing or reducing these high frequency components, the image can be transformed into the spatial domain to produce a denoised version.

The process of denoising an image using DCT can be done in several steps:

1. Apply DCT to the image to transform it from the spatial domain to the frequency domain.
2. Identify and remove or reduce the high frequency components that correspond to noise.
3. Apply the inverse DCT to the modified frequency domain image to transform it back to the spatial domain.

The decomposed image resulting from the DCT decomposition step contains the noise present in it. Hard thresholding allows you to change the image coordinates. This allows you to remove noise by changing some of the coordinate values. The process of recovering the denoised image from the decomposed output can be done using the inverse process of DCT known as the IDCT transform. In doing so, the image is re-decomposed and denoised, giving the original image in the form of a denoised image.

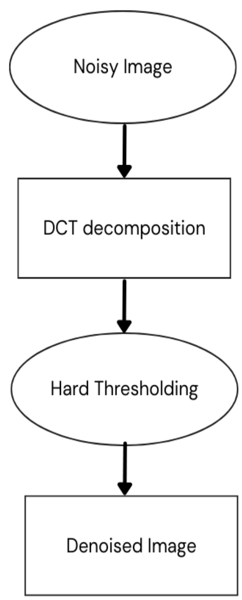

Figure 1: Block diagram of PCA denoising

6.4 Wavelet Transform

Wavelets identify features in data at various scales. Retain important signal or image features while removing noise. The basic idea behind wavelet denoising or wavelet thresholding is that the wavelet transform provides a sparse representation of many real-world signals and images. This means that wavelet transforms concentrate signal and image features into a small number of large wavelet coefficients. Small-valued wavelet coefficients are typically noise, and we can "shrink" or remove these coefficients without degrading the signal or image quality. After thresholding the coefficients, reconstruct the data using the inverse wavelet transform. In an image, edges are places where the image brightness changes abruptly. Preserving edges while denoising an image is important for perceptual quality. Traditional low-pass filtering removes noise, but it smoothes edges and often has a negative impact on image quality. Wavelets can remove noise while preserving perceptually important features.

Wavelet-based denoising methods are based on the fact that noise generally appears as fine structures in an image. Therefore, most noise tends to be represented by finer-scale wavelet coefficients. Discarding these coefficients naturally filters the noise based on scale. Coefficients of such scales also tend to be the primary carrier of edge information, so this method sets the he DWT coefficients to zero when the value falls below the threshold. Most of these coefficients correspond to noise. On the other hand, edge-related coefficients usually exceed the threshold. The inverse DWT of the threshold coefficients is the denoised image.

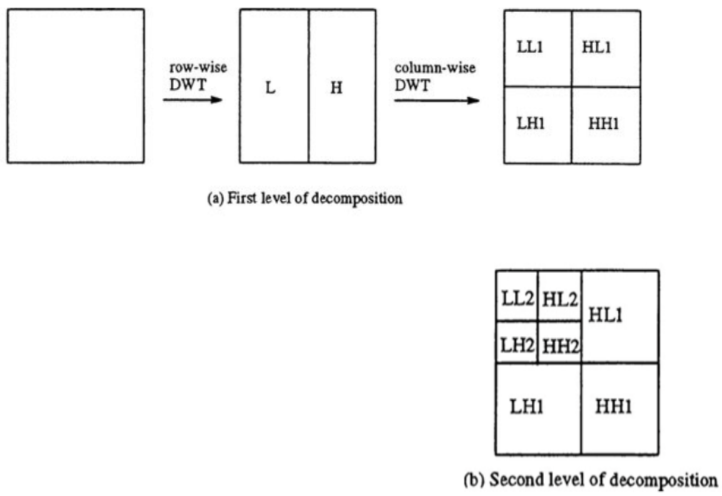

Figure 2: Block diagram of wavelet based Image Denoising System

Wavelet decomposition of an image is performed as follows. At the first stage of decomposition, the image is split into her four subbands: HH, HL, LH, and LL subbands. The HH subband shows oblique detail in the image. The HL subband represents horizontal features and the LH subband represents vertical structures. The LL subband is the low-resolution remainder consisting of the low-frequency content, and it is this subband that is further divided at higher decomposition levels.

6.5 Non-negative Matrix Factorization (NMF)

NMF is a factorization-based method that can be used to denoise images. The basic idea behind NMF is to represent an image as the product of two non-negative matrices. One represents the basis or feature image and the other the coefficients or weights.

In the context of image denoising, NMF can be used to decompose a noisy image into a low-order approximation of the clean image and a weak noise matrix. The low-rank approximation can be viewed as a denoised version of the image, and the low-noise matrix represents the residual noise.

Denoising an image using NMF typically involves the following steps:

1. Initialize the base and coefficient matrices with random nonnegative values.
2. Iteratively update the basis and coefficient matrices to best approximate the original image.
3. Multiply the basis matrix and the coefficient matrix to get a low-order approximation of the image.
4. Extract the sparse noise matrix by subtracting the low-rank approximation from the original image.

Given an $u \times v$ matrix A with nonnegative elements, we wish to find nonnegative, rank-k matrices W $(u \times k)$ and H $(k \times v)$ such that

$$A \approx WH \tag{14}$$

We typically hope that a good approximation can be achieved with

$$k \ll \operatorname{rank}(A) \tag{15}$$

The benefits range from compression (W and H are much smaller than A, which in some applications can be huge) to avoidance of "overfitting" in a prediction context. NMF has proven effective in denoising images corrupted by various types of noise. This has the advantage of preserving the non-negativity and parsimony of the image, allowing better preservation of original features and details. Additionally, it can be used for feature extraction, which is an important step in image classification, segmentation, and retrieval.

7 Experimental Results

7.1 Simulation environment and comparative analysis

The following subsections provide qualitative and quantitative analyzes of state-of-the-art Gaussian, salt-and-pepper, and speckle denoising techniques on various benchmark images at various noise densities. All decomposition techniques are implemented in MALTAB and simulated with benchmark inputs. First, all noises with different noise densities (wide range: 10% to 50%) are added to the benchmark image. The resulting noisy images are then decomposed by these decomposition techniques to extract quality metrics. Finally, both quantitative and qualitative analyzes are performed based on the extracted quality indicators and reconstructed images. Benchmark image considered for analysis and simulation results was a Pomegranate image.

Figure 3: Pomegranate image

7.1.1 For Gaussian Noise

Table 1 shows that the noise density ranges from wide (10% to 50%).It is clear from the quality index that Wavelet transform has the best noise reduction capability compared to the other methods.

For a better comparative analysis, Figure 4 shows the graphs of Table 1.Therefore, Wavelet transform has better noise rejection at not only low noise densities but also at high noise densities.

For qualitative analysis, filtered images (corrupted at 10% noise density) using different methods are shown in Figure 5. From these denoised images, we can see that the Wavelet-denoised image retains more details compared to the others. From this we can conclude that Wavelet transform has the best performance for grayscale Pomegranate images.

Table 1: Average PSNR and SSIM of Pomegranate image with gaussian noise at different noise levels

METRICS	METHOD	**10%**	**20%**	**30%**	**40%**	**50%**
PSNR	PCA	21.9965	18.5247	15.8735	13.9318	12.5189
	SVD	22.3833	18.6599	15.9400	13.9691	12.5400
	DCT	22.8177	18.8594	16.0331	14.0158	12.5674
	WAVELET	24.7507	20.0377	16.8348	14.6229	13.0634
	NMF	18.6328	15.7875	14.1767	12.7723	11.8909
SSIM	PCA	0.5720	0.7641	0.8068	0.8175	0.8165
	SVD	0.6365	0.7620	0.7897	0.7985	0.7894
	DCT	0.4614	0.5551	0.5892	0.6264	0.6681
	WAVELET	0.8335	0.8598	0.8555	0.8437	0.8271
	NMF	0.6327	0.6185	0.6837	0.6803	0.7255

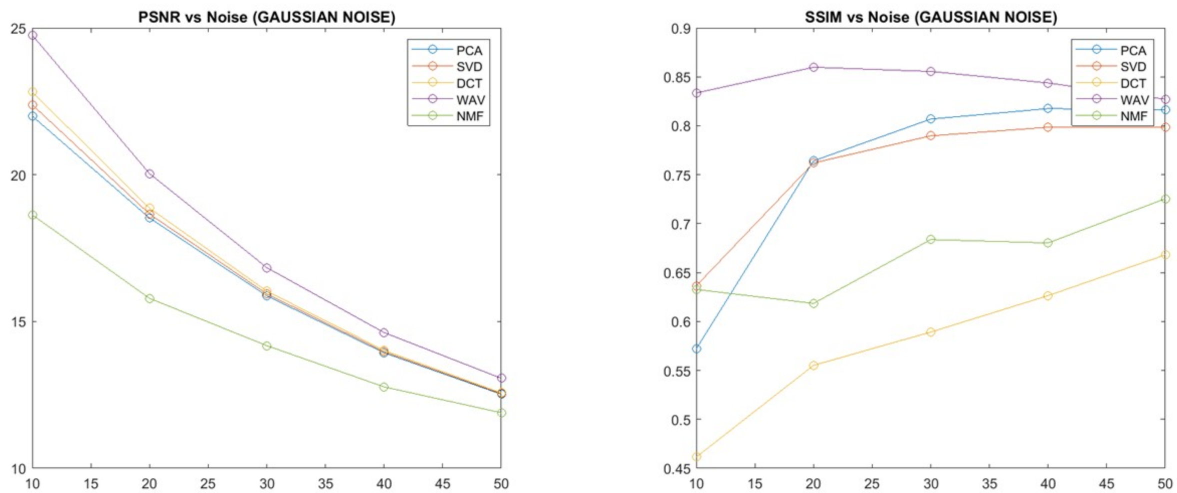

Figure 4: PSNR and SSIM value plots of Pomegranate image using various methods at different noise density

7.1.2 For Salt and Pepper Noise

Table 2 shows that the noise density ranges from wide (10% to 50%).It is clear from the quality index that Wavelet transform has the best noise reduction capability compared to the other methods.

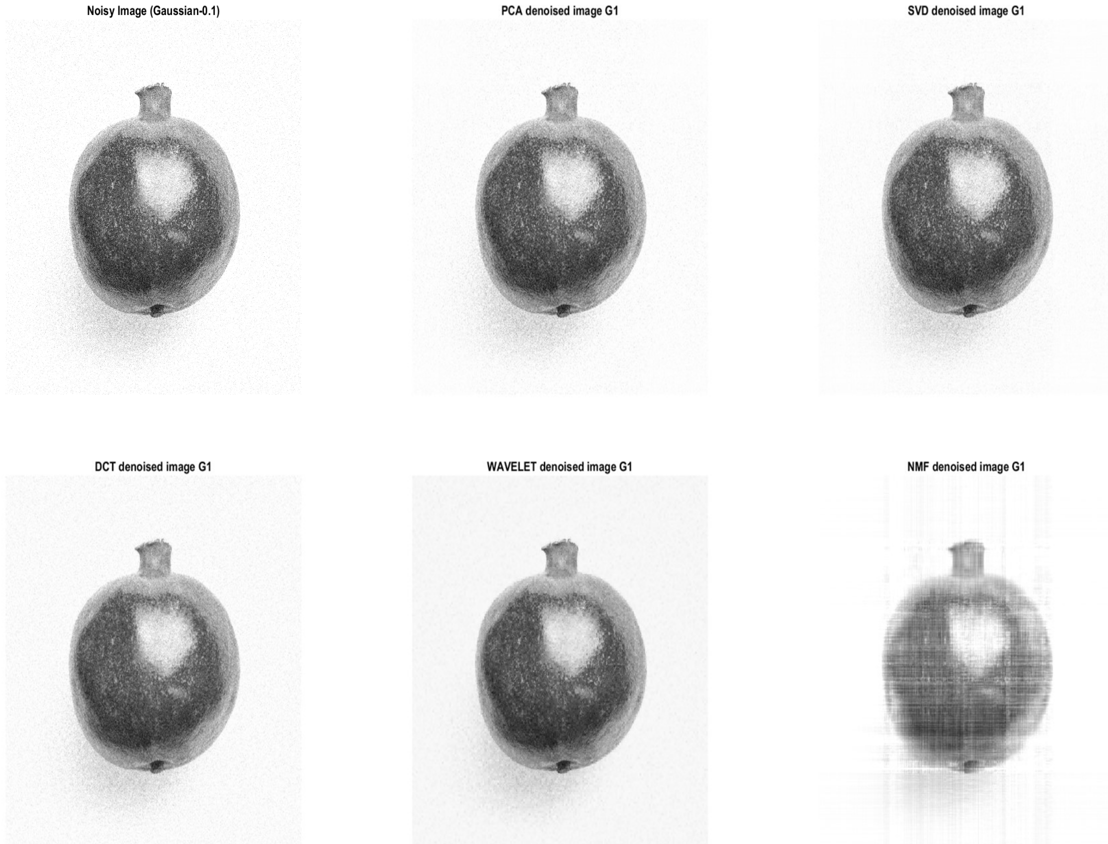

Figure 5: (a) Noisy Image (10%) (b) PCA (c) SVD (d) DCT (e) Wavelet (f) NMF

Table 2: Average PSNR and SSIM of Pomegranate image with salt and pepper noise at different noise levels

METRICS	METHOD	**10%**	**20%**	**30%**	**40%**	**50%**
PSNR	PCA	13.6068	10.6028	8.8402	7.5796	6.6387
	SVD	13.6550	10.6200	8.8496	7.5856	6.6434
	DCT	13.7193	10.6463	8.8643	7.5655	6.6505
	WAVELET	15.1626	11.9686	10.0270	8.6208	7.5595
	NMF	19.9769	17.8147	15.9585	14.1441	12.8347
SSIM	PCA	0.1253	0.0481	0.0287	0.0481	0.0140
	SVD	0.1178	0.0479	0.0286	0.0192	0.0140
	DCT	0.1112	0.0475	0.0286	0.0192	0.0140
	WAVELET	0.1478	0.0618	0.0376	0.0255	0.0183
	NMF	0.2027	0.1270	0.0948	0.0769	0.0654

For a better comparative analysis, Figure 6 shows the graphs of Table 2.Therefore, Wavelet transform has better noise rejection at not only low noise densities but also at high noise densities.

For qualitative analysis, filtered images (corrupted at 10% noise density) using different methods are shown in Figure 7. From these denoised images, we can see that the NMF-denoised image retains more details by smoothening it a bit compared to the others. From this we can conclude that NMF has the best performance for grayscale Pomegranate images.

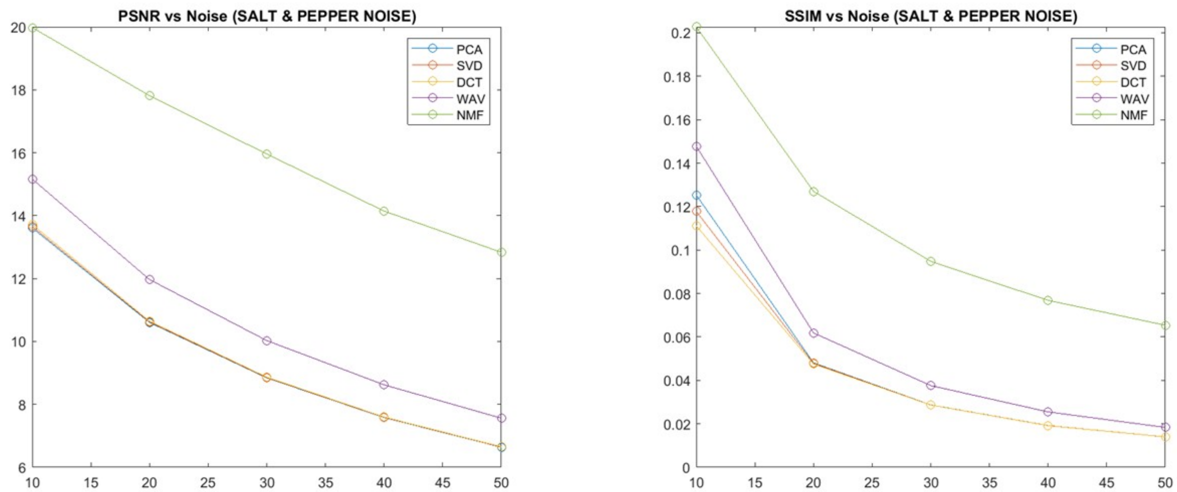

Figure 6: PSNR and SSIM value plots of Pomegranate image using various methods at different noise density

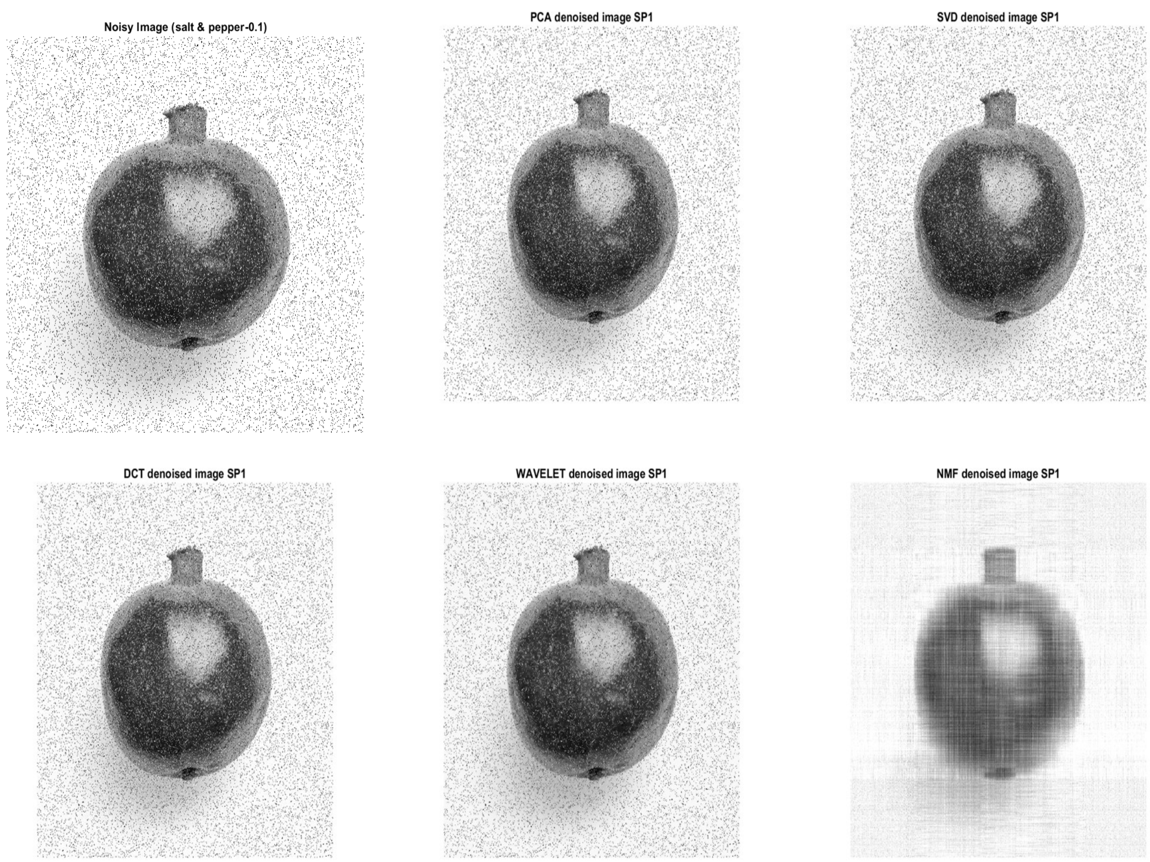

Figure 7: (a) Noisy Image (10%) (b) PCA (c) SVD (d) DCT (e) Wavelet (f) NMF

7.1.3 For Speckle Noise

Table 3 shows that the noise density ranges from wide (10% to 50%).It is clear from the quality index that Wavelet transform has the best noise reduction capability compared to the other methods.

Table 3: Average PSNR and SSIM of Pomegranate image with speckle noise at different noise levels

METRICS	METHOD	**10%**	**20%**	**30%**	**40%**	**50%**
PSNR	PCA	13.8017	10.8543	9.1166	7.9911	7.3508
	SVD	13.8571	10.8753	9.1278	7.9990	7.3571
	DCT	13.9357	11.9938	9.1466	8.0116	7.3673
	WAVELET	15.2148	19.4703	10.0931	8.8673	8.1699
	NMF	18.0263	15.6529	14.0781	13.0508	12.3712
SSIM	PCA	0.1168	0.0739	0.0546	0.0440	0.0379
	SVD	0.1152	0.0731	0.0543	0.0439	0.0378
	DCT	0.1134	0.0725	0.0538	0.0436	0.0376
	WAVELET	0.1753	0.1106	0.0816	0.0652	0.05588
	NMF	0.2493	0.1594	0.1162	0.0998	0.0903

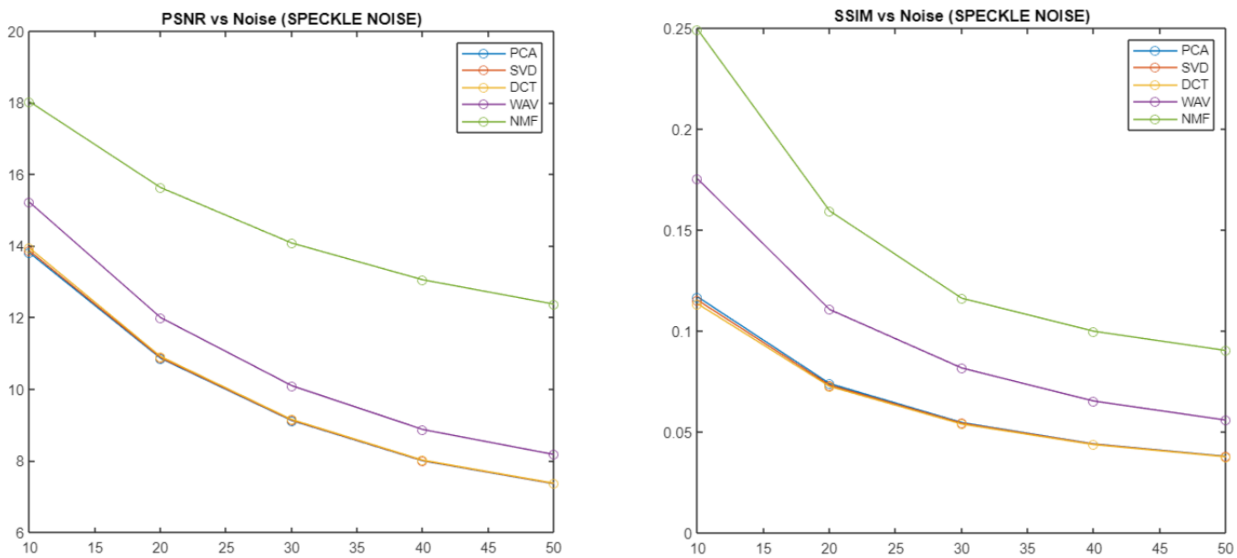

Figure 8: PSNR and SSIM value plots of Pomegranate image using various methods at different noise density

For a better comparative analysis, Figure 8 shows the graphs of Table 3.Therefore, Wavelet transform has better noise rejection at not only low noise densities but also at high noise densities.

For qualitative analysis, filtered images (corrupted at 10% noise density) using different methods are shown in Figure 9. From these denoised images, we can see that the NMF-denoised image retains more details by smoothening it a bit compared to the others. From this we can conclude that NMF has the best performance for grayscale Pomegranate images.

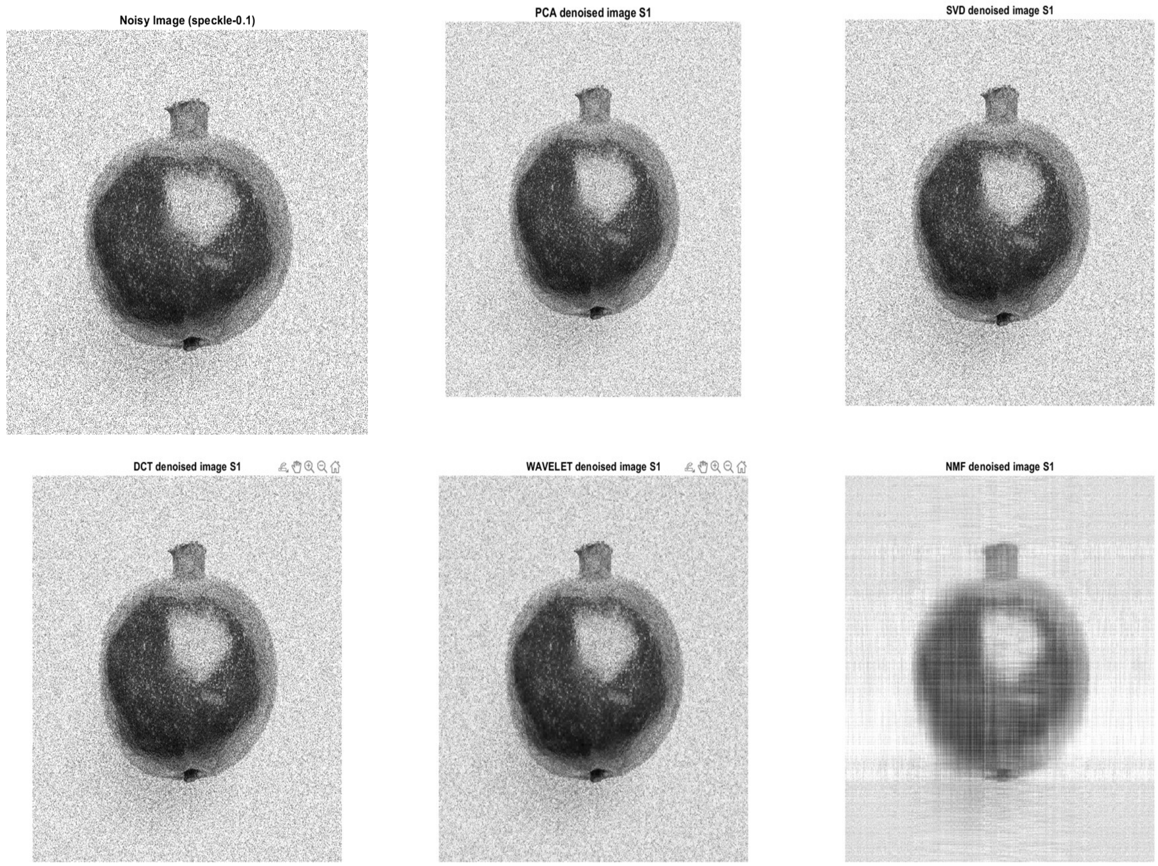

Figure 9: (a) Noisy Image (10%) (b) PCA (c) SVD (d) DCT (e) Wavelet (f) NMF

8 Conclusion

For the gaussian noise we can clearly see that Wavelet transform outperforms all the remaining algorithms. This is because of the main advantages of wavelet transforms is the ability to decompose an image into different frequency subbands. Gaussian noise is a random process spread evenly across all frequencies. By separating the image into different frequency subbands, the wavelet transform effectively separates noise from useful information in the image.

The wavelet transform is also a time-frequency representation. That is, you get a sparse representation of the image. This is especially useful for noise reduction as it can effectively separate noise from useful information in the image.

For the salt and pepper noise we can clearly see that NMF performs well. Salt-and-pepper noise is a type of impulsive noise that occurs in images. It is characterized by a small number of randomly placed, large-amplitude errors in the image. The noise is concentrated in a small number of pixels, and the remaining pixels are mostly noise-free.

NMF can effectively handle salt-and-pepper noise because it can separate the noise from the useful information in the image. By factoring the image into non-negative factors, NMF can effectively separate the noise from the useful information in the image. Additionally, by imposing non-negativity constraints on the factors, NMF can effectively remove the noise while preserving the useful information in the image.

Even for the speckle noise it's NMF as it was explained above.

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