Numerical Techiniques and statistical methods

Assignment - I

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① Find the value of $\sqrt[3]{19}$ by using Newton Raphson method.

Given

$x = \sqrt[3]{19}$

Cubing on both sides

$x^3 = 19 \Rightarrow x^3 - 19 = 0$

consider

$f(x) = x^3 - 19 = 0$

$f(0) = (0)^3 - 19 = -19$ (-ve)

$f(1) = (1)^3 - 19 = -18$ (-ve)

$f(2) = (2)^3 - 19 = -11$ (-ve)

$f(3) = (3)^3 - 19 = 8$ (+ve)

$\therefore f(2)$ is (-ve) and $f(3)$ is (+ve)

The real roots are lies b/w $(2,3)$

$\Rightarrow n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{(-11)}{3(2)^2} = 2.9167$$

$\Rightarrow n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.688$$

$\Rightarrow n = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.668.$$

$\Rightarrow n = 3$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 2.668$$

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(2) From the following data, find $y(0.373)$ by using Newton's backward formula

x	0.1	0.2	0.3	0.4	0.5
y	9.9833	4.9667	3.2836	2.9339	1.9177

We know that

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots \rightarrow (1)$$

$$u = \frac{x - x_n}{h} = \frac{0.373 - 0.5}{0.1} = -1.2730$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.1	9.9833				
		-5.017			
			3.334		
0.2	4.9667				
		-1.683		-2.501	
0.3	3.2836		0.833		
		-0.850			2.002
0.4	2.4339			-0.499	$\nabla^4 y_n$
			0.334	$\nabla^3 y_n$	
		-0.516	$\nabla^2 y_n$		
0.5	1.9177	∇y_n			

From (1)

$$y(0.373) = 1.9177 + (-1.2730)(-0.516) + \frac{(-1.2730)(-1.2730+1)}{2!} \times (0.334)$$

$$+ \frac{(-1.2730)(-1.2730+1)(-1.2730+2)}{3!} \times (0.499) + \frac{(-1.2730)(-1.2730+1)(-1.2730+2)(-1.2730+3)}{4!} \times (2.002)$$

$= 1.9177 + 0.655 + 0.057 - 0.021 + 0.036$

$= 2.645$

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(3) Evaluate $\int_0^1 \sqrt{1+x^2}\, dx$ using Simpson's 3/8th rule

Let consider $n = 6$

$$h = \frac{x_0 - x_n}{n} = \frac{1-0}{6} = 1/6$$

$x_0 = 0$	$x_1 = x_0 + h = 1/6$	$x_2 = x_0 + 2h = 2/6$	$x_3 = x_0 + 3h = 3/6$	$x_4 = x_0 + 4h = 4/6$
$y_0 = 1$	$y_1 = 1.028$	$y_2 = 1.111$	$y_3 = 1.250$	$y_4 = 1.44$

$x_5 = x_0 + 5h = 5/6$	$x_6 = x_0 + 6h = 6(\frac{1}{6}) = 1$
$y_5 = 1.694$	$y_6 = 2$

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\ldots) + 2(y_3+y_6+y_9)\right]$$

$$= \frac{3(1/6)}{8}\left[(1+2)+3(1.028+1.111+1.44)+2(1.250)\right]$$

$$= \frac{1}{16}\left(3+3(3.579)+2(1.250)\right)$$

$$= 1.015$$

4. Solve the IVP $y' = 1+2y$ for $y(1.1)$ by using Euler's series method $y(1) = 2$

We know that

$$y_{n+1}(x_{n+1}) = y_n + hf(x_0, y_n) \,;\, n = 0, 1, 2 \ldots$$

$x_0 = 1$; $y_0 = 2$

$h = \frac{x_n - x_0}{n} = \frac{(1.1-1)}{4}$

$= 0.025$

$x_1 = x_0 + h = 1 + 0.025$

$x_1 = 1.025$

$\Rightarrow n = 0$

$y_1(x_1) = y_0 + hf(x_0, y_0)$

$y_1(1.025) = 2 + 0.025\, f(1,2)$

$= 2 + 0.025(1+2(2))$

$y_1(1.025) = 2.125$

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$\Rightarrow n=1$

$y_2(x_2) = y_1 + hf(x_1, y_1)$

$x_2 = x_1 + h$
$= 1.025 + 0.025$
$= 1.050$

$y_2(1.050) = 2.125 + 0.025(1 + 2(2.125))$

$y_2(1.050) = 2.256$

$\Rightarrow n=2$

$y_3(x_3) = y_2 + hf(x_2, y_2)$

$x_3 = x_2 + h$
$= 1.050 + 0.025$
$= 1.075$

$y_3(1.075) = 2.256 + 0.025(1 + 2(2.256))$

$y_3(1.075) = 2.394$

$\Rightarrow n=3$

$y_4(x_4) = y_3 + hf(x_3, y_3)$

$x_4 = x_3 + h$
$= 1.075 + 0.025$
$= 1.100$

$y_4(1.100) = 2.394 + 0.025(1 + 2(2.394))$

$y_4(1.100) = 2.539$

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