Chi-Square Tests and the *F*-Distribution

Goodness of Fit

Objectives

- Use the chi-square distribution to test whether a frequency distribution fits a claimed distribution

Multinomial Experiments

Multinomial experiment

- A probability experiment consisting of a fixed number of trials in which there are more than two possible outcomes for each independent trial.
- A **binomial** experiment had only two possible outcomes.
- The probability for each outcome is fixed and each outcome is classified into **categories**.

Multinomial Experiments

Example:

- A radio station claims that the distribution of music preferences for listeners in the broadcast region is as shown below.

Distribution of music Preferences			
Classical	4%	Oldies	2%
Country	36%	Pop	18%
Gospel	11%	Rock	29%

Each outcome is classified into **categories**.

The probability for each possible outcome is fixed.

Chi-Square Goodness-of-Fit Test

Chi-Square Goodness-of-Fit Test

- Used to test whether a frequency distribution fits an expected distribution.
- The null hypothesis states that the frequency distribution fits the specified distribution.
- The alternative hypothesis states that the frequency distribution does not fit the specified distribution.

Chi-Square Goodness-of-Fit Test

Example:

- To test the radio station's claim, the executive can perform a chi-square goodness-of-fit test using the following hypotheses.

H_0: The distribution of music preferences in the broadcast region is 4% classical, 36% country, 11% gospel, 2% oldies, 18% pop, and 29% rock. (claim)

H_a: The distribution of music preferences differs from the claimed or expected distribution.

Chi-Square Goodness-of-Fit Test

- To calculate the test statistic for the chi-square goodness-of-fit test, the observed frequencies and the expected frequencies are used.
- The **observed frequency *O*** of a category is the frequency for the category observed in the sample data.

Chi-Square Goodness-of-Fit Test

- The **expected frequency *E*** of a category is the *calculated* frequency for the category.
  - Expected frequencies are obtained assuming the specified (or hypothesized) distribution. The expected frequency for the i^{th} category is

    $$E_i = np_i$$

    where n is the number of trials (the sample size) and p_i is the assumed probability of the i^{th} category.

Example: Finding Observed and Expected Frequencies

A marketing executive randomly selects 500 radio music listeners from the broadcast region and asks each whether he or she prefers classical, country, gospel, oldies, pop, or rock music. The results are shown at the right. Find the observed frequencies and the expected frequencies for each type of music.

Survey results (n = 500)	
Classical	8
Country	210
Gospel	72
Oldies	10
Pop	75
Rock	125

Solution: Finding Observed and Expected Frequencies

Observed frequency: The number of radio music listeners naming a particular type of music

Survey results (n = 500)	
Classical	8
Country	210
Gospel	72
Oldies	10
Pop	75
Rock	125

observed frequency

Solution: Finding Observed and Expected Frequencies

Expected Frequency: $E_i = np_i$

Type of music	% of listeners	Observed frequency	Expected frequency
Classical	4%	8	500(0.04) = 20
Country	36%	210	500(0.36) = 180
Gospel	11%	72	500(0.11) = 55
Oldies	2%	10	500(0.02) = 10
Pop	18%	75	500(0.18) = 90
Rock	29%	125	500(0.29) = 145

$n = 500$

Chi-Square Goodness-of-Fit Test

For the chi-square goodness-of-fit test to be used, the following must be true.

1. The observed frequencies must be obtained by using a random sample.
2. Each expected frequency must be greater than or equal to 5.

Chi-Square Goodness-of-Fit Test

- If these conditions are satisfied, then the sampling distribution for the goodness-of-fit test is approximated by a chi-square distribution with $k - 1$ degrees of freedom, where k is the number of categories.
- The **test statistic** for the chi-square goodness-of-fit test is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

The test is always a right-tailed test.

where O represents the observed frequency of each category and E represents the expected frequency of each category.

Chi-Square Goodness-of-Fit Test

In Words	*In Symbols*
1. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom.	d.f. = k – 1
4. Determine the critical value.	Use Table 6 in Appendix B.

Chi-Square Goodness-of-Fit Test

In Words	*In Symbols*
5. Determine the rejection region.	
6. Calculate the test statistic.	$\chi^2 = \sum \frac{(O-E)^2}{E}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Example: Performing a Goodness of Fit Test

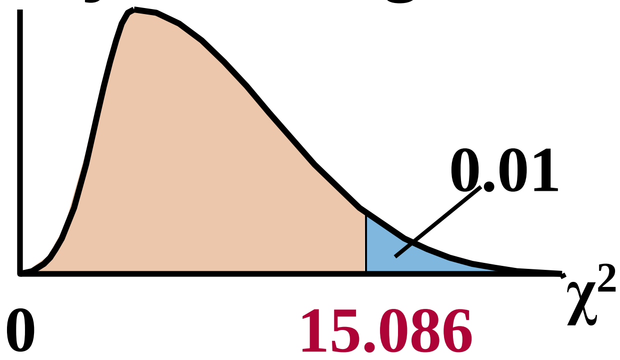
Use the music preference data to perform a chi-square goodness-of-fit test to test whether the distributions are different. Use $\alpha = 0.01$.

Distribution of music preferences	
Classical	4%
Country	36%
Gospel	11%
Oldies	2%
Pop	18%
Rock	29%

Survey results (n = 500)	
Classical	8
Country	210
Gospel	72
Oldies	10
Pop	75
Rock	125

Solution: Performing a Goodness of Fit Test

- H_0: music preference is 4% classical, 36% country, 11% gospel, 2% oldies, 18% pop, and 29% rock
- H_a: music preference differs from the claimed or expected distribution
- $\alpha = 0.01$
- d.f. = 6 – 1 = 5
- Rejection Region



- Test Statistic:
- Decision:
- Conclusion:

Solution: Performing a Goodness of Fit Test

Type of music	Observed frequency	Expected frequency
Classical	8	20
Country	210	180
Gospel	72	55
Oldies	10	10
Pop	75	90
Rock	125	145

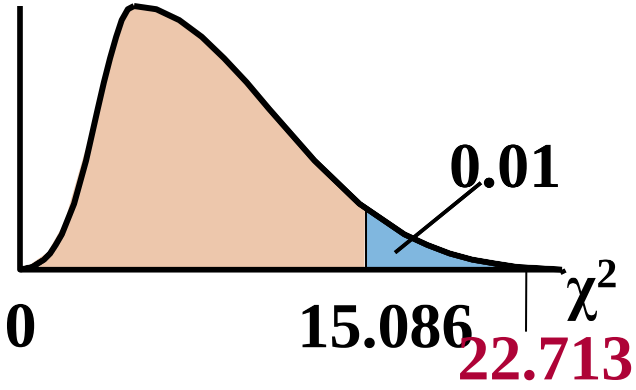
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(8-20)^2}{20} + \frac{(210-180)^2}{180} + \frac{(72-55)^2}{55} + \frac{(10-10)^2}{10} + \frac{(75-90)^2}{90} + \frac{(125-145)^2}{145}$$

$$\approx 22.713$$

Solution: Performing a Goodness of Fit Test

- ***H_0*: music preference is 4% classical, 36% country, 11% gospel, 2% oldies, 18% pop, and 29% rock**
- ***H_a*: music preference differs from the claimed or expected distribution**
- **α = 0.01**
- **d.f. = 6 – 1 = 5**
- **Rejection Region**



- **Test Statistic:**

  $\chi^2 = 22.713$

- **Decision: Reject *H_0***

There is enough evidence to conclude that the distribution of music preferences differs from the claimed distribution.

Example: Performing a Goodness of Fit Test

The manufacturer of M&M's candies claims that the number of different-colored candies in bags of dark chocolate M&M's is uniformly distributed. To test this claim, you randomly select a bag that contains 500 dark chocolate M&M's. The results are shown in the table on the next slide. Using $\alpha = 0.10$, perform a chi-square goodness-of-fit test to test the claimed or expected distribution. What can you conclude? *(Adapted from Mars Incorporated)*

Example: Performing a Goodness of Fit Test

Color	Frequency
Brown	80
Yellow	95
Red	88
Blue	83
Orange	76
Green	78

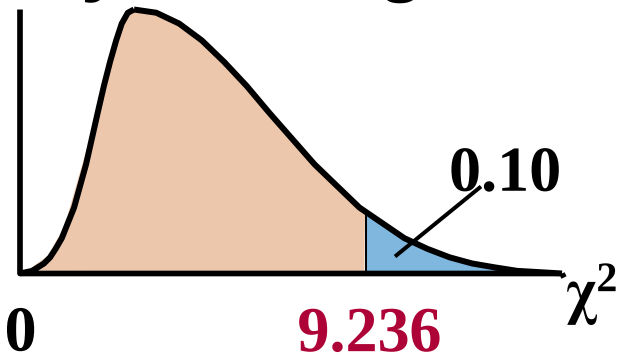
$n = 500$

Solution:

- The claim is that the distribution is uniform, so the expected frequencies of the colors are equal.
- To find each expected frequency, divide the sample size by the number of colors.
- $\boldsymbol{E = 500/6 \approx 83.3}$

Solution: Performing a Goodness of Fit Test

- H_0: **Distribution of different-colored candies in bags of dark chocolate M&Ms is uniform**
- H_a: **Distribution of different-colored candies in bags of dark chocolate M&Ms is not uniform**
- α = **0.10**
- **d.f. = 6 – 1 = 5**
- **Rejection Region**



- **Test Statistic:**
- **Decision:**
- **Conclusion:**

Solution: Performing a Goodness of Fit Test

Color	Observed frequency	Expected frequency
Brown	80	83.3
Yellow	95	83.3
Red	88	83.3
Blue	83	83.3
Orange	76	83.3
Green	78	83.3

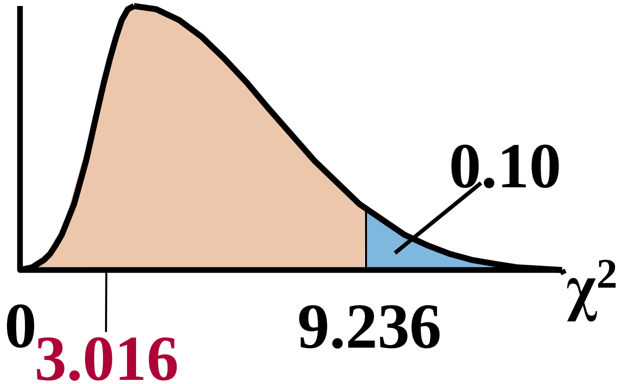
$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(80-83.3)^2}{83.3} + \frac{(95-83.3)^2}{83.3} + \frac{(88-83.3)^2}{83.3} + \frac{(83-83.3)^2}{83.3} + \frac{(76-83.3)^2}{83.3} + \frac{(78-83.3)^2}{83.3}$$

$$\approx 3.016$$

Solution: Performing a Goodness of Fit Test

- H_0: Distribution of different-colored candies in bags of dark chocolate M&Ms is uniform
- H_a: Distribution of different-colored candies in bags of dark chocolate M&Ms is not uniform
- α = 0.01
- d.f. = 6 – 1 = 5
- Rejection Region



- Test Statistic:

  $\chi^2 = 3.016$

- Decision: Fail to Reject H_0

There is not enough evidence to dispute the claim that the distribution is uniform.

Summary

- Used the chi-square distribution to test whether a frequency distribution fits a claimed distribution

Independence

Objectives

- Use a contingency table to find expected frequencies
- Use a chi-square distribution to test whether two variables are independent

Contingency Tables

$r \times c$ contingency table

- Shows the observed frequencies for two variables.
- The observed frequencies are arranged in *r* rows and *c* columns.
- The intersection of a row and a column is called a **cell**.

Contingency Tables

Example:

- The contingency table shows the results of a random sample of 550 company CEOs classified by age and size of company.*(Adapted from Grant Thornton LLP, The Segal Company)*

	Age				
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over
Small / Midsize	42	69	108	60	21
Large	5	18	85	120	22

Finding the Expected Frequency

- Assuming the two variables are independent, you can use the contingency table to find the expected frequency for each cell.
- The expected frequency for a cell $E_{r,c}$ in a contingency table is

$$\text{Expected frequency } E_{r,c} = \frac{(\text{Sum of row } r) \times (\text{Sum of column } c)}{\text{Sample size}}$$

Example: Finding Expected Frequencies

Find the expected frequency for each cell in the contingency table. Assume that the variables, age and company size, are independent.

	Age					
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over	Total
Small / Midsize	42	69	108	60	21	300
Large	5	18	85	120	22	250
Total	47	87	193	180	43	550

marginal totals

Solution: Finding Expected Frequencies

$$E_{r,c} = \frac{(\text{Sum of row } r) \times (\text{Sum of column } c)}{\text{Sample size}}$$

	Age					
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over	Total
Small / Midsize	42	69	108	60	21	300
Large	5	18	85	120	22	250
Total	47	87	193	180	43	550

$$E_{1,1} = \frac{300 \cdot 47}{550} \approx 25.64$$

Solution: Finding Expected Frequencies

	Age					
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over	Total
Small / Midsize	42	69	108	60	21	300
Large	5	18	85	120	22	250
Total	47	87	193	180	43	550

$$E_{1,2} = \frac{300 \cdot 87}{550} \approx 47.45 \qquad E_{1,3} = \frac{300 \cdot 193}{550} \approx 105.27$$

$$E_{1,4} = \frac{300 \cdot 180}{550} \approx 98.18 \qquad E_{1,5} = \frac{300 \cdot 43}{550} \approx 23.45$$

Solution: Finding Expected Frequencies

	Age					
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over	Total
Small / Midsize	42	69	108	60	21	300
Large	5	18	85	120	22	250
Total	47	87	193	180	43	550

$$E_{2,1} = \frac{250 \cdot 47}{550} \approx 21.36 \qquad E_{2,2} = \frac{250 \cdot 87}{550} \approx 39.55 \qquad E_{2,3} = \frac{250 \cdot 193}{550} \approx 87.73$$

$$E_{2,4} = \frac{250 \cdot 180}{550} \approx 81.82 \qquad E_{2,5} = \frac{250 \cdot 43}{550} \approx 19.55$$

Chi-Square Independence Test

Chi-square independence test

- Used to test the independence of two variables.
- Can determine whether the occurrence of one variable affects the probability of the occurrence of the other variable.

Chi-Square Independence Test

For the chi-square independence test to be used, the following must be true.

1. The observed frequencies must be obtained by using a random sample.
2. Each expected frequency must be greater than or equal to 5.

Chi-Square Independence Test

- If these conditions are satisfied, then the sampling distribution for the chi-square independence test is approximated by a chi-square distribution with $(r-1)(c-1)$ degrees of freedom, where r and c are the number of rows and columns, respectively, of a contingency table.
- The **test statistic** for the chi-square independence test is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

The test is always a right-tailed test.

where O represents the observed frequencies and E represents the expected frequencies.

Chi-Square Independence Test

In Words	*In Symbols*
1. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom.	d.f. = $(r - 1)(c - 1)$
4. Determine the critical value.	Use Table 6 in Appendix B.

Chi-Square Independence Test

In Words	*In Symbols*
5. Determine the rejection region.	
6. Calculate the test statistic.	$\chi^2 = \sum \frac{(O-E)^2}{E}$
7. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

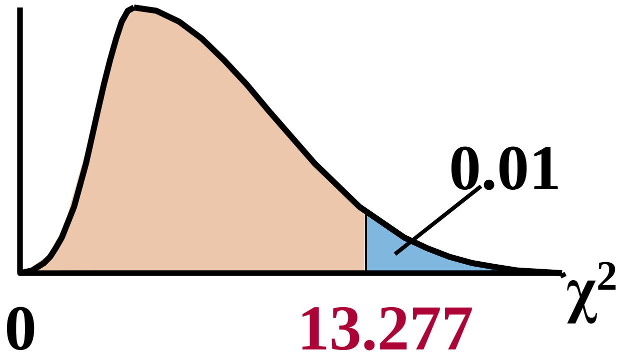
Example: Performing a χ^2 Independence Test

Using the age/company size contingency table, can you conclude that the CEOs ages are related to company size? Use $\alpha = 0.01$. Expected frequencies are shown in parentheses.

	Age					
Company size	39 and under	40 - 49	50 - 59	60 - 69	70 and over	Total
Small / Midsize	42 (25.64)	69 (47.45)	108 (105.27)	60 (98.18)	21 (23.45)	300
Large	5 (21.36)	18 (39.55)	85 (87.73)	120 (81.82)	22 (19.55)	250
Total	47	87	193	180	43	550

Solution: Performing a Goodness of Fit Test

- H_0: CEOs' ages are independent of company size
- H_a: CEOs' ages are dependent on company size
- $\alpha = 0.01$
- d.f. = $(2 - 1)(5 - 1) = 4$
- Rejection Region



- Test Statistic:
- Decision:

Solution: Performing a Goodness of Fit Test

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

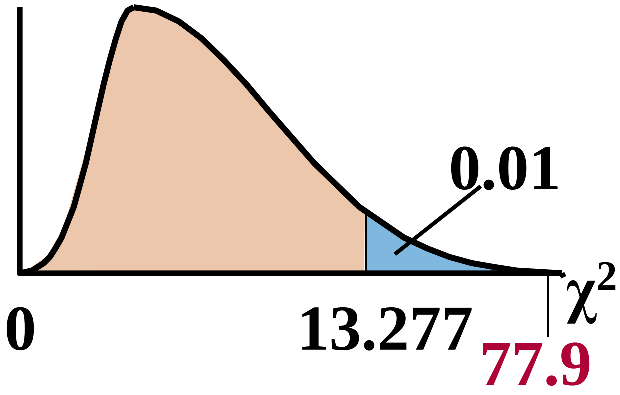
$$= \frac{(42-25.64)^2}{25.64} + \frac{(69-47.45)^2}{47.45} + \frac{(108-105.27)^2}{105.27} + \frac{(60-98.18)^2}{98.18} + \frac{(21-23.45)^2}{23.45}$$

$$+ \frac{(5-21.36)^2}{21.36} + \frac{(18-39.55)^2}{39.55} + \frac{(85-87.73)^2}{87.73} + \frac{(120-81.82)^2}{81.82} + \frac{(22-19.55)^2}{19.55}$$

$$\approx 77.9$$

Solution: Performing a Goodness of Fit Test

- H_0: CEOs' ages are independent of company size
- H_a: CEOs' ages are dependent on company size
- $\alpha = 0.01$
- d.f. = (2 – 1)(5 – 1) = 4
- Rejection Region



- Test Statistic:

  $\chi^2 = 77.9$

- Decision: Reject H_0

There is enough evidence to conclude CEOs' ages are dependent on company size.

Summary

- Used a contingency table to find expected frequencies
- Used a chi-square distribution to test whether two variables are independent

Comparing Two Variances

Objectives

- Interpret the *F*-distribution and use an *F*-table to find critical values
- Perform a two-sample *F*-test to compare two variances

F-Distribution

- Let s_1^2 and s_2^2 represent the sample variances of two different populations.
- If both populations are normal and the population variances σ_1^2 and σ_2^2 are equal, then the sampling distribution of

$$F = \frac{s_1^2}{s_2^2}$$

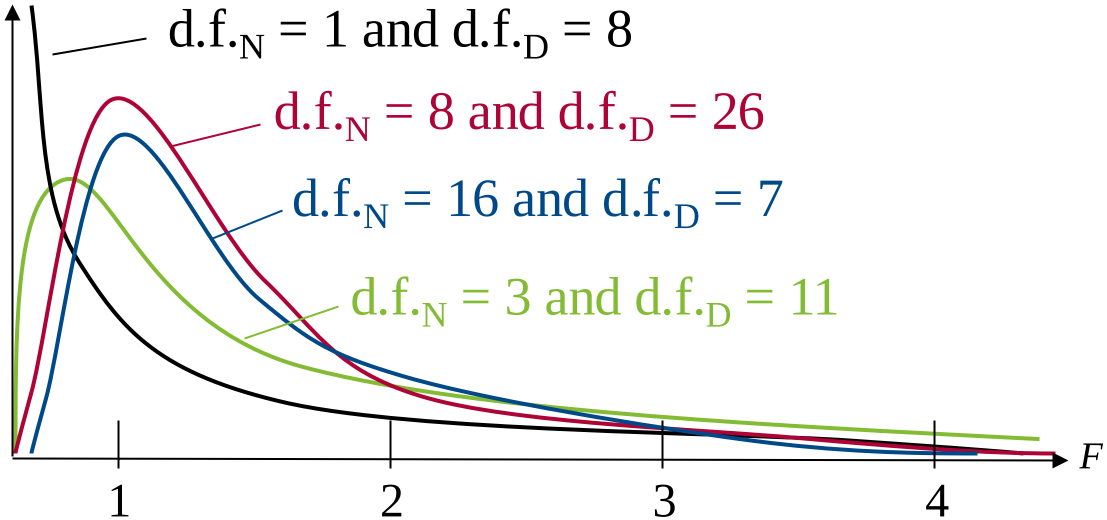
is called an ***F*-distribution**.

Properties of the *F*-Distribution

1. The *F*-distribution is a family of curves each of which is determined by two types of degrees of freedom:
   - The degrees of freedom corresponding to the variance in the numerator, denoted **d.f.$_N$**
   - The degrees of freedom corresponding to the variance in the denominator, denoted **d.f.$_D$**
2. *F*-distributions are positively skewed.
3. The total area under each curve of an *F*-distribution is equal to 1.

Properties of the *F*-Distribution

4. *F*-values are always greater than or equal to 0.
5. For all *F*-distributions, the mean value of *F* is approximately equal to 1.



Critical Values for the *F*-Distribution

1. Specify the level of significance α.
2. Determine the degrees of freedom for the numerator, d.f._N.
3. Determine the degrees of freedom for the denominator, d.f._D.
4. Use Table 7 in Appendix B to find the critical value. If the hypothesis test is
   a. one-tailed, use the α F-table.
   b. two-tailed, use the $\frac{1}{2}\alpha$ F-table.

Example: Finding Critical *F*-Values

Find the critical *F*-value for a right-tailed test when $\alpha = 0.05$, $d.f._N = 6$ and $d.f._D = 29$.

Solution:

$d.f._D$: Degrees of freedom, denominator	$\alpha = 0.05$												
	$d.f._N$: Degrees of freedom, numerator												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93

The critical value is $F_0 = 2.43$.

Example: Finding Critical *F*-Values

Find the critical *F*-value for a two-tailed test when $\alpha = 0.05$, $\text{d.f.}_N = 4$ and $\text{d.f.}_D = 8$.

Solution:

- When performing a two-tailed hypothesis test using the *F*-distribution, you need only to find the right-tailed critical value.
- You must remember to use the ½α table.

$$\frac{1}{2}\alpha = \frac{1}{2}(0.05) = 0.025$$

Solution: Finding Critical *F*-Values

½α = 0.025, $d.f._N = 4$ and $d.f._D = 8$

d.f.$_D$: Degrees of freedom, denominator	α = 0.025												
	d.f.$_N$: Degrees of freedom, numerator												
	1	2	3	4	5	6	7	8	9	10	12	15	20
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.4
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.1
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.5
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.5
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.1
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.4
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.0
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.6
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.4
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.2
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.1

The critical value is $F_0 = 5.05$.

Two-Sample *F*-Test for Variances

To use the two-sample *F*-test for comparing two population variances, the following must be true.

1. The samples must be randomly selected.
2. The samples must be independent.
3. Each population must have a normal distribution.

Two-Sample *F*-Test for Variances

- **Test Statistic**

$$F = \frac{s_1^2}{s_2^2}$$

  where s_1^2 and s_2^2 represent the sample variances with $s_1^2 \geq s_2^2$.
- The degrees of freedom for the numerator is d.f.$_N$ = $n_1 - 1$ where n_1 is the size of the sample having variance s_1^2.
- The degrees of freedom for the denominator is d.f.$_D$ = $n_2 - 1$, and n_2 is the size of the sample having variance s_2^2.

Two-Sample *F*-Test for Variances

In Words	*In Symbols*
1. Identify the claim. State the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Identify the degrees of freedom.	$\text{d.f.}_N = n_1 - 1$ $\text{d.f.}_D = n_2 - 1$
4. Determine the critical value.	Use Table 7 in Appendix B.

Two-Sample *F*-Test for Variances

In Words	*In Symbols*
5. Determine the rejection region.	
6. Calculate the test statistic.	$F = \dfrac{s_1^2}{s_2^2}$
7. Make a decision to reject or fail to reject the null hypothesis.	If F is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

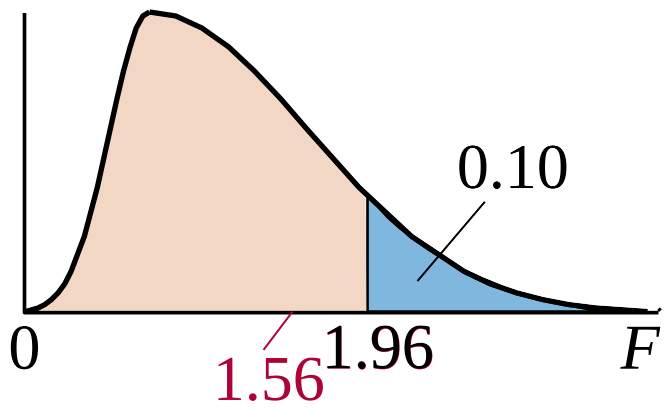
Example: Performing a Two-Sample *F*-Test

A restaurant manager is designing a system that is intended to decrease the variance of the time customers wait before their meals are served. Under the old system, a random sample of 10 customers had a variance of 400. Under the new system, a random sample of 21 customers had a variance of 256. At $\alpha = 0.10$, is there enough evidence to convince the manager to switch to the new system? Assume both populations are normally distributed.

Solution: Performing a Two-Sample *F*-Test

Because 400 > 256, $s_1^2 = 400$ and $s_2^2 = 256$

- H_0: $\sigma_1^2 \leq \sigma_2^2$
- H_a: $\sigma_1^2 > \sigma_2^2$
- $\alpha = 0.10$
- $d.f._N = 9$ $d.f._D = 20$
- **Rejection Region:**



- **Test Statistic:**

$$F = \frac{s_1^2}{s_2^2} = \frac{400}{256} \approx 1.56$$

- **Decision:** **Fail to Reject H_0**

There is not enough evidence to convince the manager to switch to the new system.

Example: Performing a Two-Sample *F*-Test

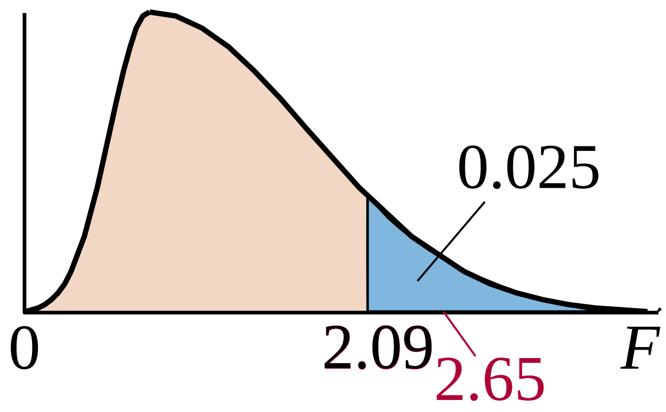
You want to purchase stock in a company and are deciding between two different stocks. Because a stock's risk can be associated with the standard deviation of its daily closing prices, you randomly select samples of the daily closing prices for each stock to obtain the results. At α = 0.05, can you conclude that one of the two stocks is a riskier investment? Assume the stock closing prices are normally distributed.

Stock A	Stock B
$n_2 = 30$	$n_1 = 31$
$s_2 = 3.5$	$s_1 = 5.7$

Solution: Performing a Two-Sample *F*-Test

Because $5.7^2 > 3.5^2$, $s_1^2 = 5.7^2$ and $s_2^2 = 3.5^2$

- **H_0: $\sigma_1^2 = \sigma_2^2$**
- **H_a: $\sigma_1^2 \neq \sigma_2^2$**
- **$\frac{1}{2}\alpha$ = 0. 025**
- **d.f.$_N$= 30 d.f.$_D$= 29**
- **Rejection Region:**



- **Test Statistic:**

$$F = \frac{s_1^{\ 2}}{s_2^{\ 2}} = \frac{5.7^2}{3.5^2} \approx 2.65$$

- **Decision: Reject H_0**

There is enough evidence to support the claim that one of the two stocks is a riskier investment.

Summary

- Interpreted the F-distribution and used an F-table to find critical values
- Performed a two-sample F-test to compare two variances