Straight Lines

11^{th} Maths - Chapter 10

The following problem is question 09 from exercise 10.4:

1. Find the value of **p** so that the three lines $3x+y-2=0, px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Solution:
Given equations can be written in the form of $\mathbf{n}^\top \mathbf{x} = c$
Therefore,

$$\begin{pmatrix} p & 2 \end{pmatrix} \mathbf{x} = 3 \tag{1}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 2 \tag{2}$$

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = 3 \tag{3}$$

Matrix form of above equations (1), (2) and (3) is

$$\begin{pmatrix} p & 2 \\ 3 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \tag{4}$$

augmented matrix is

$$\begin{pmatrix} p & 2 & 3 \\ 3 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix} \tag{5}$$

$R_1 \to R_1 - 2R_2$

$$\begin{pmatrix} p-6 & 0 & -1 \\ 3 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix} \tag{6}$$

$R_2 \to 2R_2 - 3R_3$
$R_3 \to 3R_3 - 2R_2$

$$\begin{pmatrix} p-6 & 0 & -1 \\ 0 & 5 & -5 \\ 0 & -5 & 5 \end{pmatrix} \tag{7}$$

$R_3 \to R_2 + R_3$

$$\begin{pmatrix} p-6 & 0 & -1 \\ 0 & 5 & -5 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

$R_1 \to \frac{R_1}{p-6}$
$R_2 \to \frac{R_2}{5}$

$$\begin{pmatrix} 1 & 0 & \frac{-1}{p-6} \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{9}$$

Therefore, $\mathbf{x} = \begin{pmatrix} \frac{-1}{p-6} \\ -1 \end{pmatrix}$ if the lines (1), (2) and (3) intersects at $\mathbf{x}$ then,
By solving equation (2)

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 2 \tag{10}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} \frac{-1}{p-6} \\ \\ -1 \end{pmatrix} = 2 \tag{11}$$

By solving the above equation we get,

$$p = 5 \tag{12}$$

Therefore, equation (1) can be written as

$$\begin{pmatrix} 5 & 2 \end{pmatrix} \mathbf{x} = 3 \tag{13}$$

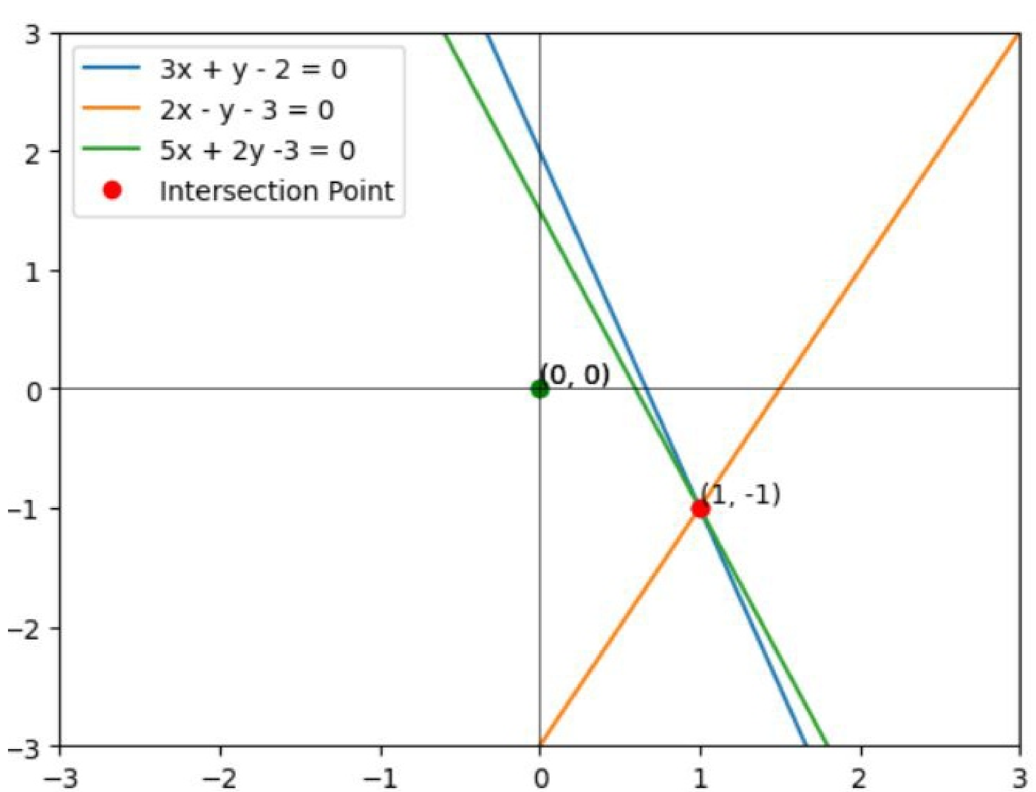

Figure 1: Straight-lines