

Straight Lines

11th Maths - Chapter 10

The following problem is question 09 from exercise 10.4:

1. Find the value of \mathbf{p} so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Solution:

Given equations can be written in the form of $\mathbf{n}^\top \mathbf{x} = c$

Therefore,

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 2 \quad (1)$$

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{x} = 3 \quad (2)$$

$$\begin{pmatrix} p & 2 \end{pmatrix} \mathbf{x} = 3 \quad (3)$$

Matrix form of above equations (1), (2) and (3) is

$$\begin{pmatrix} 3 & 1 \\ 2 & -1 \\ p & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix} \quad (4)$$

augmented matrix is

$$\begin{pmatrix} 3 & 1 & 2 \\ 2 & -1 & 3 \\ p & 2 & 3 \end{pmatrix} \quad (5)$$

$$R_1 \rightarrow R_1 + R_2$$

$$\begin{pmatrix} 5 & 0 & 5 \\ 2 & -1 & 3 \\ p & 2 & 3 \end{pmatrix} \tag{6}$$

$$R_1 \rightarrow \frac{R_1}{5}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ p & 2 & 3 \end{pmatrix} \tag{7}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ p & 2 & 3 \end{pmatrix} \tag{8}$$

$$R_3 \rightarrow R_3 - pR_1$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & 3-p \end{pmatrix} \tag{9}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 5-p \end{pmatrix} \tag{10}$$

Therefore, to satisfy Echelon form the bottom row must be zero. Then,

$$5 - p = 0 \quad (11)$$

$$p = 5 \quad (12)$$

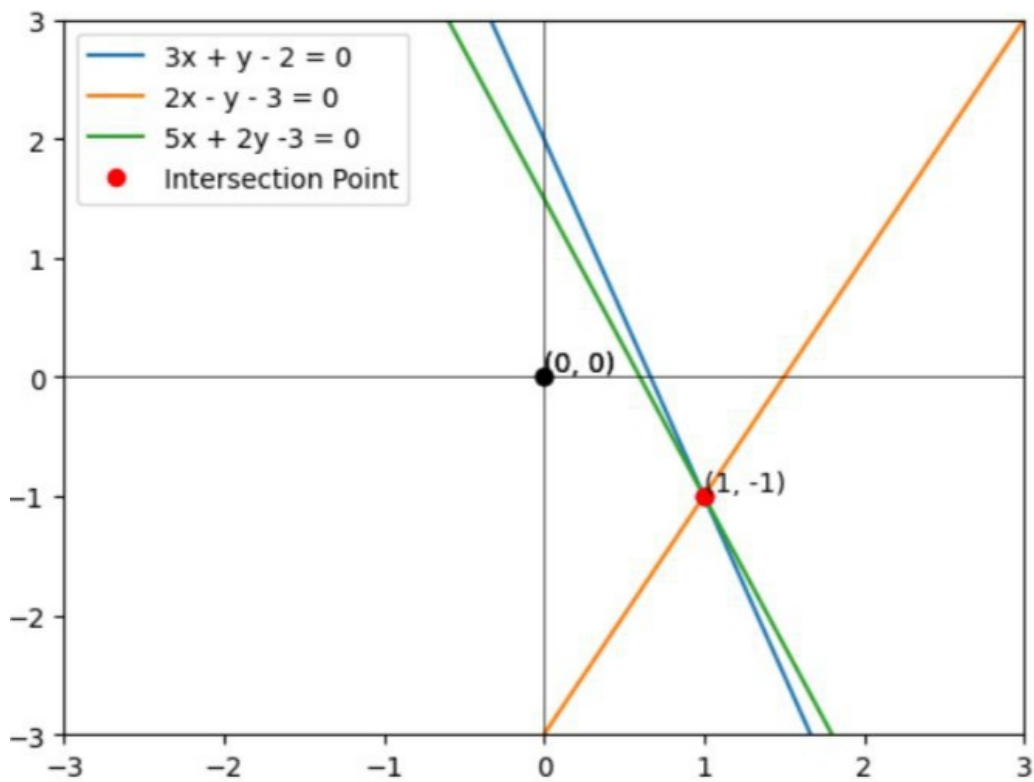


Figure 1: Straight-lines