Straight Lines

11^{th} Maths - Chapter 10

The following problem is question 09 from exercise 10.4:

1. Find the value of $\mathbf{p}$ so that the three lines $3x+y-2=0, px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Solution:
Given equations can be written in the form of $\mathbf{n}^\top\mathbf{x}=c$
Therefore,

$$\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x}=2 \tag{1}$$

$$\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{x}=3 \tag{2}$$

$$\begin{pmatrix}p & 2\end{pmatrix}\mathbf{x}=3 \tag{3}$$

Matrix form of above equations (1), (2) and (3) is

$$\begin{pmatrix}3 & 1\\ 2 & -1\\ p & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}2\\ 3\\ 3\end{pmatrix} \tag{4}$$

augmented matrix is

$$\begin{pmatrix}3 & 1 & 2\\ 2 & -1 & 3\\ p & 2 & 3\end{pmatrix} \tag{5}$$

$R_1 \rightarrow R_1 + R_2$

$$\begin{pmatrix} 5 & 0 & 5 \\ 2 & -1 & 3 \\ p & 2 & 3 \end{pmatrix} \tag{6}$$

$R_1 \rightarrow \frac{R_1}{5}$

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \\ p & 2 & 3 \end{pmatrix} \tag{7}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ p & 2 & 3 \end{pmatrix} \tag{8}$$

$R_3 \rightarrow R_3 - pR_1$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & 3-p \end{pmatrix} \tag{9}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 5-p \end{pmatrix} \tag{10}$$

Therefore, to satisfy Echelon form the bottom row must be zero. Then,

$$5 - p = 0 \tag{11}$$
$$p = 5 \tag{12}$$

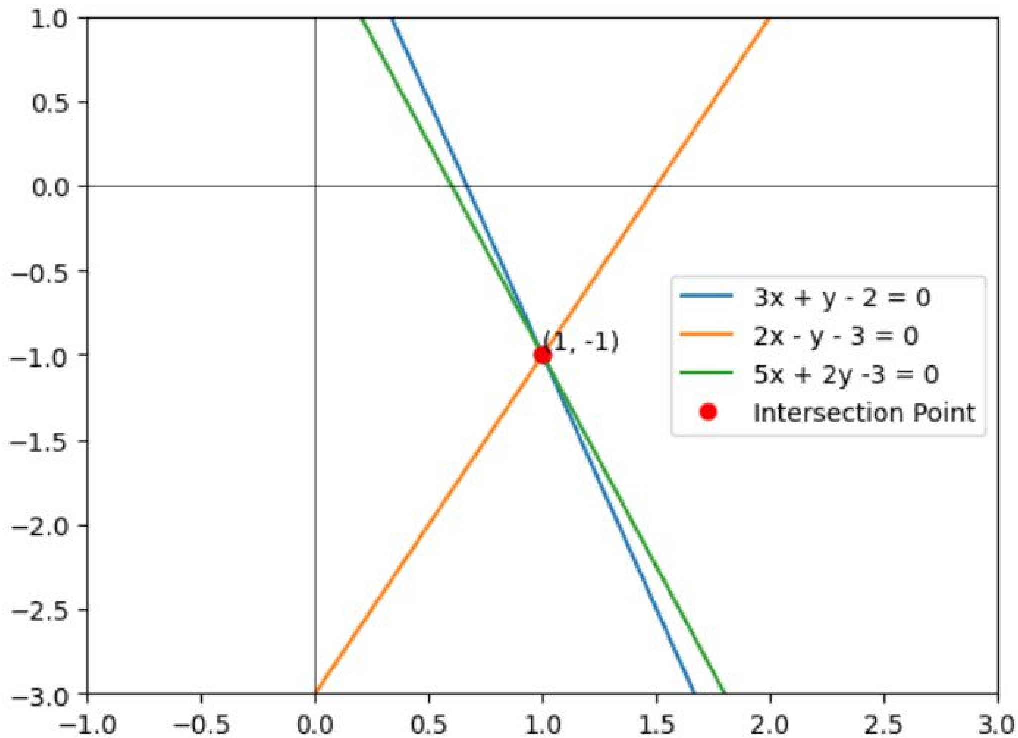

Figure 1: Straight-lines