EE 511
Fall 2018
Prof. John Silvester

Project 3 - Discrete Random Variables

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Section: Wednesday 9:00 AM
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1) Sum of Uniform RV's

Define:

$$N = \text{Min}\left\{ n : \sum_{i=1}^{n} U_i > 1 \right\}$$

where $\{U_i\}$ are iid Uniform(0,1) RV's.

Find (by simulation): $\hat{m} = E[N]$ an estimator for the mean.

Can you guess (or derive) the true value for $E[N]$?

Problem Statement:

To find the estimator for the mean for uniform random variables which are independent and identically distributed. This is done by calculating the moment of the first order for uniform random variable

Theory/Analysis:

Theory part and the true value of E[N] is written in the attachment.

References:

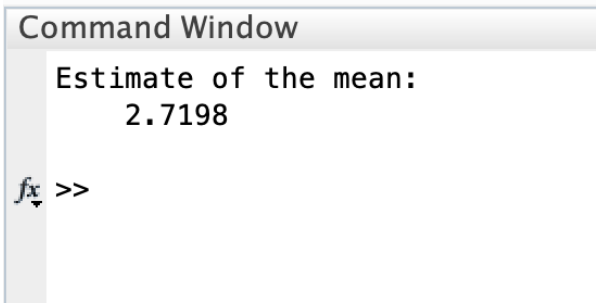
- Class Notes
- Text Book - Sheldon Ross
- Matlab Documentation

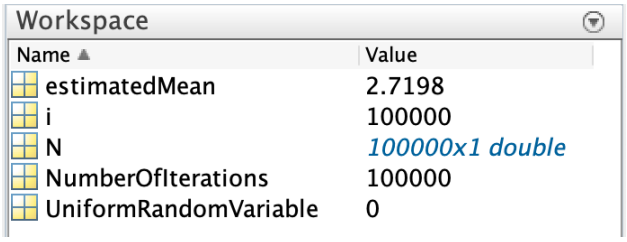
Simulation Methodology:

Uniform random variables are generated and added until the sum of the random variables is greater than or equal to one.
The mean is estimated by calculating the first order moment using the function mean() in Matlab.

Results:

The value of the estimated mean lies around 2.72 for any number of iterations.





Source Code:

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%Name: Pavan Athreya Narasimha Murthy
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%Term: Fall 2018
%Course: EE511
%Professor: John Silvester

%Clear the Workspace variables and command window for every run
clear all;
clc;

%Part 1 of Project 3
NumberOfIterations = 100000;
N = zeros(NumberOfIterations, 1);
UniformRandomVariable = 0;
for i = 1 : NumberOfIterations
   while 1
      UniformRandomVariable = UniformRandomVariable + rand;
      N(i) = N(i) + 1;
      if UniformRandomVariable >= 1
         UniformRandomVariable = 0;
         break
      end
   end
end
estimatedMean = mean(N);
disp('Estimate of the mean:');
disp(estimatedMean)
```

2) Minima of Uniform RV's

Define: $N = \text{Min}\{n : U_1 \le U_2 \le \ldots \le U_{n-1} > U_n\}$

i.e. the n^{th} term is the first that is less than its predecessor, where $\{U_i\}$ are independent identically distributed (iid) Uniform(0,1) RV's.

Find (by simulation): $\hat{m} = E[N]$ an estimator for the mean.

Can you guess (or derive) the true value for $E[N]$?

Problem Statement:

To find the estimator for the mean for uniform random variables which are independent and identically distributed on the minima of the generated uniform RV's

Theory/Analysis:

Theory part and the true value of E[N] is written in the attachment.

References:

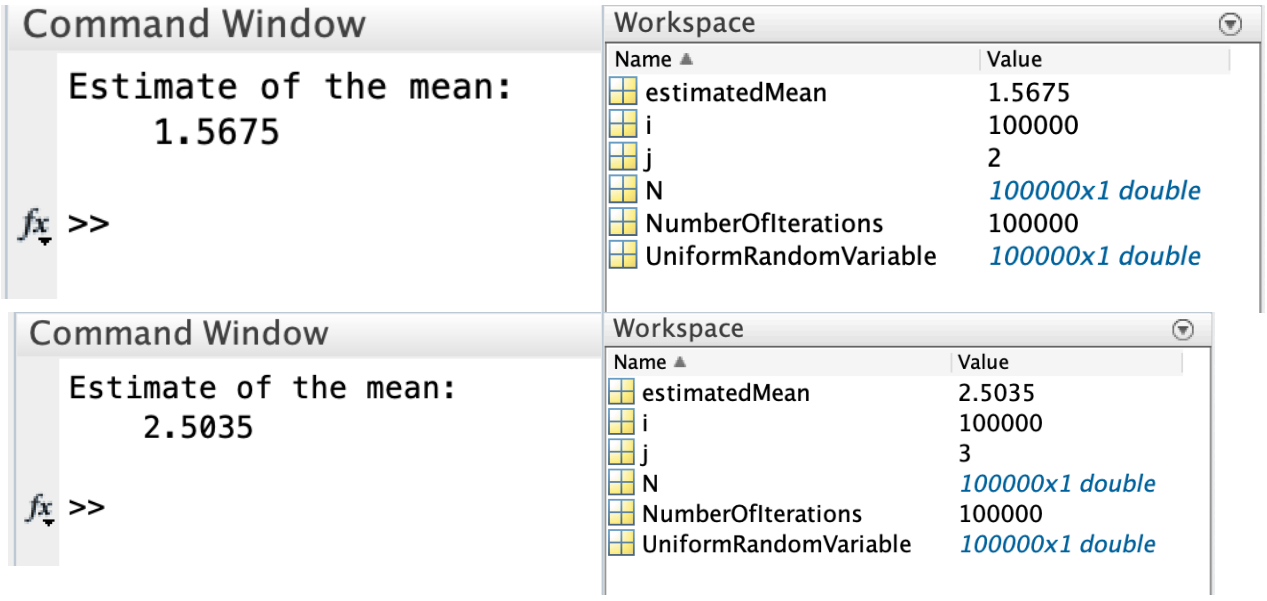
- Text Book - Sheldon Ross
- Matlab Documentation

Simulation Methodology:

Uniform random samples are added until one of the previously generated sample is less that the freshly generated random number. The mean is then calculated by the first moment.

Results:

There are two parts in this result, the first solution revolved around 1.5 estimate mean and the other solution revolved around 2.5 estimate mean.



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%Part 2 of Project 3
NumberOfIterations = 100000;
N = zeros(NumberOfIterations, 1);
N(1) = 1;
UniformRandomVariable = zeros(NumberOfIterations, 1);
UniformRandomVariable(1) = rand;
for i = 1 : NumberOfIterations
   for j = 2 : NumberOfIterations
      UniformRandomVariable(j) = rand;
      N(i) = N(i) + 1;
      if UniformRandomVariable(j-1) > UniformRandomVariable(j)
         break
      end
   end
end
estimatedMean = mean(N);
disp('Estimate of the mean:');
disp(estimatedMean);
```

3) Maxima of Uniform RV's

Consider the sequence of iid Uniform RV's $\{U_i\}$. If $U_j > \max_{i=1:j-1}\{U_i\}$ we say U_j is a record.

Example: the records are underlined.

$$\{U_i\} = \{\underline{0.2314}, \underline{0.4719}, 0.1133, \underline{0.5676}, 0.4388, \underline{0.9453}, ...\}$$

(note that the U_i are on the real line and we are just showing 4 digits of precision).

Let X_i be an RV for the distance from the $i-1^{\text{st}}$ record to the i^{th} record. Clearly $X_1 = 1$ always. In this example, $X_2 = 1, X_3 = 2, X_4 = 2$.

Distribution of Records: Using simulation, obtain (and graph) a probability histogram for X_2 and X_3 and compute the sample means.

Can you find an analytical expression for $P(X_2 = k)$? (Hint: condition on U_1 and then uncondition.) What does this say about $E[X_2]$?

Problem Statement:

To find the occurrence of the second and third maximum values in the sequence of random numbers generated and to compute the sample mean with probability histogram.

Theory/Analysis:

Theory part is written in the attachment.

References:

- Text Book - Sheldon Ross
- Matlab Documentation

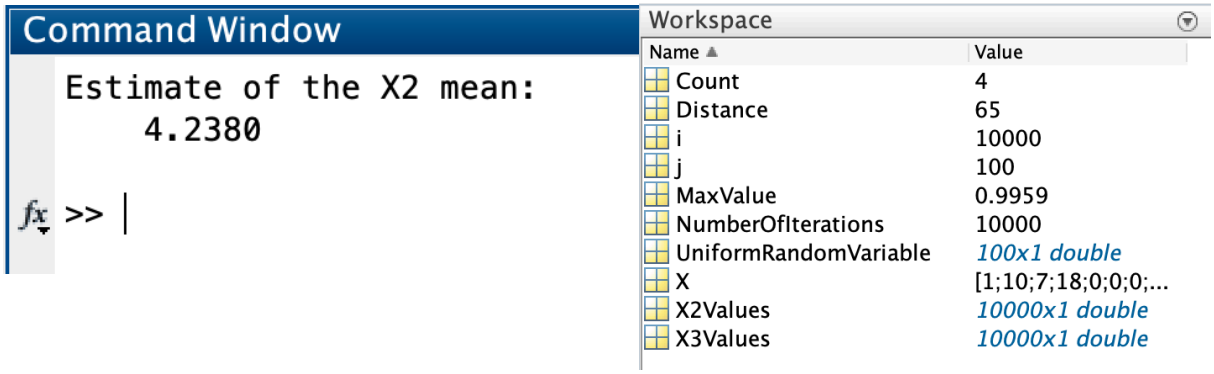
Simulation Methodology:

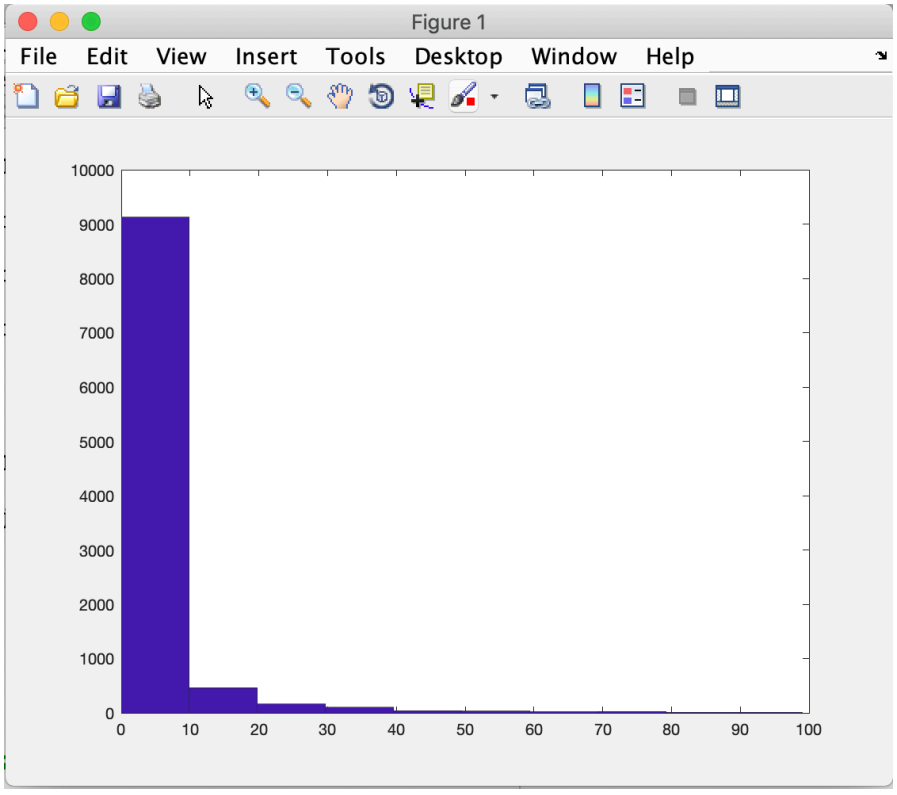
A sequence of iid uniform random variables are considered, Xi be the distance from i-1 to ith record. We generate uniform random variables and every time a greater value is obtained, the Xi value is updated. The process is repeated till we reach X4. The probability histogram of X2 and X3 are obtained. The sample means of X2 and X3 are calculated.

Results:

Mean values and histograms for X2 and X3 and shown below

X2:





X3:

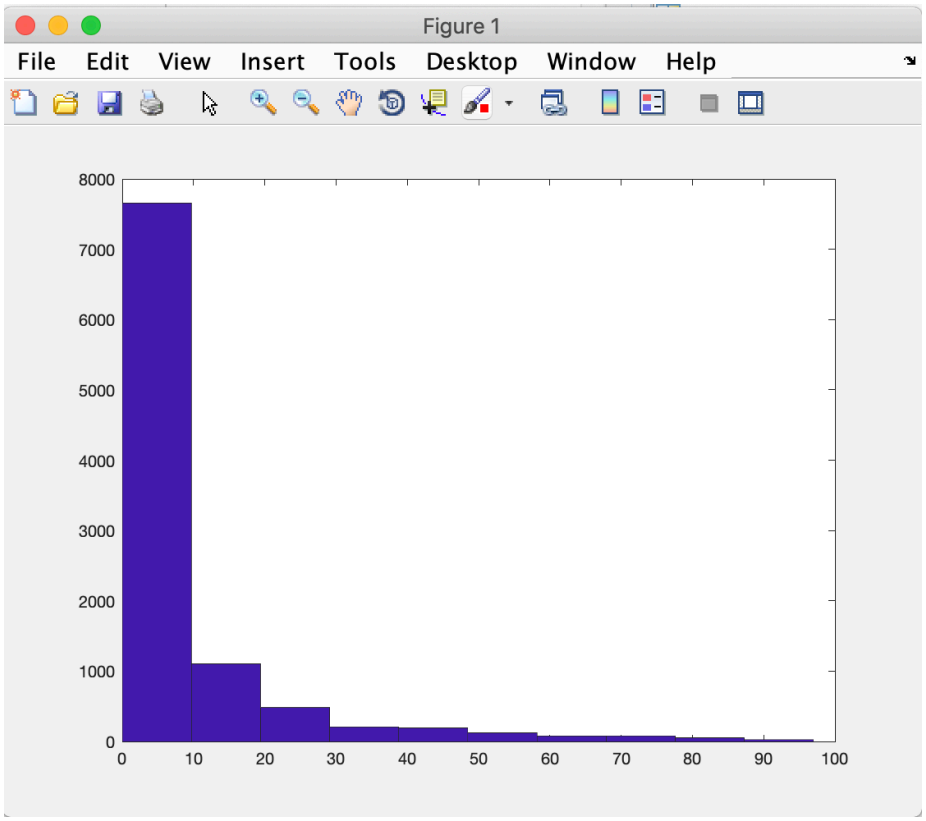
Command Window

```
Estimate of the X3 mean:
    8.7154

>>
```

Workspace

Name	Value
Count	5
Distance	85
i	10000
j	100
MaxValue	0.9915
NumberOfIterations	10000
UniformRandomVariable	*100x1 double*
X	[1;2;5;2;6;0;0;0;0...
X2Values	*10000x1 double*
X3Values	*10000x1 double*



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%Part 3 of Project 3
NumberOfIterations = 10000;
X2Values = zeros(NumberOfIterations, 1);
X3Values = zeros(NumberOfIterations, 1);
for i = 1 : NumberOfIterations
   Distance = 1;
   X = zeros(10, 1);
   X(1) = 1;
   UniformRandomVariable = zeros(100, 1);
   Count = 1;
   UniformRandomVariable(1) = rand;
   for j = 2 : 100
      MaxValue = max(UniformRandomVariable);
      UniformRandomVariable(j) = rand;
      if (MaxValue <  UniformRandomVariable(j))
         Count = Count + 1;
         X(Count) = Distance;
         Distance = 1;
      else
         Distance = Distance + 1;
      end
      if (UniformRandomVariable(j) > 0.999)
         break;
      end
   end
   X2Values(i) = X(2);
   X3Values(i) = X(3);
end
disp('Estimate of the X2 mean:');
disp(mean(X2Values));
disp('Estimate of the X3 mean:');
disp(mean(X3Values));
hist(X2Values);
hist(X3Values);
```