Project 2 Monte Carlo Methods

Due: Tuesday October 2

Some fairly straightforward Monte Carlo evaluation projects.

1. Estimate π by the area method including confidence intervals on your estimate. Draw a graph of the successive values of the estimator as the number of samples increases.

How many points do you need to use for your estimate to be within $\pm 1\%$ of the true value of π (with probability 0.95)?

2. Consider a deck of cards (for simplicity numbered $1..N$). Use a uniform random number generator to pick a card and record what card it is (if you were using actual cards, you would replace the card back into the deck – that is not necessary here since we never really take the card out of the deck). Repeat this N times, recording the number of times that each of the cards is selected. Some cards may not show up (actually, it is very likely that several card numbers will not show up) and some will show up more than once. You can use this data to estimate the following probabilities:

$$p_j = \Pr\{\text{a card will be selected } j \text{ times in the } N \text{ selections}\}$$

It is unlikely that any card will show up more than about 10 times. Run this for $N = 10, N = 52, N = 100, N = 1{,}000, N = 10{,}000$ and verify that $p_0 \simeq 1/e$. Can you also find values for the other p_j based on a mathematical analysis?

3. Use the method discussed in class to find $\hat{y}$, an estimate for Y and find a 95% confidence interval for the value of the integral.

$$Y = \int_0^{\pi} \frac{\sin(x)}{x} \, dx$$