EE511-F18 (Silvester)

Project #1: Due Thursday Sept 13

(1) Let $X \sim U(0,1)$, evaluate the mean, μ and variance, σ_X^2.

(2) Generate a sequence of $N = 100$ random numbers between [0,1] and compute the sample mean $m = \frac{1}{N}\sum_{i=1}^{N} X_i$ and sample variance $s^2 = \frac{\sum_{i=1}^{N}(X_i - m)^2}{N-1}$ and compare to μ and σ^2. Also estimate the (sample) variance of the sample mean (based on the Central Limit Theorem). Repeat for $N = 10{,}000$.

(3) The Central Limit Theorem says that $m = \frac{\sum_{i=1}^{n} X_i}{n} \to N(\mu, \sigma^2 / n)$. Repeat the experiment in (2 with $N = 100$) 50 times to generate a set of sample means $\{m_j,\ j = 1..50\}$. Do they appear to be approximately normally distributed values with mean μ and variance σ^2 / n ?

(4) We want to check whether there is any dependency between X_i and X_{i+1}

Generate a sequence of $N+1$ random numbers that are $\sim U(0,1)$ for $N = 1{,}000$

Compute

$$Z = \left[\frac{\sum_{i=1}^{N} X_i X_{i+1}}{N}\right] - \left[\frac{\sum_{i=1}^{N} X_i}{N}\right]\left[\frac{\sum_{j=2}^{N+1} X_j}{N}\right]$$

Comment on what you expect and what you find.

(5) Extra Credit for 2 bonus points. Generate 1000 samples between [0,1]. Consider that the interval [0,1] is split into 10 segments of length 0.1 and count the number of samples that fall into each interval. The resulting observations of the number in each interval should be a uniformly distributed discrete RV. Use the χ^2 Goodness of Fit test to determine whether the observations are within expectations or not.