Quantum Gates & Circuits: Submodule 1
Quantum Computing using Qbits

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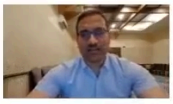
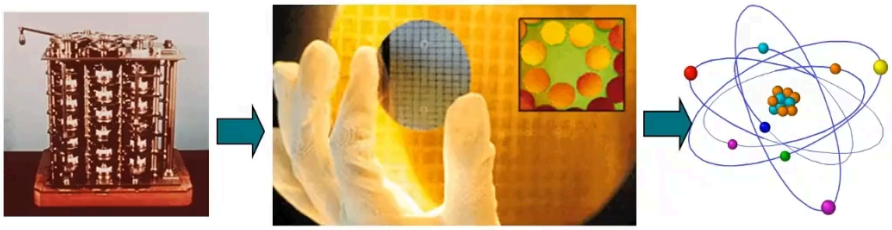
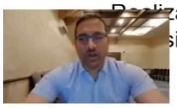


Towards Quantum



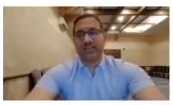


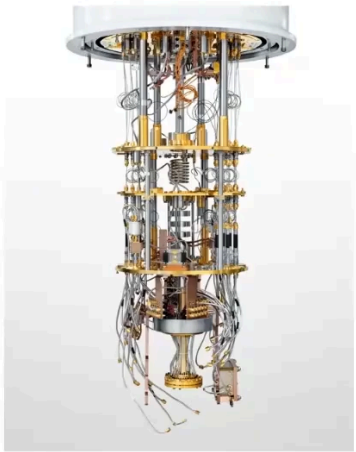
Realizations are getting smaller (and faster) and reaching a point where sical" physics is not longer a sufficient model for the laws of physics

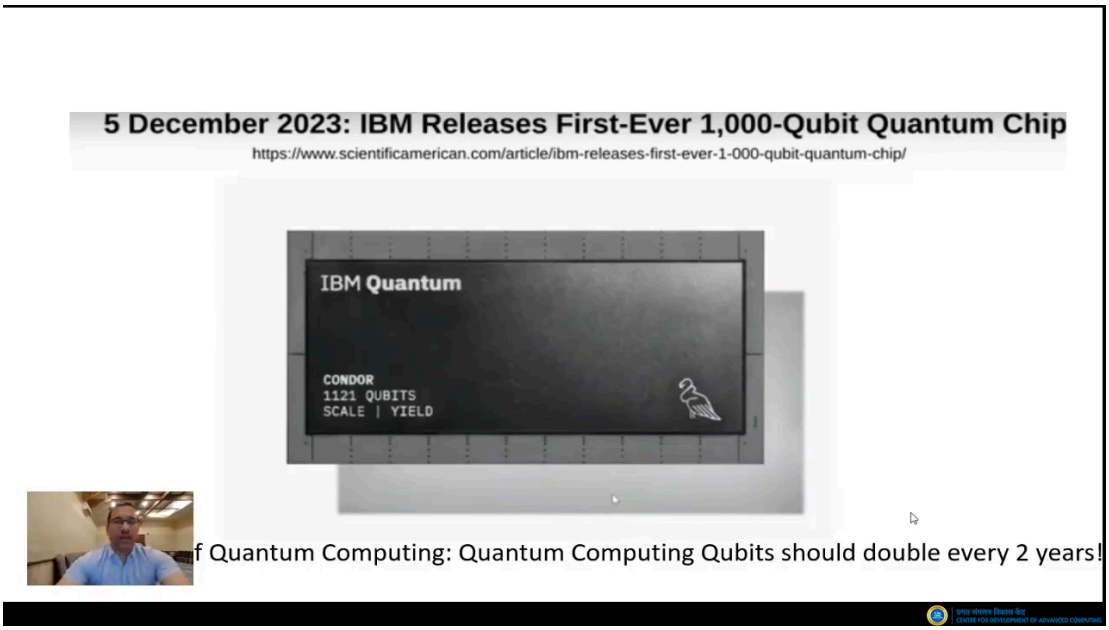
2

What is Quantum Computing?

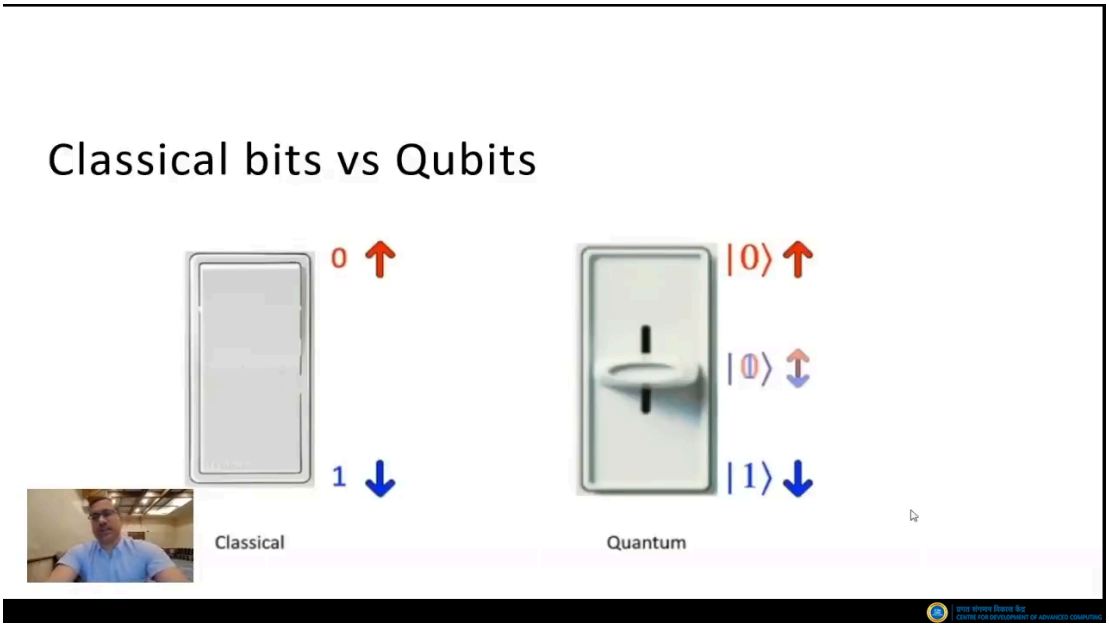
- The use of quantum mechanical principles to perform challenging computational tasks is called quantum computation.
- The quantum phenomenon like entanglement and superposition make it possible for low cost computation to happen.



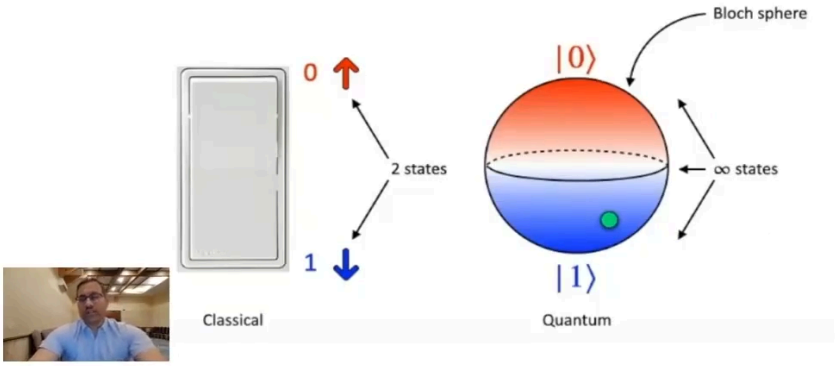




https://www.scientificamerican.com/article/ibm-releases-first-ever-1-000-qubit-quantum-chip/#:~:text=IBM%20has%20unveiled%20the%20first,error%2Dresistant%20rather%20t

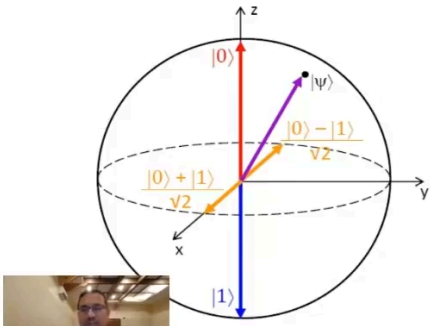


Classical bits vs Qubits



Computation-Transformation of the Memory State

1-qubit logic gates: rotations around x, y and z axes

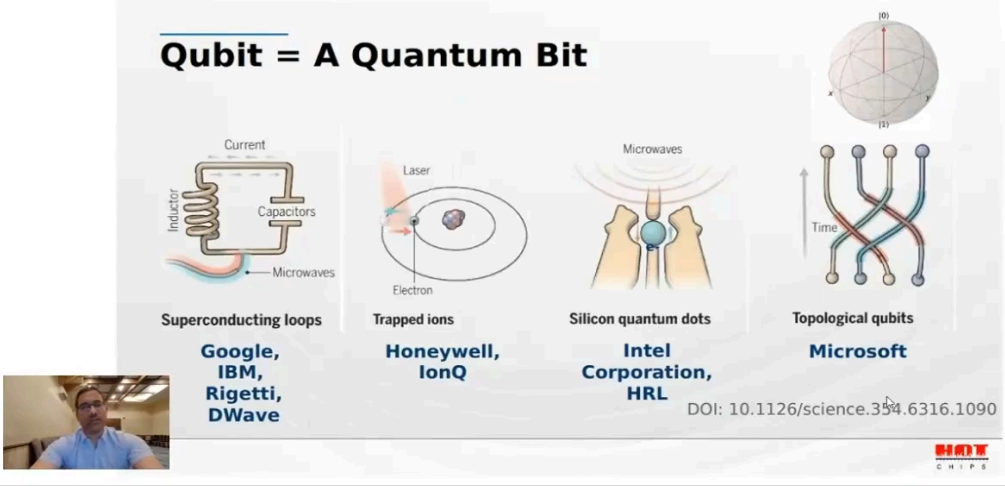


Pauli X - rotation by π around x axis
- implements NOT gate

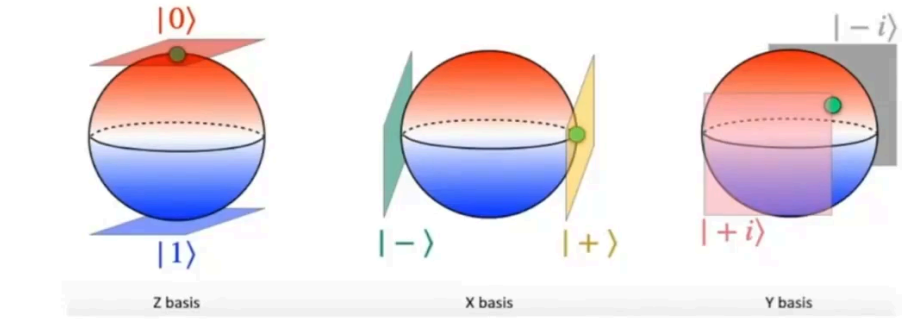
Hadamard - rotation by $\pi/2$ around y axis and then by π around x axis
- creates equal superposition when applied to $|0\rangle$ or $|1\rangle$

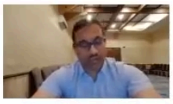
We can define infinitely many logic gates operating on a single qubit

Any unitary 2x2 matrix (rotation) is a logic gate



Most Important Basis for Measurement



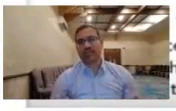


The Postulates of Quantum Mechanics

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system. The system is completely described by its *state vector*, which is a unit vector in the system's state space.

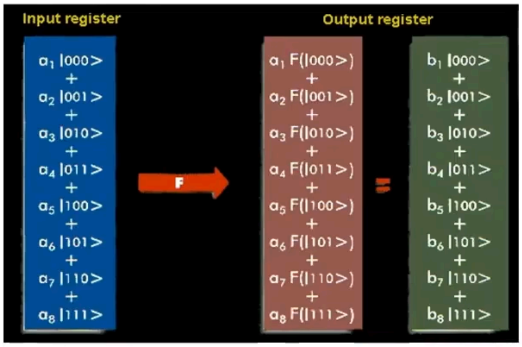
Postulate 2: The evolution of a *closed* quantum system is described by a *unitary transformation*. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2, $U|\psi\rangle = |\psi'\rangle$.

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by $p(m) = \langle\psi| M_m^\dagger M_m |\psi\rangle$, and the state of the system after the measurement is $\frac{M_m|\psi\rangle}{\sqrt{p(m)}}$. The measurement operators satisfy the *completeness equation*, $\sum_m M_m^\dagger M_m = I$.



The state space of a composite physical system is the tensor product of ces of the component physical systems Moreover, if we have systems hrough n, and system number i, is prepared in the state $|\psi_i\rangle$, then the the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

Why Is This Helpful?



- Multiple computations simultaneously
- Computing power is exponential

It's the Scaling

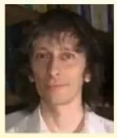
- The Quantum State Space becomes 2^n large in the number of qubits, this translates a huge energy advantage.
- The energy efficiency will scale exponentially with the increase in qubits

	CLASSICAL COMPUTING	QUANTUM COMPUTING
COMPUTING UNITS	Calculates with transistors, which can take two levels 0 and 1	Calculates with qubits, which can represent both 0 and 1 simultaneously
COMPUTING CAPACITY	Capability increased linearly (1:1) with number of transistors	Capability increased exponentially with number of qubits
ERROR RATES & ENVIRONMENT	Low error rates. Can operate at room temperate	High error rates. Some quantum systems need to be kept ultracold
SUITABILITY	Suitable for routine processing.	Suitable for complex processing

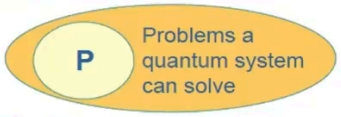
Quantum Algorithms

Richard Feynman

- Feynman (1982): there may be quantum systems that cannot be simulated efficiently on a "classical" computer
- Deutsch (1985): proposed that machines using quantum processes might be able to perform computations that "classical" computers can only perform very poorly



David Deutsch

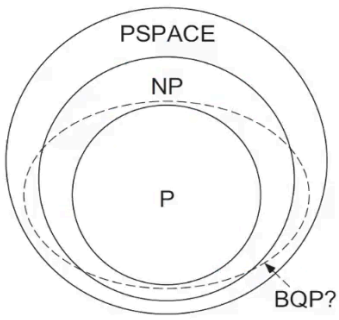


- Concept of *quantum computer* emerged as a universal device to execute such quantum algorithms

BQP (Bounded-Error Quantum Polynomial-Time): Class of problems solvable efficiently by a quantum computer

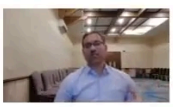
12

The Power of Quantum Computation

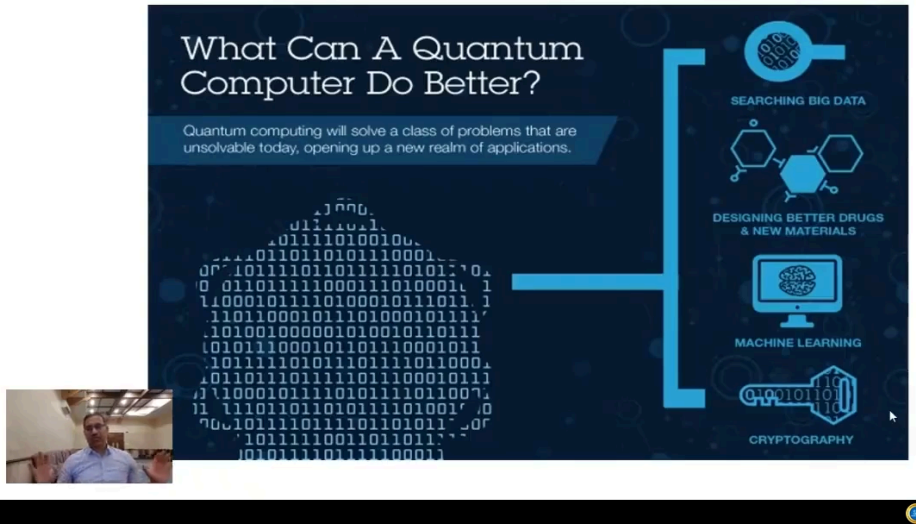


P = solved in polynomial time
NP = verified in polynomial time
PSPACE = solved in polynomial space

BQP (bounded error quantum polynomial time) is the class of decision problems solvable by a quantum computer in polynomial time, with an error probability of at most 1/3 for all instances.



Quantum Computing Can Perform Better



Quantum Computing: Thrust Areas

- Quantum Technology
- Quantum Algorithms
- Quantum Modelling and Simulation
- Quantum Communication and Cryptography

15

Quantum Gates & Circuits: Submodule 1 Basic Quantum Gates

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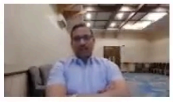
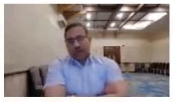


- Qubit vs. Bit:

Bit (Classical) degree of freedom that can take only two possible values.

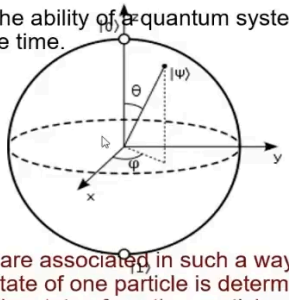
- Qubit
  - Quantum observable whose spectrum contains two values {0,1}.
  - Minimal quantum physical system.
  - The boolean observable of a qbit system is called a sharp observable , as it can have only values 0 and 1.
  - A qubit can have another observable which has an equal probability of 1 and 0, individual probabilities summed will results to unity.
  - Qubit in reality:
    - Electron spin (up or down)
    - Photon polarization (horizontal/vertical)
    - Spin of atomic nucleus
    - Current in a super conducting loop
    - Presence/absence of a particle



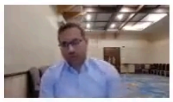
18

Quantum Phenomenon: Superposition and Entanglement

- Superposition
  - Superposition is the ability of a quantum system to be in multiple states at the same time.



- Entanglement
  - Multiple particles are associated in such a way that measurement of one quantum state of one particle is determined by the measurement of the state of another particle.



19

Computation with Qubits

How does the use of qubits affect computation?

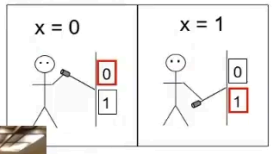
Classical Computation

Data unit: bit

● = '1' ○ = '0'

Valid states:

x = '0' or '1'



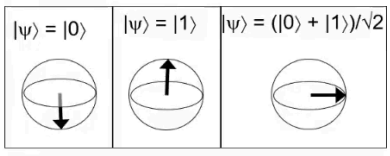
Quantum Computation

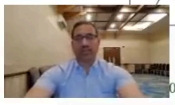
Data unit: qubit

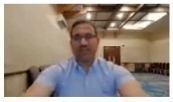
$|1\rangle$ $|0\rangle$

Valid states:

$|\psi\rangle = c_1|0\rangle + c_2|1\rangle$







21

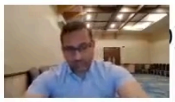
Entanglement

- An n-qubit system can exist in any superposition of the 2^n basis states

$$c_0|00\ldots00\rangle + c_1|00\ldots01\rangle + \cdots + c_{2^n-1}|11\ldots11\rangle\,, \qquad \sum_{i=0}^{2^n-1} |c_i|^2 = 1$$

- If such a state can be represented as a tensor product of individual qubit states then the qubit states are **not entangled**. For example:

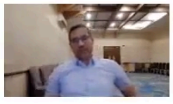
$$\underbrace{\left(\frac{1}{\sqrt{8}}|00\rangle + \frac{\sqrt{3}}{\sqrt{8}}|01\rangle + \frac{1}{\sqrt{8}}|10\rangle + \frac{\sqrt{3}}{\sqrt{8}}|11\rangle\right)}_{2^n \text{ probability amplitudes}} = \underbrace{\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle\right)}_{2n \text{ probability amplitudes}}$$



$$\left.\ldots\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) \neq (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$$

Quantum Logic Networks

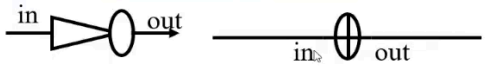
- Invented by Deutsch (1989)
  - Analogous to classical Boolean logic networks
  - Generalization of Fredkin-Toffoli reversible logic circuits
- System is divided into individual bits, or *qubits*
  - 2 orthogonal states of each qubit are designated as the *computational basis states*, "0" and "1"
- Quantum logic gates:
  - Local unitary transforms that operate on only a few state bits at a time
- Computation via predetermined sequence of gate applications to selected bits



23

Quantum Gates: NOT

- All classical input-consuming reversible gates can be represented as unitary transformations!
- *E.g.*, input-consuming NOT gate (inverter)



in	out
0	1
1	0

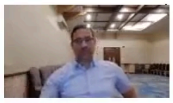
$$|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$|1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$N := \begin{matrix} 0 & 1 \end{matrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$N|0\rangle = |1\rangle$$

$$N|1\rangle = |0\rangle$$



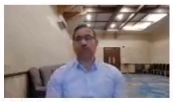
24

The Hadamard Transform

- Used frequently in quantum logic networks for generating Superpositions

$$H := \begin{matrix} 0 & 1 \end{matrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$H^2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



25

One Qbit Logic Gates

X (NOT)

$$\begin{matrix} & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$

Y

$$\begin{matrix} & 0 & 1 \\ 0 & 0 & -i \\ 1 & i & 0 \end{matrix}$$

Z

$$\begin{matrix} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{matrix}$$

S

$$\begin{matrix} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & i \end{matrix}$$

T

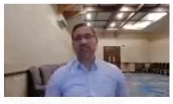
$$\begin{matrix} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & e^{i\pi/4} \end{matrix}$$

Hadamard (H)

$$\begin{matrix} & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{matrix}$$

Sqrt NOT

$$\begin{matrix} & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{matrix}$$



Identity transformation, Pauli matrices, Hadamard

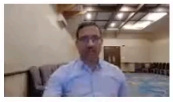
$$\delta_0 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad |\varphi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$

$$\delta_1 = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad |\varphi\rangle = \alpha_1|0\rangle + \alpha_0|1\rangle$$

$$\delta_2 = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad |\varphi\rangle = -i\alpha_1|0\rangle + i\alpha_0|1\rangle$$

$$\delta_3 = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |\varphi\rangle = \alpha_0|0\rangle - \alpha_1|1\rangle$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad |\varphi\rangle = \alpha_0 \frac{|0\rangle + |1\rangle}{\sqrt{2}} + \alpha_1 \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$



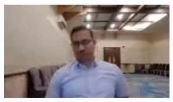
27

One qubit gates

- Transform an input qubit into an output qubit
- Characterized by a 2 x 2 matrix with complex coefficients

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle \quad \xrightarrow{G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}} \quad |\varphi\rangle = \alpha_0'|0\rangle + \alpha_1'|1\rangle$$

One-qubit gate

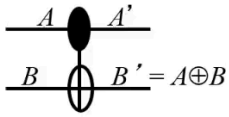


$$|\varphi\rangle = G|\psi\rangle \qquad \begin{pmatrix} \alpha_0' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

28

Controlled-NOT

- A.k.a. CNOT (or input-consuming XOR)

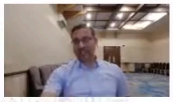


A	B	A'	B'
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

$$X := \begin{array}{c} \begin{matrix} 00 & 01 & 10 & 11 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix}$$

Example:

$$X\overset{A\ B}{|10\rangle} = \overset{A\ B}{|11\rangle}$$



In a CNOT (Controlled-NOT) gate in quantum computing, measuring one qubit can indeed provide information about the other qubit, depending on the state of the measured qubit and the entanglement between the qubits.

Explanation:

1. Entanglement in CNOT Gate:

In a CNOT gate, two qubits are involved: a control qubit (usually denoted as q_{control}) and a target qubit (usually denoted as q_{target}).

If the control qubit is in the state $|1\rangle$, it flips the state of the target qubit. If the control qubit is in the state $|0\rangle$, the target qubit remains unchanged.

2. Measurement and Entanglement:

When two qubits are entangled, measuring one qubit can instantaneously affect the state of the other qubit.

If we measure the control qubit, and it collapses to $|0\rangle$ or $|1\rangle$, we instantly know the state of the target qubit, as it will be flipped or remain the same, respectively.

3. Example:

Let's say we have a CNOT gate with qubit q_{control} as the control qubit and q_{target} as the target qubit.

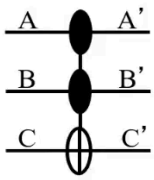
If q_{control} is in the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and q_{target} is in the state $|0\rangle$ before measurement.

If we measure q_{control} and it collapses to $|0\rangle$, then q_{target} remains $|0\rangle$. If it collapses to $|1\rangle$, then q_{target} flips to $|1\rangle$.

Conclusion:

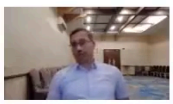
In summary, measuring one qubit in a CNOT gate can provide information about the state of the other qubit due to their entanglement. This property is fundamental in quantum computing and can be used for various quantum information processing tasks, including quantum teleportation and quantum error correction.

Toffoli Gate (CCNOT)



A B C	A' B' C'
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 0
0 1 1	0 1 1
1 0 0	1 0 0
1 0 1	1 0 1
1 1 0	1 1 0
1 1 1	1 1 1

$$X :\equiv \begin{array}{c} \begin{matrix} 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{matrix}$$



Computation with Qubits

How does the use of qubits affect computation?

Classical Computation

Operations: logical

Valid operations:

NOT = in \ 0 1 / out: 1 0 — 1-bit

ND = in \ 0 1 / in 0: 0 0 / in 1: 0 1 / out — 2-bit

Quantum Computation

Operations: unitary

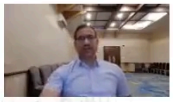
Valid operations:

1-qubit: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ $H_d = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

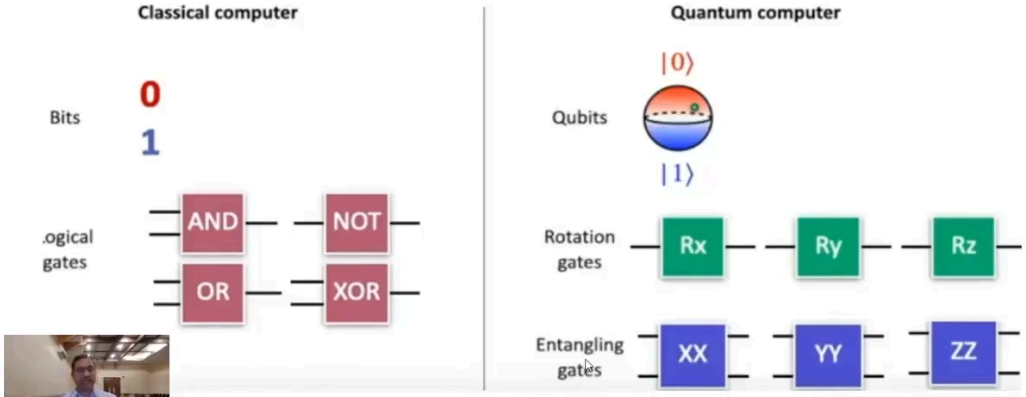
2-qubit: CNOT = $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

More than one qubit

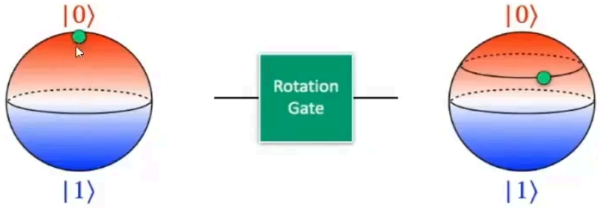
	Single qubit	Two qubits
Hilbert space	$|0\rangle, |1\rangle$ $\mathcal{H}_2 = \left\{ \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix} \right\}$	$|00\rangle, |01\rangle, |10\rangle, |11\rangle$ $\mathcal{H}_2^{\otimes 2} = \mathcal{H}_2 \otimes \mathcal{H}_2 = \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \right\}$
Arbitrary state	$\|\psi\rangle = c_1|0\rangle + c_2|1\rangle = \begin{bmatrix}c_1\\c_2\end{bmatrix}$	$\|\Psi\rangle = c_1\|00\rangle + c_2\|01\rangle + c_3\|10\rangle + c_4\|11\rangle = \begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}$
Operator	$U\|\psi\rangle = \begin{pmatrix}u_{11}u_{12}\\u_{21}u_{22}\end{pmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}$	$U\|\Psi\rangle = \begin{bmatrix}u_{11}&u_{12}&u_{13}&u_{14}\\u_{21}&u_{22}&u_{23}&u_{24}\\u_{31}&u_{32}&u_{33}&u_{34}\\u_{41}&u_{42}&u_{43}&u_{44}\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}$



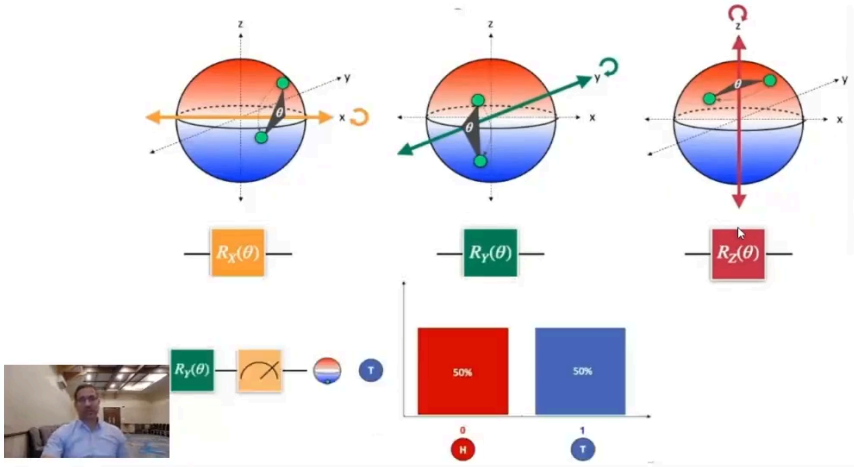
Bits, Qubits and Gates



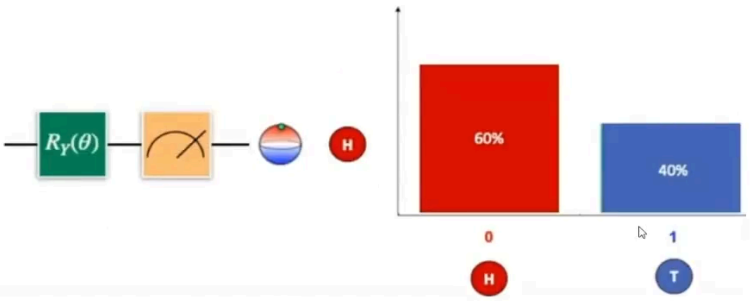
Rotation Gates: Qubit Transformations

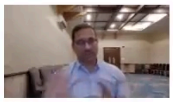


Rotation Gates: Qubit Transformations



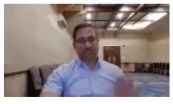
Rotation with Bias





Physical Machine Descriptions (PMDs)

- Technology for a given quantum implementation
- Quantum gate implementation is achieved by means of supported quantum operations
- Variability among the PMDs
  - primitive quantum operations
  - related cost
- Commonly known PMDs
  - Quantum dot (QD)
  - Superconducting (SC)
  - Ion trap (IT)
  - Neutral atom (NA)
  - Linear photonics (LP)
  - Nonlinear photonics (NP)



Minimal Set of 1-qubit gates

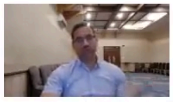
$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = P(\frac{\pi}{2})R_x(\pi) = iR_x(\pi)$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = P(\frac{\pi}{2})R_y(\pi) = iR_y(\pi)$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = P(\frac{\pi}{2})R_z(\pi) = iR_z(\pi)$$

- Any two of *{Rx,Ry,Rz} can be converted into the third one*

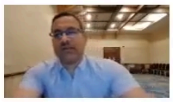
$$R_x(\theta) = R_z(-\frac{\pi}{2}) \cdot R_y(\theta) \cdot R_z(\frac{\pi}{2})$$



- Quantum Logic Circuits
  - Circuit behavior is governed explicitly by quantum mechanics
  - Signal states are vectors interpreted as a superposition of binary "qubit" vectors with complex-number coefficients

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i |i_{n-1} i_{n-1} \ldots i_0\rangle$$

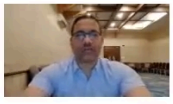
  - Operations are defined by linear algebra over Hilbert Space and can be represented by unitary matrices with complex elements
  - Severe restrictions exist on copying and measuring signals
  - Many universal gate sets exist but the best types are not obvious



40

Quantum Circuit Characteristics

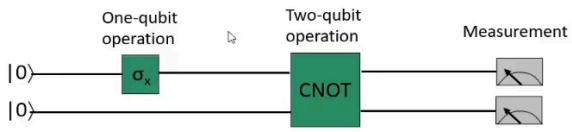
- Unitary Operations
  - Gates and circuits must be reversible (information-lossless)
    - Number of output signal lines = Number of input signal lines
    - The circuit function must be a bijection, implying that output vectors are a permutation of the input vectors
  - Classical logic behavior can be represented by permutation matrices
  - Non-classical logic behavior can be represented including state sign (phase) and entanglement



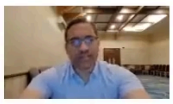
41

Quantum Circuit Model

Example Circuit



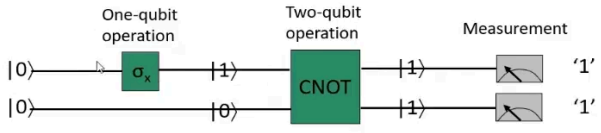
$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$



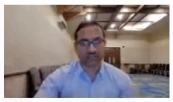
$$\sigma_x \otimes I = \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix} \qquad CNOT = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

Quantum Circuit Model

Example Circuit



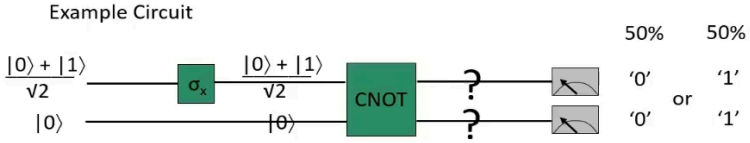
$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \rightarrow \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \rightarrow \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \rightarrow \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$



$$\sigma_x \otimes I = \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix} \qquad CNOT = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

Quantum Circuit Model

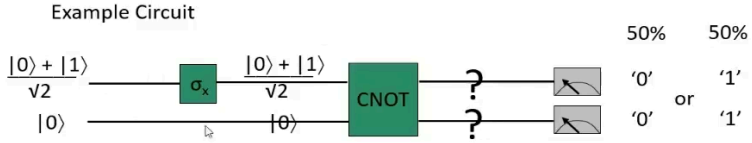
Example Circuit



$$\begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix} \rightarrow \begin{bmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix} \rightarrow \begin{bmatrix}1/\sqrt{2}\\0\\0\\1/\sqrt{2}\end{bmatrix} \rightarrow \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \text{ or } \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

Quantum Circuit Model

Example Circuit



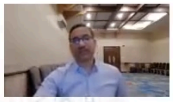
$\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Separable state: can be written as tensor product

$|\Psi\rangle = |\phi\rangle \otimes |\chi\rangle$

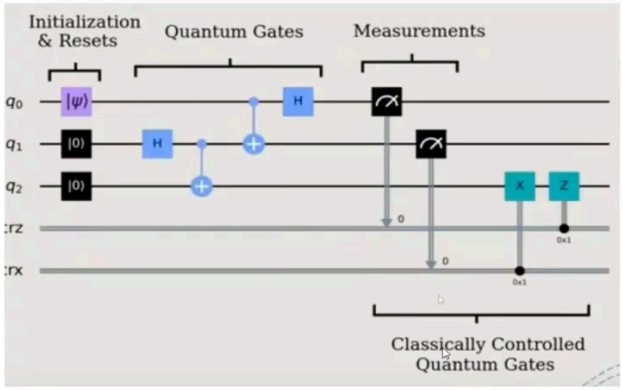
Entangled state: cannot be written as tensor product

$|\Psi\rangle \neq |\phi\rangle \otimes |\chi\rangle$



Quantum Computing Group

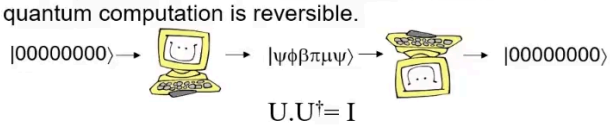
Quantum Circuits



Some Interesting Consequences

Reversibility

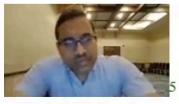
Since quantum mechanics is reversible (dynamics are unitary), quantum computation is reversible.

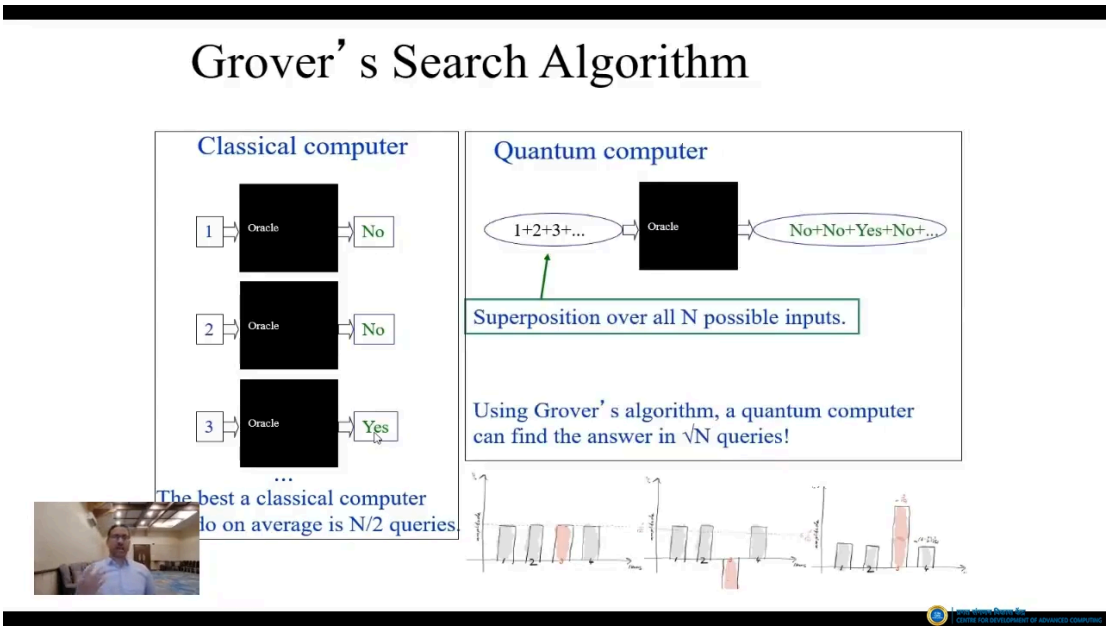


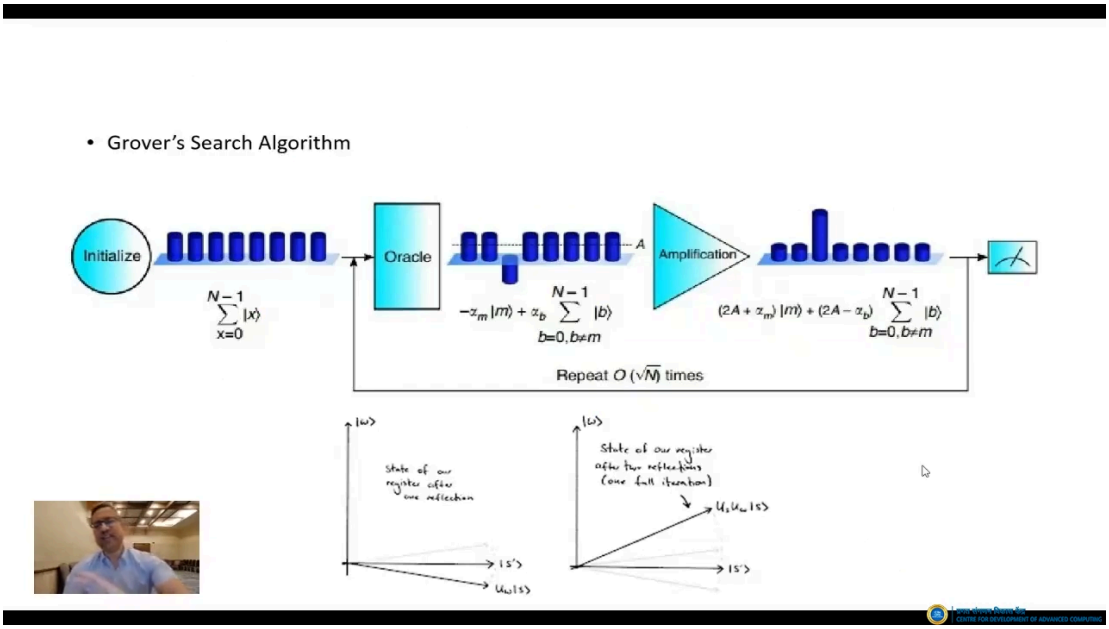
$U.U^{\dagger} = I$

No cloning theorem

It is impossible to exactly copy an unknown quantum state

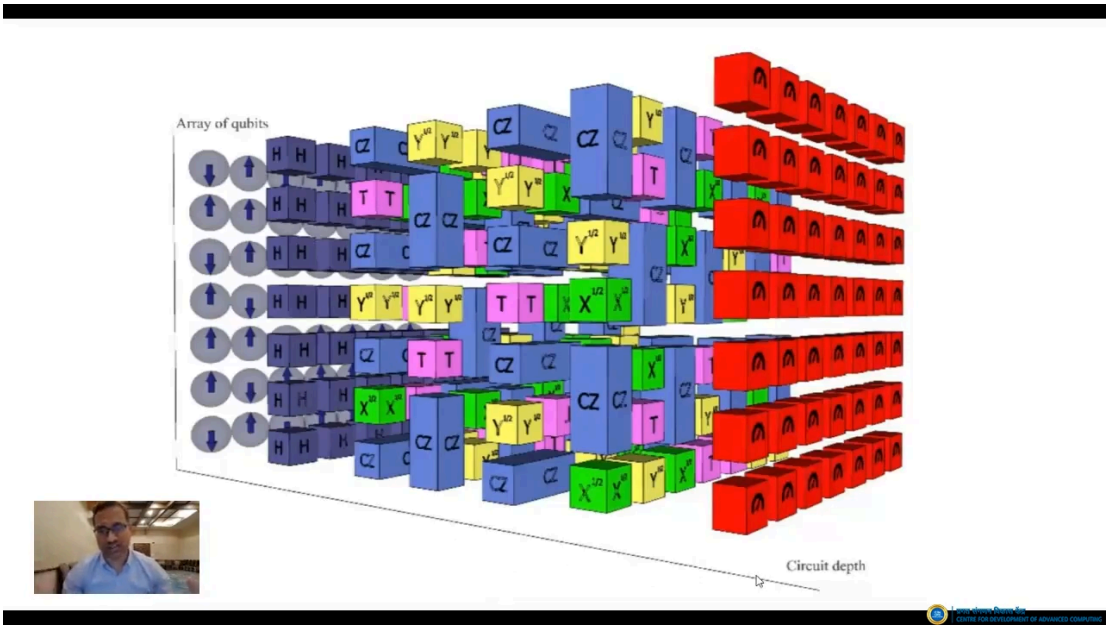






Qiskit Textbook (Algorithms) - https://github.com/Qiskit/textbook/tree/main/notebooks/ch-algorithms

Large scale Quantum Circuit



Transpilation - https://docs.quantum.ibm.com/api/qiskit/transpiler