Assignment - 3

1) Substitution Method

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$$

$T(n) = 2T(n/2) + n$

$\Rightarrow 2\left(2T(n/2^2) + n/2\right) + n$

$\Rightarrow 2^2 T(n/2^2) + 2n$

$\Rightarrow 2^2\left(2T(n/2^3) + n/2^2\right) + 2n$

$\Rightarrow 2^3 T(n/2^3) + 3n$

$\vdots$ K times

$\downarrow$

$2^K T\left(n/2^K\right) + K \cdot n$

we need add $T(n)$

$\neq (n/2) + n$
$n + n$
$\Rightarrow 2n$

$\neq (n/2) + 2n$
$\Rightarrow n + 2n$
$\Rightarrow 3n$

$n/2^K = 1 \Rightarrow n = 2^K$ $K \cdot \log 2$

$$\boxed{\log_2 n = K}$$

$\Rightarrow 2^{\log_2 n} T\left(\dfrac{n}{2^{\log_2 n}}\right) + \log_2 n \cdot n$

$n \cdot \log 2\left(\dfrac{n}{n^{\log 2}}\right) + \log_2 n * n$

$\Rightarrow n + \log_2 n * n$

Higher complexity

$\Rightarrow O(n \log_2 n) //$

$$\Rightarrow \quad T(n) = \begin{cases} 1 & n=1 \\ 8T(n/2) + n^2, & n>1 \end{cases}$$

$$T(n) = 8T\left(\frac{n}{2}\right) + n^2$$

$$\Rightarrow 8\left[8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$$

$$8\left(\frac{n^2}{4}\right) + n^2 \Rightarrow 8^2 T\left(\frac{n}{2^2}\right) + 3n^2$$

$$3n^2$$

$$\Rightarrow 8^2\left[8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 3n^2$$

$$\Rightarrow 8^2 * \frac{n^2}{16} + 3n^2 \Rightarrow 7n^2$$

$$\Rightarrow 8^3 T\left(\frac{n}{2^3}\right) + 7n^2$$

$$\vdots \quad K \text{ times}$$

$$\downarrow$$

$$8^K T\left(\frac{n}{2^K}\right) + (2^K - 1) n^2$$

$$K = \log_2 n$$

$$\Rightarrow 8^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + \left(2^{\log_2 n} - 1\right) n^2$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad\qquad \downarrow$$

$$n^{\log_2 2^3} \qquad n^{\log_2 2} \qquad n^{\log_2 2}$$

$$n^3 T\left(\frac{n}{n}\right) + (n-1) n^2$$

$$n^3 + n^3 - n^2$$

$$\Rightarrow O(n^3) //$$

1) i=1
   while

$\frac{1}{1} < 72$

i=6

i=6

2) i

We