Project 1. Fully Developed Heat Transfer in Channels with Non-circular Cross-sections

This project will focus on hydrodynamically and thermally developed convection heat transfer from non-circular flow passages with a constant rate of heat input at the walls per unit length of the channel. You may team up with a partner to work on this project.

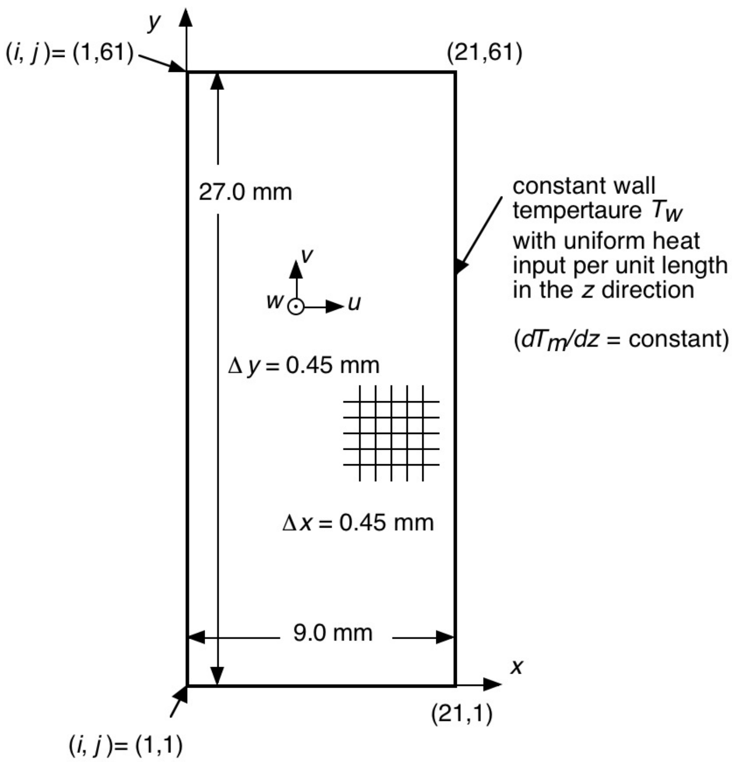


Figure 1.

Introduction

As a first step, consider flow in the rectangular passage shown above. The governing equations for fully developed flow and heat transfer are:

$$0 = \frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] - \frac{1}{\rho}\frac{\partial P}{\partial z} \tag{1}$$

$$w\frac{\partial T}{\partial z} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \tag{2}$$

Note that u and v are zero and z derivatives are zero, except for $\partial T / \partial z$, which is constant for constant heat addition at the walls per unit length. The continuity equation is satisfied for these conditions since all terms are zero. Recall that for fully developed heat transfer with uniform heat addition at the walls,

$$\frac{\partial T}{\partial z} = \frac{dT_m}{dz} \tag{3}$$

Also, the *u* and *v* momentum equations reduce to the pressure gradient in each direction equaling zero, which means the pressure is uniform across any cross section of the tube and $\partial P/\partial z$ in the flow is equal to the imposed pressure gradient dP/dz. The governing equation can therefore be written as

$$\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=\frac{1}{\rho\nu}\frac{dP}{dz} \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{w}{\alpha}\left(\frac{dT_m}{dz}\right) \tag{5}$$

These are both variations of the Poisson equation. There are a variety of methods for solving them. Here you are to adopt a numerical scheme that is flexible enough to use for a variety of non-circular geometries.

Using central differences for the second order derivatives, the above equations can be converted to the following finite-difference equations:

$$\frac{w_{i+1,j}-2w_{i,j}+w_{i-1,j}}{(\Delta x)^2}+\frac{w_{i,j+1}-2w_{i,j}+w_{i,j-1}}{(\Delta y)^2}=\frac{1}{\rho\nu}\left(\frac{dP}{dz}\right) \tag{6}$$

$$\frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{(\Delta x)^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{(\Delta y)^2}=\frac{w_{i,j}}{\alpha}\left(\frac{dT_m}{dz}\right) \tag{7}$$

Solving (6) and (7) above for $w_{i,j}$ and $T_{i,j}$, respectively, yields

$$w_{i,j}=\frac{(\Delta y/\Delta x)^2(w_{i+1,j}+w_{i-1,j})+w_{i,j+1}+w_{i,j-1}}{2(\Delta y/\Delta x)^2+2}-\frac{(dP/dz)(\Delta y)^2}{\rho\nu[2(\Delta y/\Delta x)^2+2]} \tag{8}$$

$$T_{i,j}=\frac{(\Delta y/\Delta x)^2(T_{i+1,j}+T_{i-1,j})+T_{i,j+1}+T_{i,j-1}}{2(\Delta y/\Delta x)^2+2}-\frac{w_{i,j}(dT_m/dz)(\Delta y)^2}{\alpha[2(\Delta y/\Delta x)^2+2]} \tag{9}$$

The Gauss-Seidel method is to be used in this project assignment. In this solution scheme, the computation iteratively sweeps through the array from lower *i* and *j* to higher values (bottom to top), computing improved values of *w* or *T* at each node (*i*,*j*). Values of *w* or *T* from adjacent nodes are used to evaluate the right side of the above relations. A single array is used to store the *w* or *T* values. Note that in sweeping from top to bottom, the terms on the right side of (8) and (9) with *i*-1 or *j*-1 indices will already have been updated for the current iteration. We designate values updated in the current iteration as primed variables. With this designation the relations are written as

$$w'_{i,j}=\frac{(\Delta y/\Delta x)^2(w_{i+1,j}+w'_{i-1,j})+w_{i,j+1}+w'_{i,j-1}}{2(\Delta y/\Delta x)^2+2}-\frac{(dP/dz)(\Delta y)^2}{\rho\nu[2(\Delta y/\Delta x)^2+2]} \tag{10}$$

$$T'_{i,j}=\frac{(\Delta y/\Delta x)^2(T_{i+1,j}+T'_{i-1,j})+T_{i,j+1}+T'_{i,j-1}}{2(\Delta y/\Delta x)^2+2}-\frac{w_{i,j}(dT_m/dz)(\Delta y)^2}{\alpha[2(\Delta y/\Delta x)^2+2]} \tag{11}$$

Use of some information from the new iteration values in computation of new values make the method somewhat implicit.

Task I

(a) Derive Eqs. (8) and (9) from Eqs. (6) and (7). (You must show all the steps of the work.)

(b) Construct a computer program to implement the Gauss-Seidel method for solving the equations (10) and (11). The program should follow the following sequence:

(i) Define constants and parameters for the channel geometry and flow conditions of interest.

(ii) Initialize all matrices and vectors for *w T*, *x*, *y*, etc. Set the *w* field to 0, and the temperature field to 70 °C as an initial condition. Take the following initial values for parameters:

The array should have 21 nodes in the *x* direction and 61 nodes in the *y* direction, as shown in Fig. 1.

$\Delta x = 0.45$ mm, $\Delta y = 0.45$ mm

$dP/dz = -17.0$ Pa/m, $dT_m/dz = 7.0$ °C/m, $T_w = 90$ °C

Use properties for water: $\alpha = 1.46\times10^{-7}$ m²/s, $\rho = 997$ kg/m³, $\nu = 8.26\times10^{-7}$ m²/s, $c_p = 4164$ J/kg°C, $k = 0.608$ W/m°C

(iii) The iterative calculations to determine converged *u* and *T* solutions should proceed as follows:

[1] Using the initial or previous iteration values, sweep the velocity field to compute new values of *w* for $2 \le i \le N_x - 1$, $2 \le j \le N_y - 1$, where N_x and N_x are the total number of nodes in the *x* and *y* directions, respectively. As each calculation is done, the absolute difference between the old and new value of $w_{i,j}$ should be saved if it is the largest encountered during the sweep.

[2] At the end of the sweep, the maximum absolute change of $w_{i,j}$ for the sweep should be compared to the convergence criterion $\varepsilon_w = 0.0005$ m/s. If $\left|w'_{i,j} - w_{i,j}\right|_{\max} < \varepsilon_w$, the solution is converged and iteration of the *w* field stops. If $\left|w'_{i,j} - w_{i,j}\right|_{\max}$ is not less than ε_w, return to step [1] and iterate again.

[3] Once the *w* field solution is determined, iterations of the temperature field begin. Using the initial or previous iteration values, sweep the temperature field to compute new values of *T* for $2 \le i \le N_x - 1$, $2 \le j \le N_y - 1$. As each calculation is done, the absolute difference between the old and new values of $T_{i,j}$ is saved if it is the largest encountered during that iteration.

[4] At the end of the sweep, the maximum absolute change of $T_{i,j}$ for the sweep should be compared to the convergence criterion $\varepsilon_T = 0.05$ °C. If $\left|T'_{i,j} - T_{i,j}\right|_{\max} < \varepsilon_T$, the solution is converged and iteration of the *T* field stops. If $\left|T'_{i,j} - T_{i,j}\right|_{\max}$ is not less than ε_T, return to step [3] and iterate again.

(iv) Once the field solutions are obtained, the program should do the following:

[1] Using the converged *w* and *T* field information, determine the mean *w* velocity w_m and mean temperature T_m for the flow. Note that these will involve numerically integrating appropriate quantities over the cross section of the flow.

[2] Determine the heat flux q_w from each wall element using the finite difference representation of Fourier's law. For each wall element, divide q_w by $T_w - T_m$ to get the local heat transfer coefficient. (Note: here we ignore corner element nodes.)

[3] Multiply the heat flux by the area for each wall element to get its heat transfer rate to the fluid. Sum the contributions of all wall elements to get the total heat input rate. Divide the total heat input rate by $T_w - T_m$ and the wetted wall area to determine the mean heat transfer coefficient for the flow.

[4] Generate 3-D surface plots of the *w* and *T* fields, and 2-D plots as necessary to analyze the results. Note that in matlab, if you store the *w* field in array *w*(i,j) and the *x* and *y* locations in vectors *x*(i) and *y*(j), you can, for example, hand them to the function mesh(*x*,*y*,*w*) to create a 3-D plot of the *w* field.

(c) Run the program to convergence for the parameters values indicted in part (b) above. Generate 3-D surface plots of $w(x,y)$ velocity and $T(x,y)$. Also plot the variation of the heat transfer coefficient along the short and long walls as function of *x* and *y*, respectively. At what locations are the highest and lowest local heat transfer coefficients found, and what are their values? Compute the hydraulic diameter ($D_H = 4A_o / p_w$) for this channel and convert your mean heat transfer coefficient value to a fully-developed Nusselt number. In a plot, compare this value to the trends indicated in the table on the last page for comparable aspect ratios. Also determine the Reynolds number of the flow using the mean *w* velocity.

(d) Run your program for exactly the same conditions as part (c), except shrink the passage dimensions by a factor of 3 by setting $\Delta x = 0.15$ mm, $\Delta y = 0.15$ mm. Report the values of the maximum *w* velocity and the mean heat transfer coefficient value. Does the value of the fully-developed Nusselt number change? Should it? Briefly discuss the reasons for any observed changes in these parameters compared with the results for part (c).

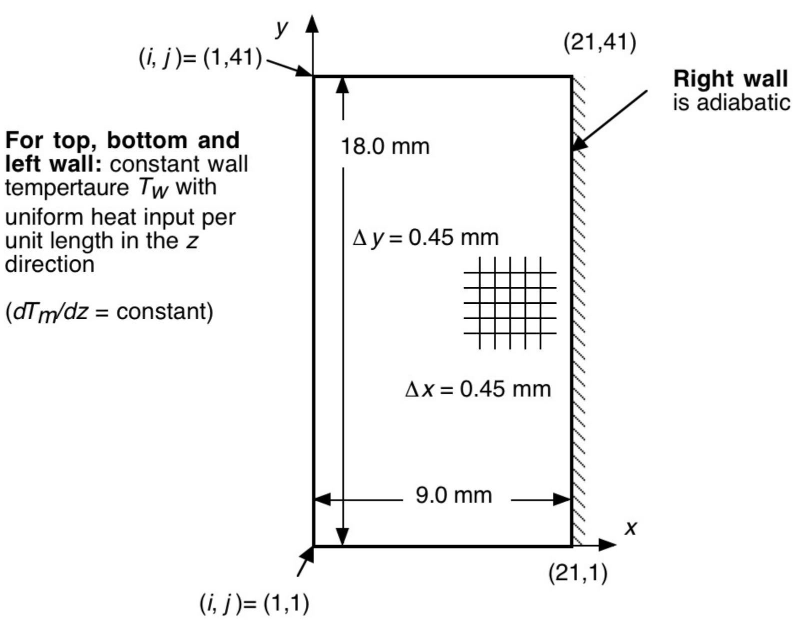
(e) Run your program for exactly the same conditions as part (c), except change the properties to those for a 50/50 mixture of ethylene glycol and water:

$\alpha = 1.08 \times 10^{-7}$ m^2/s, $\rho = 1055$ kg/m^3, $\nu = 9.00 \times 10^{-7}$ m^2/s, $c_p = 3559$ J/kg°C, $k = 0.407$ W/m°C

Report the computed values of mean heat transfer coefficient and Nusselt number. Which property most strongly affects the value of the heat transfer coefficient? (Do a sensitivity study with your code.)

Task II

Create a second copy of your program and modify it to the 2:1 aspect ratio passage shown in Fig. 2.



For this task you must make one change to the computational scheme. After each temperature iteration, the wall temperature at each node of the right wall must be set equal to the new value of temperature at the node immediately to the left of the wall node:

For all $2 \le j \le 40$: $T'_{21,j} = T'_{20,j}$

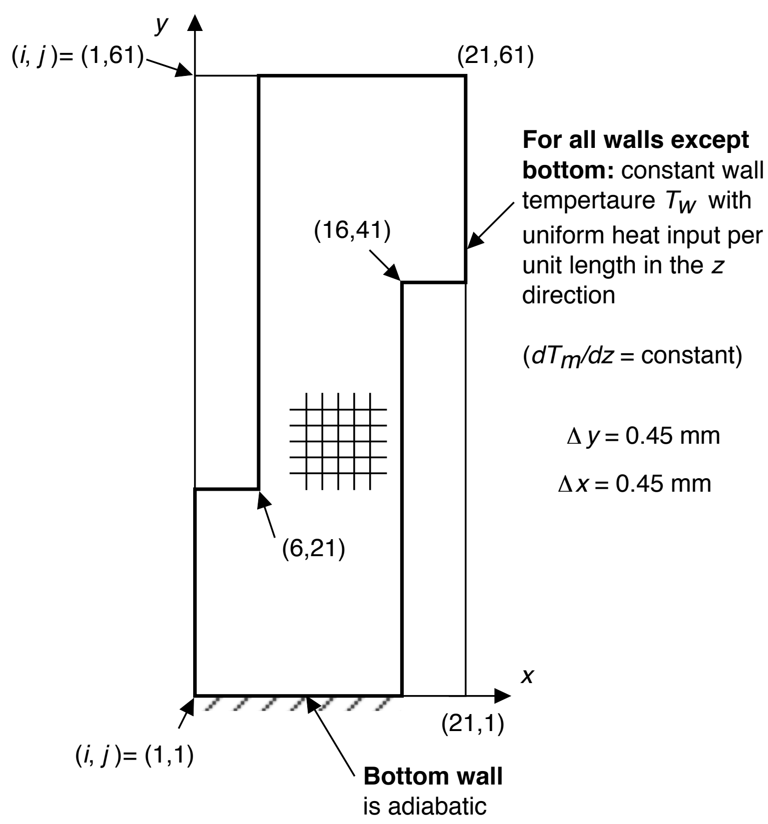
This enforces a zero gradient, zero heat flux condition at the right wall.

(a) Run the modified program to convergence for the parameters values indicted in part (b) of Task 2 above. Generate 3-D surface plots of the resulting $w(x,y)$ velocity and $T(x,y)$ fields. Compute the hydraulic diameter for this channel and use your fully developed heat transfer coefficient for this case to compute a fully-developed Nusselt number. Compare this value to that in the table on the last page for the 2:1 aspect ratio.

(b) Calculate the hydraulic diameter based on heated perimeter p_h: $D_H = 4A_o / p_h$ for this case. Compute the fully developed Nusselt number based on this hydraulic diameter definition and compare the result to that in the table on the last page for a square cross section.

Task III

Create a second copy of your program from Task I (the 3:1 aspect ratio passage) and modify it to analyze the fully developed flow and heat transfer for the passage shown in Fig. 3.



There are three key changes you must make to the Task I program to model flow in this channel. First, you must break the sweeping loop into three separate loops so you can change the x extent of the looping for each. This will be explained more fully in class. Three loops will also be needed to compute the mean velocity and mean temperature of the flow.

Second, you must evaluate the contributions of the node elements on all perimeter surfaces to determine the variation of the heat transfer coefficient for each, and sum the heat input rate from each to get the total rate of heat input to the flow. From the total rate of heat input, determine the fully developed heat transfer coefficient and Nusselt number.

Third, be sure to handle the adiabatic boundary condition on the bottom wall appropriately after each iteration.

In addition, be sure to account for the more complex geometry when computing the open area and wetted perimeter for determination of the hydraulic diameter. For the geometry shown in Fig. 3, determine the Reynolds number and fully developed Nusselt number for the flow conditions specified in Task I.

Task IV

Take a copy of your program from Task III and modify the walls to a different geometry of your choice. In doing so, make only rectilinear wall geometries with 90° corners, do not make the longest dimension of the channel cross section more that 27 mm, and keep your wall segment lengths at least $2\Delta x$ or $2\Delta y$. Present results for at least one geometry that is different from that in Task III for the flow conditions specified in Task I. A challenge to you is to find the geometry that provides a value of fully-developed Nusselt number that is as high as possible.

In considering different geometries, be sure to account for the more complex geometry when computing the open area and wetted perimeter for determination of the hydraulic diameter. For each geometry considered, determine the Reynolds number and fully developed Nusselt number for the flow conditions specified in Task I.

Tasks to be divided between partners:

(1) Equation derivations
(4) Program development (flow chart and coding)
(5) Run computations for different cases
(6) Analysis of results
(7) Write-up of results and conclusions

Deliverables:
Your submitted written report should include:

(1) Description of how work was be divided within group,.
(2) A summary of your analysis organization and computer code (including a flow chart).
(3) A description of the computed results, including plots of the heat transfer coefficient variation along the passage wall, and 3-D plots of the flow and temperature fields.

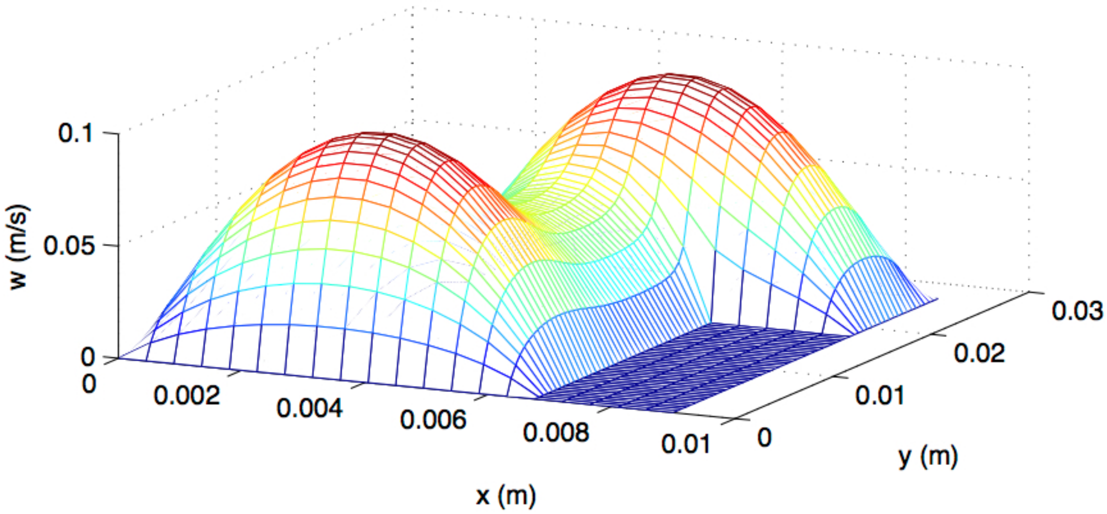
A copy of your program should be attached to the report as an appendix.

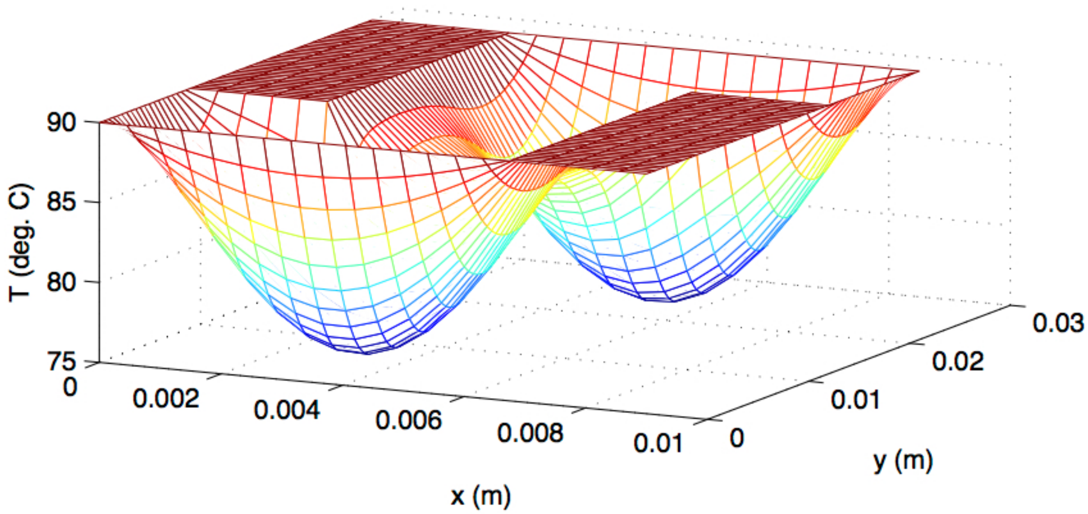
Students in ME250B: do all tasks I - IV.
Students in ME151B: do tasks I - III.

Note: Report is due Thursday 2/23/23.

Grade will be based on:

(1) Thoroughness of documentation and analysis of results.
(2) Accuracy of results and the clarity with which they support answers to questions.
(3) Presentations of results.


w (m/s)
0.1
0.05
0
0
0.002
0.004
0.006
0.008
0.01
x (m)
0
0.01
0.02
0.03
y (m)


T (deg. C)
90
85
80
75
0
0.002
0.004
0.006
0.008
0.01
x (m)
0
0.01
0.02
0.03
y (m)

$\overline{Nu}_{H1}$ = average Nusselt number for uniform heat flux in flow direction and uniform wall temperature at any cross section

$\overline{Nu}_{H2}$ = average Nusselt number for uniform heat flux both axially and circumferentially

$\overline{Nu}_{T}$ = average Nusselt number for uniform wall temperature

$f\,Re_{D_H}$ = product of friction factor and Reynolds number

TABLE 6.1 Nusselt Number and Friction Factor for Fully Developed Laminar Flow of a Newtonian Fluid Through Specific Ducts[a]

Geometry $\left(\frac{L}{D_H} > 100\right)$	$\overline{Nu}_{H1}$	$\overline{Nu}_{H2}$	$\overline{Nu}_{T}$	$f\,Re_{D_H}$	$\frac{\overline{Nu}_{H1}}{\overline{Nu}_{T}}$
60°, $2a$, $2b$; $\frac{2b}{2a} = \frac{\sqrt{3}}{2}$	3.111	1.892	2.47	53.33	1.26
$2b$, $2a$; $\frac{2b}{2a} = 1$	3.608	3.091	2.976	56.91	1.21
a, a, a, a, a, a	4.002	3.862	3.34[b]	60.22	1.20
$2b$, $2a$; $\frac{2b}{2a} = \frac{1}{2}$	4.123	3.017	3.391	62.19	1.22
(circle)	4.364	4.364	3.657	64.00	1.19
$2b$, $2a$; $\frac{2b}{2a} = \frac{1}{4}$	5.331	2.930	4.439	72.93	1.20
Insulation, $2b$, $2a$; $\frac{2b}{2a} = \frac{1}{4}$	6.279[b]	—	5.464[b]	72.93	1.15
$2b$, $2a$; $\frac{2b}{2a} = 0.9$	5.099	4.35[b]	3.66	74.80	1.39
$2b$, $2a$; $\frac{2b}{2a} = \frac{1}{8}$	6.490	2.904	5.597	82.34	1.16
$\frac{2b}{2a} = 0$	8.235	8.235	7.541	96.00	1.09
$2a$, $2b$, Insulation; $\frac{2b}{2a} = 0$	5.385	—	4.861	96.00	1.11

[a] Abstracted from Shah and London (13).
[b] Interpolated values.

from: Principles of Heat Transfer, F. Kreith and M.S. Bohn, 5th Edition.