HW-3

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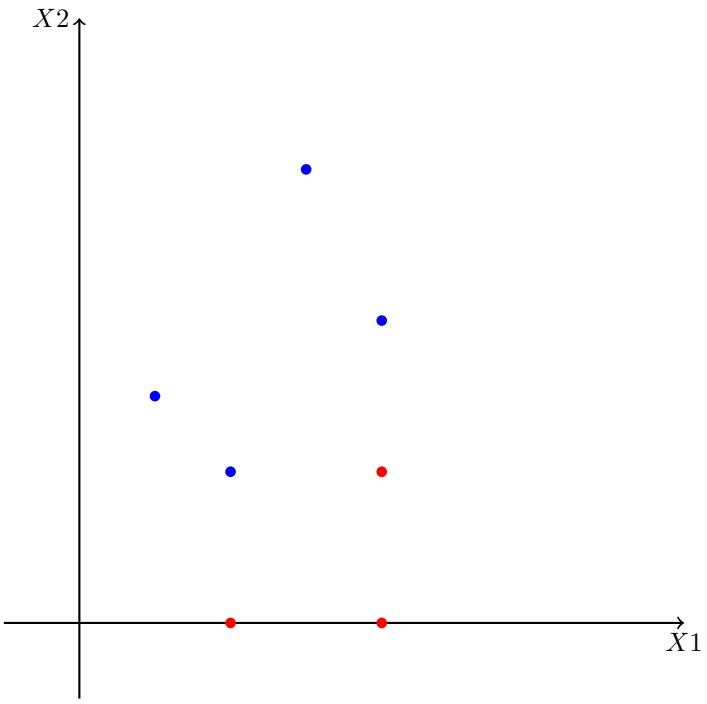
March 2024

1 Problem 1

1.1



From the plot of the observations, it is clear that the optimal separating hyperplane will be the line which would pass through the center of the pair of points (2,0) (2,2) and (4,2) (4,4).

$$P1 = (\frac{2+2}{2}, \frac{0+2}{2})$$

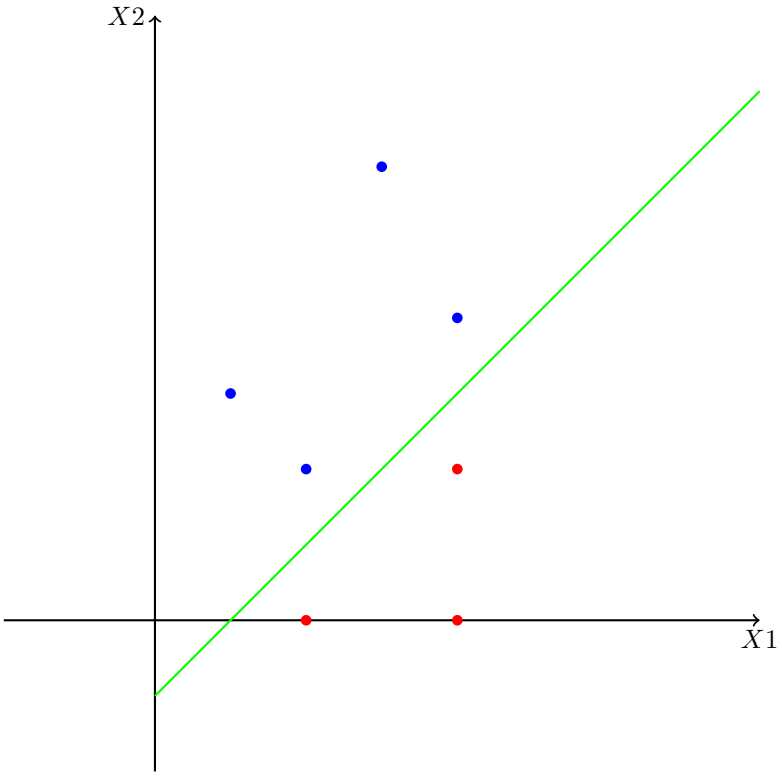
$$P1 = (2, 1)$$

$$P2 = (\frac{4+4}{2}, \frac{2+4}{2})$$

$$P2 = (4, 3)$$

The line passing through P1 and P2 will be the optimal separating hyperplane, which is indicated by the green line below

$$Y = 1 - X1 + X2$$



1.2

Classify to Red if

$$1 - X1 + X2 < 0$$

Classify to Blue if

$$1 - X1 + X2 >= 0$$

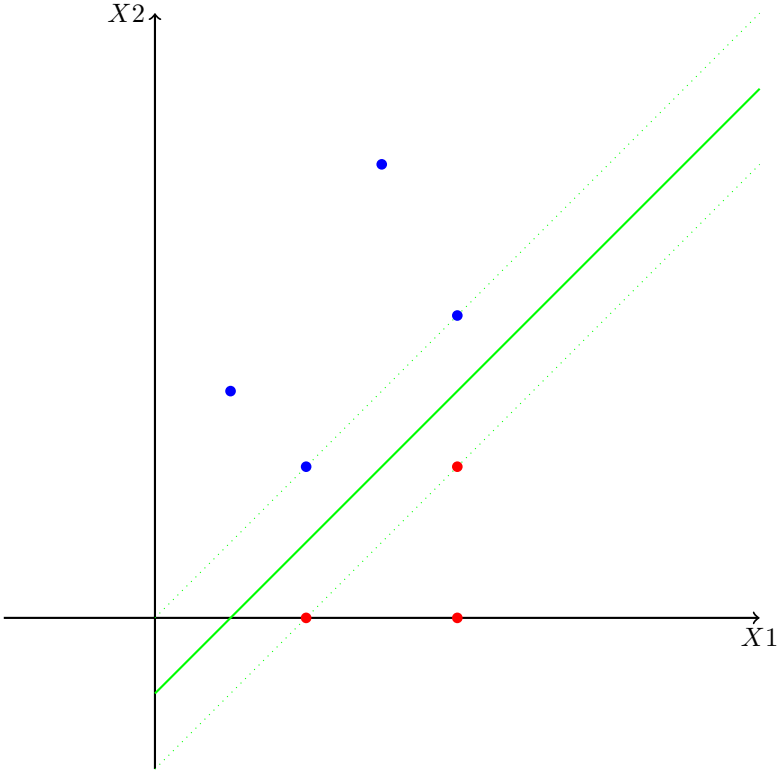
$$\beta_0 = 1$$

$$\beta_1 = -1$$

$$\beta_2 = 1$$

1.3

Maximal margin hyperplane are indicated by dotted lines



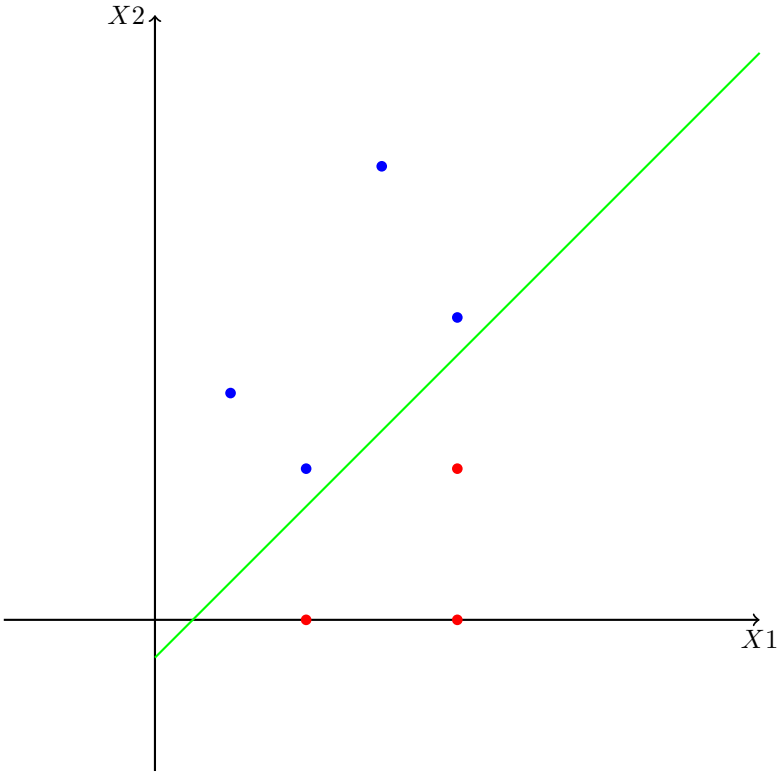
1.4

The support vectors for the maximal margin hyperplane are (2,0) (2,2) (4,2) (4,4)

1.5

Observation 7 is not part of the support vector so slight movement would not affect the maximum margin hyperplane. The point should move inside the margin to affect the results.

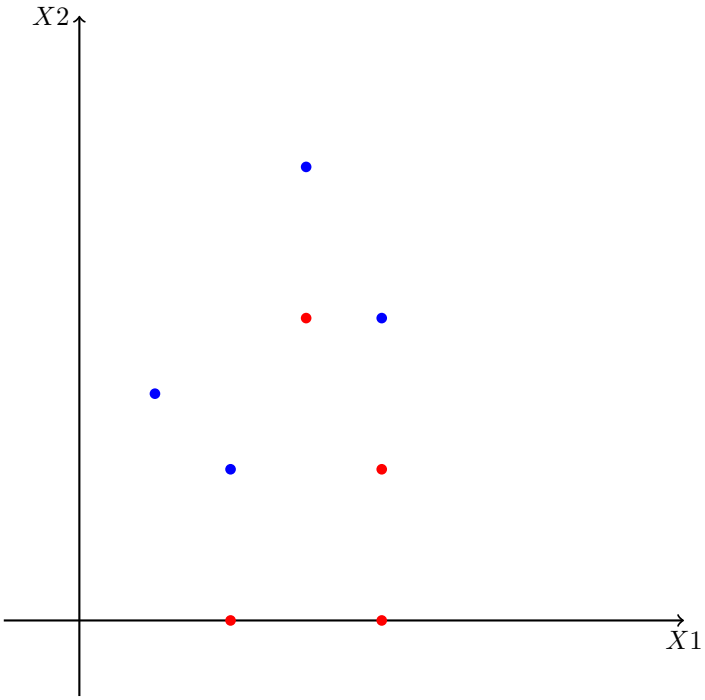
1.6



$$Y = 0.5 - X1 + X2$$

1.7

Adding point (3,4) to Red class makes the 2 classes non separable by 2D hyperplane.



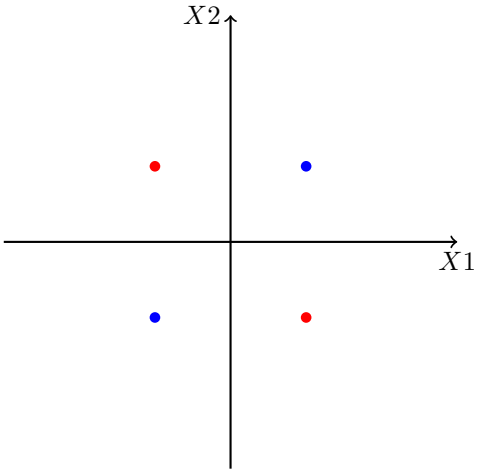
2 Problem 1

2.1

Shape of X - (4,2)
Shape of Y - (4,1)
XNOR logic gate represents the truth table

X1	X2	Y
-1	-1	1
-1	1	-1
1	-1	-1
1	1	1

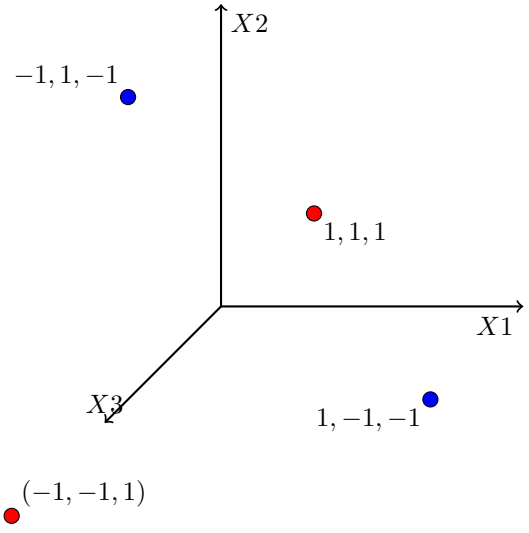
2.2



The positive and negative classes are not separable because there is no single line that can separate the positive examples from the negative ones

2.3

X1	X2	X1X2 (X3)	Y
-1	-1	1	1
-1	1	-1	-1
1	-1	-1	-1
1	1	1	1



These 4 points are linearly separable now
The equation of the maximal margin hyperplane

$$X3 = 0$$

2.4

Support Vectors - (-1,-1,1) (-1,1,-1) (1,-1,-1) (1,1,1)
Distance between points (-1,-1,1) (-1,1,-1) (1,-1,-1) (1,1,1) and plane z=0 is:

$$\frac{|1 * 1|}{\sqrt{0^2 + 0^2 + 1^2}}$$

$$1$$

Margin Size = 1 + 1 = 2