Regression Discontinuity Notes

Lee 2008[*]

S S K Pavan[†]

December 5, 2021

Theory

- RDD can be used when researcher has specific information about the rules determining the treatment of interest
- For example, treatment is scholarship awarded or not. This is determined by whether or not students scores above a threshold in a test
- We observe the following data generating process

$$Y = \begin{cases} Y_i(1) & \text{if } R_i > c \\ Y_i(0) & \text{if } R_i \leq c \end{cases}$$

$$T_i = \begin{cases} 1 & \text{if } R_i > c \\ 0 & \text{if } R_i \leq c \end{cases}$$

Treatment is deterministic and discontinuous function of an observed covariate R_i

Basic Idea:

- Compare observations just above and just below the threshold to infer treatment effect.
- Treatment is assigned close to random for individuals in the neighborhood of c. Comparing treated and nontreated near c reveals treatment effect (local)

The solid black line we can observe. The dotted lines are counterfactuals.

Assumptions:

[*]Randomized experiments from non-random selection in U.S. House elections

[†]hes207045@hss.iitd.ac.in

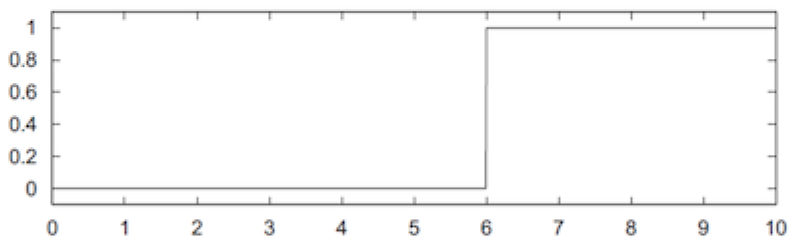


Fig. 1. Assignment probabilities (SRD).

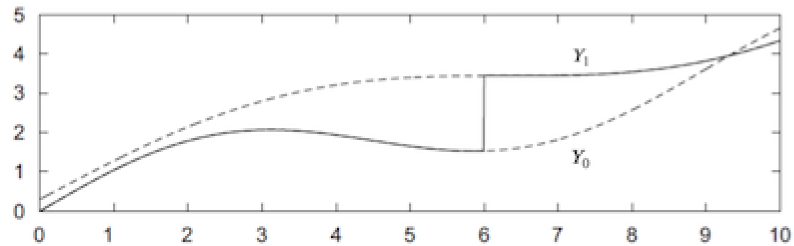


Fig. 2. Potential and observed outcome regression functions.

- Potential outcomes are smooth at the threshold

$$\lim_{r \to c^+} E[Y_i(1)|\ R_i = r] = \lim_{r \to c^-} E[Y_i(1)|\ R_i = r]$$

  i.e conditional expectation functions of potential outcomes are continuous

- The population below the threshold should not be discreetly different from the population above the threshold

Average treatment Effects:

$$ATE = \lim_{r \to c^+} E[Y_i(1)|\ R_i = r] - \lim_{r \to c^-} E[Y_i(0)|\ R_i = r]$$

From our assumption, the limit exists for both outcomes

$$\implies E[Y_i(1)|\ R_i = c] - E[Y_i(0)|\ R_i = c]$$
$$\implies E[Y_i(1) - Y_i(0)|\ R_i = c]$$

When potential outcomes are smooth around the threshold, a comparison of individuals just above and just below yields the average treatment effect for those at the threshold

Important: Till now we have not assumed anything regarding the distribution of Y or R. Only continuity assumption

Conditional Independence Assumption:

Holds trivially for RD,

$$Y_i(1), Y_i(0) \perp T_i \mid R_i$$

Because given , treatment has no variation, it is either 1 or 0 Hence we cannot do matching, or propensity score. There is no common support. Instead we extrapolate.

Estimation:

Functional form to approximate $E[Y_i \mid R_i]$ is very important. We could mistake non linearity for treatment effect. Global parametric estimation:

$$Y_i = \alpha + \beta \ 1\{R_i > c\} + \sum_{k=1}^{K} \ 1\{R_i \leq c\}(R_i - c)^k + \sum_{k=1}^{K} \ 1\{R_i > c\}(R_i - c)^k + 1\{R_i > c\}X_i + \epsilon_i$$

Incumbency Advantage in the U.S. House — Lee (2008)

Incumbecy Advantage:

Causal impact of being the incumbent party (currently in power) in a district on the votes obtained in the district's election

What is the reality?

- Incumbent parties using official power to gain unfair advantage over potential challengers OR,
- Selection effect: Successful politicians are expected to be more successful when running for re-election?

Model

$$v_{i,2} = \beta_0 v_{i,1} + \beta_1 Incumbent_{i,2} + \gamma_i X + \epsilon_i$$

- $v_{i,t}$ — vote share of Democrats in district i at time t (0 or 1)
- $incumbent_{i,2} = 1[v_{i,1} \geq 0.5]$
- X represents control variables. (some variables unobservable)

Ideally we want Incumbent to be exogeneous. Here there is no issue of endogeneity as vote share in election now cannot determine who won in the previous election.

Key assumption for RDD: density of $v_{i,1}$ conditional on X is continuous in v

<u>Problems with OLS:</u>
OVB: elements of X might be unobservable and correlated with vote share in previous year, thereby effecting incumbency. OLS estimates biased and inconsistent

Data

time frame: **1946 - 1998**

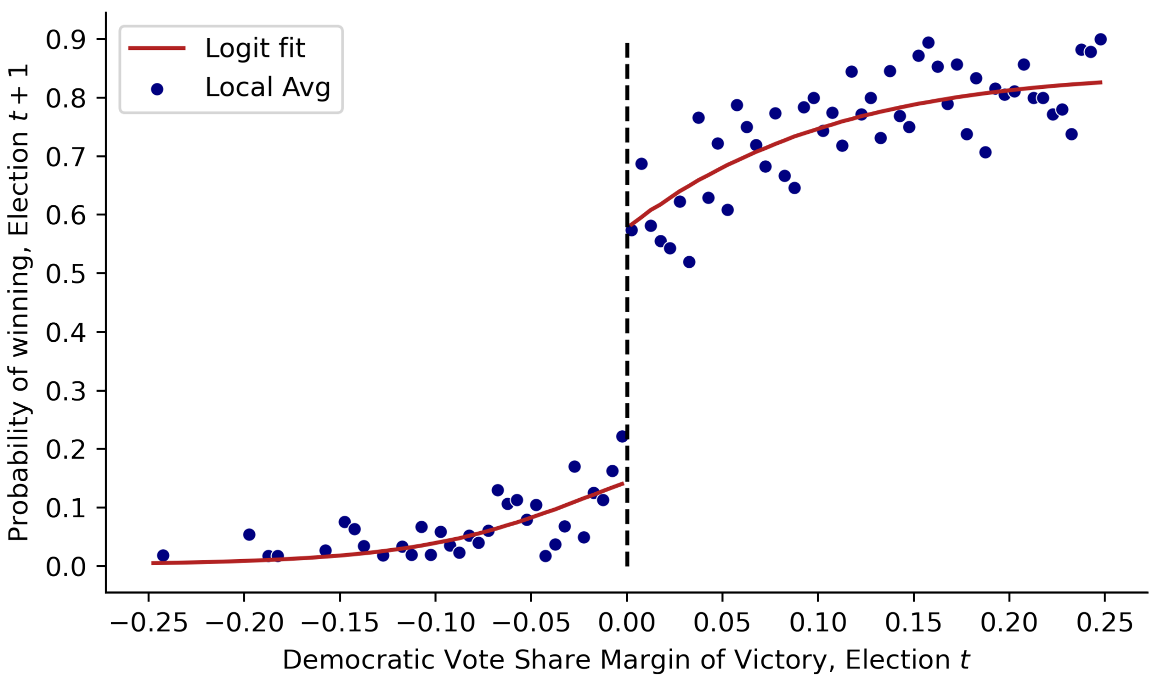
Vote share margin of victory = Democratic vote share - vote share of the strongest opponent

Democrat wins when Vote share margin of victory

Incumbency advantage analyzed at the level of party at district level. Errors clustered at the decade-district level (districts change every 10 yrs)

RDD Estimates:

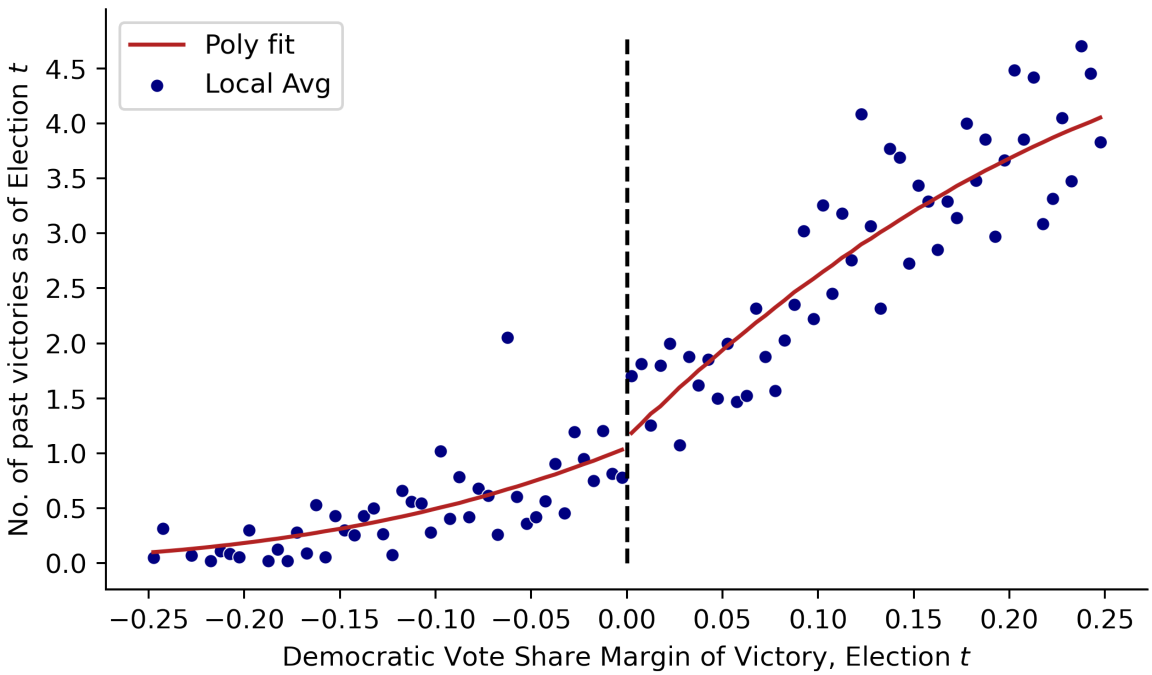
Shows prob of democrat running in and winning election t+1:



The points plotted show the following:
Between vote share margin of width 0.05,
Prob of winning election (t + 1) = Avg(winning in election t+1)

- Striking discontinuity at 0. Democrats that barely win at t, are much likely to win election at t+1
- Democrats that barely loose at t, are much likely to loose election at t+1

Here we are checking whether there is discontinuity in the covariate X at the threshold. We are doing this to ensure the continuity assumption.
It could be that savvy politicians manipulate close elections to ensure victory



Main Regression Estimates:

Covariate Balance:

Table 1: Effect of winning an election on subsequent party electoral success: alternative specifications, and refutability test, regression discontinuity estimates

	(1) Vote Share t+1	(2) Vote Share t+1	(3) Vote Share t+1	(4) Vote Share t+1	(5) Vote Share t+1	(6) Res. Vote Share t+1	(7) 1st Dif Vote Share t+1	(8) Vote Share t-1
right						0.080***		
						(0.016)		
demshareprev		0.292***			0.297***			
		(0.017)			(0.017)			
demwinprev		-0.017**			-0.007		-0.175***	0.240***
		(0.007)			(0.007)		(0.009)	(0.009)
demofficeexp			-0.001**		0.000		-0.002	0.002
			(0.001)		(0.003)		(0.003)	(0.002)
othofficeexp			0.001		-0.000		-0.008*	0.011***
			(0.001)		(0.004)		(0.004)	(0.003)
demelectexp				-0.001**	-0.003		-0.003	0.000
				(0.001)	(0.003)		(0.003)	(0.002)
othelectexp				0.001	0.003		0.011***	-0.011***
				(0.001)	(0.004)		(0.004)	(0.003)
R-squared	0.682	0.708	0.682	0.682	0.710	0.233	0.123	0.727
R-squared Adj.	0.682	0.708	0.682	0.682	0.709	0.232	0.121	0.726
No. observations	6558	6558	6558	6558	6558	6558	6558	6558

	Variable	All Winner	All Loser	(0.5) Winner	Loser	(0.05) Winner	(0.05) Loser	para Winner	para Loser	order
0	Dem vote share (t+1)	0.698	0.347	0.629	0.372	0.542	0.446	0.531	0.454	2.0
1	_se	0.003	0.003	0.003	0.003	0.006	0.006	0.008	0.008	3.0
2	_sd	0.179	0.150	0.145	0.124	0.116	0.107	NaN	NaN	4.0
3	Dem win prob (t+1)	0.909	0.094	0.878	0.100	0.681	0.202	0.611	0.253	5.0
4	_se	0.004	0.005	0.006	0.006	0.026	0.023	0.039	0.035	6.0
5	_sd	0.276	0.285	0.315	0.294	0.458	0.396	NaN	NaN	7.0
6	Dem vote share (t-1)	0.681	0.368	0.607	0.391	0.501	0.474	0.477	0.481	8.0
7	_se	0.003	0.003	0.003	0.003	0.007	0.008	0.009	0.010	9.0
8	_sd	0.189	0.153	0.152	0.129	0.129	0.133	NaN	NaN	10.0
9	Dem win prob (t-1)	0.889	0.109	0.842	0.118	0.501	0.365	0.419	0.416	11.0
10	_se	0.005	0.006	0.007	0.007	0.027	0.028	0.038	0.039	12.0
11	_sd	0.310	0.306	0.360	0.317	0.493	0.475	NaN	NaN	13.0
12	Dem pol exp	3.812	0.261	3.550	0.304	1.658	0.986	1.219	1.183	14.0
13	_se	0.061	0.025	0.074	0.029	0.165	0.124	0.229	0.145	15.0
14	_sd	3.766	1.293	3.746	1.390	2.969	2.111	NaN	NaN	16.0
15	Opp pol exp	0.245	2.876	0.350	2.808	1.183	1.345	1.424	1.293	17.0
16	_se	0.018	0.054	0.025	0.057	0.118	0.115	0.131	0.170	18.0
17	_sd	1.084	2.802	1.262	2.775	2.122	1.949	NaN	NaN	19.0
18	Dem elec exp	3.945	0.464	3.727	0.527	1.949	1.275	1.485	1.470	20.0
19	_se	0.061	0.028	0.075	0.032	0.166	0.131	0.230	0.151	21.0
20	_sd	3.787	1.457	3.773	1.550	2.986	2.224	NaN	NaN	22.0
21	Opp elec exp	0.400	3.007	0.528	2.943	1.375	1.529	1.624	1.502	23.0
22	_se	0.019	0.054	0.027	0.058	0.120	0.119	0.132	0.174	24.0
23	_sd	1.189	2.838	1.357	2.805	2.157	2.022	NaN	NaN	25.0
24	observations	3818.000	2740.000	2546.000	2354.000	322.000	288.000	3818.000	2740.000	26.0