

Principal Component Analysis Task 01

Ji Pavan Vitesh
AP21110011080
CSE-Q

Given data $X = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 3 & 5 \\ 5 & 4 \\ 5 & 6 \\ 6 & 5 \\ 8 & 7 \\ 9 & 8 \end{bmatrix}$

Centre each column on its mean $X_c = [X - \bar{X}] = \begin{bmatrix} -4 & -3 \\ -2 & -2 \\ -2 & 0 \\ 0 & -1 \\ 0 & -1 \\ 1 & 0 \\ 3 & 2 \\ 4 & 3 \end{bmatrix}$

Covariance Matrix. $S = \frac{1}{n-1} X_c^T X_c$

$$S = \frac{1}{8-1} \begin{bmatrix} -4 & -2 & -2 & 0 & 0 & 1 & 3 & 4 \\ -3 & -2 & 0 & -1 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} -4 & -3 \\ -2 & -2 \\ -2 & 0 \\ 0 & -1 \\ 0 & -1 \\ 1 & 0 \\ 3 & 2 \\ 4 & 3 \end{bmatrix}$$
$$S = \frac{1}{7} \begin{bmatrix} 50 & 34 \\ 34 & 28 \end{bmatrix}$$

Equation for eigen values and eigen vectors $[S - \lambda I]U = 0$

$$\begin{bmatrix} \frac{50}{7} - \lambda & \frac{34}{7} \\ \frac{34}{7} & 4 - \lambda \end{bmatrix} = 0$$

$$\left(\frac{50}{7} - \lambda\right)(4 - \lambda) = \frac{34}{7} \times \frac{34}{7}$$

$$\lambda = 0.468, 10.67$$

Eigen vectors are $\begin{pmatrix} -0.73 \\ 1 \end{pmatrix}, \begin{pmatrix} 1.37 \\ 1 \end{pmatrix}$

$$U = \begin{bmatrix} -0.73 & 0.37 \\ 1 & 1 \end{bmatrix}$$

New Coordinates of points are $[X - \bar{X}] U$.

$$\begin{bmatrix} -4 & -3 \\ -2 & -2 \\ -2 & 0 \\ 0 & -1 \\ 0 & 1 \\ 3 & 0 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -0.73 & 0.37 \\ 1 & 1 \end{bmatrix}$$

New Coordinates =

$$\begin{bmatrix} -0.08 & -8.48 \\ -0.54 & -4.74 \\ 1.46 & -2.74 \\ -1 & -1 \\ 1 & 1 \\ 0.73 & 1.37 \\ -0.19 & 6.11 \\ 0.08 & 8.48 \end{bmatrix}$$