1.Write the forward kinematic equation for a 2DoF planar manipulator with revolute joints.

Answer: For a 2DoF planar manipulator with revolute joints, the forward kinematic equation is

$$X=L1 \cos(\theta_1) + L2 \cos(\theta_1+\theta_2)$$

$$Y=L1 \sin(\theta_1) + L2 \sin(\theta_1+\theta_2)$$

Here X and Y are the positions of the end effectors for a 2DoF planar manipulator with revolute joints.

Here we can also find the orientation of the manipulator by adding the theta_1 and theta_2.

Theta_1: This is the angle with respect to the x axis and y axis corresponding to the first revolute.

Theta_2: These are the angles with respect to the x axis (moving frame { observer from below example's diagram}) and y axis({ observer from below example's diagram}) corresponding to the first revolute joint.

I have considered an example and explained the forward kinematic equation for a 2DoF planar manipulator with revolute joints.

NOTE : I have calculated these equations for an individual from

1. I have assumed that the length of the link is L1 and its angle is theta_1 with respect to the first revolute joint. Similarly, for the second revolute, the joint length of the link is L2, and its angle is theta 1

2. Using simple trigonometry, I have defined the end positions of the first joint and second joint as (x_1, y_1) and (x_2, y_2).

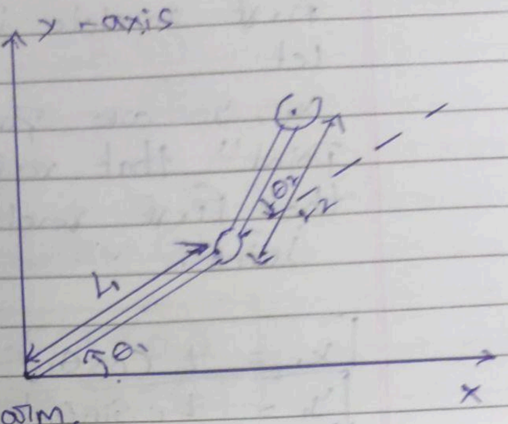
3. To get the end effector position of the 2D planar manipulator. I had just added both x_1, x_2,y_1 and y_2 to get the X and Y positions.

HERE ARE THE PICTURES OF THE EXAMPLE:

1, Write the forward Kinematic Equation for a 2DoF planar manipulator with revolute joints.

Ans

Consider an Example:-



From the diagram we can see that we have 2 Arm. (2 Joints). It has 2 DoF.

L_1 = length of first arm.
l_2 = length of second arm.
and
θ_1 = Angle of first arm wrt to x-axis.
θ_2 = angle of second arm wrt to x-axis (Base - arm)

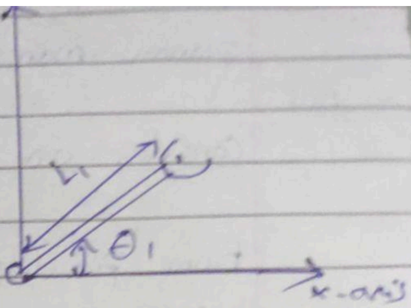
Now, we need to find the Position of ~~eff~~ End Effector in these 2D Coordinate

To determine that we need to find x and y values.

To find x & y value from Base (or) world to End effector. we need note the Joint.
Here we have 2 revolute Joints.

Initially i need to Position from Base to 1st revolute Joint.

Here we need to find
x, y value wrt to
first revolute joint
let
x_1, y_1 are position
point's that represent
the first revolute
joint.



There is first
revolute Joint

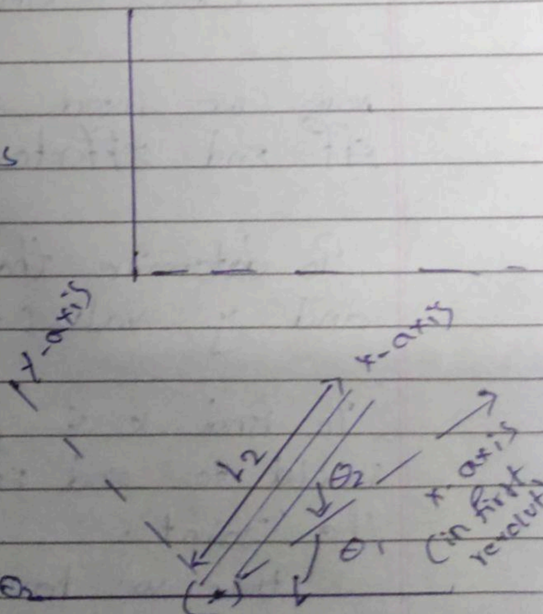
$\boxed{x_1 = L_1 \cos\theta_1}$ (according to x-axis)
$\boxed{y_1 = L_1 \sin\theta_1}$ (" " Y-axis)

now let's derive the position from
first revolute Joint to second revolut
Joint (or) End Effector.

Note:- Here we will
consider the arm as
x-axis (Because it is
an moving frame).

$$\boxed{x_2 = l_2 \cos(\theta_1 + \theta_2)}$$

$$\boxed{y_2 = l_2 \sin(\theta_1 + \theta_2)}$$



Here we got θ as $\theta_1 + \theta_2$
because the second
revolute is changing
it's direction (or) angle
So inorder to get θ we Pythagores

Therefore the position of end Effector wrt to Base is given by

$$x = x_1 + x_2 \quad -(1)$$
$$y = y_1 + y_2 \quad -(2)$$

Substituting x_1 & x_2 & y_1 & y_2 in Equation (1) & Equation (2)

$$\therefore \boxed{\begin{aligned} x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{aligned}}$$

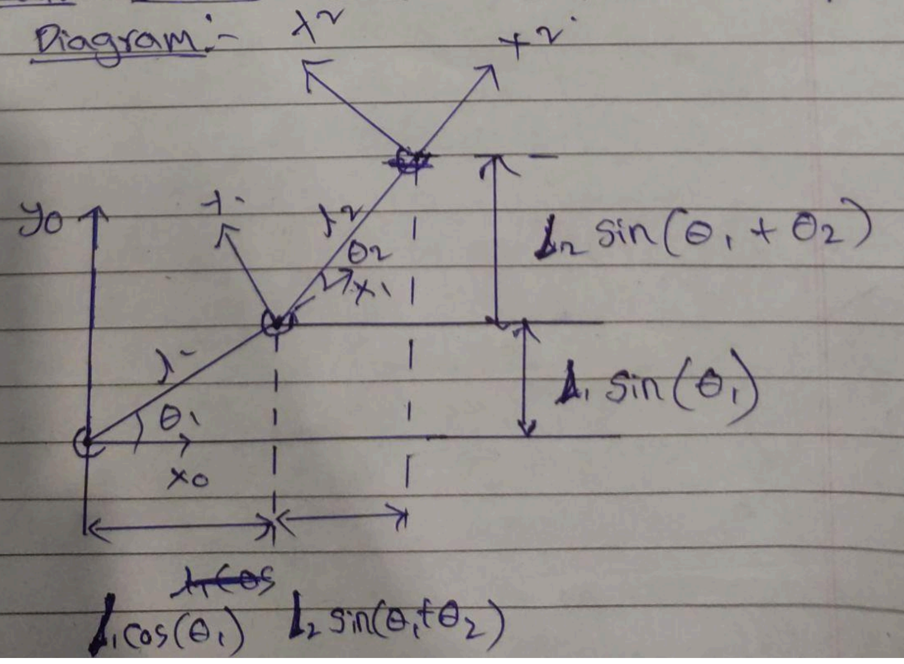
Position Points.

To determine the Orientation we will add up the angle

$$\boxed{\Theta = \theta_1 + \theta_2}$$

<u>Forward kinematic of 2DoF Planar manipulator-</u>

<u>Diagram</u>:-



These is the ultimate diagramatic representation of the 2DoF planar manipulator with revolute joints.

2.Solve the inverse kinematics problem for a 2DoF planar
manipulator with revolute joints

ANSWER :Inverse Kinematics (IK) used for finding the joint angles which are needed to achieve a desired end-effector position

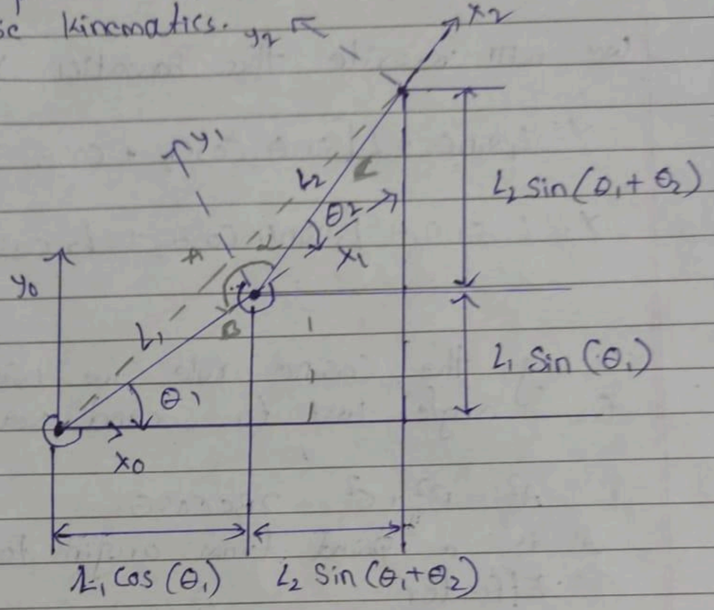
1. Let's consider the previous question as an example, and we will solve the inverse kinematic equation.
2. Initially, we assume that we don't have the position of the revolute joint. In such a case, we need to find the forward kinematic equation to determine the position."
3. Next we will find joint angles using the end effector positions

HERE IS AN EXAMPLE FOR SOLVING THIS PROBLEM

2. Solve the ~~(inserve)~~ inverse kinematics for 2 DOF planar manipulator with revolute joint:-

Ans Let's consider an example as we consider in previous question number-1 to solve.

Inverse kinematics.



Inverse kinematics:-

we need find the joint angle using the end ~~positi~~ effector positions.

Since we derived the position as x, y in forward kinematics we will use them.

$$x = L_1 \cos\theta_1 + \underbrace{L_2 \cos(\theta_1+\theta_2)}_{1} \} \rightarrow \cos(A+B)$$

using formula.

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

we will rewrite the ~~$\cos(A+B)$~~ x point. (or) x position.

In above pic i have defined all the variables

1.I have assumed that the length of the link is L1 and its angle is theta_1 with respect to the first revolute joint. Similarly, for the second revolute, the joint length of the link is L2, and its angle is theta 1

2.Using simple trigonometry, I have defined the end positions of the first joint and second joint as (x_1, y_1) and (x_2, y_2).

3. Got the end effector point as

X=L1 cos(theta_1) + L2 cos(theta_1+theta_2)

Y=L1 sin(theta_1) + L2 sin(theta_1+theta_2)

We can observe that an trigonometry formula in X ,Y

cos(A+B)= cos*A**cos*B*−sin*A**sin*B*

sin(A+B)=sinA*cosB+cosA*sinB

Now we will expand them and rewrite the X,Y values

Then

X=L1 cos(theta_1) + L2[cos(theta_1)*cos(theta_2)-sin(theta_1)sin(theta_2)]

Y=L1 sin(theta_1) + L2[sin(theta_1)*cos(theta_2)+cos(theta_1)sin(theta_2)]

NOW we need to find the theta_2 angle. So, we will the cosine rule formula

See diagram and hardwrite page for knowing we used the cosine rule

Therefore will be

By considering the big triangle from the diagram

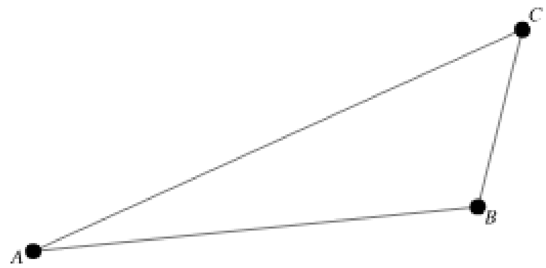
theta_2=cos^-1{ (x*2+y*2-L1^2-L2^2) / 2L1L2}

Similar by observing the diagram we can determine the

Theta_1 as beta-alpha

Where alpha look like these

Think alpha at b we have the alpha using some formula



we calculated the alpha and beta and subtract them

Beta is an angle of the big triangle

Theta_1 =beta-alpha

theta 1=tan^-1{ y*(L2.cos(theta_2+L1)- x*(L2.sin(theta_2) /

x * (L2.cos(theta_2+L1) + Y L2.sin(theta_2)}

theta_2=cos^-1{ (x*2+y*2-L1*2-L2*2) / 2L1L2}

BELOW I HAVE WRITTEN IN STEP BY STEP USING AN EXAMPLE

$X = L_1 \cos\theta_1 + L_2 \cos\theta_1 \cos\theta_2 - L_2 \sin\theta_1 \sin\theta_2$

Lepakshi Nandi™ 1686

$Y = L_1 \sin\theta_1 + L_2 \underbrace{\sin(\theta_1 + \theta_2)}_{\sin(A+B)}$

Using forumula
$\sin(A+B) = \sin A \cos B + \cos(A) \sin(B)$

We will rewrite the Equation Y position

$Y = L_1 \sin\theta_1 + L_2 [\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2]$

$Y = L_1 \sin\theta_1 + L_2 \sin\theta_1 \cos\theta_2 + L_2 \cos\theta_1 \sin\theta_2$.

Using the Cosine rule we will find θ_2 angle wrt to second revolte Joint.

$A^2 = B^2 + C^2 - 2BC\cos\theta$.

A is a point from origin to end Effector.

B is length from ~~second~~ first revolte to ~~End effector~~. second revolte

C is length from second revolte to End Effector.

So there $A^2 = x^2 + y^2$.
$B^2 = (L_1)^2$ (from diagram)
$C^2 = L_2^2$

$\cos\theta_2 = x^2 + y^2 - L_1^2 - L_2^2 / 2L_1L_2$

$$\boxed{\theta_2 = \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)}$$

θ_1 will be

$$\theta_1 = \beta - \alpha. \quad \text{(from diagram)}.$$

So, Inorder to calculate θ_1 we need to calculate α, β angle.

Calculation of α angle will be.

$$\tan\alpha = \frac{L_2 \sin\theta_2}{L_2 \cos\theta_2 + L_1} \quad -(2) \text{ (using Pythagoras formula)}$$

calculation (or) finding β angle will be

$$\tan\beta = \frac{y}{x} \quad \text{— (1)}$$

So inorder to θ_1 we will use formula.

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \tan(\beta - \alpha) = \tan\theta_1$$

$$\tan(\theta_1) = \tan(\beta - \alpha)$$

$$\tan(\theta_1) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta \tan\alpha}.$$

$$\tan(\theta_1) = \frac{\left(\frac{y}{x}\right) - \left(\frac{L_2 \sin\theta_2}{L_2 \cos\theta_2 + L_1}\right)}{1 + \left(\frac{L_2 \sin\theta_2}{L_2 \cos\theta_2 + L_1}\right)\left(\frac{y}{x}\right)} \quad \text{[from (1) \& (2)]}$$

$$\tan(\theta_1) = \frac{\dfrac{y(L_2\cos\theta_2+L_1) - x(L_2\sin\theta_2)}{(x)(L_2\cos\theta_2+L_1)}}{\dfrac{(x)(L_2\cos\theta_2+L_1) + y\,L_2\sin\theta_2}{x(L_2\cos\theta_2+L_1)}}$$

$$\therefore \tan(\theta_1) = \frac{y(L_2\cos\theta_2+L_1) - x(L_2\sin\theta_2)}{x(L_2\cos\theta_2+L_1) + y(L_2\sin\theta_2)}.$$

$$\theta_1 = \tan^{-1}\left(\frac{y(L_2\cos\theta_2+L_1) - x(L_2\sin\theta_2)}{x(L_2\cos\theta_2+L_1) + y(L_2\sin\theta_2)}\right)$$

$\therefore$ we got the θ_1 & θ_2 angles.
θ_1 & θ_2 are using for knowing the position of manipulator.

If we have space and position values then we can use then inorder to find rotation of each link in arm.

3. How many inverse kinematics solutions will exist for a 2DoF planar manipulator with revolute joints?

ANSWER :-

We will have three solutions as

FIRST SOLUTION

Theta_1 = tan^-1 (y/x) - tan^-1{(L2 sin(theta_2)) / (L1 + L2 cos(theta_2)}

Theta_2 = cos^-1 (x*2 + y*2 - L1 - L2 / 2 L1 - L2).

SECOND SOLUTION

Theta_1 = tan^-1 (y/x) - tan^-1{(L2 sin(theta_2)) / (L1 + L2 cos(theta_2)}

Theta_2 = cos^-1 (x*2 + y*2 - L1 + L2 / 2 L1 + L2)

THIRD SOLUTION

If the specified end-effector position (x, y) is outside the reachable workspace of the manipulator,then there will be no solution for valid joint angles that can be found to reach that point.

NOTE : PLEASE NOTE: I HAVE DONE A MISTAKE IN THE BELOW HANDWRITTEN IMAGES.

I'VE MENTIONED BOTH Theta_2 AND

Theta_2 = cos^-1 (x*2 + y*2 - L1 - L2 / 2 L1 - L2)

But the second theta should be

Theta_2 = cos^-1 (x*2 + y*2 - L1 + L2 / 2 L1 + L2)

~~Now~~ we need to find θ_1 angle

3, How many inverse kinematics Solution will Exists for a 2DoF Planar manipulator with revolute joints.

Ans for A 2DoF planar manipulator the inverse kinematic solution are '2'

reason:-

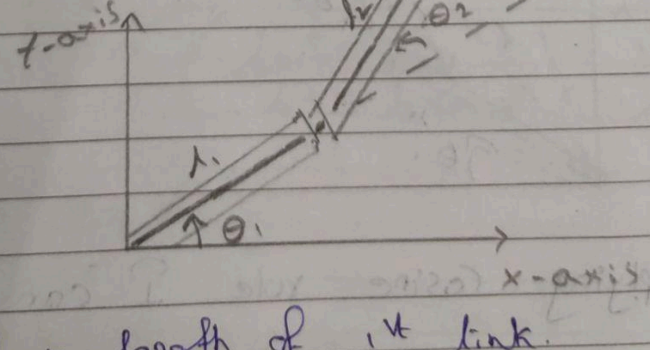
While we finding the angle of first link wrt to x-axis it depend upto on End Effector.

So, while calculating End effector angle using cosine function

Then Cosine function will have Symmetric above '0'

So, while find Effector angle ~~Ae~~ we can in positive or in negative angle. So that why we need up with two solutions.

Example:- let us consider a 2DoF planar manipulator.



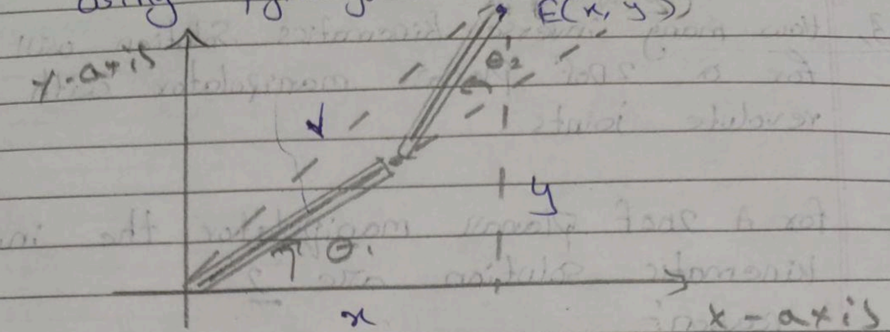
l_1 = length of 1st link.

l_2 = " " 2nd link

Similar θ_1 is angle wrt to link-1

θ_2 is angle wrt to link-2.

Initially we will find the joint angle using pythogores theorem.

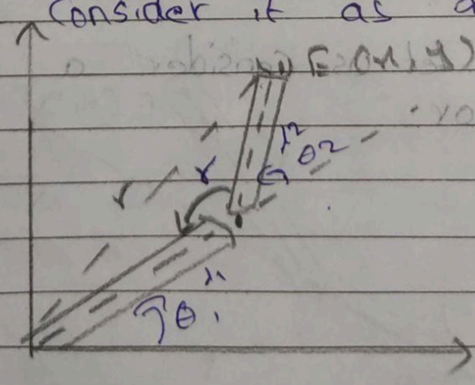


Here I have drawn the big traingle. let me consider hythosis side as r. and it's corresponding sides as x, & y

By pythogores thorem:

$$\boxed{r^2 = x^2 + y^2} \longrightarrow Eq(1)$$

Next I need to calculate angle for the rotation of 2 link. (Base & End effector). let me consider it as 'α'



By Applying cosine rule I can get

Equation as $r^2 = l_1^2 + l_2^2 - 2l_1l_2 \cos\alpha$.

$$\cos\alpha = \frac{l_1^2 + l_2^2 - r^2}{2 l_1 l_2}$$

replace r with Eq (1)

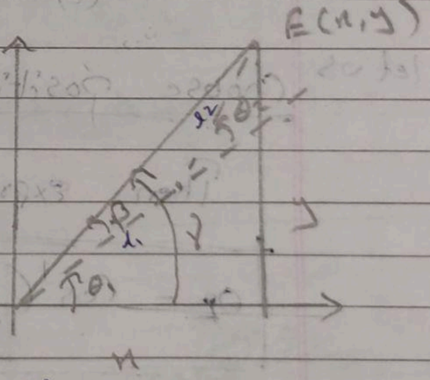
$$\cos\alpha = \frac{l_1^2 + l_2^2 - (x^2+y^2)}{2 l_1 l_2}$$

$$\cos\alpha = \frac{l_1^2 + l_2^2 - x^2 - y^2}{2 l_1 l_2}$$

using these α angle we can Easily find θ_2 angle

$$\theta_2 = 180 - \alpha.$$

By applying simple trigonometry. we will sides of θ_1



from these traingle we can get relation b/w θ_1 & θ_2 as γ

and

Angle β will be

$$\beta = \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2\cos\theta_2}\right)$$ (using pythagores theorem)

and $\tan\gamma = \frac{y}{x}$.

$$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$$

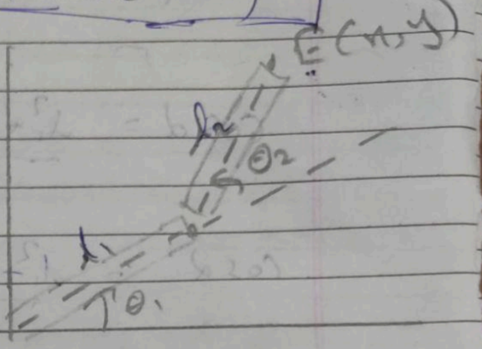
$$\boxed{\theta_1 = \gamma - \beta}$$ By oberversatio

Sub α & β in θ_1 then.

$$\boxed{\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)}$$

By cosine function we will get symmetric about '0'

So, we know cosine that has two value positive & negative



So, our θ_2 will also have '2' possible solution

(1) positive angle

(2) negative angle.

let us Choose positive angle:-

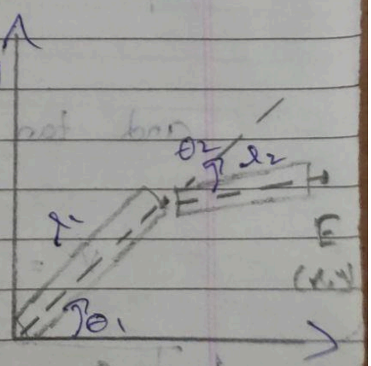
Then expression of θ_2 will be

$$\boxed{\theta_2 = \cos^{-1}\left(\frac{x^2+y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

let us Choose Negative angle:-

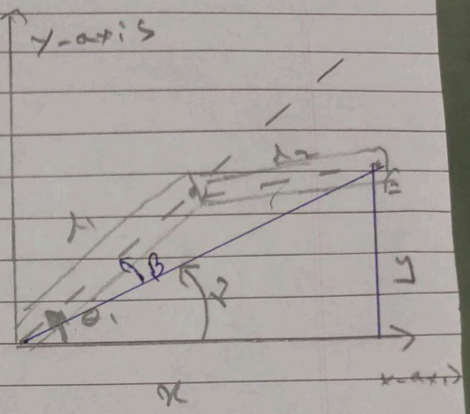
Then Expression of θ_2 will be

$$\boxed{\theta_2 = -\cos^{-1}\left(\frac{x^2+y^2 - l_1^2 + l_2^2}{2 l_1 l_2}\right)}$$



Now, we need to solve for θ_1

∴ ~~tan~~ we will introde an angle 'γ' inorder to determine θ_1



~~tan(γ)~~

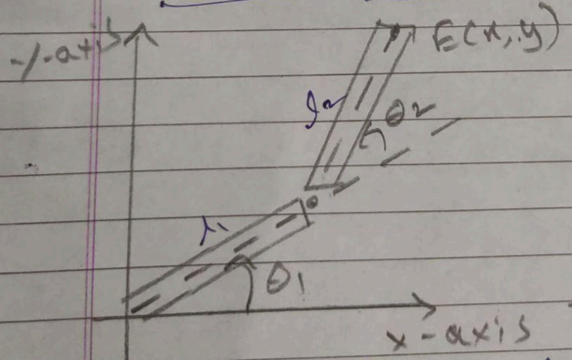
$$\gamma = \tan^{-1}\left(\frac{y}{x}\right)$$

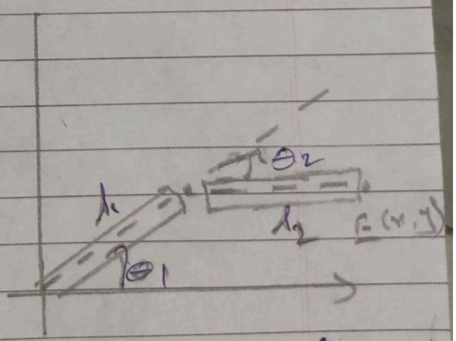
$$\therefore \theta_1 = \gamma + \beta$$

Already got the γ & β

$$\boxed{\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)}$$

∴ we obtain 2 solutions with diagram





$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta_2}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{l_2 \sin\theta_2}{l_1 + l_2 \cos\theta}\right)$$

$$\theta_2 = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$\theta_2 = -\cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

NOTE THAT:- I HAVE MENTIONED BOTH Theta_2 AS

Theta_2 = cos^-1 (x*2 + y*2 - L1 - L2 / 2 L1 . L2)

But second theta (RIGHT HAND DIAGRAM THETA_2) should be

Theta_2 = cos^-1 (x*2 + y*2 - L1 -+ L2 / 2 L1 . L2)

4.Using the manipulator file shared, find the transformation between the end effector and the base frame.

Answer:-For finding the transformation between the end effector and the base frame we need to have some formula

1 Tn=T1+T2+T3+..+Tn

2 . T=[
[R d]
[0 1]
]

3. R= Rx(theta) + Ry(theta) + Rz(theta)

4 . Rx(theta) =

```
[
    [ 1       0            0 ]
    [ 0   cos(theta)   -sin(theta)]
    [ 0       0         cos(theta) ]
]
```

5 . Ry(theta) = [

```
    [ cos(theta)    0    -sin(theta) ]
    [     0         1         0      ]
    [ -sin(theta)   0     cos(theta) ]
]
```

6 . Rz(theta) = [

```
    [ cos(theta)  -sin(theta)   0 ]
    [ sin(theta)   cos(theta)   0 ]
    [     0            0        1 ]
]
```

Similarly d = {
X

Y
Z
}

Here x= L . cos(theta). cos(Roll)

Y = L . sin(theta) .cos(Roll)

Z = L . sin(Roll)

If you are unable to understand here are images of the formula

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Similarly, for a rotation about the y-axis:

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

And for a rotation about the z-axis:

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1. Rotation around the z-axis (θ radians),
2. Rotation around the y-axis (ϕ radians),
3. Rotation around the x-axis (ψ radians).

The coordinates (x, y, z) of the endpoint can be calculated as follows:

$$x = L \cdot \cos(\theta) \cdot \cos(\phi)$$
$$y = L \cdot \sin(\theta) \cdot \cos(\phi)$$
$$z = L \cdot \sin(\phi)$$

Homogeneous Transformation Matrices

- 3x3 Rotation Matrix

$$T_1 = \begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- 3x1 Displacement Vector

$$R_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

- 4x4 Homogeneous Matrix

$$T_H = \left[\begin{array}{ccc|c} C1 & -S1 & 0 & x_1 \\ S1 & C1 & 0 & y_1 \\ 0 & 0 & 1 & z_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

January 24, 2000 Robotics 1

Pitch ➖ Here Pitch refers to the rotation around the side-to-side axis, often represented by the x-axisPitch refers to the rotation around the side-to-side axis, often represented by the x-axis.it is the tilting motion from side to side, causing one wing to move up while the other moves down

Yaw ➖Yaw is the rotation around the vertical axis, usually represented by the z-axis. It is the twisting motion.It is the twisting motion, causing the nose of an object to turn left or right

.

ROLL ➖ Roll is the rotation around the front-to-back axis, typically represented by the y-axis. It is the tilting motion from side to side, causing one wing to move up while the other moves down.

1. **Pitch (θ):**

$$\theta = \arcsin(r_{23})$$

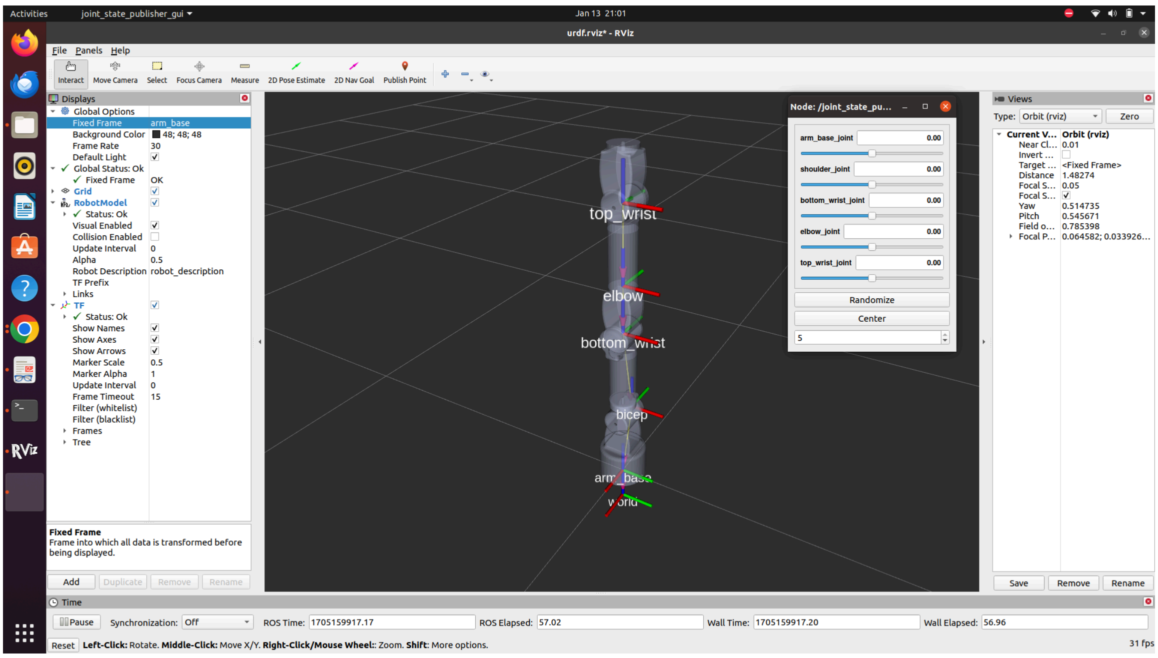
2. **Yaw (ψ):**

$$\psi = \arctan 2(-r_{13}, r_{33})$$

3. **Roll (ϕ):**

$$\phi = \arctan 2(-r_{21}, r_{22})$$

NOW FINDING TRANSFORMATION BETWEEN THE END EFFECTOR AND THE BASE FRAME.



In the above image, the manipulator is straight, and all the components are facing towards the z-axis, so the system is in z-axis coordinates.

In order to find the transformation matrix, I first need to find the rotational matrix in 3 x 3.

Homogeneous transformation matrix equation:

Tn=T1+T2+T3+..+Tn

First, we will calculate the transformation matrix for the arm_link. Let's denote arm_link as T1.

```
T1=[
        [ R  d]
        [ 0   1]
   ]
```

Calculating R

R = Rx * Ry * Rz

Rx(theta) =

[

[1 0 0]
[0 cos(theta) -sin(theta)]
[0 0 cos(theta)]

]

Ry(theta) = [

[cos(theta) 0 sin(theta)]
[0 1 0]
[-sin(theta) 0 cos(theta)]

]

Rz(theta) = [

[cos(theta) -sin(theta) 0]
[sin(theta) cos(theta) 0]
[0 0 1]

Here theta is zero because there is no orientation is involved in above figure

[[[

[1 0 0] [cos(theta) 0 -sin(theta)] [cos(theta) -sin(theta) 0]
R= [0 cos(theta) -sin(theta)] * [0 1 0] * [sin(theta) cos(theta) 0]
[0 0 cos(theta)] [-sin(theta) 0 cos(theta)] [0 0 1]

]]]

Calculation

Here we consider the pitch (theta_symbol) because

In the given manipulator, the base arm is facing towards the Z-axis, so

assumed that the rotation is done along the Z axis.

Pitch formula ➖

Theta = arcsin(r23)

r23 is value that we should need to consider it from

rotational matrix (3 x 3)

1. **Pitch (θ):**

   $\theta = \arcsin(r_{23})$

2. **Yaw (ψ):**

   $\psi = \arctan 2(-r_{13}, r_{33})$

3. **Roll (ϕ):**

   $\phi = \arctan 2(-r_{21}, r_{22})$

For arm-base finding transformation matrix

$$T_1 = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$R = R_x \cdot R_y \cdot R_z.$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here $\theta = '0'$ because there is no rotation.

$$\therefore R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now find d. $\longrightarrow$ $d = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$

$x = L \cos\theta \cdot \cos(\theta)$ Because Rotation axis facing toward the z-axis.

$y = L \sin(\theta) \cos(\theta)$ " " " "

$z = L \sin(\theta)$

Here $\theta = \arcsin(r_{23})$

$\theta = \arcsin(0)$

$$\boxed{\theta = 0}$$

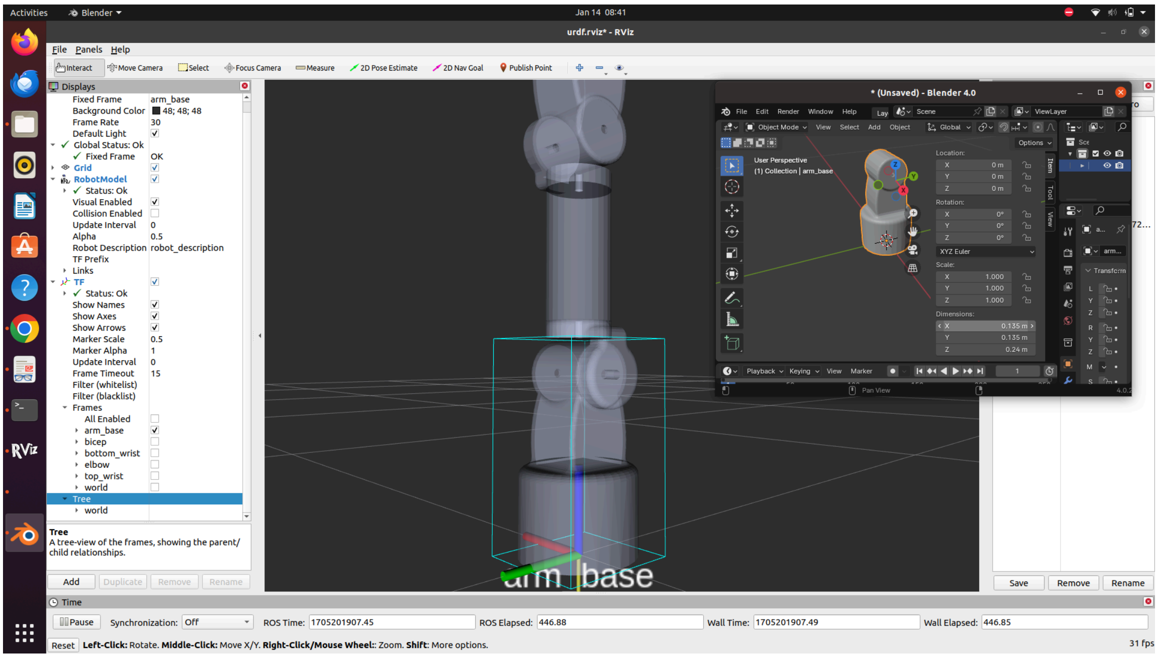
arcsin(0) value = 0

T1 is obtained by substituting theta value in x, y, and z.

In the below image, I have considered the length of the arm base (L) as 0.24.

By using Blender software, I have got the dimensions in x, y, and

Where I need the z value because the manipulator is in that direction only (as length),



Sub θ in x & y & z. & $L = (0.24)$

$x = (0.24)\cos(0)\cos(0)$

$\boxed{x = 0.24}$

$y = (0.24)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.24)\sin(0)$

$\boxed{z = 0}$

$\therefore$ Transformation matrix of Arm_Base is

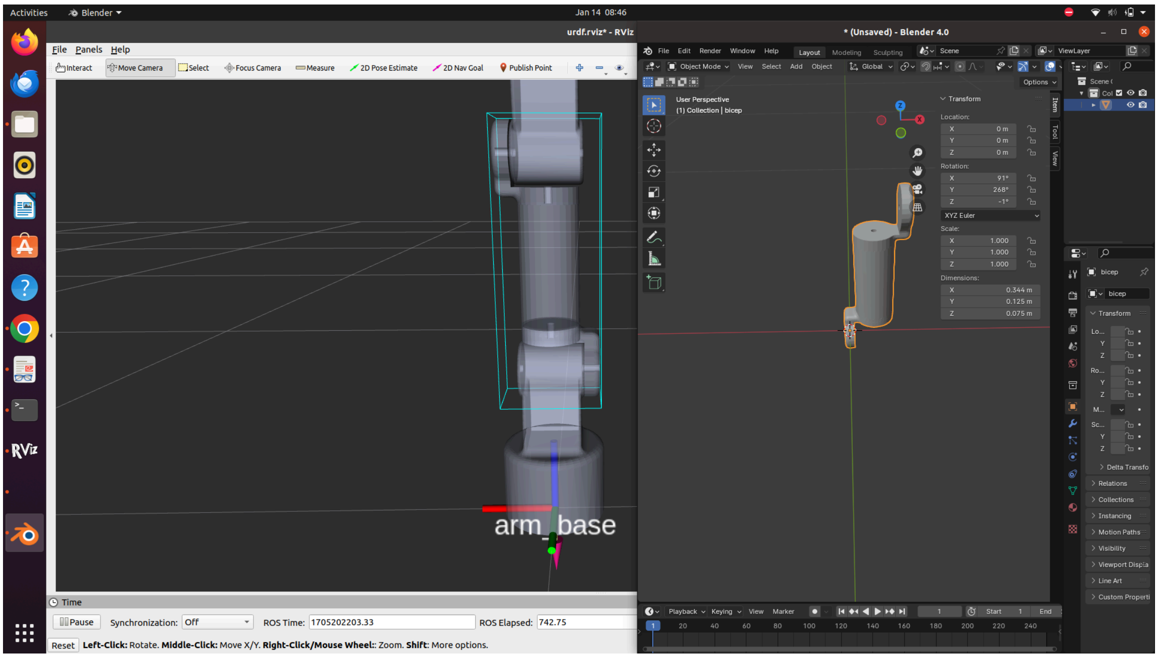
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly we need to find transformation of all components and need to multiply them

NOW CALCULATING TRANSFORMATION MATRIX OF BICEP

In the below image, I have calculated R directly, similar to the arm_base component. And I got the bicep length using the blender. Here is the picture shown in the dimension.

block to length



Transformation Matrix for Bicep Component

$$T_2 = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \longrightarrow \text{For Bicep.}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \begin{aligned} x &= L\cos(\theta)\cos(\theta) \\ y &= L\sin(\theta)\cos(\theta) \\ z &= L\sin(\theta) \end{aligned}$$

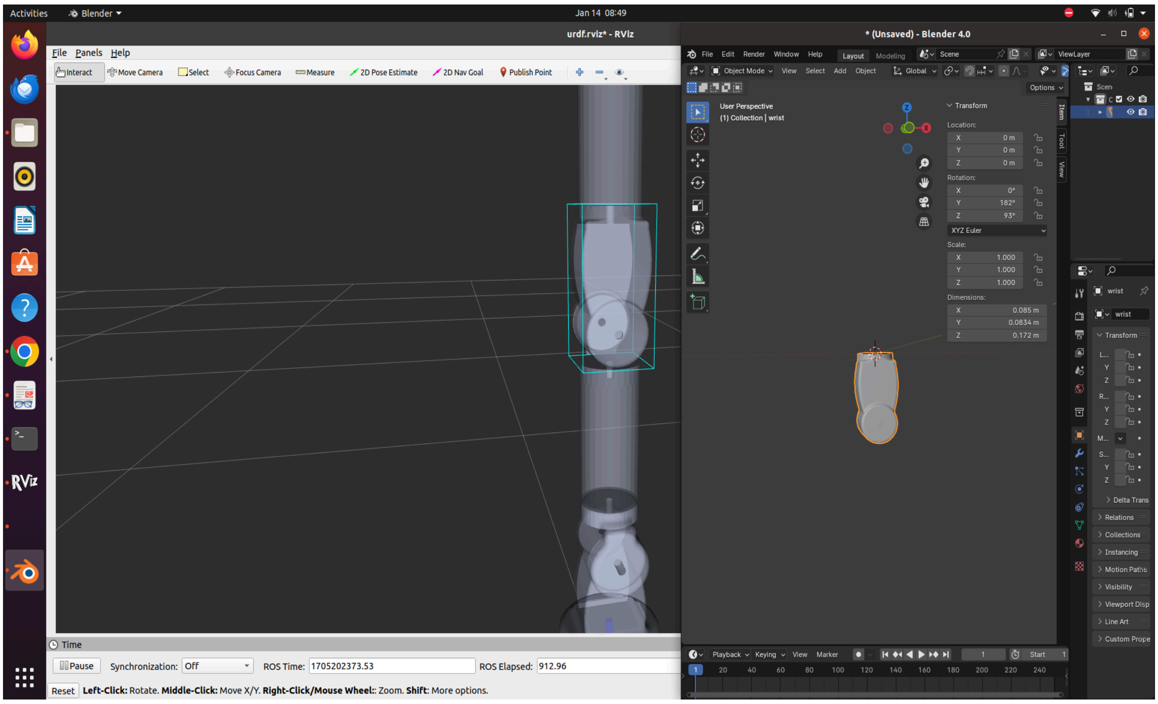
I got $L = (0.344\,m)$ from Blender S/W

$x = (0.344)\cos(0)\cos(0)$
$y = (0.344 \cdot \sin(0)\cos(0)$
$z = (0.344) \cdot \sin(0)$

$x = 0.344$
$y = 0$
$z = 0.$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0.344 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NOW CALCULATING TRANSFORMATION MATRIX FOR BOTTOM WRIST



Transformation matrix for bottom-wrist.

$$R = R_x \cdot R_y \cdot R_z$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(By Putting $\theta = 0$)

$$d = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \longrightarrow \begin{aligned} x &= L \cos\theta \cos\theta \\ y &= L \sin\theta \cos\theta \\ z &= L \sin\theta \end{aligned}$$

Pitch

$\theta = \arcsin(0)$

$L = 0.172m$

$x = 0.172 \cdot \cos(0) \cos(0)$

$\boxed{x = 0.172.}$

$$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$y = 0.72 \cdot \sin(0) \cos(0)$

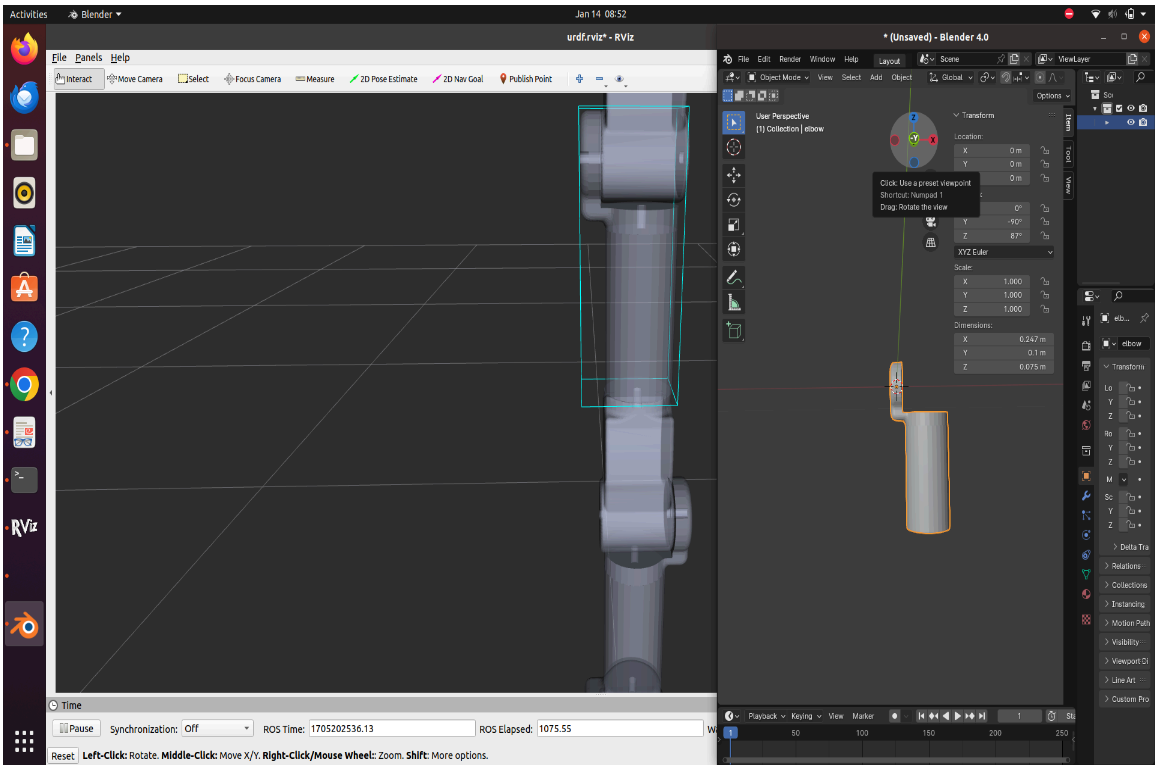
$\boxed{y = 0}$

$z = (0.172) \cdot \sin(0)$

$\boxed{z = 0}$

$$T_3 = \begin{bmatrix} 1 & 0 & 0 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NOW CALCULATING TRANSFORMATION MATRIX FOR ELBOW



Transformation for matrix for 6bl. Elbow.

$$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$R = R_x \cdot R_y \cdot R_z$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By putting $\theta = 0$ in R_x & R_y & R_z.

$d = x = L \cos\theta \cos\theta$

$y = L \sin\theta \cos\theta$

$z = L \sin\theta$.

$x = (0.247 m) \cos(0) \cos(0)$

$\boxed{x = 0.247 m}$

$\theta = \arcsin(r_{23})$

$y = (0.247m) \cos(0) \sin(0)$

$\boxed{y = 0}$

$\theta = \arcsin(0)$

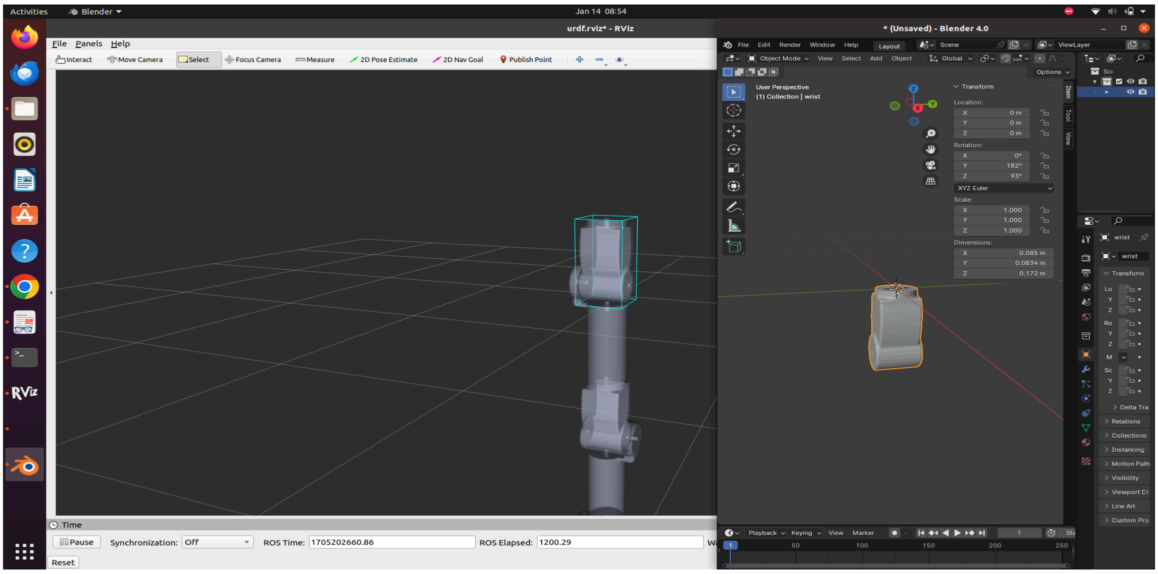
$\boxed{\theta = 0}$

$z = (0.247) \sin(0)$

$\boxed{z = 0}$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0.247 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NOW CALCULATING TRANSFORMATION MATRIX FOR TOP WRIST



Transformation matrix of Top wrist

$$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \begin{aligned} x &= L \cos\theta \cos\theta \\ y &= L \sin\theta \cos\theta \\ z &= L \sin\theta . \end{aligned}$$

$$x = (0.179\,m) \cos(0) \cos(0)$$

$$\boxed{x = 0.179}$$

$$y = (0.179\,m) \sin(0) \cos(0)$$

$$\boxed{y = 0}$$

$$\theta = \arcsin(r_{23})$$

$$z = (0.179\,m) \sin(0) \qquad \therefore \theta = 0$$

$$\boxed{z = 0}$$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0.179 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HERE IT IS NOT T4, IT IS T5 AND LENGTH IS 0.172 (MADE A SMALL MISTAKE)

Now i will multiply all transformation matrix components

T = T1 * T2 * T3 * T4 * T5

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0.344 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 1 & 0 & 0 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0.247 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = \begin{bmatrix} 1 & 0 & 0 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By using online matrix multiplication tool I got 'T' value.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

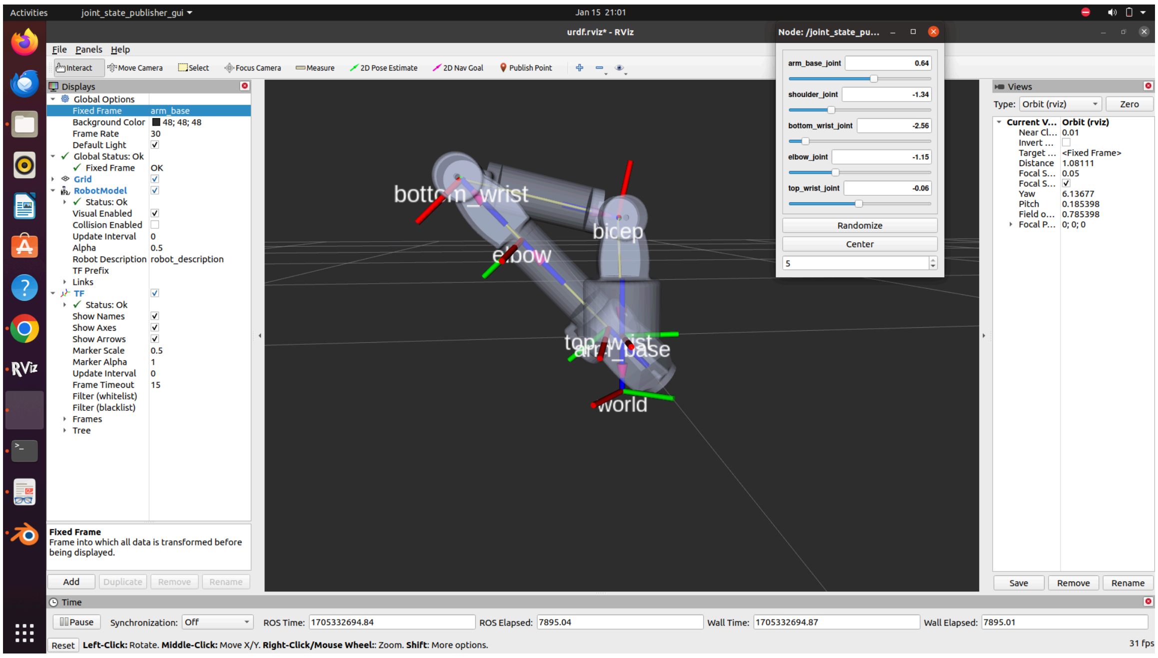
So, Above ? is the transformation ~~matrix~~ between End effector and base frame.

Therefore the transformation b/w end effector and base frame is give by

T = [[1 0 0 0.172]
[1 0 0 0]
[1 0 0 0]
[1 0 0 0]]

1.Using the Sliders, change the orientation of the manipulator and find the Transformation between the Last link and the Base link

1 st



SOLUTION

for arm-base Transformation matrix

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Since there is no rotation the Base frame is Fixed.

$$d = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{aligned} x &= l\cos\theta \cos\theta \\ y &= l\sin\theta \cos\theta \\ z &= l\sin\theta \end{aligned}$$

Here θ is pitch.

$\theta = \arcsin(r_{23})$ $\quad l = 0.24m.$

$\theta = \arcsin(0)$

$\theta = 0$

$x = (0.24)(\cos(0))\cos(0)$

$\boxed{x = 0.24}$

$y = (0.24)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.24)\sin(0)$

$\boxed{z = 0}$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation for Bicep

$\theta = -1.34$

Here Rotation is done in y axis with an $\theta = -1.34$ will be zero & x axis & z axis

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.228 & 0 & -0.973 \\ 0 & 1 & 0 \\ +0.973 & 0 & 0.228 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.228 & 0 & -0.973 \\ 0 & 1 & 0 \\ 0.973 & 0 & 0.228 \end{bmatrix}$$

$$d = \begin{aligned} x &= l \cdot \cos\theta \cos\phi \\ y &= l \cdot \sin\theta \cos\phi \\ z &= l \cdot \sin(\phi) \end{aligned}$$

$\boxed{l = 0.344\ m}$

$\phi = \arctan2(-r_{21}, r_{22})$

$\phi = \arctan2(-0, 1)$

$\boxed{\phi = 0}$

$\boxed{x = (0.344)\cos(0)\cos(0)}$

$x = 0.344$

$y = l \cdot \sin(-1.34)\cos(0)$

$= (0.344)(-0.973)(1)$

$y = (-0.334)$

$z = l \cdot \sin(0)$

$\boxed{z = 0}$

~~$\theta = -2.56$~~

$$T_2 = \begin{bmatrix} 0.228 & 0 & -0.973 & 0.344 \\ 0 & 1 & 0 & -0.33 \\ 0.973 & 0 & 0.228 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bottom wrist (T_3)

$\theta = -2.56$

Rotation is done in y-axis with an $\theta = -2.56$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-2.56) & 0 & \sin(-2.56) \\ 0 & 1 & 0 \\ -\sin(-2.56) & 0 & \cos(-2.56) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} -0.835 & 0 & -0.549 \\ 0 & 1 & 0 \\ 0.549 & 0 & -0.835 \end{bmatrix}$$ $\phi = 0$

$$d = \begin{aligned} x &= (0.172)\cos(0)\cos(0) = 0.172 \\ y &= (0.172)\sin(0)\cos(0) = 0 \\ z &= (0.172)\sin(0) = 0 \end{aligned}$$

$$T_3 = \begin{bmatrix} -0.835 & 0 & -0.549 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0.549 & 0 & -0.835 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elbow

-1.15

Here the rotation is around z, with $x = 0$ & $y = 0$. an $\theta = -1.15$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-1.15) & -\sin(-1.15) & 0 \\ \sin(-1.15) & \cos(-1.15) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.408 & -0.912 & 0 \\ +0.912 & 0.408 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\theta = \arcsin($

$\theta = 0$

$d = x = (0.247)\cos\theta\cos(0)$

$\boxed{x = 0.247}$

$y = (0.247)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.247)\sin(0)$

$\boxed{z = 0}$

$$T_4 = \begin{bmatrix} 0.408 & -0.912 & 0 & 0.247 \\ +0.912 & 0.408 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For top wrist

$\theta = -0.06$

Here the rotation is around y axis.
with angle $\theta = -0.06$.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-0.06) & 0 & \sin(-0.06) \\ 0 & 1 & 0 \\ -\sin(-0.06) & 0 & \cos(-0.06) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.998 & 0 & -0.059 \\ 0 & 1 & 0 \\ 0.059 & 0 & 0.998 \end{bmatrix}$$

$\phi = \arctan2(-r_{12}, r_{22})$

$\phi = (0, 1)$

$\phi = 0$.

$d = x = (0.172)\cos(0)\cos$

$\boxed{x = 0.172}$

$y = (0.172)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.172)\sin(0)$

$\boxed{z = 0}$

$$T_5 = \begin{bmatrix} 0.998 & 0 & -0.059 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0.059 & 0 & 0.998 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_1 = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0.228 & 0 & -0.973 & 0.344 \\ 0 & 1 & 0 & -0.33 \\ 0.973 & 0 & 0.228 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

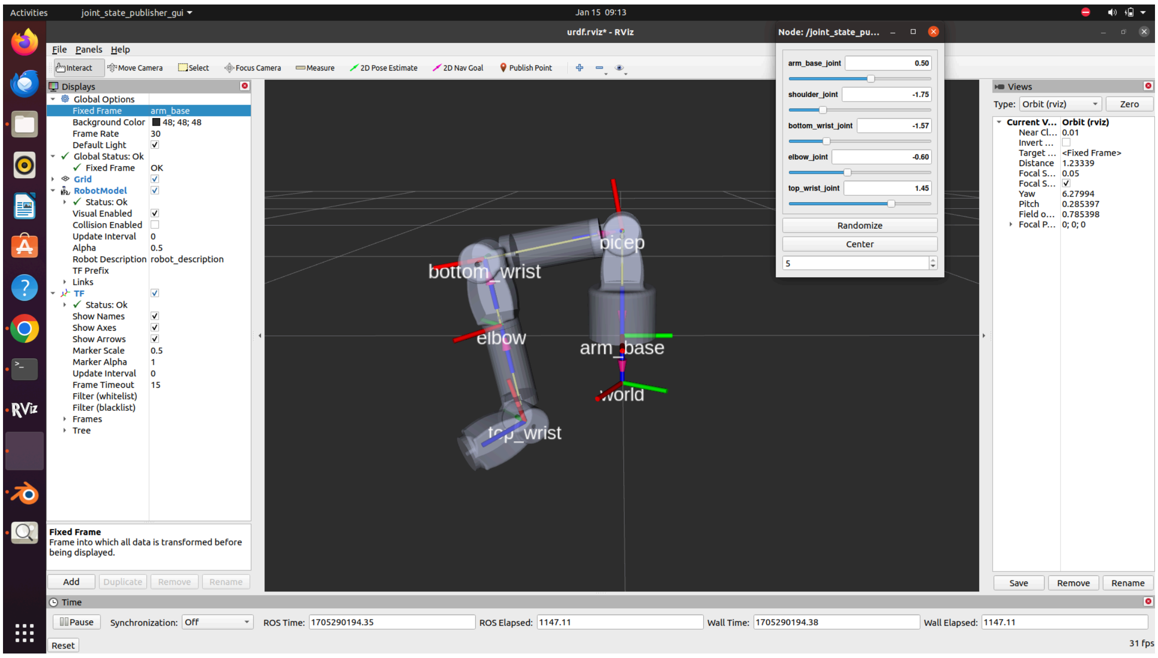
$$T_3 = \begin{bmatrix} -0.835 & 0 & -0.549 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0.549 & 0 & -0.835 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4 = \begin{bmatrix} 0.408 & -0.912 & 0 & 0.247 \\ +0.912 & 0.408 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = \begin{bmatrix} 0.998 & 0 & -0.059 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0.059 & 0 & 0.998 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

THESE IS THE TRANSFORMATION FOR THE ABOVE IMAGE

2nd



For these image the transformation is

$$T = \begin{bmatrix} -0.411 & 0.897 & -0.153 & 0.2412 \\ 0.910 & 0.4 & -0.05 & 0.32 \\ -0.014 & 0.16 & 0.986 & 0.11 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

transformation for arm base

$$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since There is no rotation so θ will be 0

$d =$ $x = l\cos\theta\cos\theta$

Here I consider θ because initial the arm base is facing toward z-axis.

$$\boxed{\begin{aligned} \theta &= \arcsin(r_{23}) \\ \theta &= \arcsin(0) \\ \theta &= 0 \end{aligned}}$$

$x = (0.24)(1)(1)$

$\boxed{x = 0.24}$

$y = (0.24)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.24)\sin(0)$

$\boxed{z = 0}$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for bicep

$$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}$$

$R = R_x \cdot R_y \cdot R_z$

Here Rotation is done around y-axis $= -1.75$

Rotation is done around x-axis $= 0$

Rotation is done around z-axis $= 0$.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-1.75) & 0 & \sin(-1.75) \\ 0 & 1 & 0 \\ -\sin(-1.75) & 0 & \cos(-1.75) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} -0.178 & 0 & -0.984 \\ 0 & 1 & 0 \\ 0.984 & 0 & -0.178 \end{bmatrix}$$

$$d = \begin{aligned} x &= l\cos\theta\cos(\phi) \\ y &= l \cdot \sin(\theta) \cdot \cos(\phi) \\ z &= l \cdot \sin(\phi) \end{aligned}$$

$\phi \rightarrow$ Roll

$\phi = \arctan2(-r_{21}, r_{22})$

$$T_2 = \begin{bmatrix} 0.228 & 0 & -0.973 & 0.344 \\ 0 & 1 & 0 & -0.33 \\ 0.973 & 0 & 0.228 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Bottom wrist

Rotation is done around y-axis is -1.57
" " " " x " 0
" " " " y-axis is 0.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-1.57) & 0 & \sin(-1.57) \\ 0 & 1 & 0 \\ -\sin(-1.57) & 0 & \cos(-1.57) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\sin(-1.57) \simeq -1$
$\cos(-1.57) \simeq 0.$

$\phi = \arcsin(r_{23})$

$\boxed{\phi = 0}$

d
$x = l \cdot \cos\theta \cdot \cos\phi$
$x = (0.172)\cos(0)\cos(0)$

$y = l \cdot \sin(\theta)\cos\phi$
$= (0.172)\sin(\overset{(-1.57)}{0})\cos(0)$

$\boxed{\cancel{y = 0}}$ $y = -0.172$

$z = (0.172)\sin(0)$

$\boxed{z = 0}$

$$T_3 = \begin{bmatrix} 0 & 0 & -1 & 0.172 \\ 0 & 1 & 0 & -0.172 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elbow

$\theta = -0.60$

z - axis $= -0.60$

x - axis $= 0$

y - axis $= 0$.

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-0.60) & -\sin(-0.60) & 0 \\ \sin(-0.60) & \cos(-0.60) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cancel{-0.564}\ 0.826 & +0.564 & 0 \\ -0.564 & 0.826 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

values

$\cos(-0.60) = 0.826$

$\sin(-0.60) = -0.564$.

$$d = \begin{cases} x = l\cos(\theta) \cdot \cos(\theta) \\ y = l \cdot \sin(\theta) \cdot \cos(\theta) \\ z = l \cdot \sin(\theta) \end{cases}$$

{since rotation around the z-axis (0 radians)}

$x = (0.247)(\cos(0))\cos(\theta)$

$\theta = \arcsin(r_{23})$

$\theta = \arcsin(0)$

$\boxed{\theta = 0}$

$\boxed{x = 0.247}$

$y = (0.247)\sin(0)\cos(0)$

$\boxed{y = 0}$

$z = (0.247)\sin(0)$

$\boxed{z = 0}$

$$T_4 = \begin{bmatrix} 0.826 & 0.564 & 0 & 0.247 \\ -0.564 & 0.826 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For Top wrist

Rotation around 3 axis

~~$y = 0.96$~~ $y = 1.45$

$x = 0$

$z = 0$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(1.45) & 0 & \sin(1.45) \\ 0 & 1 & 0 \\ -\sin(1.45) & 0 & \cos(1.45) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.137 & 0 & 0.990 \\ 0 & 1 & 0 \\ -0.990 & 0 & 0.137 \end{bmatrix}$$

$$d = \begin{cases} x = l\cos\theta\cos(\phi) \\ y = l\sin\theta\cos(\phi) \\ z = l\sin\phi) \end{cases}$$

$x = (0.179)\cos(0)\cos(\phi)$

$\phi = \arcsin(r_{23})$

$\phi = 0$

$x = (0.179)(1)(1)$

$\boxed{x = 0.179}$

$y = (0.179)\sin(1.45)\cos(0)$

$y = (0.179)(0.99)(1)$

$\boxed{y = 0.17721}$

$z = (0.179)\sin(0)$.

$\boxed{z = 0}$

$$T_5 = \begin{bmatrix} 0.137 & 0 & 0.990 & 0.179 \\ 0 & 1 & 0 & 0.177 \\ -0.99 & 0 & 0.137 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Total Transformation is

$T = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$

$$T = \begin{bmatrix} -0.287 & -0.537 & -0.780 & 0.0391 \\ -0.07 & 0.826 & -0.55 & 0.039 \\ 0.954 & -0.997 & -0.280 & 0.251 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0.24 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

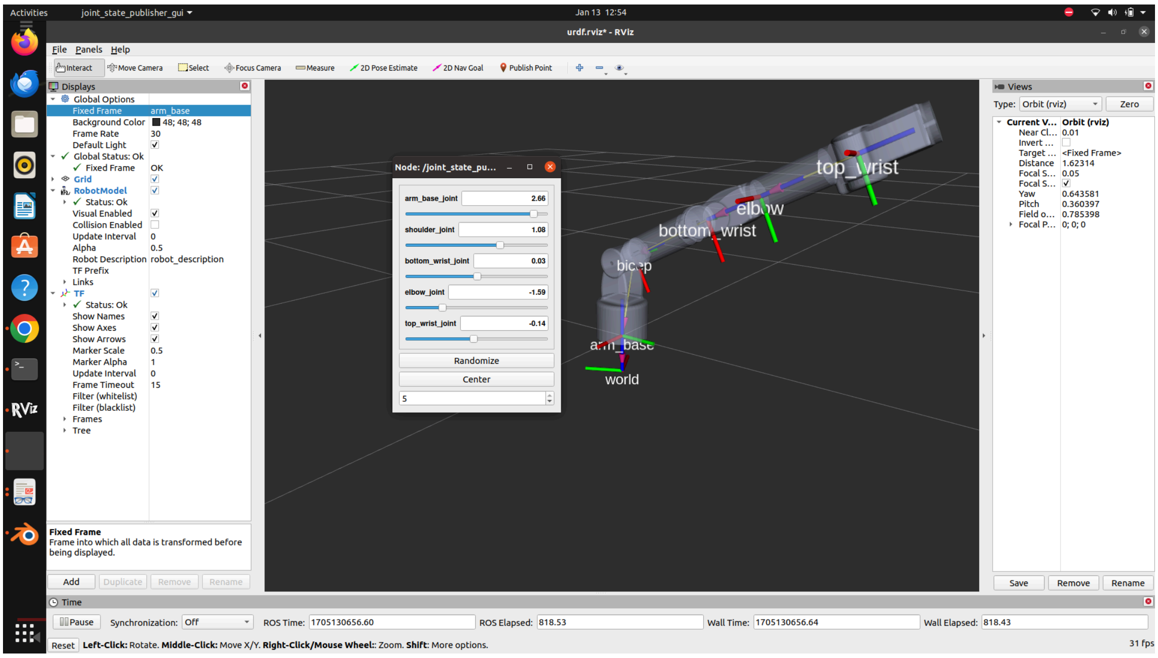
$$T_2 = \begin{bmatrix} 0.228 & 0 & -0.973 & 0.344 \\ 0 & 1 & 0 & -0.33 \\ 0.973 & 0 & 0.228 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 0 & 0 & -1 & 0.172 \\ 0 & 1 & 0 & -0.172 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4 = \begin{bmatrix} 0.408 & -0.912 & 0 & 0.247 \\ 0.912 & 0.408 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = \begin{bmatrix} 0.998 & 0 & -0.059 & 0.172 \\ 0 & 1 & 0 & 0 \\ 0.059 & 0 & 0.998 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_\phi = \begin{bmatrix} -0.411 & 0.897 & -0.153 & 0.2412 \\ 0.910 & 0.4 & -0.05 & 0.32 \\ -0.014 & 0.16 & 0.986 & 0.11 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



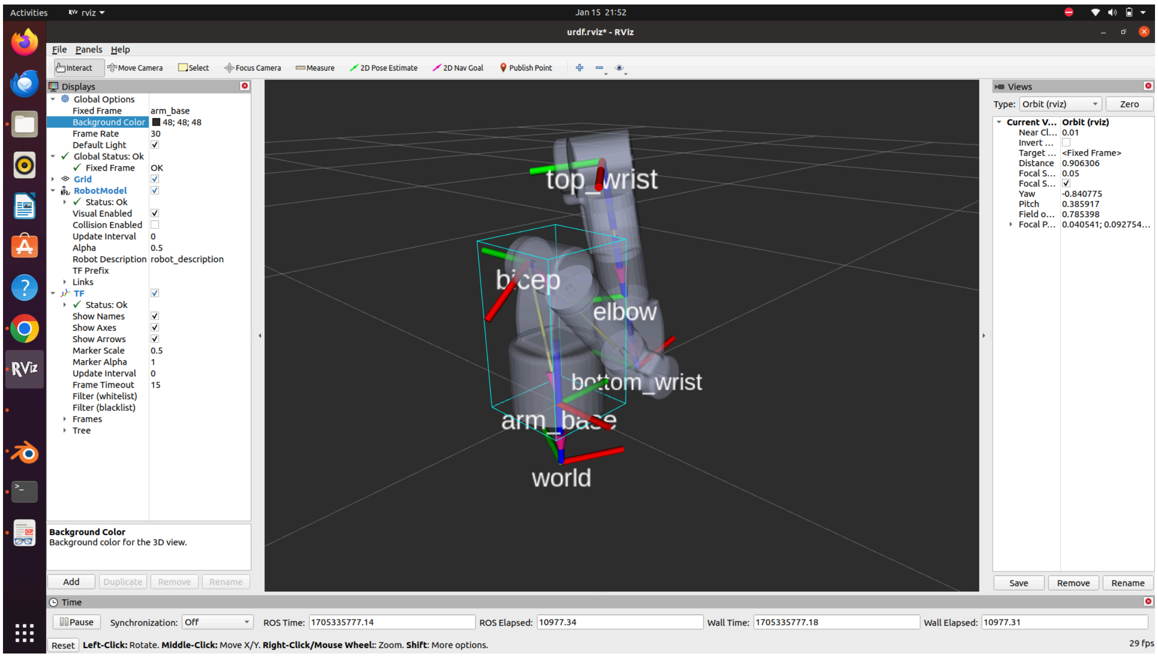
Similarly we will do transformation for the above image

2.Explain about the message that you see on finding the transformation.Does it have the correct shape as the Transformation matrix should have? If not, why?

Solution ➖

For these questions, I would say no because the shape is correct when it comes to 3D; it is ok, but when it comes to real-time applications, there will be a change in the collision of components with each other.

Here is an example image



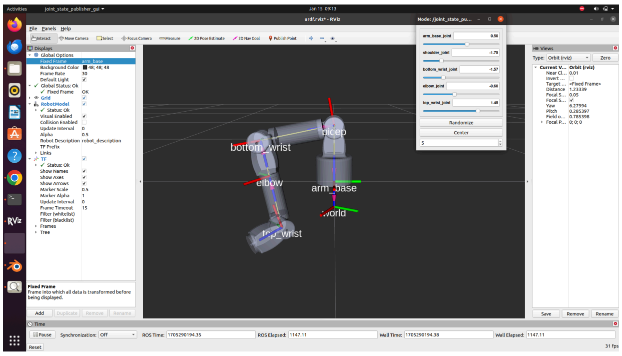
In these images, the bicep is colliding with the arm_base component and top_wrist. Because of the shape of the transformation, there are high chances of material damage in the real-time application, and it may not work effectively.

For the below image, I would yes, because the shape of the transformation looks correct and well defined.

We can observe that the end effector in the plane of xyz here is the transformation, and it is also not colliding with other components.

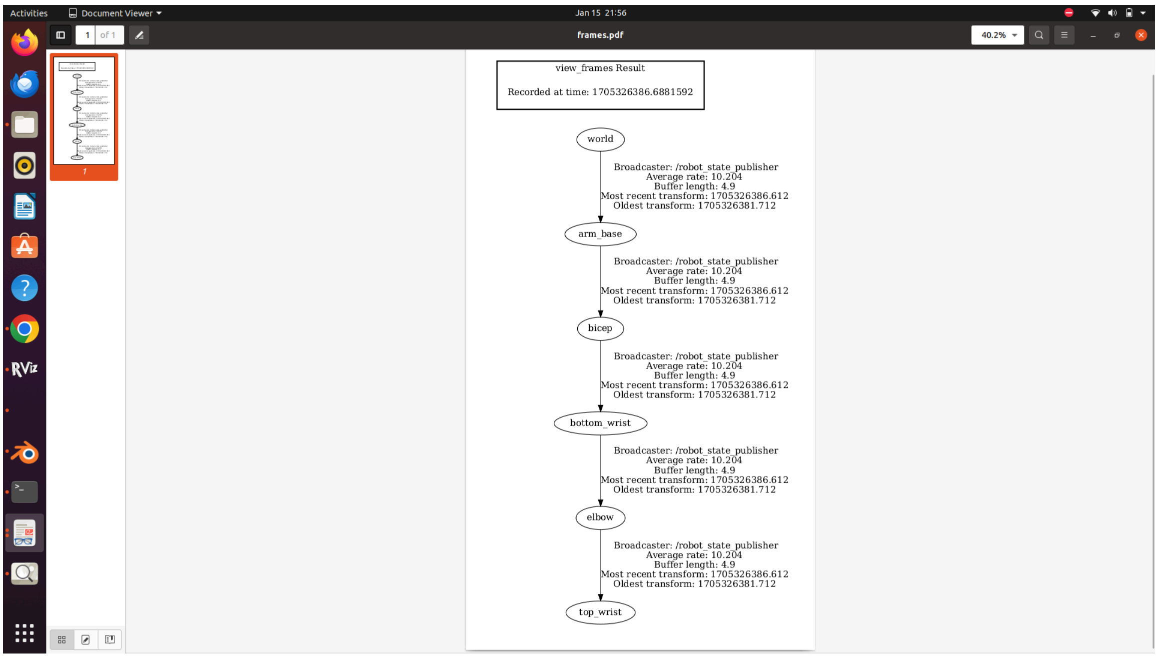
T = [[-0.411 0.897 -0.153 0.2412]
[0.910 0.4 -0.05 0.32]

[-0.014 0.16 0.986 0.11]
[0 0 0 1]]



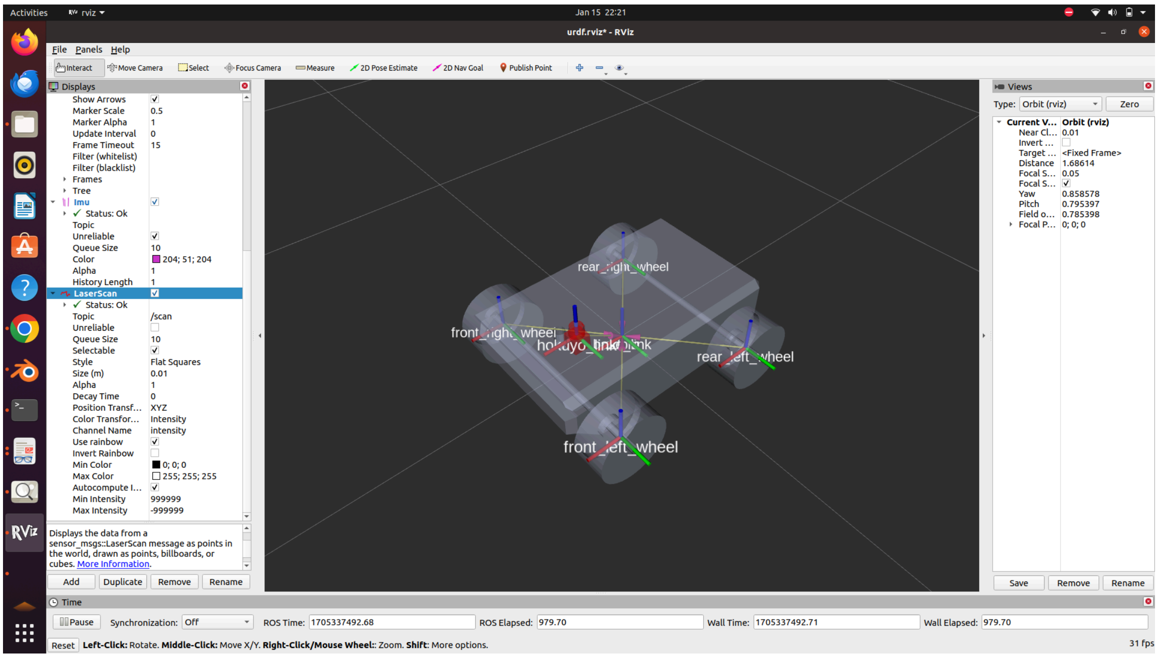
3. Attach an Image of all the links and connections of the manipulator.(Tree Structure)

HERE IS THE TREE STRUCTURE IMAGE OF MANIPULATOR

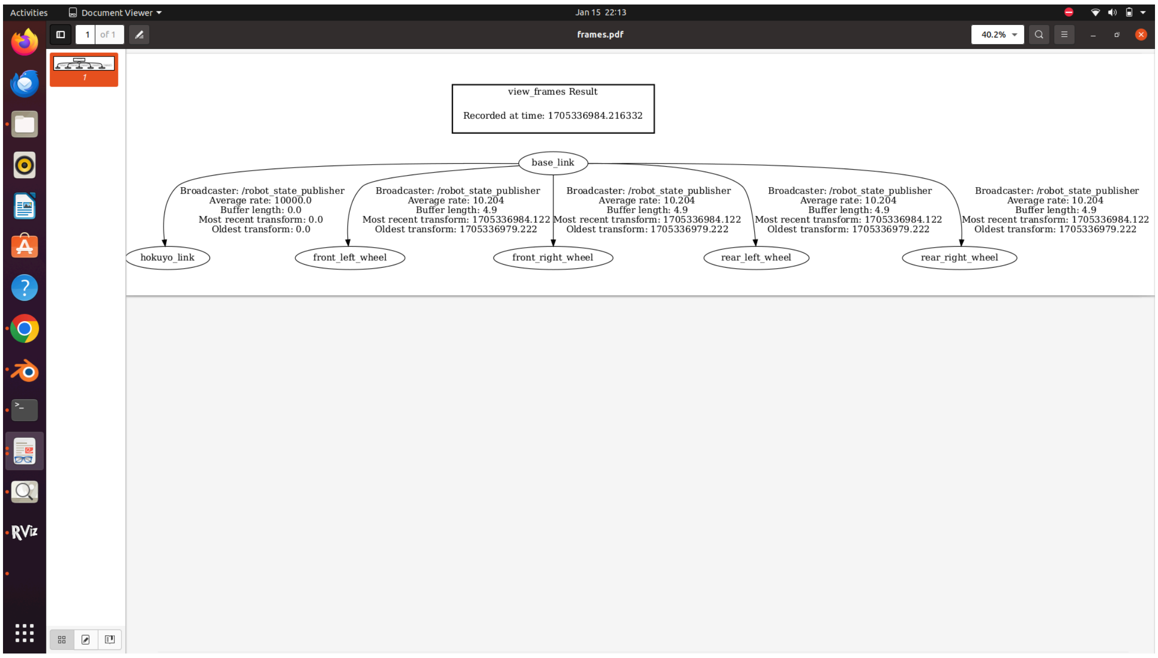


Bonus:

Add a LIDAR sensor to the Mobile Robot provided in the file.



TREE STRUCTURE OF ROBOT WITH LIDAR



While running in gazebo i am facing an issue for collecting the sensor data from the lidar