Course Title	**QUANTITATIVE ABILITY TRANING**				
Course Code	**22AM6HSQAT**	**Credits**	**3**	**L-T-P**	**2-0-1**
CIE	**50 Marks**	**SEE**	**100 Marks (50% Weightage)**		
Contact Hours /Week	**2**	**Total Lecture Hours**			**26**

UNIT – 1	**5 Hrs**
Number System: Classification of Numbers, Test of Divisibility, HCF and LCM, Fractions and Ordering Fractions, Square root and cube root.	
UNIT – 2	**5 Hrs**
Averages, Ratio – Proportion, Time-Work and Wages, Pipes and Cisterns. Highlighted concepts are there for CIE-1.	
UNIT – 3	**5 Hrs**
Time Speed and Distance: Boats and Streams, Problems on Trains. Ages.	
UNIT – 4	**5 Hrs**
Areas: Mensuration 2D, Permutation and Combination, Probability.	
UNIT – 5	**6 Hrs**
Clocks, Calendars, Data Interpretation: Tabular Data Interpretation, Bar Graphs, Venn Diagrams.	

Text Books:

1. *"Quantitative Aptitude for Competitive Examinations",* R.S Agarwal.

UNIT- 01

1. Number System:
    a. Classification of Numbers
    b. Test of Divisibility
    c. HCF and LCM
    d. Fractions
    e. Ordering Fractions.

NUMBER SYSTEM

CONCEPTS

In Hindu–Arabic system we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. This is the decimal system where we use the digits 0 to 9. Here 0 is called *insignificant digit* where as 1, ... , 9 are called *significant digits*.

• **Classification of Numbers:**

Natural Numbers: The numbers 1, 2, 3, 4, 5, 6, ... which we use in counting are known as *natural numbers.* The set of all *natural numbers* can be represented by N = {1, 2, 3, 4, 5, ...}

Whole Numbers: If we include 0 among the natural numbers then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers.* Hence, every natural number is a whole number. The set of *whole numbers* is represented by W.

Integers: All counting numbers and their negatives including zero are known as *integers*.

The set of integers can be represented by Z or I.

Z = {. . . . . . –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}

Every *natural number* is an *integer* but every *integer* is not *natural number.*

Positive Integers: The set I^+ = {1, 2, 3, 4, ...} is the set of all positive integers. Positive integers and Natural numbers are synonyms.

Negative Integers: The set I^- = {. . . , –3, –2, –1} is the set of all negative integers.

0 (zero) is neither positive nor negative.

Non Negative Integers: The set {0, 1, 2, 3, . . . } is the set of all non negative integers.

Rational Numbers: The numbers of the form $\frac{p}{q}$, where p and q are integers, p is not divisible by q and $q \neq 0$, are known as *rational numbers.*

(or) Any number that can be written in fraction form is a *rational number.* This includes *integers, terminating decimals,* and *repeating decimals* as well as *fractions.*

e.g.: $\frac{3}{7}, \frac{5}{2}, -\frac{5}{9}, \frac{1}{2}, -\frac{3}{5}$ etc

The set of rational numbers is denoted by *Q*.

Irrational Numbers: Any real number that cannot be written in fraction form is an *irrational number.* Numbers which are both *non-terminating as well as non-repeating decimals* are called irrational numbers.

e.g.: Absolute value of $\sqrt{2}, \sqrt{3}, \sqrt{10}\ldots$

Note: A *terminating decimal* will have a finite number of digits after the decimal point.

e.g.: $\frac{3}{4} = 0.75, \frac{5}{4} = 1.25, \frac{25}{16} = 1.5625.$

Repeating Decimals: A decimal number that has digits that repeat forever.

e.g.: $\frac{1}{3} = 0.333\ldots$(here, 3 repeats forever.)

Non–Repeating Decimal: A decimal that neither *terminates* nor *repeats*.

e.g.: $\sqrt{2} = 1.4142135623\ldots$

Real Numbers: The rational and irrational numbers together are called *real numbers.*

e.g.: $\frac{13}{21}, \frac{2}{5}, \frac{-3}{7}, \frac{+4}{2}$ etc are real numbers.

The set of real numbers is denoted by *R*.

Even Numbers: Any integer that can be divided exactly by 2.

e.g.: 2, 6, 0, -8, -10, ... are even numbers.

Odd Numbers: An integer that cannot be divided exactly by 2 is an Odd number.

e.g.: 1, 3, -5, -7, ... are odd numbers.

Prime Numbers: A Prime Number can be divided evenly only by 1, or itself. And it must be a whole number greater than 1.

e.g.: Numbers 2, 3, 5, 7, 11, 13, 17, ... are prime.

All primes which are greater than 3 are of the form ($6n$+1) or ($6n$–1).

Note: • 1 is not a prime number.

• 2 is the least and only even prime number.

• 3 is the least odd prime number.

• Prime numbers up to 100 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79,83, 89, 97.

There are 25 prime numbers up to 100.

Composite Number: Natural numbers greater than 1 which are not prime, are known as *composite numbers.*

The number 1 is neither *prime* nor *composite.*

Co-Prime Numbers: Two numbers which have only 1 as the common factor are called co–primes (or) relatively prime to each other.

e.g.: 3 and 5 are co primes.

Note:

Natural Numbers = 1 + Prime + Composite Numbers.

Whole Numbers = 0 (Zero) + Natural Numbers.

Integers = Negative Integers + 0 + Positive Integers.

Real Numbers = Rational + Irrational Numbers.

Any two digit number is represented as $(10a + b)$, where a is the first digit, and b is the second.
e.g.: 63 is represented as 10(6)+3;
- The sum of its digits = $(a+b)$.
- The difference of the digits = $(a-b)$.

Similarly, any three digit number is represented as $(100a + 10b + c)$ etc.

- **Test of Divisibility:**

Divisibility by 2: A number is divisible by 2 if the unit's digit is either zero or divisible by 2.
e.g.: Units digit of 76 is 6 which is divisible by 2 hence 76 is divisible by 2.
Units digit of 330 is 0 so it is divisible by 2.
Divisibility by 3: A number is divisible by 3 if sum of all digits in it is divisible by 3.
e.g.: The number 273 is divisible by 3 since $2 + 7 + 3 = 12$ which is divisible by 3.
Divisibility by 4: A number is divisible by 4, if the number formed by the last two digits in it is divisible by 4, or last two digits are zeros.
e.g.: The number 5004 is divisible by 4 since last two digits 04 is divisible by 4.
Divisibility by 5: A number is divisible by 5 if the units digit in the number is either 0 or 5.
e.g.: 375 is divisible by 5 as 5 is in the units place.
Divisibility by 6: A number is divisible by 6 if it is even and sum of all digits is divisible by 3.
e.g.: The number 6492 is divisible by 6 as it is even and sum of its digits $6 + 4 + 9 + 2 = 21$ is divisible by 3.
Divisibility by 7:
Step-1: Remove unit's digit. And double it.
Step-2: Subtract it from the rest of the number.
Step-3: Check whether the resulted number is divisible by 7 or not.
Step-4: Repeat the above steps until the resulted number is either 0 (zero) or divisible by 7.
e.g.: Consider the number 10717.
Step-1: Removing the unit's digit *i.e.* 7. Double of 7 =14.
Step-2: $1071 - 14 = 1057$.
Step-3: Now remove 7 from 1057 and double it *i.e.* 14.
Step-4: $105 - 14 = 91$.
Step-5: Now remove 1 and double it *i.e.* 2.
Step-6: $9 - 2 = 7$
The final result 7 is divisible by 7. So the given number *i.e.* 10717 is also divisible by 7.
Divisibility by 8: A number is divisible by 8, if the number formed by last 3 digits is divisible by 8.
e.g.: The number 6573392 is divisible by 8 as the last 3 digits '392' is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digit is divisible by 9.
e.g.: The number 15606 is divisible by 9 as the sum of the digits $1 + 5 + 6 + 0 + 6 = 18$ is divisible by 9.
Divisibility by 10: Last digit should be zero.
e.g.: Last digit of 4470 is zero. So, it is divisible by 10.
Divisibility by 11: A number is divisible by 11 if the difference of the sum of the digits at *odd places* and sum of the digits at the *even places* is either zero or divisible by 11. **(or)** Subtract the first digit from a number made by the other digits. If that number is divisible by 11 then the original number is also divisible by 11.
e.g.: In the number 9823, the sum of the digits at odd places is 9+2=11 and the sum of the digits at even places is 8+3=11. Difference between them is $11 - 11 = 0$. Hence, the given number is divisible by 11.
e.g.: 14641
1464 – 1 is 1463; 146 – 3 is 143; 14 – 3 = 11, which is divisible by 11, so 14641 is also divisible by 11.

- If a number 'N' is divisible by two numbers 'a' and 'b', where a, b are co primes, then 'N' is divisible by 'ab'.

Divisibility by 12: A number is divisible by 12 if it is divisible by 3 and 4.
e.g.: The number 1644 is divisible by 12 as it is divisible by 3 and 4. Here 3 and 4 because they are co-prime to each other.
Divisibility by 13: Repeatedly add 4 times the last digit to the rest until you get a number divisible by 13 .
e.g.: $7462 \Rightarrow 746 + (2\times4) = 754 \Rightarrow 75 + (4\times4) = 91$
91 is divisible by 13. So, 7462 is also divisible by 13.
Divisibility by 14: The number is divisible by 14 if the given number is divisible by 2 and 7.
e.g: $1232 = 123 - 2 \times 2 = 119$, 119 is divisible by 7. Hence 1232 is divisible by 14.
Divisibility by 15: The number is divisible by 15 if the given number is divisible by 3 and 5.
e.g: $135 = 1 + 3 + 5 = 9$, 9 is divisible by 3.
135 is divisible by 5. Hence 135 is divisible by 15.
Divisibility by 16:
With a 3 digit number: Multiply hundreds digit by 4, then add the last two digits.
e.g.: $352 \Rightarrow (3\times4)+52 = 12 + 52 = 64$
64 is divisible by 16. So, 352 is also divisible by 16.
With a more than 3 digit number: The last four digits form a number is divisible by 16.
e.g.: $38512 \Rightarrow$ Here is 8512 is divisible by 16. So, 38512 is also divisible by 16.
Divisibility by 17:
Subtract 5 times the last digit from the rest.
e.g.: $3961 \Rightarrow 396 - (1\times5) = 391 \Rightarrow 39 - (1\times5) = 34$

34 is divisible by 17. So, 3961 is also divisible by 17.

Divisibility by 18: An even number satisfying the divisibility test by 9 is also divisible by 18.

e.g.: The number 80388 is divisible by 18 as it satisfies the divisibility test of 9.

Divisibility by 19: Add twice the last digit to the rest.

e.g.: $10944 \Rightarrow 1094 + (4 \times 2) = 1102$

$\Rightarrow 110 + (2{\times}2) = 114 \Rightarrow 11 + (4 \times 2) = 11 + 8 = 19.$

Divisibility by 20: Last digit is zero & tens digit is even.

e.g.: 980; Last digit is zero. And tens digit is even.

Divisibility by 25: A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

e.g.: The number 7975 is divisible by 25 as the last two digits are divisible by 25.

• Common Factors:

A common factor of two or more numbers is a number which divides each of them exactly.

e.g.: 3 is a common factor of 6 and 15.

▪ Highest Common Factor (HCF):

Highest common factor of two or more numbers is the greatest number that *divides each of them exactly.*

e.g.: 3, 4, 6, 12 are the factors of 12 and 36. Among them the greatest is 12. Hence the HCF of 12, 36 is 12.

HCF is also called as Greatest common divisor (GCD) or Greatest Common measure (GCM).

Method of Finding HCF: Method of division

▪ HCF of Two Numbers:

Step 1: Greater no. is divided by the smaller number.

Step 2: Divisor of step - 1 is divided by its remainder.

Step 3: Divisor of step - 2 is divided by its remainder.

This could be continued until the remainder is 0.

Then HCF = Divisor of the last step.

e.g.: Find the HCF of 96 and 348.

Explanation: Here the divisor of the last step is 12.

So, HCF of 96 and 348 is 12.

```
96)348(3
   288
    60)96(1
       60
       36)60(1
          36
          24)36(1
             24
      ────▶ 12)24(2
                24
                 0
```

▪ HCF of More than Two Numbers:

Step 1: Take any two numbers and find their HCF.

Step 2: Now find the HCF of third number and HCF obtained for the previous two numbers.

Step 3: Now find the HCF of fourth number and HCF obtained in the previous step. Continue the same process till the last number. The final HCF is concluded to be the HCF of all the given numbers.

e.g.: Find the HCF of 120, 246, 100.

```
120) 246 (2
     240
  ──▶ 6) 120 (20
         120
           0
```

6 is HCF of 120, 246. Now take 3rd number (*i.e.* 100) and HCF obtained in the above step (*i.e.* 6) and find HCF.

```
6)100(16
   96
    4)6(1
      4
 ──▶ 2)4(2
       4
       0
```

▪ HCF of Decimals: e.g.: Find HCF of 3.2, 4.12, 1.3, 7.

Explanation: First eliminate the influence of decimals by multiplying it either by 10 or 100 or 1000 etc. Here multiply the numbers with 100 to make all the numbers decimal free. *i.e.* 320, 412, 130, 700.

Now, find the HCF of above numbers. We get it as 2. Did you remember we multiplied all the numbers by 100 to eliminate the influence of decimals. Hence, now we divide the answer by 100 to get HCF of the original numbers. The HCF is $\frac{2}{100} = 0.02$

▪ LCM (Least Common Multiple):

Least common multiple of two or more given numbers is the '*least or lowest number*' which is divisible by each of them exactly. In the sense without a non zero remainder.

Method of Finding LCM:

Step-1: Write numbers in a line separated by comma.

Step 2: Divide any two of the given numbers exactly with a least possible prime number then the quotients and the undivided numbers are written in the next line.

Step 3: Repeat the same process till all the numbers in the line are prime to each other *i.e.* no more common factors exist.

Conclusion: The product of all divisors and the numbers in the last line is the LCM of the numbers.

e.g.: Find the LCM of 14, 18, 24, 30.

$$\begin{array}{c|l} 2 & 14,\ 18,\ 24,\ 30 \\ \hline 3 & 7,\ 9,\ 12,\ 15 \\ \hline & 7,\ 3,\ 4,\ 5 \end{array}$$

The LCM of 14, 18, 24, 30 = $2 \times 3 \times 7 \times 3 \times 4 \times 5 = 2520$.

▪ **LCM of Decimals:** Let us observe an example.

Find the LCM of 1.6, 0.28, 3.2, 4.9.

Explanation: First eliminate the decimals by multiplying with either 10 or 100 or 1000 etc. In this case, multiply all the numbers with 100.

Then numbers will become 160, 28, 320, 490.

Now, find the LCM of the above numbers as earlier.

$$\begin{array}{c|l} 2 & 160,\ 28,\ 320,\ 490 \\ \hline 2 & 80,\ 14,\ 160,\ 245 \\ \hline 2 & 40,\ 7,\ 80,\ 245 \\ \hline 2 & 20,\ 7,\ 40,\ 245 \\ \hline 2 & 10,\ 7,\ 20,\ 245 \\ \hline 5 & 5,\ 7,\ 10,\ 245 \\ \hline 7 & 1,\ 7,\ 2,\ 49 \\ \hline & 1,\ 1,\ 2,\ 7 \end{array}$$

LCM 160, 28, 320, 490 = $2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 \times 2 \times 7 = 15680$.

Did you remember, we have multiplied all the numbers by 100 to eliminate the influence of decimals. Hence, we divide the answer by 100 to get actual LCM of the given numbers. So, the LCM is $\frac{15680}{100} = 156.80$

▪ **Finding LCM and HCF of Fractions:**

$$\text{LCM} = \frac{\text{LCM of the numbers in numerator}}{\text{HCF of the numbers in denominator}}$$

$$\text{HCF} = \frac{\text{HCF of the numbers in numerator}}{\text{LCM of the numbers in denominator}}$$

e.g.: Find the LCM of $\frac{2}{5}, \frac{81}{100}, \frac{125}{302}$.

Explanation: First find the 'LCM of the numerator'.

As there is no common number (prime) which can divide any two of the numbers hence the product itself is the LCM. *i.e.* LCM = $2 \times 81 \times 125 = 20250$.

Now find the 'HCF of the numbers in denominator'.

HCF of 5 and 100 is 5 and HCF of 5 and 302 is 1.

$\therefore$ LCM of the given fractions = $\frac{20250}{1} = 20250$

e.g.: Find the HCF of $\frac{4}{9}, \frac{10}{21}, \frac{20}{63}$

Explanation: HCF of numerators 4, 10 and 20 = 2.

LCM of denominators 9, 21 and 63 = 63.

HCF of the given fractions = $\frac{2}{63}$.

▪ **Key Points on LCM and HCF:**

1) HCF of fractions is always a fraction but LCM of fractions may be a fraction or an integer.

2) The product of any two numbers is equal to product of their LCM and HCF.

e.g.: What is LCM and HCF of 32 and 450 ?

a) 7200, 8 b) 7100, 2 c) 7800, 2 d) 7200, 2

Explanation: Product of 32 and 450 is 14400

The LCM of 32 and 450 is 7200.

The HCF of 32 and 450 is 2.

(or) You can verify from options.

Option-a: $7200 \times 8 \neq 32 \times 450$.

Option-b: $7100 \times 2 \neq 32 \times 450$.

Option-c: $7800 \times 2 \neq 32 \times 450$.

Option-d: $7200 \times 2 = 32 \times 450$.

3) To find the greatest number that will exactly divide x, y and z; Required number = HCF of x, y, z.

4) To find the greatest number that will divide x, y and z leaving remainders a, b and c respectively.

Required number = HCF of $(x-a)$, $(y-b)$ and $(z-c)$.

5) To find the least number which is exactly divisible by x, y and z. Required number = LCM of x, y and z.

6) To find the least number which when divided by x, y, z leaves the remainders a, b, c respectively.

Then it is always observed that,

$(x - a) = (y - b) = (z - c) = K$ (Assume).

Required number = (LCM of x, y and z) – K.

7) To find the least number which when divided by x, y and z leaves the same remainder 'r' in each case. Required number = (LCM of x, y and z) + r.

8) To find the greatest number that will divide x, y and z leaving the same remainder in each case,

If the value of remainder 'r' is given, then

Required number = HCF of $(x-r)$, $(y-r)$ and $(z-r)$.

If the value of remainder is not given, then

Required number = HCF of $|(x-y)|$, $|(y-z)|$, $|(z-x)|$.

• **Complete Remainder:**

A remainder obtained by dividing a given number by the method of successive division is called complete remainder.

e.g.: A certain number when successively divided by 2, 3 and 5 leave remainders 1, 2 and 4 respectively. What is the complete remainder or remainder when the same number be divided by 30?

Explanation: For example, if a number when divided by 2, leaves remainder 1 would be of form = $2n + 1$.

And a number when divided by 3, leaves remainder 2 would be of form = $3n + 2$.

So, a number when successfully divided by 2, 3, 5 leaves remainder 1, 2, 4 would be of the form = 2[3(5n+4)+2]+1

= 30n+29.

When (30n + 29) is divisible by 30, the remainder is 29.

1) When there are two divisors d_1, d_2 and two remainders r_1, r_2 the complete remainder is given by $d_1 r_2 + r_1$.

2) When there are three divisors d_1, d_2, d_3 and three remainders r_1, r_2, r_3 the complete remainder is given by $d_1 d_2 r_3 + d_1 r_2 + r_1$.

3) When there are four divisors d_1, d_2, d_3, d_4 and four remainders r_1, r_2, r_3, r_4 the complete remainder is given by $d_1 d_2 d_3 r_4 + d_1 d_2 r_3 + d_1 r_2 + r_1$.

4) In any case if there are no remainders consider them as zeros.

▪ Fractions and Ordering Fractions:

1) In the fraction $\frac{3}{4}$; bottom number (denominator) says how many parts the whole is divided into. The top number (the numerator) says how many parts we have.

2) Fraction = $\frac{\text{Numerator}}{\text{Denominator}}$

Such a fraction is known as common fraction.

3) A fraction whose denominator is 10 or 100 or 1000 etc is called a decimal fraction.

4) Fractions whose denominators are same are called like fractions. For example, $\frac{3}{7}, \frac{5}{7}$ are like fractions.

5) Fractions whose denominators are different are called unlike fractions. For example, $\frac{3}{4}, \frac{5}{13}$.

6) When two fractions have the same denominator, the greater fraction is that which has greater numerator.

7) When two fractions have the same numerator, the greater fraction is that which has the smaller denominator.

8) If the identity is not possible, convert the fraction into the convenient form.

e.g.: Arrange $\frac{3}{5}, \frac{13}{16}, \frac{5}{7}, \frac{97}{104}$ in ascending order.

Explanation: LCM of 5, 16, 7, 104 is 7280.

Now multiply the numerator and denominator of the fractions with a number such that the denominator equals 7280.

i.e. $\frac{3\times1456}{5\times1456}, \frac{13\times455}{16\times455}, \frac{5\times1040}{7\times1040}, \frac{97\times70}{104\times70}$

$= \frac{4368}{7280}, \frac{5915}{7280}, \frac{5200}{7280}, \frac{6790}{7280}$.

Now compare the fractions.

$\frac{4368}{7280} < \frac{5200}{7280} < \frac{5915}{7280} < \frac{6790}{7280}$.

The order is $\frac{3}{5} < \frac{5}{7} < \frac{13}{16} < \frac{97}{104}$.

9) To find a fraction that lies between the two fractions $\frac{a}{b}$ and $\frac{x}{y}$, use the formula $\frac{a}{b} < \frac{a+x}{b+y} < \frac{x}{y}$.

e.g.: Find three fractions between $\frac{1}{3}$ and $\frac{4}{5}$.

Explanation: $\frac{1}{3}, \left(\frac{1+4}{3+5}\right), \frac{4}{5} = \frac{1}{3}, \frac{5}{8}, \frac{4}{5}$

Now, find one fraction between $\frac{1}{3}$ and $\frac{5}{8}$ and one fraction between $\frac{5}{8}$ and $\frac{4}{5}$.

$\Rightarrow \frac{1}{3}, \left(\frac{1+5}{3+8}\right), \frac{5}{8}, \left(\frac{5+4}{8+5}\right), \frac{4}{5} \Rightarrow \frac{1}{3}, \frac{6}{11}, \frac{5}{8}, \frac{9}{13}, \frac{4}{5}$

• Finding the Square Root:

Division Method: Let us take an example 64516.

```
   2)6 45 16 (2 5 4
     4
  45) 245
      225
  50 4)20 16
       2016
          0
```

∴ Square root of 64516 is 254.

Let us observe the above working rule in words.

Step 1: Group the digits in pairs, starting with the digit in the units place.

Step 2: Think of the largest number whose square is equal to or just less than the first pair. Take this number as the divisor and also as the quotient. In the given example, largest number whose square is near to 6 is 2 (*i.e.* $2^2 = 4$). So, 2 is the divisor and quotient.

Step 3: Subtract the product of the divisor and the quotient from the first pair and bring down the next pair to the right of the remainder. This becomes the new dividend.

Step 4: Double the quotient and put a blank for a number beside it (i.e. 4[?]). Now think of a largest number (for e.g., 5) to fill in the blank in such a way that the product of a new divisor (i.e. 45) and this digit (i.e. 5) is equal to or less than new dividend (245).

Step 5: Repeat steps (2), (3) and (4) till all the pairs have been taken up. Now, the quotient so obtained is the required square root of the given number.

Observe another example below.

Square Root of 119716 is 346.

Step 1: Group two digits as pairs. 11, 97, 16.

Step 2: Largest number whose square is near to the 11 is 3. Hence, 3 is the divisor and also quotient.

$$\begin{array}{l} 3)\ \overline{11}\ \overline{97}\ \overline{16}\ (3 \\ \quad \underline{9\qquad} \end{array}$$

Step 3: Now 297 is the new dividend.

$$\begin{array}{l} 3)\ \overline{11}\ \overline{97}\ \overline{16}\ (3 \\ \quad \underline{9\qquad} \\ \quad 2\,97 \end{array}$$

Step 4: Double the quotient 3 i.e. 3×2=6 and put a blank for a number beside 6 i.e. 6[?]. Now think of a largest number (for e.g., 4) to fill in the blank in such a way that the product of a new divisor (i.e. 64) and this digit (i.e. 4) is less than or equal to new dividend (i.e. 297).

Step 5:

$$\begin{array}{rl} & 3\times2=6 \\ 3) & \overline{11}\ \overline{97}\ \overline{16}\ \ (3\,\underline{4}\ \ \underline{6} \\ & \underline{9\qquad} \\ 6\,\underline{4}) & 297 \\ & \underline{256\quad}\qquad 34\times2=68 \\ 68\,\underline{6}) & 4116 \\ & \underline{4116\quad} \\ & 0 \end{array}$$

For this type of questions, it is better to check from options in the exam.

• Key Points on Finding Square Root:

1. A number ending with 2, 3, 7, 8 cannot be a perfect square. The last digit of any perfect square must be any one among 0, 1, 4, 5, 6, 9.

2. A number ending with odd number of zeros can never be a perfect square. e.g.: 1000, 2000 etc.

3. The difference between squares of two consecutive numbers is always an odd number.

e.g.: $4^2 - 3^2 = 16 - 9 = 7$ (odd).

Finding square root of a decimal fraction:

First eliminate the decimal point by dividing and multiplying with even powers of 10 then find the square root of both numerator and denominator separately and then you can conclude the square root.

e.g.: Find the square root of 1190.25.

$$\sqrt{1190.25} = \sqrt{\frac{1190.25}{10^2} \times 10^2} = \frac{\sqrt{119025}}{10^2} = \frac{345}{10} = 34.5$$

• Simplification:

In simplification we are supposed to follow the order which is essentially demanded by Mathematics and given by a common note of remembrance as VBODMAS.

V – Vinculum (bar $\bar{x}$), B – Bracket ( ) { }, O – of
D – Division (÷), M – Multiplication (×),
A – Addition (+), S – Subtraction (–).

Use of Algebraic Identities: The following algebraic identities will be useful in simplification.

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
4. $(a + b)^2 - (a - b)^2 = 4ab$
5. $a^2 - b^2 = (a + b)(a - b)$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a + b)$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a - b)$
8. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
10. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
11. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
12. $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2)$

• Number of Divisors of a Composite Number

If N is a composite number of the form N= $a^p\, b^q\, c^r \ldots$ where a, b, c are primes, then the number of divisors of N is given by $(p+1)(q+1)(r+1) \ldots$

e.g.: Let the number be 600.

$$\begin{array}{r|l} 2 & 600 \\ 2 & 300 \\ 2 & 150 \\ 3 & 75 \\ 5 & 25 \\ & 5 \end{array}$$

$600 = 2^3 \times 3^1 \times 5^2$

∴ Number of divisors of 600 = (3+1)(1+1)(2+1) = 24.

In these 24 divisors 1 and the number itself are also included. So, number of divisors of 600 excluding 1 and its self is 24 – 2 = 22.

• Sum of Divisors of a Composite Number :

If N is a composite number of the form $a^p\, b^q\, c^r \ldots$ Where a, b, c are primes, then the sum of the divisors, S_N is given by $S_N = \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$

e.g.: Let the number be 600. $600 = 2^3 \times 3^1 \times 5^2$

Sum of the divisors $S_N = \dfrac{(2^{3+1}-1)(3^{1+1}-1)(5^{2+1}-1)}{(2-1)(3-1)(5-1)}$

$$\Rightarrow \frac{(16-1)(9-1)(125-1)}{(1)(2)(4)} \Rightarrow \frac{(15)\times(8)\times(124)}{(1)\times(2)\times(4)} = 1860$$

- **Important Key Points:**

1) Sum of natural numbers from 1 to $n = \frac{n(n+1)}{2}$.

2) Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$.

3) Sum of cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

4) Number of odd numbers from 1 to n $= \frac{\text{Last Odd Number}+1}{2}$.

5) Number of even numbers from 1 to n $= \frac{\text{Last Even Number}}{2}$.

6) Sum of even numbers from 1 to n is $k(k+1)$, where k indicates number of even numbers from 1 to n.

e.g.: Sum of even no from 1 to 80 = 40(40+1) = 1640.
Here from 1 to 80 there exists 40 even numbers.

7) Sum of odd numbers from 1 to $n = k^2$, where k is equal to number of odd numbers from 1 to n.

e.g.: Sum of odd numbers from 1 to 60 is $(30)^2 = 900$.
30 odd natural numbers exist from 1 to 60.

8) Sum of the squares of first 'n' even natural numbers = $\frac{2}{3}(n)(n+1)(2n+1)$.

9) Sum of squares of first 'n' odd natural numbers is $\frac{n(2n+1)(2n-1)}{3}$.

10) Sum of any 5 consecutive whole numbers will always be divisible by 5.

e.g.: (3 + 4 + 5 + 6 + 7) = 25 is divisible by 5.

11) XY – YX; The difference between a two digit number and its reverse is divisible 9.

e.g.: Let the two numbers be 95 and 59. Here 59 is reverse of 95. Now 95 – 59 = 36 (which is divisible by 9).

12) Products: odd × odd = odd;
odd × even = even;
even × even = even;

13) $n! = n(n-1)(n-2)(n-3). \ldots\ldots (3)(2)(1)$.

e.g.: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.

Product of 'r' consecutive integers is divisible by (r!)

14) Finding the units digit of the numbers like $(252)^{54}$.
Here the units digit of 252 is 2 and the index is 54. We know that $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$. Here units digit is repeated after each 4 indices. So divide 54 by 4 to get the remainder. Here the remainder is 2.
Hence the last digit in $(252)^{54}$ is same as 2^2 *i.e.* 4.

CONCEPTUAL EXAMPLES

1) The smallest number which when added to 4, the sum is exactly divisible by 24, 36, 48 and 60 is:
a) 700 b) 716 c) 720 d) 730

Explanation:

2	24	36	48	60
2	12	18	24	30
3	6	9	12	15
2	2	3	4	5
	1	3	2	5

∴ LCM of 24, 36, 48, 60 = 2 × 2 × 3 × 2 × 3 × 2 × 5 = 720.
∴ Required number = 720 – 4 = 716

2) Number of integral divisors of 22050 except 1 and itself is.
a) 24 b) 28 c) 36 d) 52

Explanation:

2	22050
3	11025
3	3675
5	1225
5	245
7	49
	7

$\Rightarrow 22050 = 2^1 \times 3^2 \times 5^2 \times 7^2$
Number of divisions (N) = (P + 1)(Q + 1)(R + 1)
∴ Number of divisors = (1+1) (2+1) (2+1) (2+1) = 54
∴ Number of divisors except 1 and itself = 54 –2 = 52.

3) Find the sum of first 20 multiples of 12.
a) 1830 b) 2520 c) 3494 d) None

Explanation: Sum of first 20 multiples of 12 are
= (12×1) + (12×2) + (12×3) + … + (12×19) + (12×20).
= 12 (1 + 2 + 3 + … + 20)
Use the formula for "Sum of first 'n' natural numbers".

$$\sum n = \frac{n(n+1)}{2} \Rightarrow 12 \times \sum n = \frac{12 \times (20 \times 21)}{2} = 2520$$

4) Mr. Srinivas saves one coin of ₹5 on first day of the week, three coins of ₹5 on the second day of the week. Five coins of ₹5 on third day and so on. How much money will he has at the end of the week?
a) 78 b) 125 c) 245 d) 289

Explanation: Number of ₹5 coins with him at the end of week = 5 × (1 + 3 + 5 + 7 + 9 + 11 + 13)
= 5 × (sum of first 7 odd numbers)
(Sum of Odd numbers from 1 to n = k^2 where k = number of odd numbers from 1 to n)
∴ Sum of all numbers = $5 \times 7^2 = 245$.

Sample Exercise problems

1. Numbers

1. $7372 \times 7372 + 7372 \times 628 = ?$

(a) 58976000 (b) 58967000
(c) 5897600 (d) None of these

2. $9999 + 8888 + 777 + ? = 19700$

(a) 36 (b) 16
(c) 64 (d) 26

3. $60 ? 6 \times 111 = 666666$

(a) 0 (b) 2
(c) 1 (d) 6

4. $3149 \times 1 ? 5 = 425115$

(a) 3 (b) 2
(c) 4 (d) 6

5. If the two digits of the age of Mr Manoj are reversed then the new age so obtained is the age of his wife. $\frac{1}{11}$ of the sum of their ages is equal to the difference between their ages. If Mr Manoj is older than his wife then find the difference between their ages.

(a) Cannot be determined
(b) 8 years
(c) 10 years
(d) 9 years
(e) 7 years

6. If in a long division sum, the dividend is 380606 and the successive remainders from the first to the last are 434, 125 and 413, then divisor is:

(a) 451 (b) 843
(c) 4215 (d) 3372

7. If $\frac{x}{y} = \frac{3}{4}$, then the value of $\left(\frac{6}{7} + \frac{y-x}{y+x}\right)$ equals:

(a) $\frac{5}{7}$ (b) $1\frac{1}{7}$
(c) 1 (d) 2

8. The largest natural number by which the product of three consecutive even natural numbers is always divisible, is:

(a) 16 (b) 24
(c) 48 (d) 96

9. Which number should replace both the '*'s in $\left(\frac{*}{21}\right) \times \left(\frac{*}{189}\right) = 1$?

(a) 21 (b) 63
(c) 3969 (c) 147

10. In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder be 48, then the dividend is:

(a) 240 (b) 576
(c) 4800 (d) 4848

11. What least number must be subtracted from 1294 so that the remainder when divided by 9, 11, 13 will leave in each case the same remainder 6?

(a) 0 (b) 1
(c) 2 (d) 3

12. 24 is divided into two parts such that 7 times the first part added to 5 times the second part makes 146. The first part is:

(a) 11 (b) 13
(c) 16 (d) 17

13. $\frac{1}{4}$ of a number subtracted from $\frac{1}{3}$ of the same number gives 12. The number is:

(a) 144 (b) 120
(c) 72 (d) 63

14. $\frac{4}{3}$ of a certain number is 64. Half of that number is:

(a) 32 (b) 40
(c) 80 (d) 16

15. A fraction becomes 4 when 1 is added to both the numerator and denominator; and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is:

(a) 2 (b) 3
(c) 7 (d) 15

16. Three numbers are in the ratio 3:4:5. The sum of the largest and the smallest equals the sum of the third and is 52. The smallest number is:

(a) 20 (b) 27
(c) 39 (d) 52

17. The sum of three numbers is 68. If the ratio between first and second is 2:3 and that between second and third is 5:3, then the second number is:

(a) 30 (b) 20
(c) 58 (d) 48

18. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator, the fraction becomes 1. The fraction is:

(a) $\frac{4}{7}$ (b) $\frac{5}{9}$

(c) $\frac{2}{3}$ (c) $\frac{10}{11}$

19. $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is:

(a) 30 (b) 60
(c) 90 (d) None of these

20. What is the sum of all prime numbers from 60 to 80?

(a) 361 (b) 341
(c) 351 (d) 349

Answers:

1. (a) Given Expression $= 7372 \times (7372 + 628)$
$= 7372 \times 8000$
$= 58976000.$

2. (a) Let, $9999 + 8888 + 777 + x = 19700$
$\therefore \quad x = 19700 - 19664 = 36.$

3. (a) Let, $x \times 111 = 666666$

$\Rightarrow \quad x = \frac{666666}{111} = 6006 \qquad \therefore$ Missing figure = 0.

4. (a) Let, $3149 \times x = 425115$

$\Rightarrow \quad x = \frac{425115}{3149} = 135 \qquad \therefore$ Missing digit = 3.

5. (d) Let the age of Mr Manoj be $(10x + y)$ years.
$\therefore$ His wife's age $= (10y + x)$ years

Then, $(10x + y + 10y + x)\frac{1}{11} = 10x + y - 10y - x$

or, $x + y = 9x - 9y$, or, $8x = 10y$

or, $\frac{x}{y} = \frac{5}{4}$

$\therefore$ $x = 5$ and $y = 4$

[$\because$ any other multiple of 5 will make x of two digits]

$\therefore$ Difference $= 10x + y - 10y - x$

$= 9x - 9y = 9(x - y)$

$= 9(5 - 4) = 9$ years.

6. Dividend = 380606

Remainder 1 = 434

Subtract the remainder from dividend to see which optior can be divisor.

380606 – 434 = 380, 172.

No given option can divide the above answer.

7. $x = 3/4\ y$

$\Rightarrow \frac{6}{7} + \frac{y-x}{y+x}$

$\Rightarrow \frac{6}{7} + \frac{y-3/4y}{y+3/4y}$

$\Rightarrow 6/7 + 1/7$

$\Rightarrow 7/7 = 1$

8. The smallest even natural numbers are 2, 4, 6.

Product $= 2 \times 4 \times 6 = 48$

Divisible by 48

9. **(b)** Let, $\frac{x}{21} \times \frac{x}{189} = 1$.

Then, $x^2 = 21 \times 189 = 21 \times 21 \times 3 \times 3$

$\therefore$ $x = 21 \times 3 = 63$.

10. **(d)** Let Quotient $= Q$ and remainder $= R$

Then, given $12Q = 5R$

Now, $R = 48 \Rightarrow 12Q = 5 \times 48 \Rightarrow Q = 20$

$\therefore$ Dividend $= 20 \times 240 + 48 = 4848$.

11. **(b)** The number when divided by 9, 11 and 13 leavin remainder 6 = (L.C.M. of 9, 11, 13) + 6 = 1293

$\therefore$ Required number $= 1294 - 1293 = 1$.

12. **(b)** Let these parts be x and $(24 - x)$.

Then, $7x + 5\ (24 - x) = 146 \Rightarrow x = 13$

So the first part is 13.

13. **(a)** $\frac{1}{3}x - \frac{1}{4}x = 12 \Rightarrow \frac{1}{12}x = 12 \quad \Rightarrow \quad x = 144$.

14. **(b)** $\frac{4}{5} \times x = 64 \Rightarrow x = \frac{64 \times 5}{4} = 80$

$\therefore$ $\frac{1}{2} \times x = \frac{1}{2} \times 80 = 40$.

15. **(d)** Let the required fraction be $\frac{x}{y}$

Then, $\frac{x+1}{y+1} = 4 \Rightarrow \quad x - 4y = 3$

and, $\dfrac{x-1}{y-1} = 7 \Rightarrow x - 7y = -6$

Solving these equations, we get $x = 15$, $y = 3$.

16. **(c)** Let the numbers be $3x$, $4x$ and $5x$.

Then, $5x + 3x = 4x + 52 \Rightarrow 4x = 52 \Rightarrow x = 13$

$\therefore$ The smallest number $= 3x = 3 \times 13 = 39$.

17. **(a)** Let the numbers be a, b, c.

Then, $\dfrac{a}{b} = \dfrac{2}{3}, \dfrac{b}{c} = \dfrac{5}{3} \Rightarrow \dfrac{a}{b} = \dfrac{2 \times 5}{3 \times 5} = \dfrac{10}{15}, \dfrac{b}{c} = \dfrac{5 \times 3}{3 \times 3} = \dfrac{15}{9}$

$\Rightarrow a{:}b{:}c = 10{:}15{:}9$

Let the numbers be $10x$, $15x$, $9x$,

Then, $10x + 15x + 9x = 68 \Rightarrow 34x = 68 \Rightarrow x = 2$

$\therefore$ Second number $= 15x = 15 \times 2 = 30$.

18. **(c)** Let the required fraction be $\dfrac{x}{y}$.

Then, $\dfrac{x}{y+1} = \dfrac{1}{2} \Rightarrow 2x - y = 1$

and, $\dfrac{x+1}{y} = 1 \Rightarrow x - y = -1$

Solving, $2x - y = 1$ and $x - y = -1$,

we get, $x = 2$, $y = 3$

$\therefore$ The fraction is $\dfrac{2}{3}$.

19. **(b)** Let the number be x.

Then, $\dfrac{4}{5}x - \dfrac{2}{3}x = 8 \Rightarrow \dfrac{12x - 10x}{15} = 8 \Rightarrow 2x = 120$

or, $x = 60$.

20. **(c)** $61 + 67 + 71 + 73 + 79 = 351$.

2. HCF and LCM

1. What is the H.C.F. of 27, 18 and 36?

(a) 7 (b) 11

(c) 9 (d) None of these

2. Determine the L.C.M of $\dfrac{2}{5}, \dfrac{3}{10}$ and $\dfrac{6}{25}$.

(a) $\dfrac{6}{5}$ (b) $\dfrac{11}{5}$

(c) $\dfrac{9}{5}$ (d) None of these

3. What is the L.C.M. of 25, 30, 35 and 40?

(a) 3800

(b) 4200

(c) 4400

(d) None of these

4. What is the greatest number which divides 852, 1065 and 1491 exactly?

(a) 193 (b) 183

(c) 223 (d) 213

5. What is the H.C.F. of $\dfrac{4}{9}, \dfrac{10}{21}$ and $\dfrac{20}{30}$?

(a) $\dfrac{4}{189}$ (b) $\dfrac{6}{23}$

(c) $\dfrac{2}{63}$ (d) None of these

6. Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder.

(a) 8004 (b) 13004

(c) 18004 (d) 18014

7. Area of three fields is 165 m^2, 195 m^2 and 85 m^2, respectively. In each of the fields a flower bed of equal length has to be made. If flower bed in each of the fields is 3 m wide then what is the maximum length of the flower bed in each of the fields?

(a) 7 m (b) 9 m
(c) 5 m (d) None of these

8. Find the greatest number which will divide 2112 and 2792 leaving the remainder 4 in each case.

(a) 78 (b) 68
(c) 65 (d) 63

9. The H.C.F. of two numbers is 12 and their difference is 12. The numbers are:

(a) 66, 78 (b) 70, 82
(c) 94, 106 (d) 84, 96

10. A merchant has 435 litres, 493 litres and 551 litres of three different kinds of milk. Find the least number of casks of equal size required to store all the milk without mixing.

(a) 51 (b) 61
(c) 47 (d) 45

11. Find the greatest number which will divide 25, 73 and 97 so as to leave the same remainder in each case.

(a) 12 (b) 18
(c) 24 (d) 32

12. The sum of two numbers is 216 and their H.C.F. is 27. The numbers are:

(a) 54, 162 (b) 108, 118
(c) 27, 189 (d) None of these

13. How often will five bells toll together in one hour if they start together and toll at intervals of 5, 6, 8, 12, 20 seconds, respectively?

(a) 29 (b) 30
(c) 31 (d) 120

14. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51, respectively.

(a) 71 (b) 81
(c) 61 (d) 73

15. Find the side of the largest square slabs which can be paved on the floor of a room 5 m 44 cm long and 3 m 74 cm broad.

(a) 56 (b) 42
(c) 38 (d) 34

16. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds, respectively. If they all change simultaneously at 8:20:00 hours; then they will again change simultaneously at:

(a) 8:27:12 hours (b) 8:27:24 hours
(c) 8:27:36 hours (d) 8:27:48 hours

17. The product of two numbers is 6760 and their H.C.F. is 13. How many such pairs can be formed?

(a) 2 (b) 3
(c) 4 (d) only one

18. Find the greatest number of four digits which when divided by 10, 15, 21 and 28 leaves 4, 9, 15 and 22 as remainders, respectively.

(a) 9654 (b) 9666
(c) 9664 (d) 9864

19. The number of prime factors in the expression $(6)^{10} \times (7)^{17} \times (11)^{27}$ is:

(a) 54 (b) 64
(c) 71 (d) 81

20. Find the greatest number which will divide 3962, 4085 and 4167 leaving the same remainder in each case.

(a) 37 (b) 39
(c) 41 (d) 43

21. A wholesale tea dealer has 408 kilograms, 468 kilograms and 516 kilograms of three different qualities of tea. He wants it all to be packed into boxes of equal size without mixing. Find the capacity of the largest possible box.

(a) 50 (b) 36
(c) 24 (d) 12

22. A room is 4 m 37 cm long and 3 m 23 cm broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose.

(a) 485 (b) 431
(c) 391 (d) 381

23. The least perfect square number which is divisible by 3, 4, 5, 6 and 8:

(a) 900 (b) 1200
(c) 2500 (d) 3600

24. Find the least number of five digits which when divided by 12, 16, 21, 36 and 40 leaves remainder 8 in each case.

(a) 10088 (b) 10072
(c) 10080 (d) None of these

25. Three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?

(a) 7 m (b) 14 m (c) 42 m (d) 63 m

Solutions

1. **(c)** H.C.F. of 27, 18 and 36

```
18) 27 (1
    18
    ------
     9) 18 (2
        18
        ---
         ×
        ---
```

$\therefore$ H.C.F. of 27 and 18 is 9

Now, H.C.F. of 9 and 36

```
9) 36 (4
   36
   ---
    ×
   ---
```

$\therefore$ H.C.F. of 9 and 36 is 9

Therefore, the required H.C.F of 27, 18 and 36 is 9.

2. **(a)** L.C.M. of $\frac{2}{5}, \frac{3}{10}$ and $\frac{6}{25}$

$= \frac{\text{L.C.M. of 2, 3 and 6}}{\text{H.C.F. of 5, 10 and 25}}$

$\because$ L.C.M. of 2, 3 and 6 = 6

and, H.C.F. of 5, 10 and 25 = 5

$\therefore$ Required L.C.M. = $\frac{6}{5}$.

3. **(b)**

2	25, 30, 35, 40
5	25, 15, 35, 20
	5, 3, 7, 4

$\therefore$ Required L.C.M. = $2 \times 5 \times 5 \times 3 \times 7 \times 4$

$= 4200$.

4. **(d)** H.C.F. of 852 and 1065 is 213.

H.C.F. of 213 and 1491 is 213.

5. **(c)** H.C.F. of $\frac{4}{9}, \frac{10}{21}$ and $\frac{20}{63}$

$= \frac{\text{H.C.F. of 4, 10 and 20}}{\text{L.C.M. of 9, 21 and 63}}$

$\because$ H.C.F. of 4, 10 and 20 = 2

and L.C.M. of 9, 21 and 63 = 63

$\therefore$ Required H.C.F. = $\frac{2}{63}$.

6. **(c)** L.C.M. of 16, 18, 20 and 25 is 3600.

Required number $= 3600 \times K + 4$

$= (7 \times 514 + 2)K + 4$

$= (7 \times 514)K + 2K + 4$

Now $(2K + 4)$ is divisible by 7 for $K = 5$.

$\therefore$ Required number $= 5 \times 3600 + 4$

$= 18004.$

7. **(c)** H.C.F. of 165, 195 and 85 will be maximum area of each of the flower beds.

H.C.F. of 165 and 195:

$$\begin{array}{l} 165\,)\,195\,(\,1 \\ \quad\;\; \underline{165} \\ \quad\;\; 30\,)\,165\,(\,2 \\ \qquad\quad\; \underline{150} \\ \qquad\quad\; 15\,)\,35\,(\,2 \\ \qquad\qquad\;\; \underline{30} \\ \qquad\qquad\;\; \underline{\times} \end{array}$$

$\therefore$ H.C.F. of 165 and 195 is 15.

Also, now, H.C.F. of 15 and 85 is 5.

8. **(b)** Subtract 4 from each of the numbers 2112 and 2792 and then take the H.C.F, i.e., H.C.F of 2108 and 2788.

9. **(d)** The difference of requisite numbers must be 12 and each one must be divisible by 12. So, the numbers are 84, 96.

10. **(a)** Since minimum number of casks are required, the size of the cask is greatest. Also the cask in three cases are of equal size. The size of the cask is the H.C.F. of 435, 493 and 551 which is 29.

Now, the number of casks required for storing the milk $= (493 + 435 + 551) \div 29 = 51.$

11. **(c)** $73 - 25 = 48$

$97 - 73 = 24$

$97 - 25 = 72$

H.C.F. of 48, 24 and 72 is 24.

12. **(c)** Let the numbers be $27a$ and $27b$

Then, $27a + 27b = 216$ or, $27(a + b) = 216$

or, $a + b = \dfrac{216}{27} = 8$

$\therefore$ Values of co-primes (with sum 8) are (1, 7) and (3, 5)

So, the numbers are $(27 \times 1, 27 \times 7)$,

i.e., (27, 189).

13. **(c)** The time after which the bells will ring together is the L.C.M. of 5, 6, 8, 12 and 20 seconds, i.e., 120 seconds. The number of times they will toll together in one hour

$= (3600 \div 120) + 1$

$= 30 + 1 = 31.$

14. **(a)** $964 - 41 = 923$

$1238 - 31 = 1207$

$1400 - 51 = 1349$

H.C.F. of 923 and 1207 is 71.

H.C.F. of 71 and 1349 is 71.

15. **(d)** The side of the square slab is the H.C.F. of 544 and 374 cm, i.e., 34.

16. **(a)** Interval of change = (L.C.M. of 48, 72, 108) seconds

$= 432$

So, the lights will change after every 432 seconds, i.e., 7 minutes and 12 seconds.

So, the next simultaneous change will take place at 8:27:12 hours.

17. **(a)** Let the numbers be $13x$ and $13y$.

$$13x \times 13y = 6760$$

$\therefore$ $x \times y = 6760 \div (13 \times 13) = 40$

Possible values of (x, y) are

(1, 40); (2, 20); (4, 10); (5, 8)

Only two acceptable values are (1, 40) and (5, 8).

18. **(a)** First, find the greatest number of four digits that is divisible by the L.C.M. of 10, 15, 21 and 28 and then subtract 6 from it to get the required number.

19. **(b)** Since 2, 3, 7, 11 are prime numbers and the given expression is $2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$, the number of prime factors in the given expression is $(10 + 10 + 17 + 27) = 64$.

20. **(c)** $4085 - 3962 = 123$

$4167 - 4085 = 82$

$4167 - 3962 = 205$

H.C.F. of 123, 82 and 205 is 41.

21. **(d)** The capacity of the box is H.C.F. of 408, 468 and 516, i.e., 12.

22. **(c)** Length = 437 cm

Breadth = 323 cm.

The side of the square slab is the H.C.F. of 437 and 323, i.e., 19 cm.

$\therefore$ Area of square slab $= 19 \text{ cm} \times 19 \text{ cm} = 361 \text{ cm}^2$

The number of slabs $= \dfrac{\text{Area of the room}}{\text{Area of the slab}}$

$= \dfrac{437 \times 323 \text{ cm}^2}{361 \text{ cm}^2}$

$= 391.$

23. **(d)**

$$\begin{array}{r|l} 2 & 3,\ 4,\ 5,\ 6,\ 8 \\ \hline 2 & 3,\ 2,\ 5,\ 3,\ 4 \\ \hline 3 & 3,\ 1,\ 5,\ 3,\ 2 \\ \hline & 1,\ 1,\ 5,\ 1,\ 2 \end{array}$$

L.C.M. of 3, 4, 5, 6, 8 = 120

Required number $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 3600.$

24. **(a)** Required number = the least number of 5 digits divisible by the L.C.M. of 12, 16, 21, 36, 40 + the remainder 8.

25. **(a)** Greatest possible length of each plank

$$= (\text{H.C.F. of } 42, 49, 63)\ \text{m} = 7\ \text{m}.$$

Square root and cube root

1. Find the square root of 4356.
(a) 68 (b) 64
(c) 66 (d) None of these

2. Find the square root of 104976.
(a) 324 (b) 424
(c) 326 (d) None of these

3. Find the square root of 211600.
(a) 460 (b) 440
(c) 480 (d) None of these

4. Find the value of $\sqrt{6492304}$.
(a) 2384 (b) 2484
(c) 2548 (d) 2684

5. Find the least number, which, when multiplied with 74088 will make it a perfect square.
(a) 42 (b) 44
(c) 46 (d) 48

6. $\sqrt{10} \times \sqrt{250} = ?$
(a) 46.95 (b) 43.75
(c) 50.25 (d) 50.00

7. $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = ?$
(a) $20\sqrt{5}$ (b) $25\sqrt{2}$
(c) $15\sqrt{2}$ (d) None of these

8. $\dfrac{250}{\sqrt{?}} = 10$
(a) 25 (b) 250
(c) 625 (d) 2500

9. If $\sqrt{256}\,\sqrt{\sqrt{x}} = 2$, then x is equal to:
(a) 64 (b) 128
(c) 512 (d) 1024

10. Find the smallest number by which 216 should be divided to make the result a perfect square.
(a) 4 (b) 3
(c) 6 (d) 2

11. $\dfrac{\sqrt{?}}{200} = 0.02$
(a) 0.4 (b) 4
(c) 16 (d) 1.6

12. $\dfrac{\sqrt{6727}}{\sqrt{7}} = ?$
(a) 30.79 (b) 32.29
(c) 31 (d) None of these

13. $\sqrt{.09} = ?$
(a) 0.3 (b) 0.03
(c) 0.003 (d) None of these

14. $\dfrac{14}{3+\sqrt{2}} = ?$
(a) 3.172 (b) 4.586
(c) 8.828 (d) None of these

15. Find the smallest number, which, when added to 3579 gives a perfect square.
(a) 27 (b) 24
(c) 21 (d) 18

16. If $\sqrt{\left(1+\dfrac{27}{169}\right)} = 1+\dfrac{x}{13}$, then x equals:
(a) 1 (b) 3
(c) 5 (d) 7

17. $\dfrac{\sqrt{4375}}{\sqrt{7}} = ?$
(a) 24.75 (b) 27.75
(c) 25 (d) 35

18. If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then the value of $\dfrac{a}{b}$ is:
(a) 0.016 (b) 1.60
(c) 0.16 (d) None of these

19. $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt[3]{\sqrt{3}}}}}}} = ?$

(a) $3^{31/64}$ (b) $3^{31/32}$
(c) $3^{1/64}$ (d) None of these

20. $\dfrac{\sqrt{1296}}{?} = \dfrac{?}{2.25}$

(a) 6 (b) 7
(c) 8 (d) 9

21. $\sqrt{176 + \sqrt{2401}} = ?$

(a) 14 (b) 15
(c) 18 (d) 24

22. $\sqrt{10} \times \sqrt{15} = ?$

(a) $5\sqrt{6}$ (b) $6\sqrt{5}$
(c) 5 (d) $\sqrt{30}$

23. $\sqrt{\dfrac{4}{3}} - \sqrt{\dfrac{3}{4}} = ?$

(a) $\dfrac{1}{2\sqrt{3}}$ (b) $-\dfrac{1}{2\sqrt{3}}$
(c) 1 (d) $\dfrac{5\sqrt{3}}{6}$

24. $\sqrt{248 + \sqrt{52 + \sqrt{144}}} = 1$

(a) 14 (b) 16
(c) 16.6 (d) 18.8

25. $\sqrt{0.0009} \surd \sqrt{0.01} = ?$

(a) 3 (b) 0.3
(c) $\dfrac{1}{3}$ (d) None of these

26. $\dfrac{1}{\sqrt{9} - \sqrt{8}} = ?$

(a) $\dfrac{1}{2}(3 - 2\sqrt{2})$
(b) $\dfrac{1}{3 + 2\sqrt{2}}$
(c) $3 - 2\sqrt{2}$
(d) $3 + 2\sqrt{2}$

27. If $\sqrt{\dfrac{x}{169}} = \dfrac{54}{39}$, then x is equal to:

(a) 108 (b) 324
(c) 2916 (d) 4800

Solutions:

1. **(c)** The prime factors of 4356 are

$2 \times 2 \times 3 \times 3 \times 11 \times 11$

$4356 = 2 \times 2 \times 3 \times 3 \times 11 \times 11$

2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

$\sqrt{4356} = \sqrt{2^2 \times 3^2 \times 11^2}$
$= 2 \times 3 \times 11 = 66.$

2. **(a)**

$$\begin{array}{r|l} & 3\quad 2\quad 4 \\ \hline 3 & \overline{10}\ \ \overline{49}\ \ \overline{76} \\ & 9 \\ \hline 62 & 149 \\ & 124 \\ \hline 644 & 2576 \\ & 2576 \\ \hline & \times \end{array}$$

$\therefore$ Square root of 104976 is 324.

3. (a)

$$\begin{array}{r|l} & \quad 460 \\ \hline 4 & \overline{21}\ \ \overline{16}\ \ \overline{00} \\ & \underline{16} \\ 86 & 516 \\ & \underline{516} \\ & \times \end{array}$$

$\therefore$ Square root of 211600 is 460.

4. (c)

$$\begin{array}{r|l} & \quad 2548 \\ \hline 2 & 6\ \ \overline{49}\ \ \overline{23}\ \ \overline{04} \\ & 4 \\ \hline 45 & 249 \\ & 225 \\ 644 & \ \ 2423 \\ \hline & \ \ 2016 \\ 5088 & \quad 40704 \\ & \quad 40704 \\ \hline & \qquad \times \end{array}$$

$\therefore \sqrt{6492304} = 2548.$

5. (a) $74088 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$= (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times (2 \times 3 \times 7)$

Therefore, required number $= 2 \times 3 \times 7 = 42$.

2	74088
2	37044
2	18522
3	6261
3	3087
3	1029
7	343
7	49
7	7
	1

6. (d) $\sqrt{10} \times \sqrt{250} = \sqrt{2500} = 50.$

7. (a) $\sqrt{80} + 3\sqrt{245} - \sqrt{125} = 4\sqrt{5} + 21\sqrt{5} - 5\sqrt{5}.$

$= 20\sqrt{5}.$

8. (c) Let, $\dfrac{250}{\sqrt{x}} = 10$. Then, $\sqrt{x} = \dfrac{250}{10} = 25$

$\therefore \quad x = (25)^2 = 625.$

9. (a) $\dfrac{\sqrt{256}}{\sqrt{x}} = 2 \quad$ or, $\quad \dfrac{16}{\sqrt{x}} = 2$

$\therefore \quad 16 = 2\sqrt{x} \quad \Rightarrow \quad \sqrt{x} = 8$ or, $x = 64.$

10. (c) We know that

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^2 \times 6$

Thus, $\dfrac{216}{6} = 2^2 \times 3^2 = 6^2.$

Therefore, 216 should be divided by 6, so that the result is a perfect square.

11. (c) Let, $\dfrac{\sqrt{x}}{200} = 0.02$. Then,

$\sqrt{x} = 200 \times 0.02$ or, $\sqrt{x} = 4$

So, $x = 16.$

12. (c) $\dfrac{\sqrt{6727}}{\sqrt{7}} = \dfrac{\sqrt{6727}}{\sqrt{7}} = \sqrt{961} = 31.$

13. (a) $\sqrt{0.09} = \sqrt{\dfrac{9}{100}} = \dfrac{3}{10} = 0.3.$

14. (a) $\dfrac{14}{3+\sqrt{2}} = \dfrac{14(3-\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \dfrac{14(3-\sqrt{2})}{9-2}$

$= 2(3 - \sqrt{2}) = 2(3 - 1.414)$

$= 2 \times 1.586 = 3.172.$

15. (c) The number nearest to 3579 which is a perfect square is 3600.

$\therefore$ Required number $= 60^2 - 3579 = 21.$

16. (a) $\sqrt{\left(1+\dfrac{27}{169}\right)} = 1 + \dfrac{x}{13}$

$\therefore \quad \sqrt{\dfrac{196}{169}} = 1 + \dfrac{x}{13}$

or, $\dfrac{14}{13} = 1 + \dfrac{x}{13} \quad$ or, $\quad \dfrac{x}{13} = \dfrac{14}{13} - 1$

or, $\dfrac{x}{13} = \dfrac{1}{13} \quad$ or, $\quad x = 1.$

17. (c) $\dfrac{\sqrt{4375}}{\sqrt{7}} = \dfrac{\sqrt{4375}}{\sqrt{7}} = \sqrt{625} = 25.$

18. (a) $\sqrt{0.016a} = 0.016 \times \sqrt{b}$

$\Rightarrow \quad \sqrt{\dfrac{a}{b}} = \dfrac{0.016}{\sqrt{0.016}} \Rightarrow \sqrt{\dfrac{a}{b}} = \sqrt{0.016} \Rightarrow \dfrac{a}{b} = 0.016.$

19. (b) $\sqrt{\sqrt[3]{\sqrt[3]{\sqrt[3]{3.3^{1/2}}}}} = \sqrt{\sqrt[3]{\sqrt[3]{3.3^{3/4}}}} = \sqrt{\sqrt[3]{3.3^{7/8}}}$

$= \sqrt{3.3^{15/16}} = 3^{31/32}.$

20. (d) Let, $\dfrac{\sqrt{1296}}{x} = \dfrac{x}{2.25}$

Then, $\dfrac{36}{x} = \dfrac{x}{2.25}$

or, $x^2 = 36 \times \dfrac{225}{100}$

$\therefore \quad x = \sqrt{\dfrac{36 \times 225}{100}} = \dfrac{6 \times 15}{10} = 9.$

21. (b) $\sqrt{176 + \sqrt{2401}} = \sqrt{176 + 49} = \sqrt{225} = 15.$

22. (a) $\sqrt{10} \times \sqrt{15} = \sqrt{150} = \sqrt{25 \times 6}$

$= \sqrt{25} \times \sqrt{6} = 5\sqrt{6}.$

23. (a) $\dfrac{\sqrt{4}}{\sqrt{3}} - \dfrac{\sqrt{3}}{\sqrt{4}} = \dfrac{2}{\sqrt{3}} - \dfrac{\sqrt{3}}{2} = \dfrac{4-3}{2\sqrt{3}} = \dfrac{1}{2\sqrt{3}}.$

24. (b) $\sqrt{248+\sqrt{52+\sqrt{144}}} = \sqrt{248+\sqrt{52+12}}$
$= \sqrt{248+\sqrt{64}}$
$= \sqrt{248+8}$
$= \sqrt{256} = 16.$

25. (b) Given expression $= \dfrac{\sqrt{0.0009}}{\sqrt{0.01}} = \sqrt{\dfrac{0.0009}{0.0100}}$
$= \sqrt{\dfrac{9}{100}} = \dfrac{3}{10} = 0.3.$

26. (d) $\dfrac{1}{\sqrt{9}-\sqrt{8}} = \dfrac{1}{\sqrt{9}-\sqrt{8}} \times \dfrac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \dfrac{3+2\sqrt{2}}{9-8}$
$= 3 + 2\sqrt{2}.$

27. (b) $\sqrt{\dfrac{x}{169}} = \dfrac{54}{39} \Rightarrow \dfrac{x}{169} = \dfrac{54}{39} \times \dfrac{54}{39}$
$\therefore\ x = \dfrac{54}{39} \times \dfrac{54}{39} \times 169 = 324.$

Fractions

1. Simplify:

$\dfrac{3}{10} \div \dfrac{3}{7}$ of $\left(2\dfrac{3}{10} + 2\dfrac{3}{5}\right) + \dfrac{1}{5} \div 1\dfrac{2}{5} - \dfrac{2}{7}$

(a) 1 (b) 2
(c) 0 (d) 3

2. $1+1 \div \left\{1+1 \div \left(1-\dfrac{1}{3}\right)\right\} = ?$

(a) $\dfrac{7}{5}$ (b) $\dfrac{2}{3}$
(c) $\dfrac{4}{5}$ (d) None of these

3. $48 \div 12 \times \left(\dfrac{9}{8} \text{ of } \dfrac{4}{3} + \dfrac{3}{4} \text{ of } \dfrac{2}{3}\right) = ?$

(a) 9 (b) 12
(c) 15 (d) None of these

4. Simplify:

2 ÷ [2 + 2 ÷ {2 + 2 ÷ (2 + 2 ÷ 3)}]

(a) 13/15 (b) 17/15
(c) 11/15 (d) None of these

5. $7\dfrac{1}{2} - \left[2\dfrac{1}{4} \div \left\{1\dfrac{1}{4} - ?\left(1\dfrac{1}{2} - \dfrac{1}{3} - \dfrac{1}{6}\right)\right\}\right] = 3$

(a) $\dfrac{1}{4}$ (b) $\dfrac{3}{4}$
(c) $\dfrac{4}{3}$ (d) None of these

6. The simplification of $\dfrac{0.8 \times 0.8 \times 0.8 - 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 + 0.8 \times 0.5 + 0.5 \times 0.5}$ gives:

(a) 0.8 (b) 0.4
(c) 0.3 (d) 0.13

7. The simplification of $\left[\dfrac{1}{2} + \dfrac{1}{2}\left\{\dfrac{3}{4} - \dfrac{1}{2}\left(\dfrac{7}{8} - \dfrac{3}{4}\right)\right\}\right]$ yields:

(a) $\dfrac{27}{16}$ (b) $\dfrac{27}{32}$
(c) $\dfrac{27}{64}$ (d) $\dfrac{107}{112}$

8. Simplify: $1 - [2 - \{5 - (4 - \overline{3-2})\}]$

(a) 1 (b) 2
(c) 3 (d) 4

9. $3 \div \left[(8-5) \div \left\{(4-2) \div \left(2 + \dfrac{8}{13}\right)\right\}\right] = ?$

(a) $\dfrac{33}{71}$ (b) $\dfrac{55}{17}$
(c) $\dfrac{13}{17}$ (d) None of these

10. $\dfrac{69842 \times 69842 - 30158 \times 30158}{69842 - 30158} = ?$

(a) 100000 (b) 69842
(c) 39684 (d) 30158

11. Simplify $\dfrac{2\frac{1}{7} - 2\frac{1}{2}}{2\frac{1}{4} + 1\frac{1}{7}} \div \dfrac{1}{2 + \dfrac{1}{2 + \dfrac{1}{2 - \frac{1}{2}}}}$

(a) $\dfrac{-1}{2}$ (b) $-\dfrac{1}{8}$
(c) $-\dfrac{1}{6}$ (d) $-\dfrac{1}{4}$

12. The value of $\dfrac{2.75 \times 2.75 \times 2.75 - 2.25 \times 2.25 \times 2.25}{2.75 \times 2.75 + 2.75 \times 2.25 + 2.25 \times 2.25}$ is:

(a) 0.30 (b) 0.50
(c) 3.00 (d) 5.00

Solutions

1. (c) Given expression

$= \frac{3}{10} \div \frac{3}{7} \text{ of } \left(\frac{23}{10} + \frac{13}{5}\right) + \frac{1}{5} \times \frac{5}{7} - \frac{2}{7}$

$= \frac{3}{10} \div \frac{3}{7} \text{ of } \frac{49}{10} + \frac{1}{7} - \frac{2}{7} = \frac{3}{10} \div \frac{21}{10} - \frac{1}{7}$

$= \frac{3}{10} \times \frac{10}{21} - \frac{1}{7} = \frac{1}{7} - \frac{1}{7} = 0.$

2. (a) Given expression

$= 1 + 1 \div \left\{1 + 1 \div \left(\frac{2}{3}\right)\right\}$

$= 1 + 1 \div \left\{1 + 1 \times \frac{3}{2}\right\}$

$= 1 + 1 \div \left\{1 + \frac{3}{2}\right\} = 1 + 1 \div \frac{5}{2}$

$= 1 + 1 \times \frac{2}{5} = 1 + \frac{2}{5} = \frac{7}{5}.$

3. (b) Given expression

$= 48 \div 12 \times \left(\frac{9}{8} \text{ of } \frac{4}{3} \div \frac{3}{4} \text{ of } \frac{2}{3}\right)$

$= \frac{48}{12} \times \left\{\left(\frac{9}{8} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{2}{3}\right)\right\}$

$= \frac{48}{12} \times \left(\frac{3}{2} \times 2\right) = 4 \times 3 = 12.$

4. (c) Given expression

$= 2 \div \left[2 + 2 \div \left\{2 + 2 \div \left(2 + \frac{2}{3}\right)\right\}\right]$

$= 2 \div \left[2 + 2 \div \left\{2 + 2 \times \frac{3}{8}\right\}\right]$

$= 2 \div \left[2 + 2 \div \frac{11}{4}\right] = 2 \div \left[2 + 2 \times \frac{4}{11}\right]$

$= 2 \div \frac{30}{11} = 2 \times \frac{11}{30} = \frac{11}{15}.$

5. (b) Let the missing figure $= x$.

$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right] = 3$

$\frac{15}{2} - \left[\frac{9}{4} \div \left\{\frac{5}{4} - x\right\}\right] = 3$

$\frac{15}{2} - 3 = \frac{9/4}{5/4 - x}$

$9/2 = \frac{9}{5 - 4x}$

$5 - 4x = 2$

$x = 3/4.$

6. (c) We know that $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$

$\therefore$ The given expression $= 0.8 - 0.5 = 0.3.$

8. (a) Given expression

$= 1 - [2 - \{5 - (4 - 1)\}]$

$= 1 - [2 - \{5 - 3\}]$

$= 1 - [2 - 2] = 1 - 0 = 1.$

9. (c) Given expression

$= 3 \div \left[3 + \left\{2 \div \frac{34}{13}\right\}\right]$

$= 3 \div \left[3 + \left\{2 \times \frac{13}{34}\right\}\right] = 3 \div \left[3 \times \frac{17}{13}\right]$

$= 3 \div \frac{51}{13} = 3 \times \frac{13}{51} = \frac{13}{17}.$

10. (a) Given expression

$= \frac{(69842)^2 - (30158)^2}{69842 - 30158}$

$= \frac{(69842 - 30158)(69842 + 30158)}{69842 - 30158}$

$= 100000.$

11. (d) Given expression

$= \frac{\frac{15}{7} - \frac{5}{2}}{\frac{9}{4} + \frac{8}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{2}{3}}}$

$= \frac{-5}{14} \times \frac{28}{95} \div \frac{1}{2 + \frac{3}{8}}$

$= \frac{-2}{19} \div \frac{8}{19} = \frac{-2}{19} \times \frac{19}{8} = \frac{-1}{4}.$

12. (b) The given expression

$= 2.75 - 2.25 = 0.50.$

AVERAGES

1. The daily earnings of a taxi driver during a week are: ₹60, ₹65, ₹70, ₹52.50, ₹63, ₹73 and ₹68. What is his average daily earning for the week?
(a) ₹74.50 (b) ₹54.50
(c) ₹64.50 (d) ₹84.50

2. The average of 10 numbers is 7. What will be the new average if each of the numbers is multiplied by 8?
(a) 45 (b) 52
(c) 56 (d) 55

3. The average weight of 5 persons, sitting in a boat, is 38 Kg. If the average weight of the boat and the persons sitting in the boat is 52 Kg, what is the weight of the boat?
(a) 228 Kg (b) 122 Kg
(c) 232 Kg (d) 242 Kg

4. There are 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by ₹42 per day while the average expenditure per head decreased by ₹1. Find out the actual expenditure of the mess.
(a) ₹480 (b) ₹440
(c) ₹520 (d) ₹420

5. The daily maximum temperature in Delhi, for 7 consecutive days in May 1988, were 42.7°C, 44.6°C, 42.0°C, 39.1°C, 43.0°C, 42.5°C and 38.5°C. Find out the average daily maximum temperature.
(a) 42.63° C (b) 45.65° C
(c) 41.77° C (d) 39.60° C

6. The average salary per head of all the workers in a workshop is ₹850. If the average salary per head of 7 technicians is ₹1000 and the average salary per head of the rest is ₹780, find out the total number of workers in the workshop.
(a) 26 (b) 24
(c) 28 (d) 22

7. An aeroplane travels 2500 Km, 1200 Km and 500 Km at 500 Km/h, 400 Km, and 250 Km/h, respectively. The average speed is:
(a) 420 Km/h (b) 410 Km/h
(c) 405 Km/h (d) 575 Km/h

8. In an examination, out of 20 students in a class, in Mathematics 2 students scored 100 marks, 3 students scored 0, and average marks for rest of the students was 40. What is the average mark of the whole class?
(a) 40 marks (b) 35 marks
(c) 32 marks (d) 45 marks

9. The average weight of 24 students in section A of a class is 58 Kg, whereas the average weight of 26 students in section B of the same class is 60.5 Kg. Find out the average weight of all the 50 students of the class.
(a) 57.4 Kg (b) 59.3 Kg
(c) 58.9 Kg (d) 59.7 Kg

10. The average age of 5 members is 21 years. If the age of the youngest member be 5 years, find out the average age of the family at the birth of the youngest member.
(a) 24 years (b) 25 years
(c) 20 years (d) 28 years

11. The average of 7 numbers is 5. If the average of first six of these numbers is 4, the seventh number is:
(a) 14 (b) 12
(c) 11 (d) 15

12. Three years ago the average age of a family of 5 members was 27 years. On addition of a child to the family, the present average age of the family is still 27 years. Find out the present age of the child.
(a) 16 years (b) 12 years
(c) 24 years (d) 20 years

13. The average weight of 10 students is increased by half a Kg when one of the students weighing 50 Kg is replaced by a new student. Find out the weight of the new student.
(a) 55 Kg (b) 60 Kg
(c) 45 Kg (d) 40 Kg

14. The average monthly salary of a staff of 9 persons is ₹2450. One member of the staff whose monthly salary is ₹2650 is transferred. Find out the average salary of the remaining 8 persons of the staff.
(a) ₹2425 (b) ₹2625
(c) ₹3025 (d) ₹2825

15. The mean marks of 10 boys in a class is 70%, whereas the mean marks of 15 girls is 60%. The mean marks of all the 25 students is:
(a) 64% (b) 60%
(c) 55% (d) 52%

1. **(c)** Total earning for 7 days

= ₹(60 + 65 + 70 + 52.50 + 63 + 73 + 68)

= ₹451.50

Average daily earning = ₹ $\frac{451.50}{7}$ = ₹64.50.

Alternative Solution

Let base value = 60

Net average change $= \frac{0+5-7.5+3+13+8}{7}$

= 4.5

$\therefore$ Average = 60 + 4.5 = 64.5

2. **(c)** The average of 10 numbers = 7

Total of 10 numbers = 10 × 7 = 70

New total of 10 numbers after each of given numbers is multiplied by 8 = 70 × 8 = 560

$\therefore$ New average = $\frac{560}{10}$ = 56.

Alternative Solution

If each number is multiplied by 8 then average is also multiplied by 8.

New average = 7 × 8 = 56

3. **(b)** Average weight of 5 persons = 38 Kg

$\therefore$ Total weight of these five persons

= 38 × 5 = 190 Kg

Now, average weight of (the boat + 5 persons)

= 52 Kg

$\therefore$ Total weight of (the boat + 5 persons)

= 52 × 6 = 312 Kg

$\therefore$ Weight of the boat = 312 − 190 = 122 Kg.

4. **(d)** Let, the original expenditure = ₹x

Original average expenditure = $\frac{x}{35}$

New average expenditure = $\frac{x+42}{42}$

$\Rightarrow \frac{x}{35} - \frac{x+42}{42} = 1 \Rightarrow x = 420$

$\therefore$ Original expenditure = ₹420.

5. **(c)** Average daily maximum temperature

$= \frac{42.7+44.6+42.0+39.1+43.0+42.5+38.5}{7}$

$= \frac{292.4}{7} = 41.77°C.$

6. **(d)** Let, the total number of workers be x.

$\Rightarrow 850 \times x = 7 \times 1000 + (x - 7) \times 780 \Rightarrow x = 22.$

7. **(a)** The total time taken can be calculated as shown below:

Distance	*Speed*	*Time*
2500 Km	500 Km/h	5 hrs
1200 Km	400 Km/h	3 hrs
500 Km	250 Km/h	2 hrs
Total 4200 Km		10 hrs

Average speed = $\frac{4200}{10}$ = 420 Km/h

8. **(a)** Marks scored by 2 students = 100 × 2 = 200

Marks scored by 3 students = 3 × 0 = 0

Marks scored by 15 students = 15 × 40 = 600

$\therefore$ Marks scored by 20 students

= 200 + 0 + 600 = 800

$\therefore$ Average marks = $\frac{800}{20}$ = 40.

9. **(b)** Average weight of 24 students of section A = 58 Kg

Total weight of 24 students of section A = 58 × 24 = 1392 Kg

Average weight of 26 students of section B = 60.5 Kg

Total weight of 26 students of section B = 60.5 × 26 = 1573 Kg

Total weight of 50 students = (1392 + 1573) Kg

= 2965 Kg

Average weight of the students in the class

$= \frac{2965}{50}$ = 59.3 Kg.

10. **(c)** Total age of 5 members = 21 × 5 = 105 years.

Total age of 4 members at the birth of the youngest member, that is, 5 years ago

= 105 − (5 × 5) = 80 years

Before the birth of the youngest member, the family consisted of only 4 members.

Average age of 4 members 5 years ago

$= \frac{80}{4}$ = 20 years.

11. **(c)** Sum of seven numbers = 7 × 5 = 35

Sum of first six numbers = 6 × 4 = 24

Therefore, the seventh number = 35 − 24 = 11.

12. **(b)** Present average age of 5 members

= 27 + 3 = 30 years

Sum of present age of 5 members

= 30 × 5 = 150 years

Let, the present age of the child be x years.

Present average age of 6 members

$= \frac{150+x}{6}$ and this is equal to 27 years.

So, $\frac{150+x}{6} = 27$

or, $x = 27 \times 6 - 150$ or, $x = 12$ years.

Alternative Solution

Old average = 27

No. of years added = 3 × 5 = 15 years.

Age of Child = 27 − 15 =12 years

13. **(a)** Weight of the new student

= 50 + 10 × 0.5 = 55 Kg.

14. **(a)** Average salary of 9 persons = ₹2450

Total salary of 9 persons

= ₹2450 × 9 = ₹22050

Total salary of the person who is transferred

= ₹2650

Thus, the total salary of remaining 8 persons

= ₹22050 − ₹2650 = ₹19400

The average salary of the remaining 8 persons

= ₹$\frac{19400}{8}$ = ₹2425.

15. **(a)** The mean marks of 10 boys = 70%

Total marks of 10 boys = 70% × 10 = 700%

The mean marks of 15 girls = 60%

Total marks of 15 girls = 60% × 15 = 900%

$\therefore$ Sum of the total marks of 25 students

= 700 + 900 = 1600%

$\therefore$ The mean marks of all the 25 students

$= \frac{1600}{25}$ = 64%

16. **(d)** Income for 6th day in rupees

= 15 × 70 − 5 × 60 − 9 × 80 = 30.