

# Linear Kalman Filter Summary

## System Model:

$$\mathbf{x}_k = \mathbf{F}_{k-1} \mathbf{x}_{k-1} + \mathbf{G}_{k-1} \mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

## Assumptions:

$$\left. \begin{array}{l} \mathbf{w}_k \sim N(0, \mathbf{Q}_k) \\ \mathbf{v}_k \sim N(0, \mathbf{R}_k) \end{array} \right\} \begin{array}{l} E(\mathbf{w}_k \mathbf{w}_j^T) = \mathbf{Q}_k \delta_{k-j} \\ E(\mathbf{v}_k \mathbf{v}_j^T) = \mathbf{R}_k \delta_{k-j} \end{array} \quad \begin{array}{l} \text{Not correlated} \\ \text{with time} \end{array}$$

Gaussian Distribution with zero mean and given covariance matrix

$$E(\mathbf{w}_k \mathbf{v}_j^T) = 0 \quad \begin{array}{l} \text{Process noise and} \\ \text{Measurement noise are} \\ \text{independent} \end{array}$$

## Predict:

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1}^+ + \mathbf{G}_{k-1} \mathbf{u}_{k-1}$$

$$\mathbf{P}_k^- = \mathbf{F}_{k-1} \mathbf{P}_{k-1}^+ \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

## Update:

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-$$

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_k^- \mathbf{H}_k^T + \mathbf{R}_k$$

$$\mathbf{K}_k = \mathbf{P}_k^- \mathbf{H}_k^T \mathbf{S}_k^{-1}$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k \tilde{\mathbf{y}}_k$$

$$\mathbf{P}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^-$$