Introduction to noise :-

Noise is defined as any undesirable electrical energy that falls with in the passband of message signal. This gives rise to an audible noise in a system. The presence of noise degrades the performance of communication systems. In this chapter we analyze the noise in continuous wave modulation system. For such analysis we first define the receiver model. Then we analyze the noise in AM receivers namely DSBSC, SSB. Finally, we discuss the noise in FM receivers.

Receiver model :-

The Fig. shows the receiver model in its most basic form. Modulated signal is s(t) and noise is w(t). Signal w(t) is known as front end receiver noise. The receiver input signal is the sum of s(t) and w(t). The output of band-pass filter is x(t). The band-width of a bandpass filter is kept just wide enough to pass the modulated signal s(t) without distortion.

The demodulation process represented by the block demodulator depends on the modulation used.

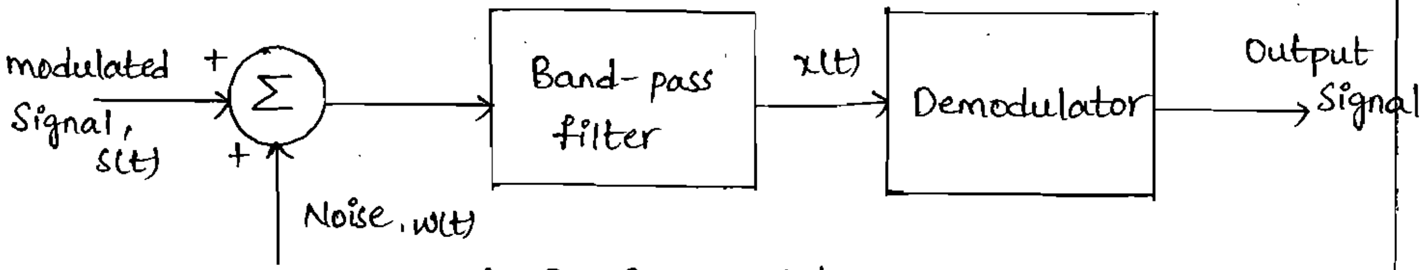


Fig. Receiver model.

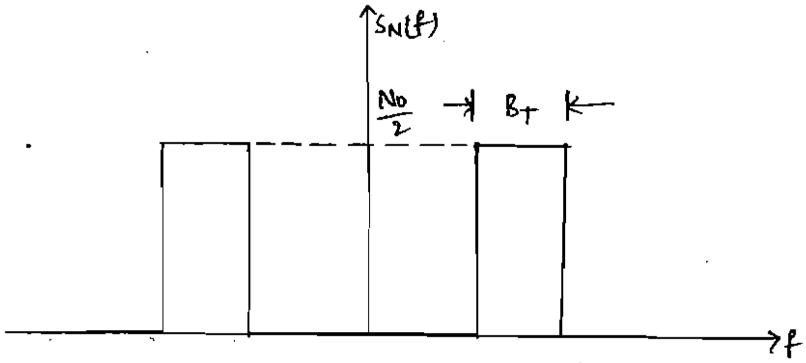


Fig. Idealized characteristic of bandpass filtered noise.

For receiver model, we may denote and define the following things.

* we denote $N_0/2$ as the power spectral density of noise w(t) for both +ve and -ve frequencies.
* 'N_0' is the average noise power per unit bandwidth.
* Bandwidth of bandpass filter is equal to transmission bandwidth of the modulated signal and is denoted as 'B_T'.
* Midband frequency is equal to the carrier frequency and it is denoted as f_c.

* Typically, the carrier frequency $f_c >> B_T$ and therefore we may consider the filtered noise $n(t)$ as narrow band noise and it is defined in canonical form as

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t.$$

* The filtered signal available for demodulation is defined by $x(t) = s(t) + n(t)$

* The average noise power is given as,

avg. noise power = avg. noise power per unit bandwidth X Bandwidth

$= N_0 B_T$

* Input signal to noise ratio is given by

$$(SNR)_I = \frac{\text{Average power of the modulated signal, } s(t)}{\text{Average power of filtered noise, } n(t).}$$

* Output signal to noise ratio is given by

$$(SNR)_0 = \frac{\text{Average power of the demodulated msg signal}}{\text{Average power of the noise.}}$$

* Channel Signal to noise ratio is given by

$$(SNR)_c = \frac{\text{Average power of the modulated signal}}{\text{Avg. power of noise in message bandwidth}}$$

* Figure of merit $= \dfrac{(SNR)_0}{(SNR)_c}$

* Higher the value of figure of merit, better the performance of the receiver.
* The value of figure of merit also depends upon the type of modulation used.

Noise in DSBSC Receivers :-

The below figure shows the model of a DSBSC receiver using a coherent detector. As shown in the figure, the filtered signal is applied to coherent detector $x(t)$. It is multiplied with a locally generated sinusoidal wave $\cos 2\pi f_c t$ using product modulator. The product is then filtered using low pass filter.

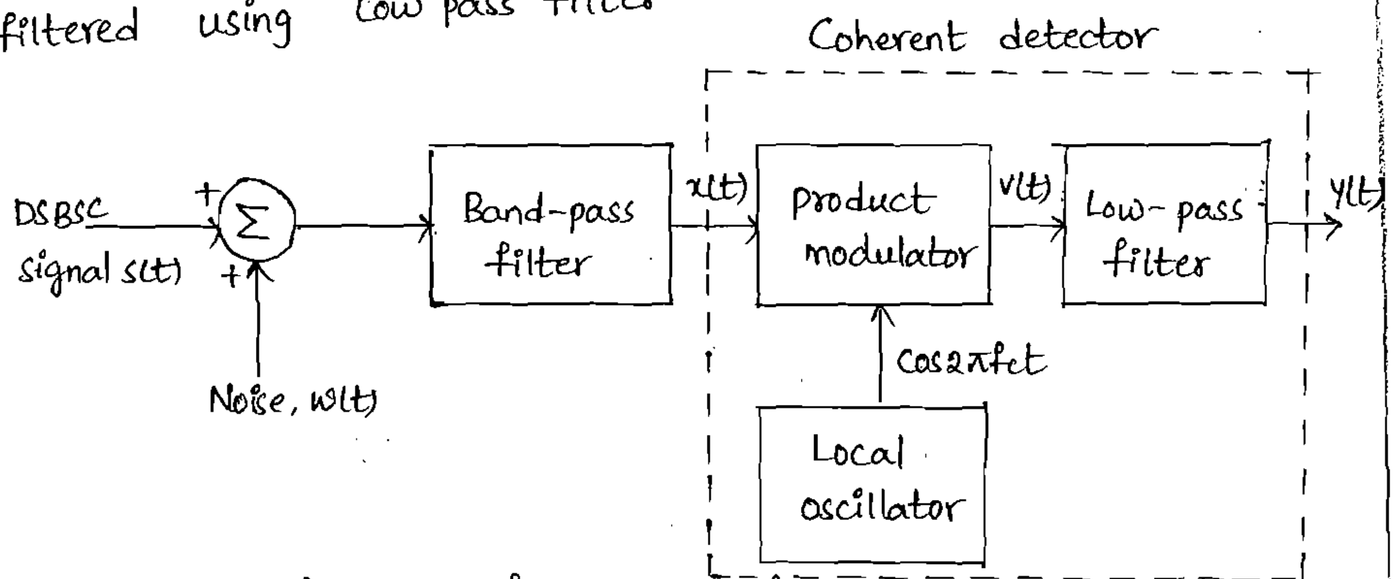


Fig. Model of DSBSC receiver

The time-domain expression of DSB-SC wave is given as,

$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\, A_c \cos 2\pi f_c t$$

Channel Signal-to-noise ratio :-

It is given as,

$$(SNR)_c = \frac{\text{Avg. power of modulated signal } s(t)}{\text{Avg. power of noise in msg bandwidth}}$$

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of modulated signal $= \overline{s^2(t)}$

$$= \overline{[m(t) A_c \cos 2\pi f_c t]^2}$$

$$= \overline{m^2(t)}\ \overline{[A_c \cos 2\pi f_c t]^2}$$

$$= A_c^2 \cdot P \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2 P}{2}$$

where, 'P' represents average power of m(t).

Average power of noise in message bandwidth is given as $\overline{n_w^2(t)}$ = avg. noise power per unit bandwidth x bandwidth

$$= \frac{N_0}{2} \cdot 2W$$

$$= N_0 W$$

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}} = \frac{A_c^2 P}{2 N_0 W}$$

Output Signal-to-noise ratio :-

It is given as,

$$(SNR)_0 = \frac{\text{Average power of demodulated msg signal}}{\text{Average power of noise}}$$

$$(SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of band pass filter is

$$x(t) = s(t) + n(t)$$

$$x(t) = m(t) A_c \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

This $x(t)$ is passed through product modulator. Another input to product modulator is $\cos 2\pi f_c t$. The output of product modulator is

$$v(t) = x(t) \cos 2\pi f_c t$$

$$v(t) = m(t) A_c \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$v(t) = m(t) A_c \left(\frac{1+\cos 4\pi f_c t}{2}\right) + n_I(t)\left(\frac{1+\cos 4\pi f_c t}{2}\right) - \frac{n_Q(t)}{2} \sin 4\pi f_c t$$

$$v(t) = \frac{m(t)A_c}{2} + \frac{n_I(t)}{2} + \left(\frac{m(t)A_c}{2} + \frac{n_I(t)}{2}\right)\cos 4\pi f_c t - \frac{n_Q(t)}{2} \sin 4\pi f_c t.$$

This $v(t)$ is passed through Low-pass filter.

The output of LPF is

$$y(t) = \frac{m(t) A_c}{2} + \frac{n_I(t)}{2}$$

$$= m_d(t) + n_d(t)$$

where,

$m_d(t) = \frac{m(t) A_c}{2}$ is the desired signal component

$n_d(t) = \frac{n_I(t)}{2}$ is the noise component.

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[\frac{m(t) A_c}{2}\right]^2}$$

$$= \frac{A_c^2}{4} \overline{m^2(t)} = \frac{A_c^2 P}{4}$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

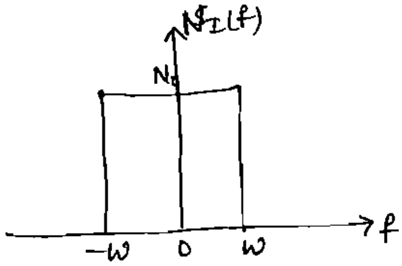
$$= \overline{\left[\frac{n_I(t)}{2}\right]^2}$$

$$= \frac{1}{4} \overline{n_I^2(t)}$$

$$= \frac{1}{4} \times \text{Area under PSD curve}$$

$$= \frac{1}{4} \times N_0 \times 2W$$

$$= \frac{N_0 W}{2}$$



$$(SNR)_o = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}} = \frac{A_c^2 P/4}{N_0 W/2} = \frac{A_c^2 P}{2 N_0 W}$$

Figure of merit (γ) is given as,

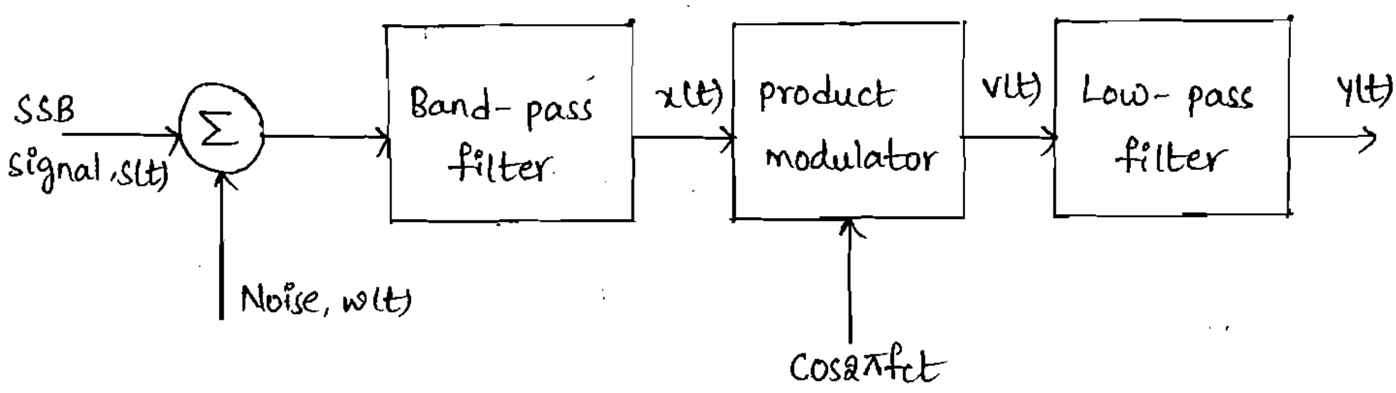
$$\gamma = \frac{(SNR)_o}{(SNR)_c}$$

$$\gamma = \frac{\frac{A_c^2 P}{2N_o W}}{\frac{A_c^2 P}{2N_o W}} = 1$$

Thus, the figure of merit of DSB-SC system is 1.

Noise in an SSB-SC System :-

The block-diagram of the SSB-SC system is just as like DSB-SC system except for the fact that bandwidth of BPF of SSB-SC receiver is exactly half of that required for DSB-SC.



Channel Signal-to-noise ratio,

$$(SNR)_c = \frac{\text{Avg. power of modulated signal}}{\text{Avg. power of noise in msg bandwidth.}}$$

$$= \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

The expression of s(t) for an SSB is given as

$$s(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

Average power of s(t) = $\overline{s^2(t)}$

$$= \overline{\left[\frac{A_c m(t)}{2} \cos 2\pi f_c t\right]^2} + \overline{\left[\frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t\right]^2}$$

$$\overline{s^2(t)} = \frac{A_c^2}{4} \overline{m^2(t)} \, \overline{\cos^2 2\pi f_c t} + \frac{A_c^2}{4} \overline{\hat{m}^2(t)} \, \overline{\sin^2 2\pi f_c t}$$

$$= \frac{A_c^2 P}{4} \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{A_c^2 P}{4} \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2 P}{4}$$

Average power of $n_\omega(t) = \overline{n_\omega^2(t)}$

= Power Spectral density x bandwidth

$= N_0 W$

$$\therefore (SNR)_c = \frac{A_c^2 P}{4 N_0 W}$$

Output Signal-to-noise ratio :-

$$(SNR)_0 = \frac{\text{Avg. power of demodulated msg signal}}{\text{Average power of noise.}}$$

$$= \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of band pass filter is

$$x(t) = s(t) + n(t).$$

Here, the type of modulation used is SSB, So the center frequency of band pass filter changes to $(f_c - w/2)$

Therefore, noise signal is expressed as

$$n(t) = n_I(t)\cos 2\pi(f_c - w/2)t - n_Q(t)\sin 2\pi(f_c - w/2)t$$

The output of product modulator is

$$v(t) = x(t)\cos 2\pi f_c t$$

$$= [s(t) + n(t)]\cos 2\pi f_c t$$

$$= s(t)\cos 2\pi f_c t + n(t)\cos 2\pi f_c t$$

$$v(t) = \left[\frac{A_c}{2} m(t)\cos 2\pi f_c t + \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t +$$

$$\left[n_I(t)\cos 2\pi(f_c - w/2)t - n_Q(t)\sin 2\pi(f_c - w/2)t\right]\cos 2\pi f_c t.$$

$$v(t) = \frac{A_c m(t)}{2}\cos^2 2\pi f_c t + \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t \cos 2\pi f_c t +$$

$$n_I(t)\cos 2\pi(f_c - w/2)t\cos 2\pi f_c t - n_Q(t)\sin 2\pi(f_c - w/2)t\cos 2\pi f_c t$$

$$v(t) = \frac{A_c m(t)}{2}\left\{\frac{1 + \cos 4\pi f_c t}{2}\right\} + \frac{A_c \hat{m}(t)}{4}\sin 4\pi f_c t +$$

$$\frac{n_I(t)}{2}\left\{\cos 2\pi(2f_c - w/2)t + \cos 2\pi(w/2)t\right\} -$$

$$\frac{n_Q(t)}{2}\left\{\sin 2\pi(2f_c - w/2)t - \sin 2\pi(w/2)t\right\}$$

$$v(t) = \frac{A_c m(t)}{4} + \frac{n_I(t)}{2}\cos\pi wt + \frac{n_Q(t)}{2}\sin\pi wt + \frac{A_c m(t)}{4}\cos 4\pi f_c t + \frac{A_c \hat{m}(t)}{4}\sin 4\pi f_c t + \frac{n_I(t)}{2}\cos 2\pi(2f_c - w/2)t - \frac{n_Q(t)}{2}\sin 2\pi(2f_c - w/2$$

This v(t) is passed through LPF, In LPF high frequencies are attenuated and only low frequencies are allowed. So, the output of LPF is

$$y(t) = \frac{A_c m(t)}{4} + \frac{n_I(t)}{2}\cos\pi wt + \frac{n_Q(t)}{2}\sin\pi wt$$

$$= m_d(t) + n_d(t)$$

where,

$$m_d(t) = \frac{A_c m(t)}{4}$$

$$n_d(t) = \frac{n_I(t)}{2}\cos\pi wt + \frac{n_Q(t)}{2}\sin\pi wt$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[\frac{A_c m(t)}{4}\right]^2}$$

$$= \frac{A_c^2}{16}\overline{m^2(t)} = \frac{A_c^2 P}{16}$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

$$= \overline{\left[\frac{n_I(t)}{2}\cos\pi wt\right]^2} + \overline{\left[\frac{n_Q(t)}{2}\sin\pi wt\right]^2}$$

$$\overline{n_d^2(t)} = \frac{\overline{n_I^2(t)}}{4}\ \overline{\cos^2 \pi w t} + \frac{\overline{n_Q^2(t)}}{4}\ \overline{\sin^2 \pi w t}$$

$$\overline{n_d^2(t)} = \frac{N_0 W}{4}\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{N_0 W}{4}\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{N_0 W}{4}$$

$$\therefore (SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}} = \frac{A_c^2 P/16}{N_0 w/4} = \frac{A_c^2 P}{4 N_0 w}$$

Figure of merit (r) is given as

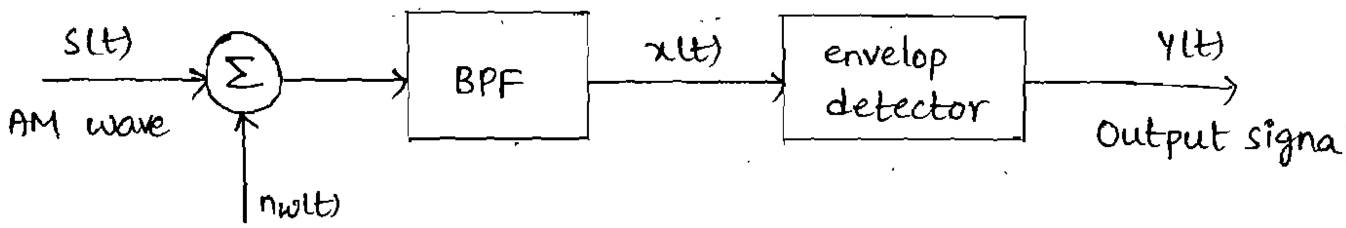
$$\text{Figure of merit} = \frac{(SNR)_0}{(SNR)_c}$$

$$= \frac{\frac{A_c^2 P}{4 N_0 w}}{\frac{A_c^2 P}{4 N_0 w}} = 1$$

Thus, the figure of merit of SSB-SC system is 1.

Noise in AM system :-

The block diagram of AM receiver is as shown.



The time-domain expression for AM wave is given by

$$s(t) = A_c[1+k_a m(t)] \cos 2\pi f_c t$$

Channel Signal-to-noise ratio :-

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of $s(t) = \overline{s^2(t)}$

$$= \left[\overline{A_c \cos 2\pi f_c t}\right]^2 + \left[\overline{A_c k_a m(t) \cos 2\pi f_c t}\right]^2$$

$$= A_c^2 \left(\frac{1}{\sqrt{2}}\right)^2 + A_c^2 k_a^2 P \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2}{2}\left[1 + k_a^2 P\right]$$

Average power of $n_w(t) = \overline{n_w^2(t)}$

$$= N_0 W$$

$$(SNR)_c = \frac{A_c^2 (1+k_a^2 P)}{2 N_0 W}$$

Output Signal-to-Noise ratio -

$$(SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of Band-pass filter is $x(t)$

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c\left[1+k_a m(t)\right]\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t.$$

$$x(t) = \left[A_c + A_c k_a m(t) + n_I(t)\right]\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

$$e(t) = \sqrt{(\text{In phase comp.})^2 + (\text{Quadrature comp.})^2}$$

$$e(t) = \sqrt{\left[A_c + A_c k_a m(t) + n_I(t)\right]^2 + (n_Q(t))^2}$$

In this case,

$$A_c(1+k_a m(t)) >> n(t)$$

Thus,

$$A_c(1+k_a m(t)) >> n_I(t) \text{ (or) } n_Q(t)$$

$$\therefore e(t) = A_c(1+k_a m(t)) + n_I(t)$$

$$= A_c + A_c k_a m(t) + n_I(t)$$

The output of envelop detector is $y(t)$

$$y(t) = A_c k_a m(t) + n_I(t)$$

$$= m_d(t) + n_d(t)$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \left(\overline{A_c k_a m(t)}\right)^2$$

$$= A_c^2 k_a^2 \cdot P$$

Average power of noise is given as $\overline{n_d^2(t)}$

$$= \overline{n_I^2(t)}$$

$$= N_0(2W)$$

$$= 2N_0W$$

$$(SNR)_0 = \frac{A_c^2 k_a^2 P}{2N_0W}$$

Figure of merit (γ) is given as

$$\gamma = \frac{(SNR)_0}{(SNR)_c}$$

$$\gamma = \frac{\frac{\cancel{A_c^2} k_a^2 P}{2\cancel{N_0W}}}{\frac{\cancel{A_c^2}(1+k_a^2P)}{2\cancel{N_0W}}} = \frac{k_a^2 P}{1+k_a^2 P}$$

we know that 'p' is the average power of the message signal and it is given as

$$P = \frac{1}{2}A_m^2$$

$$\gamma = \frac{\frac{k_a^2 A_m^2}{2}}{1+\frac{k_a^2 A_m^2}{2}} = \frac{k_a^2 A_m^2}{2+k_a^2 A_m^2}$$

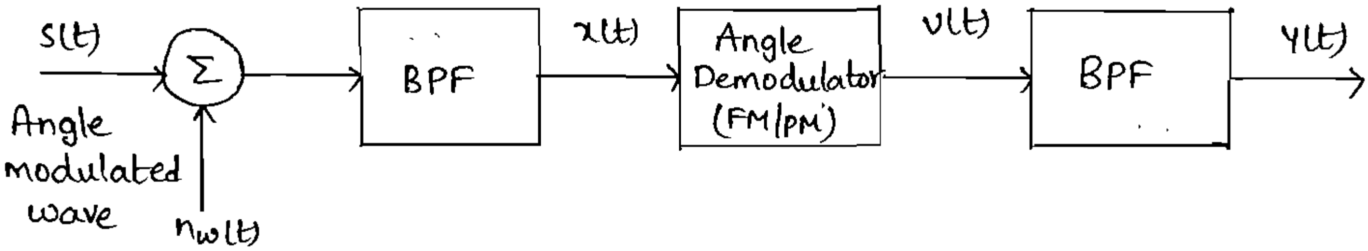
$$\gamma = \frac{\mu^2}{2+\mu^2} \quad [\because \mu = k_a A_m]$$

For 100% modulation i.e $\mu = 1$ we get

$$\gamma = \frac{1}{2+1} = \frac{1}{3}.$$

Noise in Angle modulated System :-

The block diagram of an angle modulated system is as shown,



The time-domain expression for an angle modulated carrier is given by,

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

where $\phi(t)$ represents the instantaneous phase angle and is given as,

$$\phi(t) = k_p m(t) \quad \text{[For phase modulation]}$$

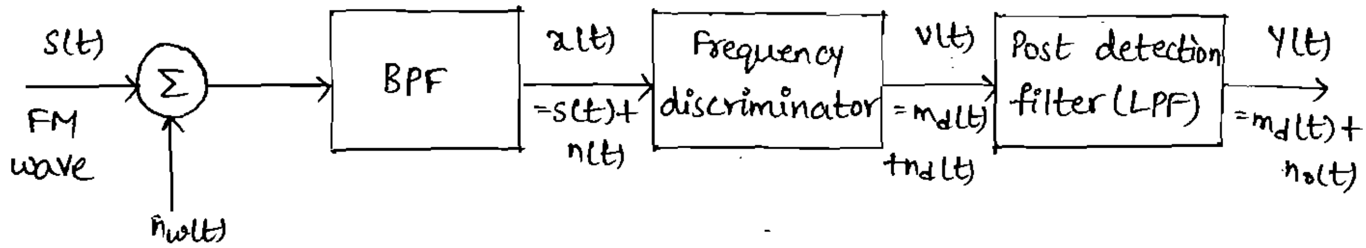
$$\phi(t) = 2\pi k_f \int_0^t m(t)\, dt \quad \text{[For frequency modulation]}$$

Here k_p and k_f represents the sensitivities of phase & frequency respectively. The transmission bandwidth B_T in angle modulated system determined by Carson's rule is

$$B_T = 2(\Delta f + W)$$

where W represents the Bandwidth of msg signal and Δf is the peak frequency deviation.

Noise in FM Receivers :-



The time domain expression for frequency modulated carrier is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

The noise can be expressed in terms of inphase and quadrature components as,

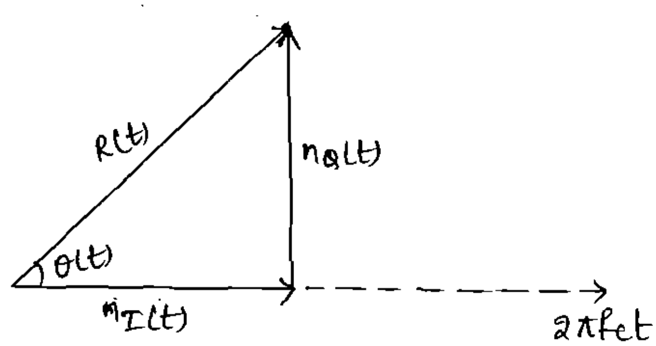
$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$= R(t) \cos(2\pi f_c t + \theta(t))$$

where, $R(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

$$\theta(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

The phasor diagram of n(t) drawn by taking reference as phase of unmodulated carrier is as shown.



Channel Signal-to-noise ratio.

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of $s(t) = \overline{s^2(t)}$

$$= \left[\overline{A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t) dt\right]}\right]^2$$

$$= A_c^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2}{2}$$

Average power of noise $n_w(t) = \overline{n_w^2(t)}$

$$= N_0 W$$

Thus, $(SNR)_c = \dfrac{A_c^2}{2N_0 W}$

Output Signal-to-noise ratio -

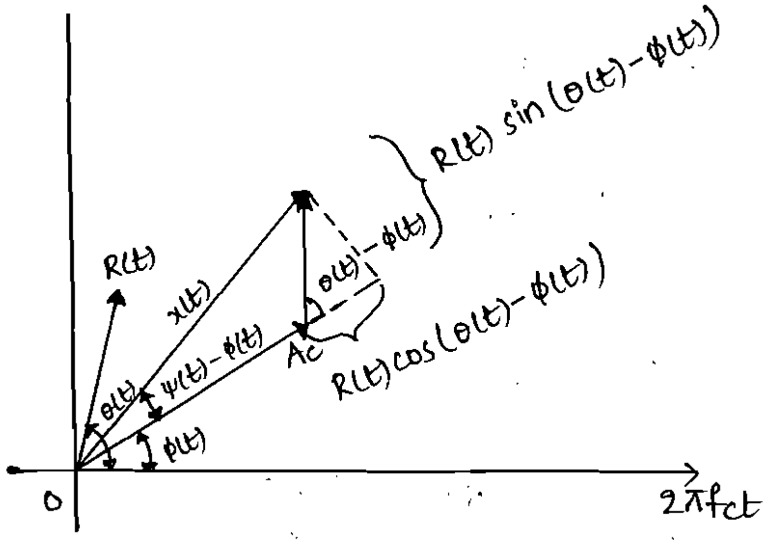
$$(SNR)_o = \frac{\text{Average power of demodulated signal}}{\text{Average power of the noise}}$$

The output of band-pass filter is

$$x(t) = s(t) + n(t)$$

$$= A_c \cos(2\pi f_c t + \phi(t)) + R(t) \cos(2\pi f_c t + \theta(t))$$

where, $\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$



The relative phase $\psi(t)$ of the resulting $x(t)$ can be obtained from the figure as,

$$\psi(t) - \phi(t) = \tan^{-1}\left\{ \frac{R(t)\sin(\theta(t)-\phi(t))}{A_c + R(t)\cos(\theta(t)-\phi(t))} \right\}$$

While drawing phasor diagram, it is assumed that $\theta(t) > \phi(t)$ and $A_c > R(t)$.

We can write,

$$\tan(\psi(t)-\phi(t)) \approx \frac{R(t)\sin\theta(t)}{A_c}$$

Referring the phasor diagram of narrow band noise, we get

$$R(t)\sin\theta(t) = n_Q(t)$$

Hence,

$$\tan(\psi(t)-\phi(t)) = \frac{n_Q(t)}{A_c}$$

Since $A_c >> n_Q(t)$ we can write

$$\psi(t) - \phi(t) \approx \frac{n_Q(t)}{A_c}$$

$$\psi(t) = \phi(t) + \frac{n_Q(t)}{A_c}$$

The output of frequency discriminator is

$$v(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t) + \frac{n_Q(t)}{A_c}\right]$$

$$v(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f\int_0^t m(t)\,dt\right] + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= k_f m(t) + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= m_d(t) + n_d(t)$$

where,

$$m_d(t) = k_f m(t)$$

$$n_d(t) = \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= k_f^2\,\overline{m^2(t)}$$

$$= k_f^2 \cdot P$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

By applying fourier transformation, we get

$$N_d(f) = \frac{1}{2\pi A_c}\left[j\, 2\pi f\, N_Q(f)\right]$$

$$= \frac{jf}{A_c} N_Q(f)$$

If $S_{nd}(f)$ and $S_{nQ}(f)$ be the power spectral densities of $N_d(f)$ and $N_Q(f)$ then relation is given as,

$$S_{nd}(f) = \frac{f^2}{A_c^2} S_{nQ}(f)$$

If the signal is passed through LPF, the value of $S_{nQ}(f)$ will be N_0.

$$\therefore S_{no}(f) = \frac{f^2}{A_c^2} N_0$$

The average power of noise signal in demodulated signal is given as

$$\overline{n_o^2(t)} = \int_{-w}^{w} S_{No}(f)\, df$$

$$= \int_{-w}^{w} \frac{N_0}{A_c^2} f^2\, df$$

$$= \frac{N_0}{A_c^2}\int_{-w}^{w} f^2\, df = \frac{N_0}{A_c^2}\left(\frac{f^3}{3}\right)_{-w}^{w}$$

$$= \frac{N_0}{A_c^2}\left[\frac{w^3}{3} + \frac{w^3}{3}\right] = \frac{2 N_0 w^3}{3 A_c^2}$$

$$\therefore (SNR)_o = \frac{k_f^2 P}{\frac{2 N_0 w^3}{3 A_c^2}} = \frac{3 A_c^2 k_f^2 P}{2 N_0 w^3}$$

Figure of merit $(r) = \dfrac{(SNR)_o}{(SNR)_c}$

$$r = \frac{\frac{3A_c^2 k_f^2 P}{2N_o W^3}}{\frac{A_c^2}{2N_o W}} = \frac{3k_f^2 P}{W^2}$$

$$\boxed{\therefore\ r = \frac{3k_f^2 P}{W^2}}$$

we know that p is the average power of message signal and it is given as,

$$P = A_m^2/2$$

$$r = \frac{3k_f^2}{W^2} \cdot \frac{A_m^2}{2} = \frac{3}{2W^2} \Delta f^2$$

$$r = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2$$

$$\boxed{r = \frac{3}{2}\beta^2}$$ where, $\frac{\Delta f}{W} = \beta$ (modulation index)

Now Let us compare the figure of merit of FM w.r.t AM

For 100% modulation the figure of merit of AM $= \frac{1}{3}$

The figure of merit of FM $= \frac{3}{2}\beta^2$

To have less noise in FM when compared to AM we have to take

$$\frac{3}{2}\beta^2 > \frac{1}{3}$$

$$\Rightarrow \beta > \frac{\sqrt{2}}{3}$$

$$\beta > 0.47 \approx 0.5$$

The value of $\beta = 0.47$ (or) $\beta = 0.5$ actually the transition point between the narrow-band FM and wide-band FM.

If $\beta < 0.5$, the FM is considered as narrow band FM in which there is no improvement in noise when compared to AM.

Capture Effect :-

In the frequency modulation, the signal can be affected by another frequency modulated signal whose frequency content is close to carrier frequency of the desired FM wave. The receiver may lock such an interference signal and suppress the desired FM signal when interference signal is stronger than desired signal.

When the strength of interference and desired signal are nearly equal, the receiver fluctuates back and forth between them i.e receiver locks interference signal for some time and desired signal for some time and this goes randomly. This phenomenon is Capture effect.

Threshold effect in angle modulation System:-

The threshold effect in FM is much more pronounced than in AM. The figure of merit of FM is valid if the carrier-to-noise is high compared to unity (i.e, CNR>>1).

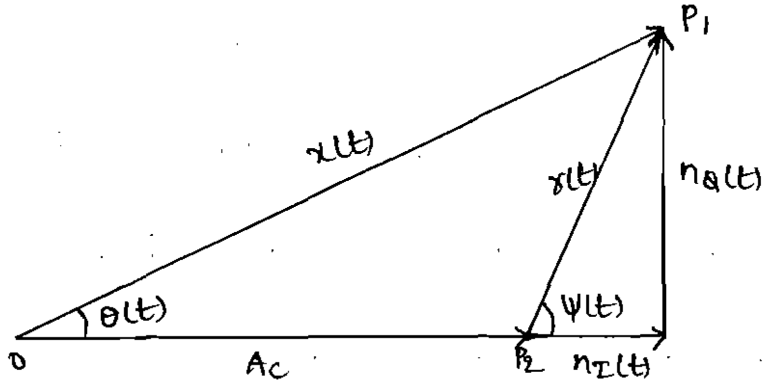
Suppose if the input noise power is increased or the carrier power is decreased, the CNR decreases consequently demodulator discriminator output becomes more and more corrupted by noise. Spikes comes out FM receiver and if CNR further decreases, continuous spikes comes out of FM receiver. The FM receiver is said to breakdown when clicks are heard. This phenomenon is called as threshold effect.

The threshold effect is defined as the minimum carrier to noise ratio that gives the output signal to noise ratio not less than the value predicted by the usual signal to noise formula assuming a small noise power.

At the frequency discriminator input is given by

$$x(t) = [A_c + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

where $n_I(t)$ and $n_Q(t)$ are in-phase and quadrature components of narrow band noise signal $n(t)$ w.r.t carrier respectively. The relationships defined by this equation are represented by phasor diagram as shown.



Let us derive the conditions for positive clicks to occur and negative clicks to occur are as follows:

<u>Conditions for positive clicks:</u>

$$r(t) > A_c$$

$$\psi(t) < \pi < \psi(t) + d\psi(t)$$

$$\frac{d\psi(t)}{dt} > 0$$

<u>conditions for negative clicks:</u>

$$r(t) > A_c$$

$$\psi(t) > -\pi > \psi(t) + d\psi(t)$$

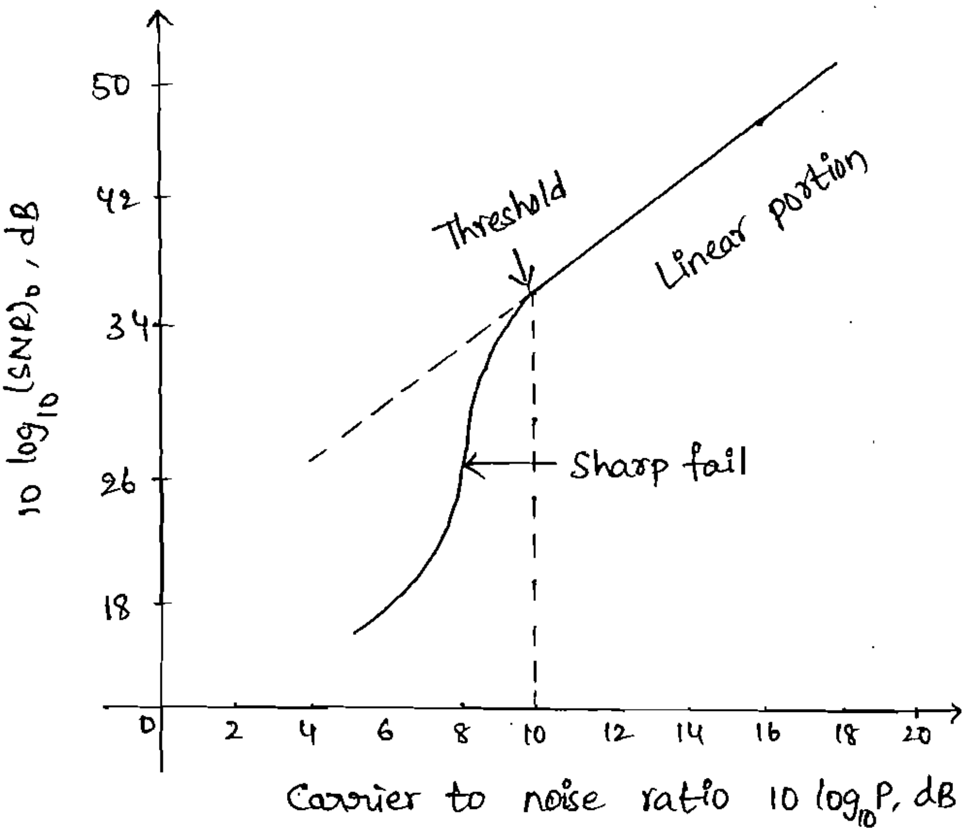
$$\frac{d\psi(t)}{dt} < 0$$

The conditions for positive clicks ensure that $\theta(t)$ changes by 2π radians and for negative clicks ensure that $\theta(t)$ changes by -2π radians during the time increment dt.

Carrier-to-noise ratio is defined by

$$P = \frac{A_c^2}{2 B_T N_0}$$

The average no. of clicks per unit time is inversely proportional to P. It is seen that $(SNR)_0$ ratio is a linear function of P when P is greater than 10 dB. However, it falls sharply for lower values of P than 10 dB. This is shown below.



Threshold can be avoided by keeping $P > 20$ i.e 13 dB

$$\frac{A_c^2}{2 B_T N_0} \geq 20 \quad \Rightarrow \quad \frac{A_c^2}{2} \geq 20 B_T N_0$$

$$\gamma = \frac{3k_f^2 P}{W^2}$$

As $m(t) = A_m \cos[2\pi f_m t]$

$$\Rightarrow P = \overline{m^2(t)} = A_m^2/2$$

$$\Rightarrow \gamma = \frac{3k_f^2 A_m^2}{2W^2}$$

The expression for frequency deviation is given by

$$\Delta f = |k_f\ m(t)|_{max} = |k_f A_m \cos(2\pi f_m t)|_{max}$$

$$\Rightarrow \cancel{\Delta f = k_f m(t)} \Rightarrow \Delta f = k_f A_m$$

$$\therefore \gamma = \frac{3}{2}\left[\frac{\Delta f}{W}\right]^2 \Rightarrow \boxed{\gamma = 1.5\beta^2}$$

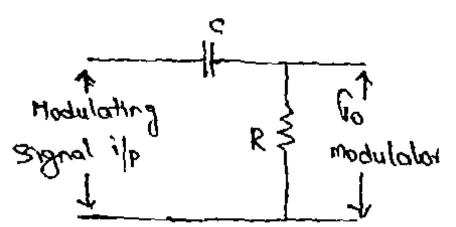
Where β is modulation index of FM given by $\beta = \frac{\Delta f}{W}$

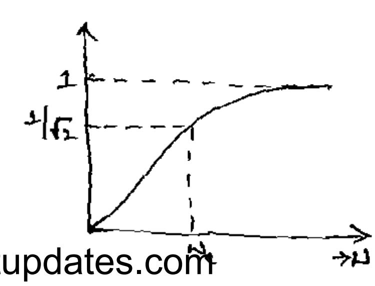
PRE-EMPHASIS AND DE-EMPHASIS

Noise produced in electronic circuits is low in low AF range, but at higher frequencies it increases. So, for information signals with a uniform signal level, a non-uniform signal-to-noise ratio is obtained. The higher modulating signal frequencies have a lower signal-to-noise ratio than the lower frequencies. To overcome this a high frequency modulating signals are emphasized (or) boosted in amplitude in the FM transmitter before modulation. This is known as pre-emphasis. The de-emphasis circuit restores the original amplitude - frequency characteristics of the information signal. Pre-emphasis and De-emphasis give a more (or) less uniform signal-to-noise ratio over the whole AF range.

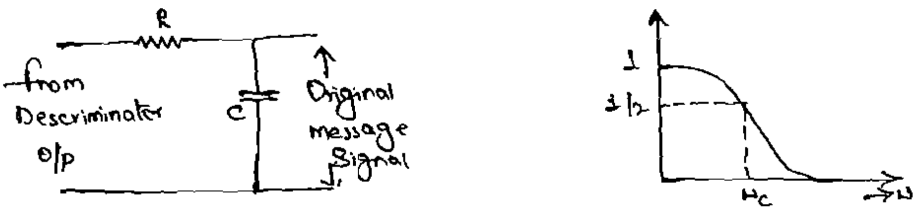
A Pre-emphasis circuit is a high pass filter i.e. ~~an~~ RC ~~circuit~~ ~~with a~~ high frequency components are boosted up at the output, because the capacitor C offers a low reactance at high frequencies.

The circuit and frequency response curve of the Pre-emphasis are shown as





A de-emphasis circuit is low-pass filter, i.e high frequency components are attenuated because the capacitor short circuits such components. The circuit and frequency response curve of de-emphasis are shown as



UNIT-5
NOISE

Sources of noise:-
→ Natural
→ Manmade
→ fundamental

Classification

→ Shot noise:- It is produced due to shot effect. It produced in all the amplifying devices rather than in all the active devices.

* Shot noise is produced because of the random variations in the arrival of electrons & holes at the o/p electrode of an amplifying device. It sounds like a shower of lead shots falling on a metal sheet. The mean square shot noise current equation for diode is given as $I_n^2 = 2(I + 2I_0) q B$ (Amp²)

$I \rightarrow$ direct current across the junction (Amps)
$I_0 \rightarrow$ reverse saturation current (Amps)
$q \rightarrow$ electronic charge $= 1.6 \times 10^{-19}$ Columbs
$B \rightarrow$ effective noise Bandwidth (Hz)

For the amplifying devices the shot noise is inversely proportional to the transconductance of the device & directly proportional to the direct current.

→ Partition Noise:- It is generated when the current gets divided b/w two or more paths it is generated due to the random fluctuations in the division. Therefore the partition noise in a transistor will be higher than that in a diode.

→ Low frequency / flicker Noise:- It will appear at frequencies below a few KHz. It is sometimes called as $\frac{1}{f}$ noise. In the semiconductor devices flicker noise is generated due to the fluctuations in the carrier density. These fluctuations in the carrier density will cause the fluctuations in the conductivity of the material. This will produce a fluctuation voltage drop when a direct current flows through a device.

This fluctuating voltage is known as flicker noise voltage. The mean square value of the flicker noise voltage is proportional to the square of direct current flowing through the device.

→ **Thermal / Johnson / white noise**:- The free electrons within a conductor are always in random motion. This random motion is due to the thermal energy received by them. The distribution of this free electrons with in a conductor at a given instant of time is not uniform. It is possible that an excess ~~the~~ no of electrons may appear at one end or the other of the conductor. The average voltage resulting from this non-uniform distribution is zero, but the avg power is not zero. At this power results from the thermal energy. It is called as the thermal noise power.

Avg thermal noise power is given by $P_n = KTB$ watts

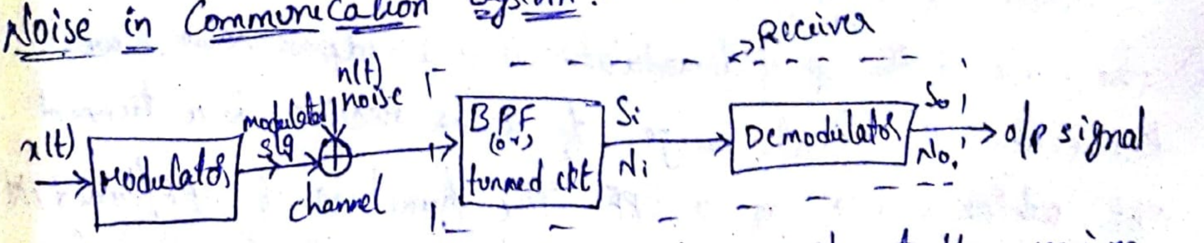
K – Boltzmann constant 1.38×10^{-23} J/K

B – B.W of noise spectrum (Hz)

T – temp of conductor °K

→ **High frequency / Transit time noise**:- If the time taken by an electron to travel from the emitter to the collector of a transistor becomes comparable to the period of the s/g which is being amplified then the transit time effect takes places. This effect is obsved at very high frequencies. Due to transit time effect some of the carriers diffused back to the emitter. This gives rise to a i/p admittance, the conductance component of which increases with frequency. The minute currents & induced in input of device by the random fluctuations in o/p current, will create ~~at~~ random noise at high frequencies.

Noise in Communication System :-



The message s/g travels from the transmitter to the receiver through a medium called channel. Noise is present in every communication system. The channel introduces a negative additive noise in the message s/g, and thus the msg which is received at the receiver is distorted.

Since the receiver detects both msg & noise signals. It will reproduce a msg s/g which contains noise. A noise calculation in a communication s/m is carried out ~~by~~ in the form of a parameter called figure of merit. It is noted by letter (γ).

figure of merit is defined as the ratio of o/p SNR to i/p SNR of a receiver.

$$\gamma = \frac{(SNR)_o}{(SNR)_i}$$

→ few assumptions to calculate the figure of merit for various communications systems.

1) Channel noise is always white & Gaussian :- we assume that the noise of channel n(t) is always a white noise. This means that it is uniformly distributed over the entire band of frequencies hence the PSD of channel noise will be uniform.

The total noise power may be obtained by taking the product of noise power spectrum density No/2 with the Bandwidth ~~with the bandwidth~~.

Total noise power N = white noise PSD × B.W

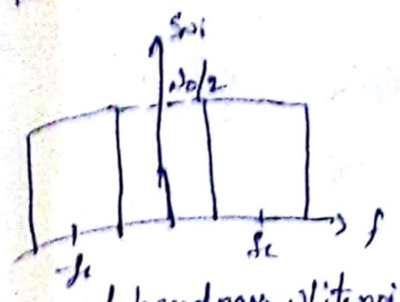
$$N = \frac{N_o}{2} \times B.W$$

Thus the noise has a Gaussian distribution.

2) Channel noise is always additive :- we assume that the disturbing effect of channel noise is always additive. This means that the effect of channel noise may be obtained by simple addition of s/g x(t) + noise n(t).

3) The noise at the i/p of demodulator is a bandpass noise :- we know that the first range of each receiver is a turned ckt which works as a BPF. The function of BPF/turned ckt is to allow only an narrow band s/g centered about $\pm f_c$ and reject all other frequencies. This means that the noise s/g lying ~~at~~ outside this range is also rejected. And thus the B.W of noise s/g at the i/p of detector as same as that of the incoming modulated s/g.

we assumed that the channel noise is white in nature so the PSD of white noise at i/p of demodulator is $S_{ni} = \frac{N_0}{2}$

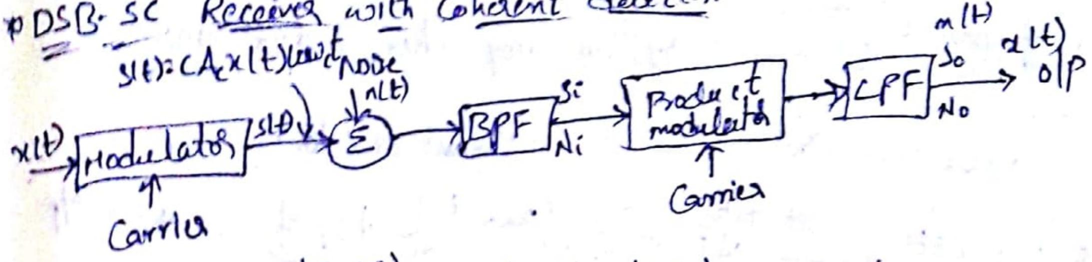


$r(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$

PSD of bandpass whitenoise

(i) The noise i/p power N_i: to calculate the figure of merit we evaluate the noise i/p power for passband / B.W of ~~i/p~~ incoming modulating signal. Total noise power $N = \frac{N_0}{2} \times 2f_m = N_0 f_m$ (for AM s/g)

DSB-SC Receiver with coherent detector:-



$$\gamma = \frac{(SNR)_o}{(SNR)_i} \qquad (SNR)_i = (SNR)_c$$

$$(SNR)_i = \frac{\text{Avg s/g power at the receiver i/p}}{\text{Avg noise power at the receiver i/p}}.$$

$$\text{Avg signal power} = \frac{c^2 A_c^2 P}{2}$$

$$\text{Avg noise power} = 2 \times \frac{N_0}{2} \times 2f_m = 2N_0 f_m$$

$$(SNR)_i = \frac{c^2 A_c^2 P}{2 \times 2N_0 f_m} = \frac{c^2 A_c^2 P}{4N_0 f_m} \quad — (1)$$

After LPF i.e., receiver o/p.

$$S_0 = (CA_c x(t)\cos\omega_c t + n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t)\cos\omega_c t$$

$$= CA_c x(t)\cos^2\omega_c t + n_I(t)\cos^2\omega_c t - n_Q(t)\sin\omega_c t\cos\omega_c t$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin 2A = 2\sin A\cos A.$$

$$= CA_c x(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) + n_I(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) - \frac{n_Q(t)}{2}\sin 2\omega_c t$$

$$S_0 = \frac{CA_c x(t)}{2} + \frac{CA_c x(t)}{2}\cos 2\omega_c t + \frac{n_I(t)}{2} + \frac{n_I(t)}{2}\cos 2\omega_c t - \frac{n_Q(t)}{2}\sin 2\omega_c t.$$

After LPF

$$m(t) = \frac{CA_c x(t)}{2} + \frac{n_I(t)}{2}$$

$m(t) = \frac{CA_c}{2} x(t) + \frac{1}{2} n_I(t)$

Avg signal power $= \left(\frac{1}{2}\right)^2 \frac{c^2A_c^2 P}{2} = \frac{c^2A_c^2P}{8}$

Noise $\left(\frac{1}{2}\right)^2 2\times\frac{N_0}{2}\times 2f_m = \frac{1}{2}N_0 f_m = \frac{N_0 f_m}{2}$

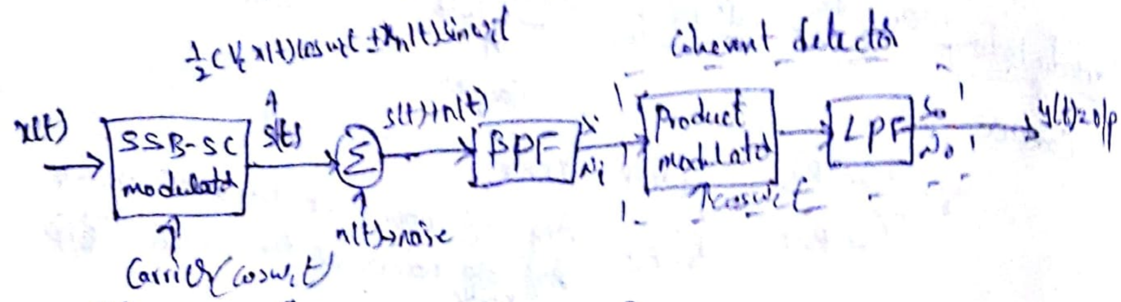
$(SNR)_0 = \frac{2c^2A_c^2P}{8 N_0 f_m} = \frac{c^2A_c^2P}{4N_0f_m}$ — ②

Sub ① & ② in γ

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\frac{c^2A_c^2P}{4N_0f_m}}{\frac{c^2A_c^2P}{4N_0f_m}} = 1$$

$\boxed{\gamma = 1}$

* figure of merit for SSBSC system using coherent detection:-



$s(t) = \frac{1}{2} cV_c \left[x(t)\cos w_c t \pm x_h(t)\sin w_c t\right]$

$S_i = \left(\frac{1}{2}cV_c\right)^2 \frac{P}{2} + \left(\frac{1}{2}cV_c\right)^2 \frac{P}{2}$

$= \frac{1}{4}c^2V_c^2\frac{P}{2} + \frac{1}{4}c^2V_c^2\frac{P}{2}$

$= \frac{c^2V_c^2P}{8} + \frac{c^2V_c^2P}{8}$

$S_i = \frac{c^2V_c^2P}{4}$

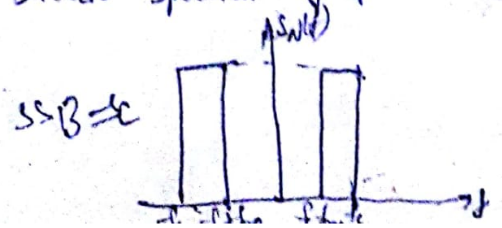
$N_i = 2\times\frac{N_0}{2}\times f_m = N_0 f_m$

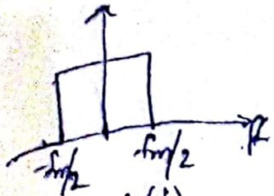
$(SNR)_i = \frac{S_i}{N_i} = \frac{c^2V_c^2P}{4N_0f_m}$ — ①

1) Shift the spectrum left $\frac{f_m}{2}$

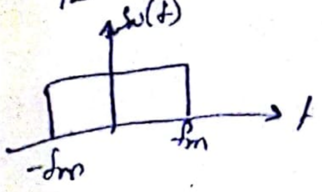
2) Shift the spectrum right $f_m/2$

3) Divide spectrum by 4





$$\frac{f_m}{2} - f_m = -\frac{f_m}{2}$$
$$-\frac{f_m}{2} + f_m = \frac{f_m}{2}$$



$$n(t) = n_I(t)\cos\left(2\pi\left(f_c - \frac{f_m}{2}\right)t\right] + n_Q(t)\sin\left(2\pi\left(f_c - \frac{f_m}{2}\right)t\right]$$

$$\underbrace{s(t)+n(t)}_{m(t)} = \left[\left[\left(\frac{1}{2}cV_c x(t)\cos\omega_c t + \frac{1}{2}cV_c x(t)\sin\omega_c t\right) + \left[n_I(t)\cos 2\pi\left(f_c - \frac{f_m}{2}\right)t\right] + n_Q(t)\sin\left[2\pi\left(f_c - \frac{f_m}{2}\right)t\right]\right]\right. \cos\omega_c t.$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \sin 2A = 2\sin A\cos B$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]; \quad \sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$m(t) = \frac{1}{2}V_c c x(t)\cos\omega_c t\cos\omega_c t + \frac{1}{2}cV_c x(t)\sin\omega_c t\cos\omega_c t + n_I(t)\cos 2\pi\left(f_c - \frac{f_m}{2}\right)t\cos\omega_c t + n_Q(t)\sin\left(2\pi f_c - \frac{f_m}{2}t\right)\cos\omega_c t.$$

$$= \frac{1}{4}V_c c x(t)\left[\cos(2\omega_c t] + \cos(0)\right] + \frac{1}{4}cV_c x(t)\left[\sin(2\omega_c t) + \sin(0)\right] + \frac{1}{2}n_I(t)\left[\cos\left(\omega_c - \frac{\omega_m}{2} + \cos\omega_c t\right) + \cos\left(\omega_c - \frac{\omega_m}{2} - \cos\omega_c t\right)\right] + \frac{1}{2}n_Q(t)\left[\sin\left(\omega_c - \frac{\omega_m}{2} + \cos\omega_c\right)t + \sin\left(\omega_c - \frac{\omega_m}{2} - \omega_c t\right)\right]$$

$$\Rightarrow \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}V_c c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\left[\cos 2\omega_c - \frac{\omega_m}{2}\right] + \frac{1}{2}n_I(t)\cos\left(\frac{\omega_m}{2}\right) + \frac{1}{2}n_Q(t)\left[\sin\left(2\omega_c - \frac{\omega_m}{2}\right)\right] + \frac{1}{2}n_Q(t)\left[\sin\frac{\omega_m}{2}\right]$$

$$= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\cos 2\omega_c t - \frac{1}{2}n_I(t)\frac{2\pi f_m}{2} + \frac{1}{2}n_I(t)\cos\left|\frac{2\pi f_m}{2}\right|t + \frac{1}{2}n_Q(t)\sin 2\omega_c t + \frac{1}{2}n_Q(t)\frac{2\pi f_m}{2} - \frac{1}{2}n_Q(t)\sin\left(\frac{2\pi f_m}{2}\right)$$

$$\cancel{= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t - \frac{1}{2}n_I(t)\pi f_m t + \frac{1}{2}n_I(t)\cos\pi f_m t + \frac{1}{2}n_Q(t)\sin 2\omega_c t - \frac{1}{2}n_Q(t)\pi f_m t - \frac{1}{2}n_Q(t)\sin\pi f_m t}$$

$$= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\cos 2\omega_c t - \frac{1}{2}n_I(t)\pi f_m t + \frac{1}{2}n_I(t)\cos\pi f_m t + \frac{1}{2}n_Q(t)\sin 2\omega_c t - \frac{1}{2}n_Q(t)\pi f_m t - \frac{1}{2}n_Q(t)\sin\pi f_m t$$

The above expression is passed through a Low pass filter. ~~After~~ The filter attenuates unwanted expression & allows wanted expression. After LPF o/p is

$$y(t) = \frac{1}{4}cV_c x(t) + \frac{1}{2}n_I(t)\cos(\pi f_m t) + \frac{1}{2}n_Q(t)\sin(\pi f_m t)$$

This is the ~~eq~~ required o/p of LPF.

$$y(t) = \frac{1}{4}C\,V_c\,x(t) + \frac{1}{2}n_I(t)\cos(\pi f_m t) + \frac{1}{2}n_Q(t)\sin(\pi f_m t)$$

S_o = Avg signal power at the o/p

$$S_o = \left(\frac{1}{4}CA_c\right)^2\frac{P}{2} = \frac{c^2}{16}V_c^2\frac{P}{2} = \frac{c^2V_c^2P}{32} \quad -②$$

$$N_o = \left(\frac{1}{2}\right)^2\frac{N_o}{4}f_m + \left(\frac{1}{2}\right)^2\frac{N_o}{4}f_m$$

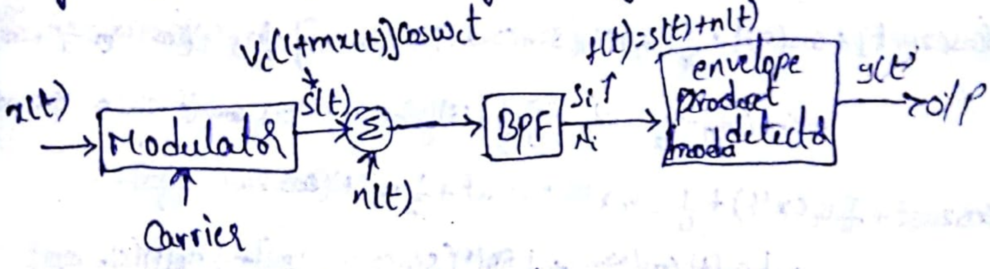
$$N_o = \frac{1}{4}\frac{N_of_m}{4} + \frac{1}{4}\frac{N_o}{4}f_m = \frac{N_of_m}{8}$$

$$(SNR)_o = \frac{S_o}{N_o} = \frac{c^2V_c^2P}{32} \times \frac{8}{N_of_m} = \frac{c^2V_c^2P}{4N_of_m} \quad -③$$

$$\gamma = \frac{(SNR)_o}{(SNR)_i} = \frac{\frac{c^2V_c^2P}{4N_of_m}}{\frac{c^2V_c^2P}{4N_of_m}} = 1$$

$$\boxed{\gamma = 1}$$

* figure of merit for AM system using ~~coherent~~ Envelope detection:-



$$s(t) = V_c[1+mx(t)]\cos\omega_c t$$

$$(SNR)_i = ? \qquad V_c\cos\omega_c t = \frac{V_c^2}{2}$$

$$V_c m x(t)\cos\omega_c t = \frac{V_c^2}{2}\frac{m^2}{V_m^2}P$$

$$S_i = \frac{V_c^2}{2} + \frac{V_c^2}{2}\frac{m^2}{V_m^2}P$$

$$S_i = \frac{V_c^2}{2}\left[1+\left(\frac{m}{V_m}\right)^2P\right]$$

$$N_i = 2 \times \frac{N_o}{2} \times 2f_m$$

$$N_i = 2N_of_m$$

$$(SNR)_i = \frac{S_i}{N_i} = \frac{\frac{V_c^2}{2}\left[1+\left(\frac{m}{V_m}\right)^2P\right]}{2N_of_m} \rightarrow ①$$

$(SNR)_o = ?$

$$f(t) = s(t) + n(t) = V_c[1+mx(t)]\cos\omega_c t + \overbrace{n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t}^{n(t)}$$

$$f(t) = \cos\omega_c t\,[V_c + V_c m x(t) + n_I(t)] - n_Q(t)\sin\omega_c t$$

i/p for envelope detector.

o/p $y(t) = \sqrt{(V_c + V_c m x(t) + n_I(t))^2 + n_Q(t)^2}$

$n_Q(t)$ is zero, because envelope detector allows only inphase components.

$y(t) \cong V_c + V_c m x(t) + n_I(t)$

$y(t) = V_c m x(t) + n_I(t)$ & $V_c = 0$

$$S_0 = \frac{V_c^2}{2} P \frac{m^2}{V_m^2}$$

$$N_0 = 2 \times \frac{N_0}{2} \times 2 f_m$$

$$N_0 = 2 N_0 f_m$$

$$\frac{S_0}{N_0} = (SNR)_0 = \frac{\frac{V_c^2}{2} P \left(\frac{m}{V_m}\right)^2}{2 N_0 f_m}$$

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\frac{\frac{V_c^2}{2} P \left(\frac{m}{V_m}\right)^2}{2 N_0 f_m}}{\frac{\frac{V_c^2}{2}\left[1 + \left(\frac{m}{V_m}\right)^2 P\right]}{2 N_0 f_m}}$$

$$\boxed{\gamma = \frac{P\left(\frac{m}{V_m}\right)^2}{1 + \left(\frac{m}{V_m}\right)^2 P}}$$

1) Calculate the figure of merit for AM using envelope detector for single tone AM.

A. Single tone AM.

$x(t) = V_m \cos \omega_m t$

$x(t) = P$

Avg power of $x(t) = P = \frac{V_m^2}{2}$

$$(SNR)_i = \frac{\frac{V_c^2}{2}\, 1 + \left(\frac{m^2}{V_m^2}\right) \frac{V_m^2}{2}}{2 N_0 f_m} = \frac{\frac{V_c^2}{2} \cdot 1 + \frac{m^2}{2}}{2 N_0 f_m}$$

$$(SNR)_0 = \frac{\frac{V_c^2}{2} \frac{V_m^2}{2} \frac{m^2}{V_m^2}}{2 N_0 f_m} = \frac{\frac{V_c^2}{2} \cdot \frac{m^2}{2}}{2 N_0 f_m}$$

for 100% modulation $m = 1$

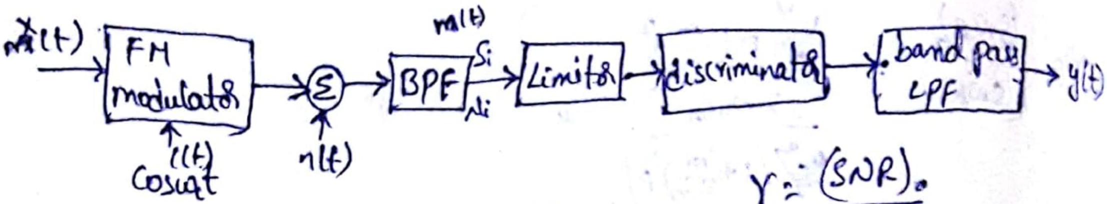
$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\frac{\frac{V_c^2}{2} \frac{m^2}{2}}{2 N_0 f_m}}{\frac{\frac{V_c^2}{2} \cdot (1 + \frac{m^2}{2})}{2 N_0 f_m}} = \frac{\frac{V_c^2}{2} \cdot \frac{m^2}{2}}{\frac{V_c^2}{2} \cdot 1 + \frac{m^2}{2}}$$

$$\gamma = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} \qquad \text{Sub } m=1$$

$$= \frac{1/2}{1+\frac{1}{2}} = \frac{1}{2+1}$$

$$\boxed{\gamma = 1/3}$$

* Noise in FM Receivers:-



$$\gamma = \frac{(SNR)_o}{(SNR)_i}$$

$$n(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$$

$$r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$$

$$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

$$n(t) = r(t)\cos(\omega_c t + \psi(t)) \quad -①$$

$$s(t) = A_c\cos\left[\omega_c t + 2\pi k_f\int_0^t x(t)\,dt\right]$$

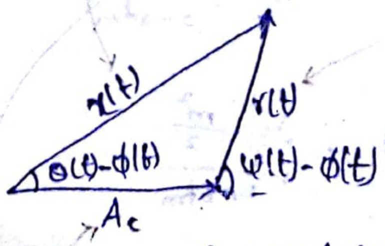
Assume $\phi(t) = 2\pi k_f \int_0^t x(t)\,dt$

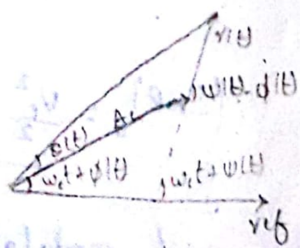
$$s(t) = A_c\cos(\omega_c t + \phi(t)) \quad -②$$

Add ① & ②

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c\cos(2\pi f_c t + \phi(t)) + r(t)\cos(2\pi f_c t + \psi(t))$$





$$\theta(t) - \phi(t) = \tan^{-1}\left[\frac{r(t)\sin(\psi(t)-\phi(t))}{A_c + r(t)\cos(\psi(t)-\phi(t))}\right]$$

$$= \frac{r(t)}{A_c}\sin(\psi(t)-\phi(t))$$

independent of x(t)

$$\theta(t) = \frac{r(t)}{A_c}\sin(\psi(t)-\phi(t)) + \phi(t)$$

$$\theta(t) = \frac{r(t)}{A_c}\sin\psi(t) + \phi(t)$$

$$\phi(t) = 2\pi K_f \int_0^t x(t)\,d(t) + \frac{r(t)}{A_c}\sin\psi(t)$$

$$\frac{1}{2\pi}\frac{d\phi(t)}{dt} = \frac{1}{2\pi}2\pi K_f\, x(t) + \frac{1}{2\pi A_c}\frac{d}{dt}\underbrace{r(t)\sin\psi(t)}_{n_Q(t)}$$

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = K_f x(t) + \frac{1}{2\pi A_c}[n_Q(t)]$$

$$v(t) = K_f x(t) + n_d(t)$$

$$S_0 = K_f^2 P \quad \text{— (3)}$$

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j2\pi f\, X(F)$$

$$H(F) = \frac{1}{2\pi A_c}\times j2\pi f = \frac{jf}{A_c}$$

$$|H(f)|^2 = \frac{f^2}{A_c^2}$$

$$P = \int_{-f_m}^{f_m} S_{N_0}(f)\,df$$

$$N_0 = P = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\int_{-f_m}^{f_m} f^2\,d(t) = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\left[\frac{f^3}{3}\right]_{-f_m}^{f_m} = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\frac{2f_m^3}{3} \quad \text{— (4)}$$

sub (3) & (4) in $(SNR)_0$

$$(SNR)_0 = \frac{S_0}{N_0} = \frac{3K_f^2 P A_c^2}{2N_0 f_m^3} \quad \text{— (5)}$$

$$s(t) = A_c\cos(\omega_c t + \phi(t))$$

$$S = \frac{A_c^2}{2} \quad \text{— (6)}$$

$$n_i = \frac{N_0}{2}\times 2f_m = N_0 f_m \quad \text{— (7)}$$

sub (6), (7) in $(SNR)_i$

$$(SNR)_i = \frac{A_c^2}{2N_0 f_m} \quad \text{— (8)}$$

sub (5), (8) in γ

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{3K_f^2 P A_c^2 / 2N_0 f_m^3}{\frac{A_c^2}{2N_0 f_m}}$$

$$= \frac{3K_f^2 P A_c^2}{\cancel{2N_0} f_m^3}\times\frac{\cancel{2N_0}f_m}{A_c^2}$$

$$\boxed{\gamma = \frac{3K_f^2 P}{f_m^2}}$$

Calculate figure of merit for single tone FM:-

$$A_c\cos\left(\omega_c t + \frac{\Delta f}{f_m}\sin\omega_m t\right)$$

$$\frac{\Delta f}{f_m}\sin\omega_m t = 2\pi K_f\int_0^t x(t)\,dt$$

Diff on b.s

$$\omega_m \frac{\Delta f}{f_m} \cos\omega_m t = 2\pi k_f x(t)$$

$$2\pi f_m \cdot \frac{\Delta f}{f_m} \cos\omega_m t = 2\pi k_f x(t)$$

$$\Delta f \cos\omega_m t = k_f x(t)$$

$$x(t) = \frac{\Delta f}{k_f} \cos\omega_m t$$

Avg power of the modulating signal across 1Ω resistor is

$$P = \left(\frac{\Delta f}{k_f}\right)^2 \frac{1}{2}$$

$$k_f^2 P = \frac{(\Delta f)^2}{2}$$

$$(SNR)_o = \frac{3}{2}\left(\frac{k_f^2 P A_c^2}{N_0 f_m^3}\right) = \frac{3}{2}\left[\frac{\frac{(\Delta f)^2}{2} \cdot A_c^2}{N_0 f_m^3}\right]$$

$$(SNR)_i = \frac{A_c^2}{2N_0 f_m}$$

$$\gamma = \frac{(SNR)_o}{(SNR)_i} = \frac{\frac{3}{2}\left[\frac{(\Delta f)^2}{2} \cdot A_c^2\right]}{\frac{2N_0 f_m^3}{\frac{A_c^2}{2N_0 f_m}}} = \frac{3\left[\frac{(\Delta f)^2}{2} \cdot A_c^2\right]}{2N_0 f_m^3} \times \frac{2N_0 f_m}{A_c^2}$$

$$\gamma = \frac{3\frac{(\Delta f)^2}{2}}{\frac{f_m^2}{1}} = \frac{3(\Delta f)^2}{2} \times \frac{1}{f_m^2}$$

$$= \frac{3}{2}\left(\frac{\Delta f}{f_m}\right)^2$$

$$\boxed{\gamma = \frac{3}{2}(m_f)^2}$$