Discriminant Function

$R(\alpha_i/x) \rightarrow$ minimum then it is the good thing to take

so $-R(\alpha_i/x)$ is maximum then it is good to take

$g_i(x) \leftrightarrow \omega_i$

if $g_i(x)$ is maximum then ω_i is the prediction

$g_i(x) = P(\omega_i/x) = P(x/\omega_i)\,P(\omega_i)$

$= -R(\alpha_i/x)$

$g_i(x) = f\{P(\omega_i/x)\}$

$\uparrow$ monotically increasing ~~function~~ function

if $P(\omega_i/x) > P(\omega_j/x)$

monotonically increasing $\Leftrightarrow f\{P(\omega_i/x)\} > f(P\{\omega_j/x\})$

$\ln(x) \rightarrow$ monotinically increasing so it is also better option to take as f

$$g_i(x) = \ln\{P(\omega_i/x)\}$$
$$= \ln\{P(x/\omega_i)\,P(\omega_i)\}$$
$$= \ln\left(P\left(\frac{x}{\omega_i}\right)\right) + \ln(P(\omega_i))$$
$$= \ln P(x/\omega_i) + \ln P(\omega_i)$$

lets assume $P(x/\omega_i)$ follows multivariate Normal distribution:

$$p(x) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right]$$

$$P(x/\omega_i) = \frac{1}{(2\pi)^{d/2}|\Sigma_i|^{1/2}} \exp\left[-\frac{1}{2}(x-\mu_i)^t \Sigma_i^{-1}(x-\mu_i)\right]$$

because for different classes different covariance matrix

$$g_i(x) = \ln P(\omega_i) - \underbrace{\frac{d}{2}\ln(2\pi)}_{} - \frac{1}{2}\ln|\Sigma_i| - \frac{1}{2}(x-\mu_i)^t\Sigma_i^{-1}(x-\mu_i)$$

independent of class ω_i as there is no subscript i. so for comparing different classes it does not matter.

$$g_i(x) = -\frac{1}{2}\ln|\Sigma_i| - \frac{1}{2}(x-\mu_i)^t\Sigma_i^{-1}(x-\mu_i) + \ln P(\omega_i)$$

if lets say

(1) $\leftarrow \Sigma_i = \sigma^2 I$ {all the colums are independent & having same variance}

(2) $\leftarrow \Sigma_i = \Sigma$

$\hookrightarrow$ all classes have same covariance matrix

(3) $\leftarrow \Sigma_i$.

Case (1)

$$g_i(x) = -\frac{1}{2}\ln|\Sigma_i| - \frac{1}{2}(x-\mu_i)^t \Sigma_i^{-1}(x-\mu_i) + \ln p(\omega_i)$$

as we are taking same for all classes

its not matters at all

$$g_i(x) = -\frac{1}{2}(x-\mu_i)^t \Sigma_i^{-1} (x-\mu_i) + p(\omega_i)$$

$$\downarrow \frac{1}{\sigma^2}$$

$$g_i(x) = \frac{-1}{2\sigma^2}(x-\mu_i)^T(x-\mu_i)$$

class independent

$$= -\frac{1}{2\sigma^2}\{\cancel{x^t x} - 2\mu_i^t x + \mu_i^t \mu_i\} + \ln p(\omega_i)$$

$$g_i(x) = -\frac{1}{2\sigma^2}\{-2\mu_i^t x + \mu_i^t \mu_i) + \ln p(\omega_i)$$

$$= \frac{1}{\sigma^2}\mu_i^t x - \frac{1}{2\sigma^2}\mu_i^t \mu_i + \ln p(\omega$$

$$\simeq w_i^t X + w_{i0}$$

$$w_i = \frac{1}{\sigma^2}\mu_i$$

$$w_{i0} = \frac{-1}{2\sigma^2}\mu_i^t \mu_i + \ln\{p(\omega_i)\}$$

$$g(x) = g_i(x) - g_j(x) = 0$$

$$-\frac{1}{2\sigma^2}(x-\mu_i)^t(x-\mu_i) + \ln p(\omega_i) + \frac{1}{2\sigma^2}(x-\mu_j)^t(x-\mu_j) - \ln(p(\omega_j)) = 0$$

$$\frac{1}{\sigma^2}(\mu_i-\mu_j)^t x - \frac{1}{2\sigma^2}\{\mu_i^t\mu_i - \mu_j^t\mu_j\} + \ln(p(\omega_i)) - \ln(p(\omega_j)) = 0$$

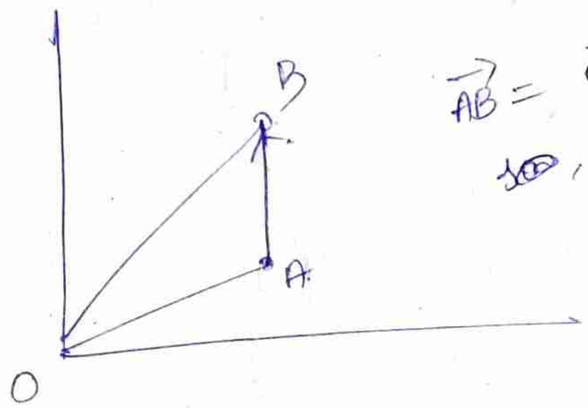
$$\frac{1}{\sigma^2}(\mu_i-\mu_j)^t x - \frac{1}{2\sigma^2}(\mu_i^t\mu_i - \mu_j^t\mu_j) + \ln\left\{\frac{p(\omega_i)}{p(\omega_j)}\right\} =$$

$$(\mu_i-\mu_j)^t x - \frac{1}{2}\underbrace{(\mu_i^t\mu_i - \mu_j^t\mu_j)}_{(\mu_i-\mu_j)^t(\mu_i+\mu_j)} + \sigma^2\ln\left\{\frac{p(\omega_i)}{p(\omega_j)}\right\} =$$

$w^t(x-x_0) = 0 \rightarrow$ Boundary b/w class i & j is a straight line.

$w = \mu_i - \mu_j$

$$x_0 = \frac{1}{2}(\mu_i+\mu_j) - \frac{\sigma^2 \ln\left\{\frac{p(\omega_i)}{p(\omega_j)}\right\}}{\|\mu_i-\mu_j\|^2}(\mu_i-\mu_j)$$



$\vec{AB} = \vec{OB} - \vec{OA}$

so, since μ_i, μ_j are vectors so w is also a vector joining μ_i & μ_j.

$\{x - x_0\}$ is also an vector.

since $w^T(x-x_0) = 0$

i.e, these 2 are $\perp^{er}$ to each other

Boundary is $\perp^{er}$ to the vector w i.e, line joining μ_i, μ_j

and line also passes through x_0.

where
$$x_0 = \frac{1}{2}(\mu_i + \mu_j) - \frac{\sigma^2}{\|\mu_i - \mu_j\|^2} \ln\left\{\frac{P(\omega_i)}{P(\omega_j)}\right\}(\mu_i - \mu_j)$$

lets say $P(\omega_i) = P(\omega_j)$

$$x_0 = \frac{1}{2}(\mu_i + \mu_j)$$

so Boundary passes trough midpoint of line joining μ_i, μ_j and perpendicular to it

<u>case (2) :-</u>

$$\Sigma_i = \Sigma \neq \sigma^2 I$$

$$g_i(x) = \underbrace{\cancel{-\frac{1}{2}\ln|\Sigma|}}_{\text{independent of class}} - \frac{1}{2}(x-\mu_i)^t \Sigma^{-1} (x-\mu_i) + \ln(P(\omega_i))$$

$$g_i(x) = -\frac{1}{2}(x-\mu_i)^t \Sigma^{-1} (x-\mu_i) + \ln\{P(\omega_i)\}$$

$$= -\frac{1}{2}\left[\cancel{x^t \Sigma x} - 2\mu_i^t \Sigma^{-1} x + \mu_i^t \Sigma^{-1} \mu_i\right] + \ln\{P(\omega_i)\}$$

independen-dent of class

$$g_i(x) = \mu_i^t \Sigma^{-1} x - \frac{1}{2}\mu_i^t \Sigma^{-1} \mu_i + \ln\{P(\omega_i)\}$$

$$= w_i^t x + w_{i0}$$

$$w_i = \Sigma^{-1} \mu_i$$
$$w_{i0} = \ln\{P(\omega_i)\} - \frac{1}{2}\mu_i^t \Sigma^{-1} \mu_i$$

$$g(x) = g_i(x) - g_j(x) = 0$$

$$g_i(x) = \mu_i^t \Sigma^{-1} x - \frac{1}{2} \mu_i^t \Sigma^{-1} \mu_i + \ln\{p(\omega_i)\}$$

$$g_j(x) = \mu_j^t \Sigma^{-1} x - \frac{1}{2} \mu_j^t \Sigma^{-1} \mu_j + \ln\{p(\omega_j)\}$$

$$g(x) = (\mu_i^t \Sigma_i^{-1} - \mu_j^t \Sigma^{-1}) x - \frac{1}{2}\{\mu_i^t \Sigma^{-1} \mu_i - \mu_j^t \Sigma^{-1} \mu_j\} + \ln\left\{\frac{p(\omega_i)}{p(\omega_j)}\right\} = 0$$

$$= w^T (x - x_0) = 0$$

$$W = \Sigma^{-1}(\mu_i - \mu_j)$$

$$x_0 = \frac{1}{2}(\mu_i + \mu_j) - \frac{\ln\left\{\frac{p(\omega_i)}{p(\omega_j)}\right\}(\mu_i - \mu_j)}{(\mu_i - \mu_j)^t \Sigma^{-1} (\mu_i - \mu_j)}$$

Case (3) :-

$$\Sigma = \Sigma_i$$

$$g_i(x) = \cancel{-\frac{d}{2} \ln|2\pi|}^{\,0} - \frac{1}{2} \ln|\Sigma_i| - \frac{1}{2}(x - \mu_i)^t \Sigma_i^{-1} (x - \mu_j) + \ln\{p(\omega_i)\}$$

no i

$$g_i(x) = -\frac{1}{2} \ln|\Sigma_i| - \frac{1}{2}(x - \mu_i)^t \Sigma_i^{-1} (x - \mu_i) + \ln\{p(\omega_i)\}$$

$$g_i(x) = \underbrace{x^T A_i x + B_i^T x + C}_{\text{not linear} \rightarrow \text{Quadratic}}$$

$$A_i = -\frac{1}{2} \Sigma_i^{-1} \qquad C_i = -\frac{1}{2} \mu_i^t \Sigma_i^{-1} \mu_i - \frac{1}{2} \ln|\Sigma_i| + \ln\{p(\omega_i)\}$$

$$B_i = \Sigma_i^{-1} \mu_i$$

$$g(x) = g_i(x) - g_j(x) = 0$$