

$$5) \tilde{\theta}_4 = x_1 + \frac{1}{n-1} \sum_{i=2}^n x_i$$

$$M\tilde{\theta}_4 = Mx_1 + \frac{1}{n-1} \sum_{i=2}^n Mx_i = M\xi + \frac{1}{n-1} \sum_{i=2}^n M\xi = \frac{\theta}{2} + \frac{\theta}{2} = \theta \Rightarrow [\tilde{\theta}_4 - \text{несмещ.}]$$

$$D\tilde{\theta}_4 = D\xi + \frac{1}{(n-1)^2} \sum_{i=2}^n D\xi = \frac{\theta^2}{12} + \frac{1}{n-1} \cdot \frac{\theta^2}{12} = \frac{\theta^2}{12} \left(\frac{n}{n-1} \right) \xrightarrow{n \rightarrow \infty} \theta^2$$

рост усл. не выпол.

$$x_1 + \frac{1}{n-1} \sum_{i=2}^n x_i$$

$$\begin{array}{c} \xi_n \xrightarrow{P} \xi \\ \eta_n \xrightarrow{P} \eta \end{array} \Rightarrow \begin{array}{c} \xi_n + \eta_n \xrightarrow{P} \xi + \eta \\ \xi_n \eta_n \xrightarrow{P} \xi \eta \end{array}$$

$$x_1 \xrightarrow{P} \xi$$

Исп. ЗБЧ Хинчина: $\{\xi_n\}$ - независ., равномер.

$$\exists M\xi \Rightarrow \frac{1}{n} \sum_{i=1}^n \xi_i \xrightarrow{P} M\xi$$

$$\xRightarrow{\text{ЗБЧ}} \frac{1}{n-1} \sum_{i=1}^n x_i \xrightarrow{P} \frac{\theta}{2}$$

$$\Rightarrow \tilde{\theta}_4 \xrightarrow{P} \xi + \frac{\theta}{2} \Rightarrow \boxed{\text{не явл. } \tilde{\theta}_4 \text{ состоят.}}$$

⑥ $\tilde{\theta}_2$ и $\tilde{\theta}_4$ - не сост

Состоят. явл только $\tilde{\theta}_1$ и $\tilde{\theta}_3'$

$$D\tilde{\theta}_1 = \frac{\theta^2}{3n} = D_1$$

$$D\tilde{\theta}_3' = \frac{\theta^2}{n(n+2)} = D_3$$

$$D_3 < D_1 \forall n$$

$\Rightarrow \tilde{\theta}_3'$ более эффективнее $\tilde{\theta}_1$