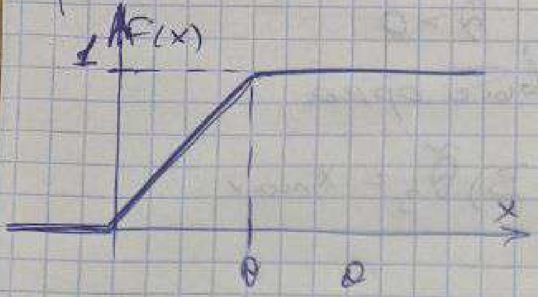
$$\varphi(x) = \Phi'(x) = n\left(1-\frac{x}{\theta}\right)^{n-1} F'(x) = n\left(1-\frac{x}{\theta}\right)^{n-1}\frac{1}{\theta}\{(0,\theta)\}$$



$$Mx_{min} = \int_0^\theta x n \Big(\underbrace{1-\frac{x}{\theta}}_{t}\Big)^{n-1}\frac{1}{\theta}\,dx = -\int_1^0 n(1-t)t^{n-1}\theta\,dt =$$

$$= n\theta\int_0^1 (t^{n-1}-t^n)\,dt = n\theta\left(\frac{1}{n} - \frac{1}{n+1}\right) = \frac{\theta}{n+1}$$

$\longrightarrow \tilde\theta_2$ - смещ.

$\tilde\theta_2' = (n+1)x_{min}$

$\Rightarrow M\tilde\theta_2' = (n+1)Mx_{min} = \theta \Rightarrow \boxed{\tilde\theta_2' \text{ - несмещ.}}$

$$Mx^2_{min} = \int_0^\theta x^2 n\Big(\underbrace{1-\frac{x}{\theta}}_{t}\Big)^{n-1}\frac{1}{\theta}\,dx = \int_0^1 \theta^2(1-t)^2 n t^{n-1}\,dt =$$

$$= n\theta^2\int_0^1 (t^{n-1}-2t^n+t^{n+1})\,dt = n\theta^2\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right) =$$

$= \frac{n\theta^2}{2+n} \not\xrightarrow[n\to\infty]{} 0 \Rightarrow$ достаточ. усл. не работа

Докажем по опр-ю:

$$P(|\tilde\theta_2'-\theta| \ge \varepsilon) \ge P(\tilde\theta_2' \ge \theta+\varepsilon) =$$

$$= P((n+1)x_{min} \ge \theta+\varepsilon) = P\left(x_{min} \ge \frac{\theta+\varepsilon}{n+1}\right)$$

$$= 1-\Phi\left(\frac{\theta+\varepsilon}{n+1}\right) = (*) \quad \{\Phi(x) = 1-(1-F(x))^n\}$$

$$(*) = 1-\left(1-\left(1-\frac{\theta+\varepsilon}{\theta(n+1)}\right)^n\right) =$$

$$= \left(1-\frac{\theta+\varepsilon}{\theta(n+1)}\right)^n \xrightarrow[n\to\infty]{} e^{-\frac{\theta+\varepsilon}{\theta}} > 0$$

$\Rightarrow P(|\tilde\theta_2'-\theta| \ge \varepsilon) > 0 \Rightarrow \boxed{\tilde\theta_2' \text{ не явл. состоят.}}$