№1

$\xi \sim R(0,\theta)$ – вер. модель, $\theta > 0$

$\theta \in \Theta = (0, +\infty)$; $\overline{x}_n$ – выборка

a)

А) $\tilde{\theta}_1 = 2\overline{x}$ Б) $\tilde{\theta}_2 = x_{min}$ В) $\tilde{\theta}_3 = x_{max}$

Г) $\tilde{\theta}_4 = x_1 + \frac{1}{(n-1)} \sum_{i=2}^{n} x_i$

Предварит. вычисления:

$$\underline{M\xi} = \int_{-\infty}^{\theta} x\, dF(x,\theta) = \int_0^{\theta} x \frac{1}{\theta} dx = \frac{\theta}{2}$$

$p(x,\theta) = \frac{1}{\theta}\{(0,\theta)\}$ $\xi \sim R(a,b)$

$$M\xi^2 = \int_0^{\theta} x^2 \frac{1}{\theta} dx = \frac{\theta^2}{3}$$

$$\underline{D\xi} = M\xi^2 - (M\xi)^2 = \frac{\theta^2}{12}$$

А) $\forall \theta > 0$ $M\tilde{\theta}_1 = M\left(2\frac{1}{n}\sum x_i\right) =$

$= \{x_i$ – независ. сл. вел.: $x_i \sim R(0,\theta)\} =$

$= \frac{2}{n}\sum M x_i = \frac{2}{n} n M\xi = \theta \Rightarrow \boxed{\tilde{\theta}_1 \text{ – несмещ.}}$

$$D\tilde{\theta}_1 = D\left(2\frac{1}{n}\sum x_i\right) = \frac{4}{n^2}\sum D x_i = \frac{4}{n^2} n D\xi = \frac{\theta^2}{3n} \xrightarrow[n\to\infty]{} 0$$

$\forall \theta > 0$

$\Rightarrow$ выпол. достаточ. усл.

$\Rightarrow \boxed{\tilde{\theta}_1 \text{ – состоятельная}}$

Б) $\tilde{\theta}_2 = x_{min}$

$\forall \theta > 0$ $M\tilde{\theta}_2 = M x_{min}$

$\xi \sim F(x)$

$\Rightarrow \xi_{min} \sim \underbrace{1 - (1 - F(x))^n}_{\Phi(x)}$