(*) Док-ем сост-ть по опр.:

$\tilde{\theta}_3$: $P(|\tilde{\theta}_3 - \theta| \geq \varepsilon) = P(x_{max} \leq \theta - \varepsilon) =$

$= (F(\theta - \varepsilon))^n = \left(\frac{\theta - \varepsilon}{\theta}\right)^n = \underbrace{\left(1 - \frac{\varepsilon}{\theta}\right)}_{q}{}^n$

для $\varepsilon < \theta$: $0 < q < 1$

$\Rightarrow \lim\limits_{n \to \infty} q^n = 0$

для $\varepsilon \geq \theta$: $\lim q^n = 0$

$\Rightarrow \forall \varepsilon > 0$ и $\theta \in \Theta \hookrightarrow P(|\tilde{\theta}_3 - \theta| \geq \varepsilon) \xrightarrow[n \to \infty]{} 0$

$\Rightarrow$ состоят.

$\tilde{\theta}_3'$: $P\left(\left|\frac{n+1}{n} x_{max} - \theta\right| \geq \varepsilon\right)$

нер-во Δ-ка:

$\left|\frac{n+1}{n} x_{max} - \theta\right| \leq \frac{n+1}{n} |x_{max} - \theta| + \frac{\theta}{n}$

$\Rightarrow$

$P\left(\left|\frac{n+1}{n} x_{max} - \theta\right| \geq \varepsilon\right) \leq P\left(\frac{n+1}{n} |x_{max} - \theta| + \frac{\theta}{n} \geq \varepsilon\right)$

$|x_{max} - \theta| > \left(\varepsilon - \frac{\theta}{n}\right) \frac{n}{n+1} =$

$= \frac{n}{n+1} \varepsilon - \frac{\theta}{n+1} \xrightarrow[n \to \infty]{} \varepsilon$

т.е.

$|x_{max} - \theta| \geq \varepsilon$

$P(|x_{max} - \theta| \geq \varepsilon) \longrightarrow 0$

– доказали ранее

$\Rightarrow P\left(\left|\frac{n+1}{n} x_{max} - \theta\right| \geq \varepsilon\right) \xrightarrow[n \to \infty]{} 0$

$\Rightarrow \tilde{\theta}_3'$ – сост. тоже.