ИДЗ 1

В-13

$$p_{\xi,\eta}(x,y) = C\cdot \exp\left(-\frac{1}{2}\left(2x^2-4xy+5y^2-4x+16y+14\right)\right)$$

$$2(x^2-2xy-2x+1+y^2+2y) =$$

$$= 2(x-y-1)^2 \cancel{+(3y^2+12y+12)}$$

$$2x^2-4xy+5y^2-4x+16y+14 =$$

$$= 2(x-y-1)^2 + (3y^2+12y+12) =$$

$$= 2(x-y-1)^2 + 3(y+2)^2 =$$

$$= 2(x+1)^2 - 4(x+1)(y+2) + 5(y+2)^2$$

$$a_1 = -1 \qquad a_2 = -2$$

$$E_{\xi,\eta} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x+1 \\ y+2 \end{pmatrix}^T \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} x+1 \\ y+2 \end{pmatrix}$$

$$R^{-1} = \begin{pmatrix} +2 & -2 \\ -2 & 5 \end{pmatrix} \qquad R = \frac{1}{6}\begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix} = Var_{\xi,\eta}$$

$$\sigma_1^2 = 5 \qquad \sigma_2^2 = 2$$

$$\rho\sigma_1\sigma_2 = 2 \qquad \rho = \frac{2}{\sqrt{10}}$$

$$C = \frac{1}{2\pi\sqrt{10}\cdot\sqrt{1-\frac{4}{10}}} = \frac{1}{2\sqrt{6}\,\pi}$$

№2

$$2x^2 - 4xy + 5y^2 - 4x + 16y + 14 =$$
$$= 2(x-y-1)^2 + 3(y+2)^2$$

$$\begin{pmatrix} \xi\sqrt{2} - \eta\sqrt{2} - \sqrt{2} \\ \eta\sqrt{3} + 2\sqrt{3} \end{pmatrix} = A\vec{\nu} + B$$

$$A = \begin{pmatrix} \sqrt{2} & -\sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix} \qquad B = \begin{pmatrix} -\sqrt{2} \\ 2\sqrt{3} \end{pmatrix}$$

проверка:

$$E\vec{\nu} = A\begin{pmatrix} E\xi \\ E\eta \end{pmatrix} + B = \begin{pmatrix} \sqrt{2} & -\sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix}\begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} -\sqrt{2} \\ 2\sqrt{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathrm{Var}\,\vec{\nu} = A \cdot R \cdot A^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

№3

$$R^{-1} = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$$

собственные числа:

$$\begin{vmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda_1 = 6 \qquad \lambda_2 = 1$$

1) $\lambda = 6$

$$\begin{pmatrix} -4 & -2 \\ -2 & -1 \end{pmatrix} \longrightarrow \text{с.в. } x = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{5}}$$

2) $\lambda = 1$

$$\begin{pmatrix} 1 & -2 \\ -2 & -1 \end{pmatrix} \longrightarrow \text{с.в. } x = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \frac{1}{\sqrt{5}}$$

$$Q = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \frac{1}{\sqrt{5}}$$ – матрица ортогонального преобразования

№4

$\mu_1 = -4\xi + \eta$

$\mu_2 = \xi + 2\eta$

$$E\vec{\mu} = \begin{pmatrix} -4 & 1 \\ 1 & 2 \end{pmatrix} E_{\xi,\eta} = \begin{pmatrix} -4 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

$$\mathrm{Var}\,\vec{\mu} = \begin{pmatrix} -4 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 1 & 2 \end{pmatrix} \cdot \frac{1}{6} =$$

$$= \begin{pmatrix} 66 & -30 \\ -30 & 21 \end{pmatrix}\frac{1}{6} = \begin{pmatrix} 22 & -10 \\ -10 & 7 \end{pmatrix}\frac{1}{2}$$

$\cancel{\sigma_1^2 = 66/2}$ $\quad \cancel{\sigma_2^2 = 21/2}$

$\cancel{\rho\sigma_1\sigma_2 = -5}$

$\cancel{\sigma_1 = \sqrt{33}}$ $\quad \cancel{\sigma_2 = \sqrt{21/2}}$ $\quad \cancel{\rho = -\frac{5\sqrt{2}}{693}}$

$\sigma_1^2 = 11$ $\quad \sigma_2^2 = 7/2$ $\quad \rho\sigma_1\sigma_2 = -5$

$\sigma_1 = \sqrt{11}$ $\quad \sigma_2 = \sqrt{7/2}$ $\quad \rho = -\frac{5\sqrt{2}}{\sqrt{77}}$

$$p_{\vec{\mu}}(x,y) = \frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2}\sqrt{1-\rho^2}} \cdot \exp\left(-\frac{1}{2(1-\rho^2)} \cdot \left(\frac{(x-E\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x-E\mu_1)(y-E\mu_2)}{\sigma_1\sigma_2} + \frac{(y-E\mu_2)^2}{\sigma_2^2}\right)\right) =$$

$$= \frac{1}{2\pi \frac{\sqrt{77}}{\sqrt{2}} \cdot \frac{\sqrt{27}}{\sqrt{77}}} \cdot \exp\left(-\frac{1}{2\left(\frac{27}{77}\right)} \cdot \left(\frac{(x-6)^2}{11} + \frac{\overset{2}{\cancel{10\sqrt{2}}}(x-6)(y-1)}{\sqrt{77} \cdot \cancel{\frac{\sqrt{77}}{\sqrt{2}}}\,77} + \right.\right.$$

$$\left.\left. + \frac{(y-1)^2}{(7/2)}\right)\right) = \frac{\sqrt{2}}{2\pi\sqrt{27}} \cdot \exp\left(-\frac{1}{54}\left(7(x-6) + 20(x-6)(y-1) + \right.\right.$$

$$\left.\left. + 22(y-1)^2\right)\right)$$

№5.

$p_{\xi|\eta}$ - ?

$$p_{\xi|\eta}(x) = C_1(y) \exp\left(-\frac{1}{2}\left(2x^2 - 4xy + 5y^2 - 4x + 16y + 14\right)\right) =$$

$$= \cancel{\{2(x^2 - 2xy + y^2}\ C_1(y) \exp\left(-\frac{1}{2}\left(2(x-y-1)^2 + C_2(y)\right)\right) =$$

$$= C_3(y)\, e^{-\frac{(x-y-1)^2}{2 \cdot (1/\sqrt{2})^2}}$$

$$E(\xi \mid \eta = y) = \eta + 1$$

$$D(\xi \mid \eta = y) = 1/2 \quad , \quad \sigma = 1/\sqrt{2}$$

$$C_3 = \frac{1}{\sqrt{2\pi}\,\sigma} = \frac{1}{\sqrt{\pi}}$$

$$p_{\xi|\eta=y}(x) = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{1}{2} \cdot \frac{1}{(1/\sqrt{2})^2}(x-y-1)^2\right)$$