дз от 17.10

№5.

$x_1 \dots x_n \sim N(0, \sigma^2)$. ДИ для $\theta = 2\log\sigma$

$\xi = \sqrt{n}\,\frac{\bar{x}}{\sigma} \sim N(0,1) \Rightarrow \xi^2 \sim \chi^2_1$

$\Rightarrow$

$P_\theta\left(\frac{n\bar{x}^2}{\sigma^2} < x\right) = K_1(x)$

$x_{1\alpha}: K_1(x_{1\alpha}) = \alpha/2 \quad x_{2\alpha}: K_1(x_{2\alpha}) = 1 - \alpha/2$

$P_\theta\left(x_{1\alpha} \le \frac{n\bar{x}^2}{\sigma^2} \le x_{2\alpha}\right) =$

$= P_\theta\left(\frac{n\bar{x}^2}{x_{2\alpha}} \le \sigma^2 \le \frac{n\bar{x}^2}{x_{1\alpha}}\right) =$

$= P_\theta(\log n + 2\log\bar{x} - \log x_{2\alpha} \le 2\log\sigma \le$

$\le \log n + 2\log\bar{x} - \log x_{1\alpha}) =$

$= 1 - \alpha$

$\Rightarrow$

ДИ для $\theta = 2\log\sigma$:

$[\log n + 2\log\bar{x} - \log x_{2\alpha};\ \log n + 2\log\bar{x} - \log x_{1\alpha}]$

№6.

$x_1 \ldots x_n$ — выб. из $N(a, 3\sigma^2)$

$y_1 \ldots y_m$ — выб. из $N(b, \sigma^2)$

ДИ для $\theta = a + b$

I. $\sqrt{n}\,\frac{\bar{x} - a}{\sigma} \sim N(0, 3)$ $\qquad$ $\sqrt{m}\,\frac{\bar{y} - b}{\sigma} \sim N(0, 1)$

$\frac{\bar{x} - a}{\sigma} \sim N(0, 3/n)$ $\qquad$ $\frac{\bar{y} - b}{\sigma} \sim N(0, 1/m)$

$\Rightarrow$

$$\frac{\bar{x} + \bar{y} - \theta}{\sigma} \sim N\left(0, \frac{3}{n} + \frac{1}{m}\right)$$

$$\frac{\bar{x} + \bar{y} - \theta}{\sigma} \cdot \sqrt{\frac{nm}{3m+n}} \sim N(0, 1)$$

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II. $\frac{n s_x^2}{\sigma^2} \sim \chi^2_{n-1}$ $\qquad$ $\frac{m s_y^2}{\sigma^2} \sim \chi^2_{m-1}$

$$\frac{n s_x^2 + m s_y^2}{\sigma^2} \sim \chi^2_{n+m+2}$$

III.
$$\frac{\sqrt{\frac{nm}{3m+n}} \cdot \frac{\bar{x} + \bar{y} - \theta}{\sigma}}{\sqrt{\frac{1}{m+n-2} \cdot \frac{n s_x^2 + m s_y^2}{\sigma^2}}} \sim S_{n+m-2}$$

$$\sqrt{\frac{nm(n+m-2)}{3m+n}} \cdot \frac{\bar{x}+\bar{y}-\theta}{\sqrt{nS_x^2+mS_y^2}}$$

$$P_{a,b}\ P(\theta,\delta) = \left(\sqrt{\dots}\ \frac{\bar{x}+\bar{y}-\theta}{\sqrt{nS_x^2+mS_y^2}} < x\right) = S_{n+m-2}(x)$$

$$x_\alpha : S_{n+m-2}(x_\alpha) = 1 - \alpha/2 .$$

$$\Rightarrow$$

$$P(\theta,\delta) = \left(-x_\alpha \le \sqrt{\dots} \cdot \frac{\bar{x}+\bar{y}-\theta}{\sqrt{nS_x^2+mS_y^2}} \le x_\alpha\right) =$$

$$= 1 - \alpha$$

$\Rightarrow$ ДИ для $\theta = a + b$:

$$\left[-x_\alpha \frac{\sqrt{nS_x^2+mS_y^2}}{\sqrt{\dots}} + \bar{x}+\bar{y};\ x_\alpha \frac{\sqrt{nS_x^2+mS_y^2}}{\sqrt{\dots}} + \bar{x}+\bar{y}\right]$$

№7

$x_1 \ldots x_n \sim N(a, \sigma_1^2)$

$y_1 \ldots y_m \sim N(b, b_2^2)$

ДИ для $\theta = \log \sigma_1 - \log \sigma_2$

№8.

$x_1 \ldots x_n$ выборка из

$$p(x) = \begin{cases} \frac{1}{\theta} e^{x/\theta}, & x \leq 0 \\ 0, & x > 0 \end{cases} \qquad \theta > 0$$

] $x' = -x$, $\Rightarrow$

$$p(x) = \begin{cases} \frac{1}{\theta} e^{-x'/\theta}, & x' > 0 \\ 0, & x' \leq 0 \end{cases}$$

$$p(x) = \frac{1}{\theta} e^{-x'/\theta} \, \mathbb{1}_{\{x' > 0\}}$$

$$x'_1 \sim E(\cancel{a}\, 1/\theta) = \Gamma(1, \theta)$$

$\Rightarrow$

$$\sum_{i=1}^{n} x'_i \sim \Gamma(n, \theta)$$

$\Rightarrow$

$$1/\theta \sum_{i=1}^{n} x'_i \sim \Gamma(n, 1)$$

$x_{1\alpha}: G_{n,1}(x_{1\alpha}) = \alpha/2$

$x_{2\alpha}: G_{n,1}(x_{2\alpha}) = 1 - \alpha/2$

$\Rightarrow$

$$P_\theta\left(x_{1\alpha} \le \frac{1}{\theta}\sum_{i=1}^{n} x_i' \le x_{2\alpha}\right) =$$

$$= P_\theta\left(\frac{n\bar{x}'}{x_{2\alpha}} \le \theta \le \frac{n\bar{x}'}{x_{1\alpha}}\right) = 1-\alpha$$

$\Rightarrow$

ДИ для θ:

$$\left[\frac{n\bar{x}}{x_{2\alpha}}; \frac{n\bar{x}}{x_{1\alpha}}\right]$$