НРМА для $\theta = b^2$ по $x_1 \dots x_n$ из

$$p_\theta(x) = \frac{2\sqrt{x}\, e^{-x/b}}{\sqrt{b^3 \pi}} \mathbb{1}\{x > 0\}$$

$$L(x, \theta) = \prod_{i=1}^{n} p_\theta(x_i) \mathbb{1}\{x_i > 0\} =$$

$$= \frac{2^n \prod x_i^{1/2} e^{-\sum x_i / b}}{(b^3 \pi)^{n/2}} \mathbb{1}\{x_{(1)} > 0\}$$

$$= \frac{\prod (2\sqrt{x_i}) \exp(-\sum x_i / b)}{(b^3 \pi)^{n/2}} \mathbb{1}_{\{x_{(1)} > 0\}} \to T(x) = \left\{ \prod x_i, x_{(1)} \right\}$$

- МВС, полная

$$LL(x, \theta) = \log \left(\prod_{i=1}^{n} 2\sqrt{x_i} \right) - \frac{\sum_{i=1}^{n} x_i}{b} - \frac{3n}{2} \log b + -$$

$$- \frac{n}{2} \log \pi$$

$$\frac{d}{d\theta} LL = \frac{\sum x_i}{b^2} - \frac{3n}{2b} = 0$$

$$3nb = \sum x_i \cdot 2$$

$$\hat{b} = \frac{2 \sum x_i}{3n} = \frac{2\bar{x}}{3}$$

$$\hat{\theta} = \hat{b}^2 = \frac{4\bar{x}^2}{9} - \text{ОМП}$$

$$\sum x_i \sim \Gamma(n, b) \Rightarrow$$

$$E\left(\frac{4(\sum x_i)^2}{(3n)^2} \right) = \frac{4}{(3n)^2} b^2 \frac{\Gamma(n+2)}{\Gamma(n)} =$$

$$= \frac{4b^2}{(3n)^2} (n+1) n = \frac{4b^2 (n+1)}{9n}$$

$\Delta C = \pm 360$ ед/Т

корректируем:

$$\tilde{\theta} = \frac{q_n \hat{\theta}}{(n+1)4}$$

$$\Rightarrow$$

$$E\tilde{\theta} = \theta^2 \longrightarrow \tilde{\theta} - \text{НРМД} - \text{оценка}$$

№1.

$x_1 \dots x_n$ ц $N(a, 2\delta^2)$

$y_1 \dots y_n$ ц $N(b, \delta^2)$

$z_1 \dots z_n$ ц $N(c, \delta^2)$

ВЧ для $a+b-c \neq 1$

$$\sqrt{n}\,\frac{\bar{x}-a}{\delta} \sim N(0,2)$$

$$\sqrt{n}\,\frac{\bar{y}-b}{\delta} \sim N(0,1)$$

$$\sqrt{n}\,\frac{\bar{z}-c}{\delta} \sim N(0,1)$$

$$\Rightarrow$$

$$\sqrt{n}\,\frac{\bar{x}+\bar{y}-\bar{z}-(a+b-c)}{\delta} \sim N(0,2)$$

$$\Rightarrow$$

$$\sqrt{\frac{n}{2}}\,\frac{\bar{x}+\bar{y}-\bar{z}-(a+b-c)}{\delta} \sim N(0,1)$$