№4.

$X_1 \dots X_n$ – выб. из $Bi(m_0, p)$, m_0 – известно

АРН $\theta = p$ – ? х3 сл.

$$p_\theta(x) = \frac{m_0!}{(m_0-x)!\,x!} p^x q^{m_0 - x}$$

$$L(\theta; x) = \prod_{i=1}^{n} \left(\frac{m_0!}{(m_0-x_i)!\,x_i!} p^{x_i} q^{m_0-x_i} \right)$$

$$LL(\dots) = \log\left(\prod \frac{\dots}{\dots}\right) + \sum x_i \log p + (nm_0 - \sum x_i) \cdot \log q$$

$$\frac{dLL}{dp} = \frac{\sum x_i}{p} - \frac{nm_0 - \sum x_i}{1-p} = 0$$

$$(1-p)\sum x_i = p(nm_0 - \sum x_i)$$

$$\sum x_i = p(nm_0)$$

$$\hat{p} = \frac{\bar{x}}{m_0} \text{ – ОМП}$$

$$E_p x_1 = m_0 p$$

$$D_p x_1 = m_0 p(1-p)$$

используя ЦПТ

$$\frac{\sum x_i - nm_0 p}{\sqrt{nm_0 p(1-p)}} = \sqrt{n}\, \frac{\bar{x} - m_0 p}{\sqrt{m_0 p(1-p)}} \Rightarrow N(0,1)$$

$\exists\, x_\alpha : \Phi(x_\alpha) = 1 - \alpha/2$

I способ:

$$\ldots = \sqrt{n m_0}\, \frac{\bar{x}/m_0 - p}{\sqrt{p(1-p)}} \quad ?$$

II способ:

$$\hat{p} = \frac{\bar{x}}{m_0}$$

$$\sqrt{n m_0}\, \frac{\bar{x}/m_0 - p}{\sqrt{\bar{x}/m_0 - (\bar{x}/m_0)^2}} = \sqrt{n}\, \frac{\bar{x}/m_0 - p}{\sqrt{\bar{x} - \frac{\bar{x}^2}{m_0}}}$$

$$P_p(-x_\alpha \le \ldots \le x_\alpha) =$$

$$= \cancel{P_p\left(x_\alpha \frac{\bar{x}/m_0 - (\bar{x}/m_0)^2}{n m_0}\right)}$$

$$= P_p\Big(x_\alpha \sqrt{\frac{\bar{x} - \bar{x}^2/m_0}{n}} + \bar{x}/m_0 \ge p \ge -x_\alpha \sqrt{\frac{\cdots}{\cdots}} + \bar{x}/m_0\Big) = 1 - \alpha$$

$\Rightarrow$

$$\left[-x_\alpha \sqrt{\frac{\bar{x} - \bar{x}^2/m_0}{n}} + \bar{x}/m_0;\ x_\alpha \sqrt{\frac{\bar{x} - \bar{x}^2/m_0}{n}} + \bar{x}/m_0\right]$$

АДИ

III способ

$\tilde{p} = s$

$$\sqrt{n m_0}\,\frac{\bar{x}/m_0 - p}{\sqrt{s - s^2}} \Rightarrow N(0,1)$$

$$P_p(-x_\alpha \le \ldots \le x_\alpha) =$$

$$= P_p\left(x_\alpha\sqrt{\frac{s-s^2}{n m_0}} + \bar{x}/m_0 \ge p \ge -x_\alpha\sqrt{\frac{s-s^2}{n m_0}} + \frac{\bar{x}}{m_0}\right) =$$

$$= 1 - \alpha$$

$$\Rightarrow \left[-x_\alpha\sqrt{\frac{s-s^2}{n m_0}} + \frac{\bar{x}}{m_0};\ x_\alpha\sqrt{\frac{s-s^2}{n m_0}} + \frac{\bar{x}}{m_0}\right] - \text{АДИ}$$

№5

$X_1 \ldots X_n$ – выб. из $U(0,\theta)$

построить АДИ для θ с исп. стат. $\bar{X}$ (ЦПТ)

$$p_\theta = \frac{1}{\theta}\,\mathbb{1}_{\{x>0\}}\,\mathbb{1}_{\{x<\theta\}}$$

используя ЦПТ

$$\sqrt{n}\,\frac{\bar{x} - \theta/2}{\sqrt{\theta^2/12}} = \sqrt{n/3}\,\frac{2\bar{x} - \theta}{\theta} \Rightarrow N(0,1)$$

$$P_\theta\left(-x_\alpha \le \sqrt{\frac{n}{3}}\,\frac{2\bar{x}}{\theta} - \sqrt{\frac{n}{3}} \le x_\alpha\right) =$$

$$= P_\theta\left(-x_\alpha\sqrt{\frac{3}{n}}+1 \le \frac{2\bar{x}}{\theta} \le x_\alpha\sqrt{\frac{3}{n}}+1\right) =$$

$$=\left(\frac{2\bar{x}}{1-x_\alpha\sqrt{\frac{3}{n}}} \geq \theta \geq \frac{2\bar{x}}{1+x_\alpha\sqrt{\frac{3}{n}}}\right)$$

$$\Rightarrow \left[\frac{2\bar{x}}{1+x_\alpha\sqrt{\frac{3}{n}}} ; \frac{2\bar{x}}{1-x_\alpha\sqrt{\frac{3}{n}}}\right] \text{- ADU}$$