К/р 1

№2.

ОМП для θ по $x_1 \dots x_n$ при

$$p_\theta(x) = \frac{1}{x\theta} \mathbb{1}_{\{x \in [1, e^\theta]\}}$$

$$LL = \prod_{i=1}^{n} \frac{1}{x_i \theta} \mathbb{1}_{\{x_i \in [1, e^\theta]\}} =$$

$$= \frac{1}{\prod x_i \cdot \theta^n} \mathbb{1}_{\{x_{(1)} \geq 1\}} \mathbb{1}_{\{x_{(n)} \leq e^\theta\}}$$

$$\begin{cases} \prod x_i \cdot \theta^n \to \min \\ x_{(1)} \geq 1 \\ x_{(n)} \leq e^\theta \end{cases} \quad \begin{cases} \theta^n \prod x_i \to \min \\ x_{(1)} \geq 1 \\ \theta \geq \log x_{(n)} \end{cases}$$

$$x_{(1)} \geq 1 \Rightarrow \log x_{(n)} \geq \log x_{(1)} \geq 0$$

$$\Rightarrow$$

$$\hat{\theta} = \log x_{(n)}$$

№3

НРМД для $\theta = b^2$ по $x_1 \dots x_n$ из

$$p_\theta(x) = \frac{2\sqrt{x}\, e^{-x/b}}{\sqrt{b^3 \pi}} \mathbb{1}\{x>0\}$$

$$L(x,\theta) = \prod_{i=1}^{n} p_\theta(x_i) \mathbb{1}\{x_i>0\} =$$

$$= \cancel{\frac{2 / x^{n/2} e^{-\frac{\sum x_i}{b}}}{(b^3\pi)^{n/2}}} \mathbb{1}\{x_{(1)} \cancel{b\, 0}\}$$

$$= \frac{\prod (2\sqrt{x_i}) \exp(-\sum x_i / b)}{(b^3 \pi)^{n/2}} \mathbb{1}_{\{x_{(1)}>0\}} \to T(x) = \left\{ \prod x_i,\ x_{(1)} \right\}$$

– МВС, полная

$$LL(x,\theta) = \log\left(\prod_{i=1}^{n} 2\sqrt{x_i}\right) - \frac{\sum_{i=1}^{n} x_i}{b} - \frac{3n}{2} \log b - \frac{n}{2}\log \pi$$

$$\frac{d}{d\theta} LL = \frac{\sum x_i}{b^2} - \frac{3n}{2b} = 0$$

$$3nb = \sum x_i$$

$$\hat{b} = \frac{\sum x_i}{3n} = \frac{\bar{x}}{3}$$

$$\hat{\theta} = \hat{b}^2 = \frac{\bar{x}^2}{9} \text{ – ОМП}$$

$$\sum x_i \sim \Gamma(n, b) \Rightarrow$$

$$E\left(\frac{(\sum x_i)^2}{(3n)^2}\right) = \frac{1}{(3n)^2} b^2 \frac{\Gamma(n+2)}{\Gamma(n)} =$$

$$= \frac{b^2}{(3n)^2} (n+1) n = \frac{b^2}{9n}(n+1)$$

корректируем:

$$\tilde{\theta} = \frac{9n\hat{\theta}}{n+1}$$

$\Rightarrow$

$E\tilde{\theta} = b^2 \longrightarrow \tilde{\theta}$ – НРМД – оценка

№1.

$x_1 \dots x_n$ из $N(a, 2\sigma^2)$

$y_1 \dots y_n$ из $N(b, \sigma^2)$

$z_1 \dots z_n$ из $N(c, \sigma^2)$

ДИ для $e^{a+b-c+1}$

$$\sqrt{n}\,\frac{\bar{x}-a}{\sigma} \sim N(0,2)$$

$$\sqrt{n}\,\frac{\bar{y}-b}{\sigma} \sim N(0,1)$$

$$\sqrt{n}\,\frac{\bar{z}-c}{\sigma} \sim N(0,1)$$

$\Rightarrow$

$$\sqrt{n}\,\frac{\bar{x}+\bar{y}-\bar{z}-(a+b-c)}{\sigma} \sim N(0,2)$$

$\Rightarrow$

$$\sqrt{\frac{n}{2}}\,\frac{\bar{x}+\bar{y}-\bar{z}-(a+b-c)}{\sigma} \sim N(0,1)$$

$$\frac{n S_x^2 + n S_y^2 + n S_z^2}{\sigma^2} \sim \chi^2_{3n-3}$$

$$\frac{\sqrt{\frac{n}{2}} \cdot \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\sigma}}{\sqrt{\frac{1}{3n-3} \cdot \frac{n(S_x + S_y + S_z)}{\sigma^2}}} \sim S_{3n-3}$$

$$\|$$

$$\sqrt{\frac{3n-3}{2}} \; \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\sqrt{S_x + S_y + S_z}}$$

$$P_{(a,b,c,\sigma)}(\ldots < x) = S_{3n-3}(x)$$

$$\exists\, x_\alpha : S_{3n-3}(x_\alpha) = 1 - \alpha/2$$

$$\Rightarrow$$

$$P_{(a,b,c,\sigma)}(-x_\alpha \le \ldots \le x_\alpha) =$$

$$= P_{(a,b,c,\sigma)}\left(x_\alpha \sqrt{\frac{2(S_x + S_y + S_z)}{3n-3}} + \bar{x} + \bar{y} - \bar{z} - 1 \geq \right.$$

$$\left. \geq \theta \geq -x_\alpha \sqrt{\frac{2(S_x + S_y + S_z)}{3n-3}} + \bar{x} + \bar{y} - \bar{z} - 1\right)$$

$$\Rightarrow \text{ДИ} = [\ldots ; \ldots]$$