

K/p 1

N2.

OMП гдѣ  $\theta$  по  $x_1 \dots x_n$  нрн

$$p_{\theta}(x) = \frac{1}{x^{\theta}} \mathbb{1}_{\{x \in [1, e^{\theta}]\}}$$

$$LL = \prod_{i=1}^n \frac{1}{x_i^{\theta}} \mathbb{1}_{\{x_i \in [1, e^{\theta}]\}} =$$

$$= \frac{1}{\prod x_i \cdot \theta^n} \mathbb{1}_{\{x_{(1)} \geq 1\}} \mathbb{1}_{\{x_{(n)} \leq e^{\theta}\}}$$

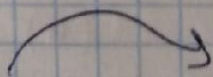
$$\begin{cases} \prod x_i \cdot \theta^n \rightarrow \min \\ x_{(1)} \geq 1 \\ x_{(n)} \leq e^{\theta} \end{cases} \quad \begin{cases} \theta^n \prod x_i \rightarrow \min \\ x_{(1)} \geq 1 \\ \theta \geq \log x_{(n)} \end{cases}$$

$$x_{(1)} \geq 1 \Rightarrow \log x_{(n)} \geq \log x_{(1)} \geq 0$$

$\Rightarrow$

$$\hat{\theta} = \log x_{(n)}$$

N3



ИРМТ где  $\theta = b^2$  по  $x_1, \dots, x_n$  и

$$p_{\theta}(x) = \frac{2\sqrt{x} e^{-x/b}}{\sqrt{b^3 \pi}} \mathbb{1}_{\{x > 0\}}$$

$$L(x, \theta) = \prod_{i=1}^n p_{\theta}(x_i) \mathbb{1}_{\{x_i > 0\}} =$$

$$= \frac{2^n x^{n/2} e^{-\sum x_i/b}}{(b^3 \pi)^{n/2}} \mathbb{1}_{\{x_i > 0\}}$$

$$= \frac{\prod (2\sqrt{x_i}) \exp(-\sum x_i/b)}{(b^3 \pi)^{n/2}} \mathbb{1}_{\{x_i > 0\}} \rightarrow T(x) = \left\{ \prod x_i, \sum x_i \right\}$$

- МВС, полный

$$LL(x, \theta) = \log \left( \prod_{i=1}^n 2\sqrt{x_i} \right) - \frac{\sum x_i}{b} - \frac{3n}{2} \log b +$$

$$- \frac{n}{2} \log \pi$$

$$\frac{d}{d\theta} LL = \frac{\sum x_i}{b^2} - \frac{3n}{2b} = 0$$

$$\forall \quad 3nb = \sum x_i$$

$$\hat{b} = \frac{\sum x_i}{3n} = \frac{\bar{x}}{3}$$

$$\hat{\theta} = \hat{b}^2 = \frac{\bar{x}^2}{9} - \text{ОМТ}$$

$$\sum x_i \sim \Gamma(n, b) \Rightarrow$$

$$E\left(\frac{(\sum x_i)^2}{(3n)^2}\right) = \frac{1}{(3n)^2} b^2 \frac{\Gamma(n+2)}{\Gamma(n)} =$$

$$= \frac{b^2}{(3n)^2} (n+1)n = \frac{b^2(n+1)}{9n}$$



корректируем:

$$\tilde{\theta} = \frac{q_n \hat{\theta}}{n+1} \leftarrow \theta \theta^2$$

$\Rightarrow$

$$E \tilde{\theta} = \theta^2 \rightarrow \tilde{\theta} - \text{КРМД-оценка}$$

Н1.

$$x_1, \dots, x_n \text{ из } N(a, 2\sigma^2)$$

$$y_1, \dots, y_n \text{ из } N(b, \sigma^2)$$

$$z_1, \dots, z_n \text{ из } N(c, \sigma^2)$$

или где  ~~$a+b-c$~~   $a+b-c \neq 1$

$$\sqrt{n} \frac{\bar{x} - a}{\sigma} \sim N(0, 2)$$

$$\sqrt{n} \frac{\bar{y} - b}{\sigma} \sim N(0, 1)$$

$$\sqrt{n} \frac{\bar{z} - c}{\sigma} \sim N(0, 1)$$

$\Rightarrow$

$$\sqrt{n} \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\sigma} \sim N(0, 2)$$

$\Rightarrow$

$$\sqrt{\frac{n}{2}} \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\sigma} \sim N(0, 1)$$

$$\frac{n S_x^2 + n S_y^2 + n S_z^2}{\delta^2} \sim \chi_{3n-3}^2$$

$$\frac{\sqrt{\frac{n}{2}} \cdot \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\delta}}{\sqrt{\frac{1}{3n-3} \cdot \frac{n(S_x + S_y + S_z)}{\delta^2}}} \sim t_{3n-3}$$

$$\parallel$$

$$\sqrt{\frac{3n-3}{2}} \frac{\bar{x} + \bar{y} - \bar{z} - (a+b-c)}{\sqrt{S_x + S_y + S_z}}$$

$$\Downarrow$$

$$P_{(a,b,c,\delta)}(\dots \leq x) = S_{3n-3}(x)$$

$$\exists x_L: S_{3n-3}(x_L) = 1 - \alpha/2$$

$$\Rightarrow$$

$$P_{(a,b,c,\delta)}(-x_L \leq \dots \leq x_L) =$$

$$= P_{(a,b,c,\delta)}\left(x_L \sqrt{\frac{2(S_x + S_y + S_z)}{3n-3}} + \bar{x} + \bar{y} - \bar{z} - 1\right)$$

$$\geq 0 \geq -x_L \sqrt{\frac{2(S_x + S_y + S_z)}{3n-3}} + \bar{x} + \bar{y} - \bar{z} - 1$$

$$\Rightarrow BU = [\dots, \dots]$$

