

# Circle fractaloids and Pi constant screening method in quantum mechanics

Pavel Florián<sup>\*1,a</sup>

## Affiliations:

<sup>1</sup> J. Heyrovsky Institute of Physical Chemistry, Dolejškova 2155/3, 182 23 Prague 8, Czech Republic

<sup>a</sup> Author to whom correspondence should be addressed: [florian43@seznam.cz](mailto:florian43@seznam.cz)

## Abstract

The Dirac constant, also known as the reduced Planck constant, is the Planck constant divided by  $2\pi$ . This constant is utilized in the quantization of angular momentum and spin in quantum mechanics. Although it has not been directly measured as the regular Planck constant  $h$ , which was first used by Max Planck in [1] and [2] to explain the quantization of direct motion and the radiation of a black body, it is believed that circular motion can be accurately described by this constant.

In this study, a new method was developed for multi-nucleus electron systems with non-zero inter-electron integrals, where the Dirac constant is squared and the influence of the  $\pi$  value is not entirely eliminated from the description of the system's energy, electron energy levels and wavefunctions.

This method involves using alternative  $\pi$  constant values for circle fractaloids instead of the regular circle  $\pi$  for highly precise quantum chemistry computations, with the goal of determining the most accurate  $\pi$  value. The  $\pi$  values for circle fractaloids are found to be greater than the regular circle  $\pi$ , and the results suggest that particles may be moving in a fractal space-time for inner movements.

## Introduction

The circle  $\pi$  constant is used in quantum mechanics in various physical equations for the Bohr radius:

$$a = (\epsilon_0 h^2) / (\pi m_e e^2), \quad (1)$$

where  $a$  is Bohr radius,  $\epsilon_0$  is permittivity of vacuum,  $\pi$  is a circle constant,  $m_e$  is mass of electron and  $e$  is the elementary charge, for nucleus-nucleus potential energy

$$V = -\frac{e^2 Z_1 Z_2}{4\pi\epsilon_0 R}, \quad (2)$$

where  $Z_1$  and  $Z_2$  are charges of the nucleuses and  $R$  is inter-nucleuses distance in Bohr radiiuses, for electron-nucleus potential energy

$$V = -\frac{e^2 Z}{4\pi\epsilon_0} \left\langle \psi_i \left| \frac{1}{R} \right| \psi_i \right\rangle, \quad (3)$$

where  $\psi_i$  is wavefunction of i-th electron and  $R$  is electron-nucleus distance in Bohr radiiuses, for electron-electron potential energy:

$$V = \frac{e^2}{4\pi\epsilon_0} \left\langle \psi_i \psi_i \left| \frac{1}{R} \right| \psi_j \psi_j \right\rangle, \quad (4)$$

where  $\psi_j$  is wavefunction of j-th electron,  $R$  is inter-electron distance in Bohr radiuses, for electron-electron resonance integrals

$$V = \frac{e^2}{4\pi\epsilon_0} \left\langle \psi_i \psi_j \left| \frac{1}{R} \right| \psi_i \psi_j \right\rangle, \quad (5)$$

and for electron-electron kinetic energy (nucleuses with electrons-electron kinetic energies are described from a virial theorem as negatively taken one half of electron-nucleus potential energy in equilibrium)

$$T = \frac{\hbar^2}{8\pi^2 m_e} * \psi_i \nabla^2 \psi_j, \quad (6)$$

where  $\nabla^2$  is Laplace operator.

Thus, the  $\pi$  constant is entirely eliminated, including for inter-electron kinetic energy integrals, which depend on the second power of the Bohr radius via wavefunctions.

The screened values of the  $\pi$  constant for fractaloids and their impact on the precision of quantum chemistry computations can be preferably studied in molecules with a strong spin-orbit coupling effect.

$$\Delta E = Z^4 \frac{\mu_0}{4\pi} g_s \mu_B^2 \frac{1}{a_0^3 l(l+1/2)(l+1)} j(j+1)l(l+1)s(s+1), \quad (7)$$

where  $\mu_0$  is permeability of vacuum,  $g_s$  is g-factor,  $\mu_B$  is the Bohr magneton,  $a_0$  is Bohr radius,  $j$  is total angular momentum quantum number,  $l$  is magnetic quantum number and  $s$  is spin quantum number and this quantum numbers are quantified by reduced Planck constant – Dirac constant.

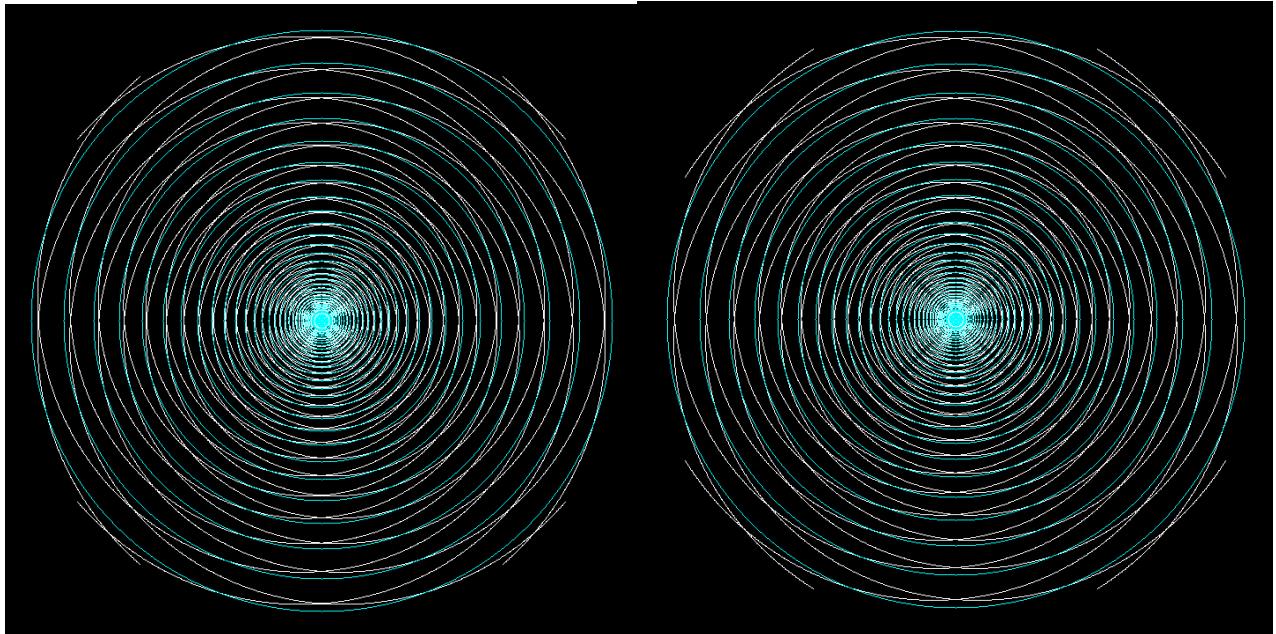


Fig. 1: Spiral fractaloid for 8 Golden spirals (left) and spiral fractaloid for spirals increased by 1.66 for each level (right). The approximated circles are shown by cyan.

In recent years, a constant  $\pi$  value has been developed or discovered for fractal-like circle structures composed of spirals. This study investigates the fractal-spiral motion in quantum mechanics, focusing on two types of spiral circle fractaloids. These fractaloids consist of 8 spirals with phases at  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  starting angles, and both clockwise and counterclockwise directions.

The two types of spiral circle fractaloids examined are:

1. Spiral fractaloids formed from Golden spirals, where the ratio of distances between neighboring levels is  $\phi = (\sqrt{5}+1)/2$ , corresponding to  $\pi = 4/\sqrt{\phi}$ , (See Fig. 1 left.).
2. Spiral fractaloids formed from spirals with half-circles between edge of circumscribed squares and circumference of a circle, corresponding to  $\pi = (14 - \sqrt{2})/4$ , (See Fig. 1 right).

Mathematic proofing for both values of Pi are available from [3] and [4].

## Derivation of the value $4/\sqrt{\phi}$

- Recently, in 11-dimensional string theories, it is possible to have balls of eight strings with the same center and the particles, which are too composed from strings.
- These strings vibrate at a set of harmonic frequencies, which are mutually harmonic.
- The frequencies of oscillations on the strings are mutually added and subtracted, resulting in a distribution that follows the Fibonacci sequence (where each number is the sum of the two preceding numbers).
- The limit of the Fibonacci sequence for the difference between two neighboring numbers in the sequence is the Golden ratio, and thus these strings are in a Golden spiral conformation.
- The optimized shape of this unit is a fractaloid consisting of 8 spirals with phases at  $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  starting angles, and both clockwise and counterclockwise directions.
- In this fractaloid, there are circle levels with  $\pi = 4/\sqrt{\phi}$ , as mentioned in Fig. 1.
- The particles in M-theory with 8 supersymmetries are a vibrating string circles and from this fractaloid can be folded a deformed circle levels and inter-levels of vibrating strings vibrating on tones with Fibonacci sequence distribution ( $\text{AdS}_4 \times S^7$  correspondence) [5].

This is a base model for a string loop effects for anomalous magnetic moments in spin-orbit coupling. The true model for multi-loop effect is more complex and probably not impressionable only by simple fractal structure.

## References

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