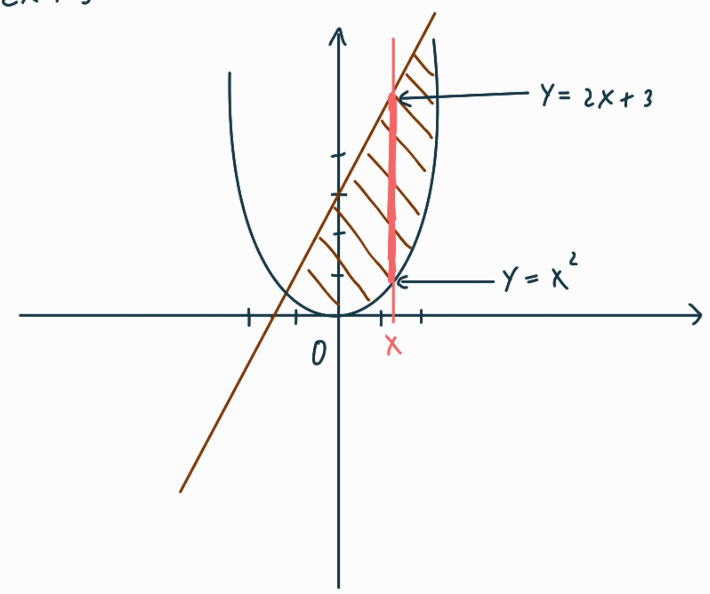
1) $I=\iint_A (x+2y)\,dx\,dy$ where A is the set bounded by the parabola $y=x^2$ and the line $y=2x+3$



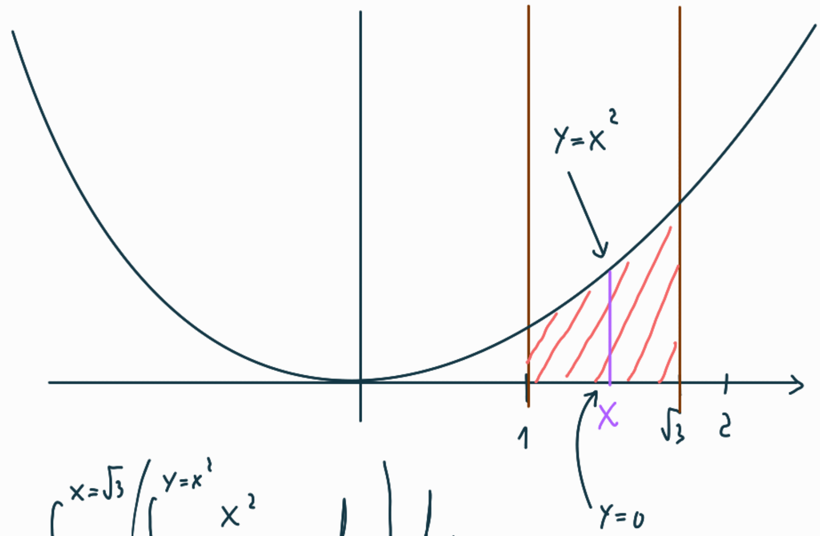
$$\left.\begin{aligned} y&=x^2\\ y&=2x+3\end{aligned}\right\} \Rightarrow x^2-2x-3=0$$

$$x_1=-1$$
$$x_2=3$$

$$\int_{x=-1}^{x=3}\left(\int_{y=x^2}^{y=2x+3} x+2y\,dy\right)dx=\int_{x=-1}^{x=3}(2x^2+3x-x^3+4x^2+9+12x-x^3-x^4)\,dx$$

$$=\int_{-1}^{3}(-x^4-x^3+6x^2+15x+9)\,dx$$

2) $I=\iint_A \frac{x^2}{x^2+y^2}\,dx\,dy$, A is curvilinear trapezoid bounded by the lines $x=1$, $x=\sqrt{3}$, $y=0$ and by the parabola $y=x^2$

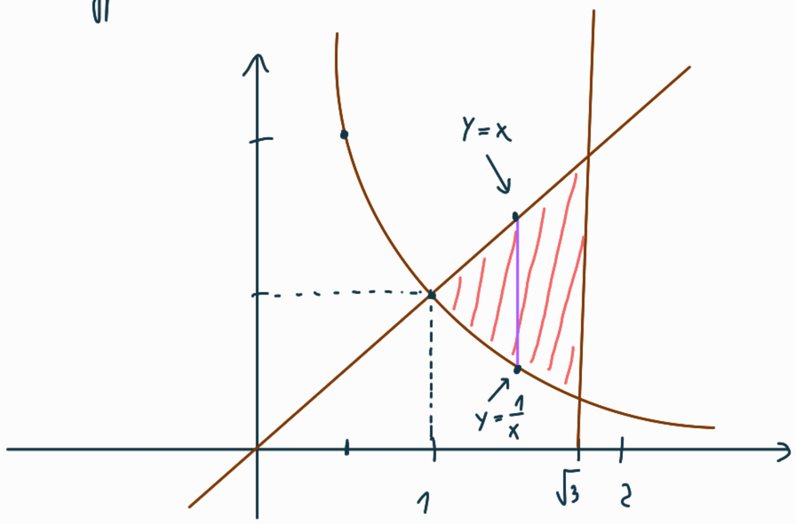


$$I=\int_{x=1}^{x=\sqrt{3}}\left(\int_{y=0}^{y=x^2}\frac{x^2}{x^2+y^2}\,dy\right)dx=$$

$$=\int_{x=1}^{x=\sqrt{3}}\left(x^2\cdot\frac{1}{x}\operatorname{arctg}\frac{y}{x}\Big|_{y=0}^{y=x^2}\right)dx=\int_1^{\sqrt{3}}x\cdot\operatorname{arctg}x\,dx=\int_1^{\sqrt{3}}\left(\frac{x^2}{2}\right)'\cdot\operatorname{arctg}x\,dx=$$

$$=\frac{x^2}{2}\operatorname{arctg}x\Big|_1^{\sqrt{3}}-\int_1^{\sqrt{3}}\frac{x^2}{2}\cdot\frac{1}{x^2+1}\,dx=\frac{3}{2}\cdot\frac{\pi}{3}-\frac{1}{2}\cdot\frac{\pi}{4}-\frac{1}{2}\int_1^{\sqrt{3}}\frac{x^2+1-1}{x^2+1}\,dx=\dots$$

3) $I=\iint_A \frac{x}{y^2+1}\,dx\,dy$ A is the set bounded by the lines $x=\sqrt{3}$, $y=x$ and the hyperbola $xy=1$



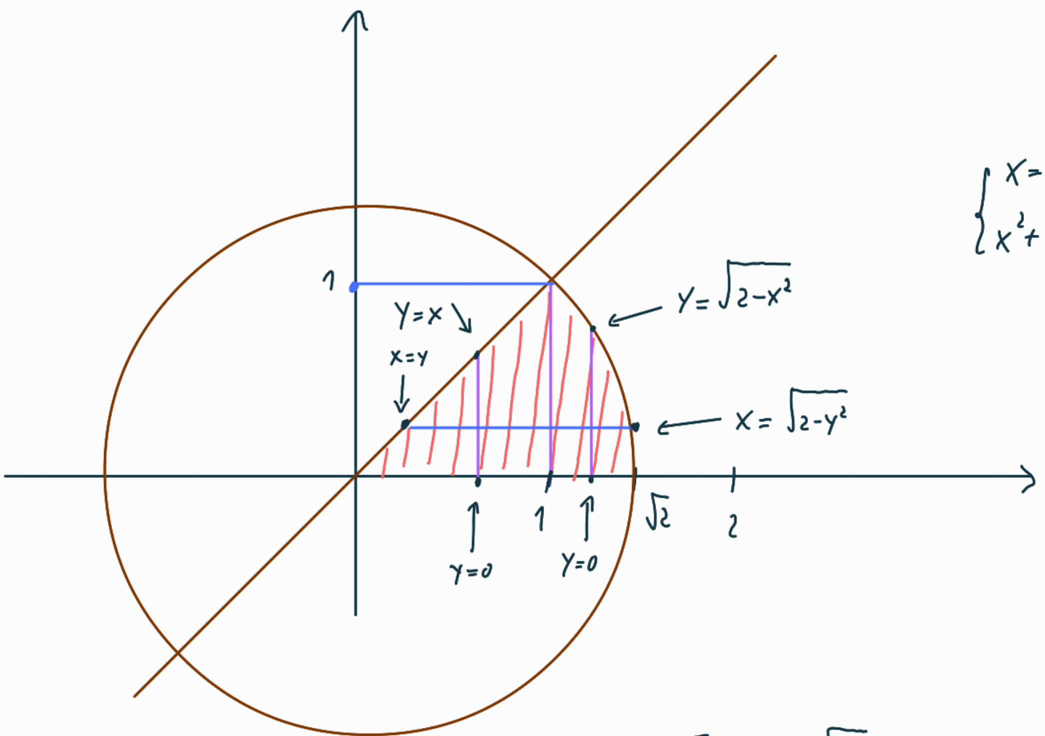
$$\operatorname{arctg} x+\operatorname{arctg}\frac{1}{x}=\begin{cases}\frac{\pi}{2}, & x>0\\ -\frac{\pi}{2}, & x<0\end{cases}$$

$$I=\int_{x=1}^{x=\sqrt{3}}\int_{y=\frac{1}{x}}^{y=x}\frac{x}{y^2+1}\,dx=\int_{x=1}^{x=\sqrt{3}}x\cdot\left(\operatorname{arctg} y\Big|_{y=\frac{1}{x}}^{y=x}\right)dx=\int_1^{\sqrt{3}}x\Big(\operatorname{arctg} x-\underbrace{\operatorname{arctg}\frac{1}{x}}_{\frac{\pi}{2}-\operatorname{arctg} x}\Big)\,dx=$$

$$=\int_1^{\sqrt{3}}x\left(2\operatorname{arctg} x-\frac{\pi}{2}\right)dx=\int_1^{\sqrt{3}}2x\operatorname{arctg} x\,dx-\left(\frac{\pi}{2}\,\frac{x^2}{2}\Big|_1^{\sqrt{3}}\right)=$$

$$=\int_1^{\sqrt{3}}(x^2)'\cdot\operatorname{arctg} x\,dx-\left(\frac{3\pi-1\pi}{4}\right)=\ \dots\ =1-\sqrt{3}+\frac{\pi}{3}$$

4) $I=\iint_A\frac{x}{y^2+1}\,dx\,dy$ $A=\{(x,y)\in\mathbb{R}^2 \mid x\geq y\geq 0,\ x^2+y^2\leq 0\}$



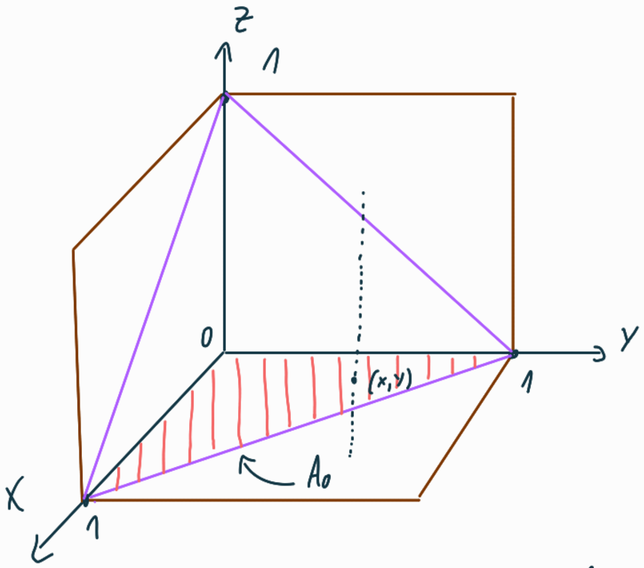
$$\begin{cases}x=y\\ x^2+y^2=2\end{cases}\Rightarrow x=1$$

$$I=\int_{x=0}^{x=1}\left(\int_{y=0}^{y=x}\frac{x}{y^2+1}\,dy\right)dx+\int_1^{\sqrt{2}}\left(\int_{y=0}^{y=\sqrt{2-x^2}}\frac{x}{y^2+1}\,dy\right)dx=$$

$$= \int_{y=0}^{y=1} \left(\int_{x=y}^{x=\sqrt{2-y^2}} \frac{x}{y^2+1}\, dx \right) dy = \int_{y=0}^{y=1} \frac{1}{1+y^2} \left(\frac{x^2}{2} \Big|_{x=y}^{x=\sqrt{2-y^2}} \right) dy =$$

$$= \int_0^1 \frac{1}{1+y^2} \cdot \frac{2-2y^2}{2}\, dy = \int_0^1 \frac{-1-y^2+2}{1+y^2}\, dy = (-y + 2\,\mathrm{arctg}\, y)\Big|_0^1 = \frac{\pi}{2} - 1$$

5) $I = \iiint_A \frac{1}{(x+y+z+1)^2}\, dx\, dy\, dz$ if A is the set bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$



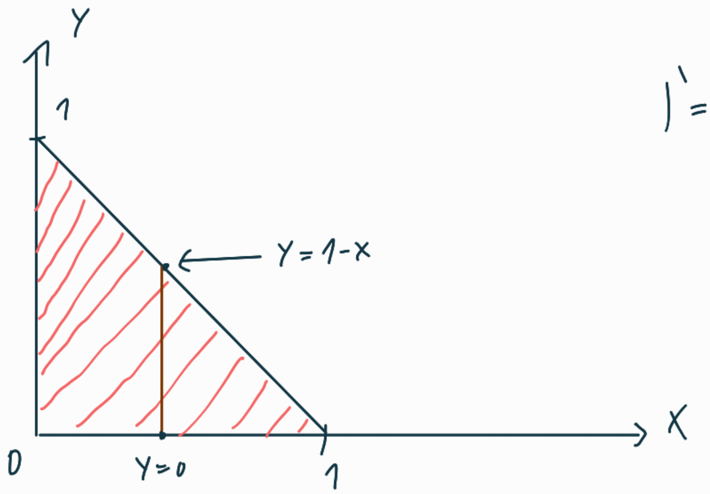
$$I = \iint_{A_0} \left(\int_{z=0}^{z=1-x-y} \frac{1}{(x+y+z+1)^2}\, dz \right) dx\, dy =$$

$$= \iint_{A_0} \frac{-1}{x+y+z+1} \Big|_{z=0}^{z=1-x-y} dx\, dy =$$

$$= \iint_{A_0} \left(-\frac{1}{2} + \frac{1}{x+y+1} \right) dx\, dy =$$

$$= -\frac{1}{2} \underbrace{\iint_{A_0} dx\, dy}_{\frac{1}{2}} + \underbrace{\iint_{A_0} \frac{1}{x+y+1}\, dx\, dy}_{I'}$$

$$= -\frac{1}{4} - I'$$



$$I' = \int_{x=0}^{x=1} \left(\int_{y=0}^{y=1-x} \frac{1}{x+y+1}\, dy \right) dx =$$

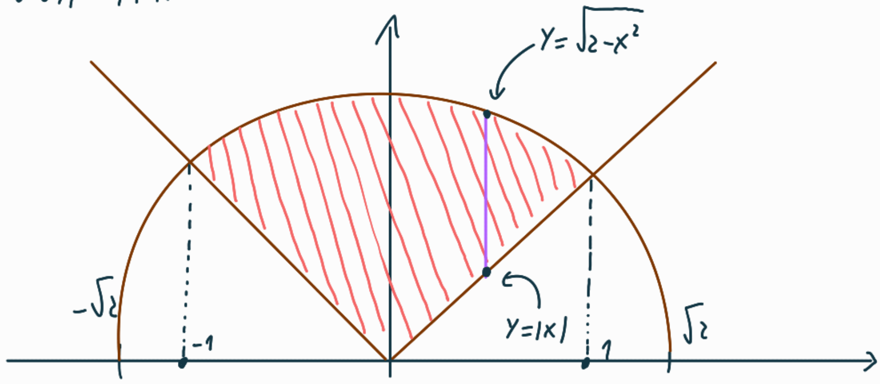
$$= \int_{x=0}^{x=1} \ln(x+y+1) \Big|_{y=0}^{y=1-x} dx =$$

$$= \int_0^1 \ln 2 - \ln(x+1)\, dx = \ln 2 - \int_0^1 (x+1)' \ln(x+1)\, dx$$

$$= \dots$$

$$\Rightarrow I = \frac{3}{4} - \ln 2$$

6) $I = \iint_A \frac{y}{1+x^2}\, dx\, dy$, $A = \{ (x,y) \in \mathbb{R}^2 \mid y \geq |x|,\ x^2+y^2 \leq 2 \}$



$$I = \int_{x=-1}^{x=1} \left(\int_{y=|x|}^{y=\sqrt{2-x^2}} \frac{y}{x^2+1}\, dy \right) dx = \int_{x=-1}^{x=1} \left(\frac{1}{x^2+1} \cdot \frac{y^2}{2} \Big|_{y=|x|}^{y=\sqrt{2-x^2}} \right) dx =$$

$$= \int_{-1}^{1} \frac{1}{1+x^2} \cdot \frac{2(1+x^2)}{2}\, dx = \ldots$$