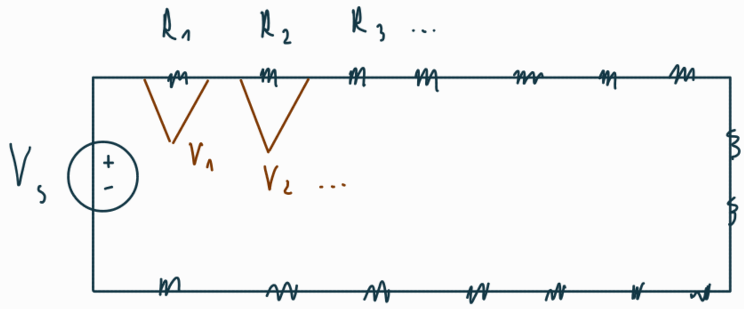
$$\sum_{i=1}^{n} p_i = 0 \qquad n - \text{nr. of elements in the circuit}$$

$$\sum_{j=1}^{n} i_j = 0 \qquad n - \text{nr. of branches that are connected to the node} = KCL$$

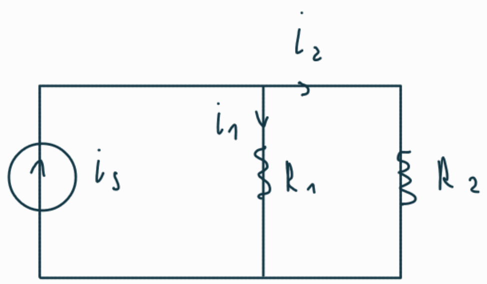
$$\sum_{i=1}^{n} V_i = 0 \qquad n - \text{nr. of element in the loop} = KVL$$

$$I = \frac{U}{R}$$

$$P = U \cdot I = I^2 \cdot R = \frac{U^2}{R} \quad ; \text{ in AC } \; P = \frac{1}{2} I^2 Z$$



$$V_i = V_s \cdot \frac{R_i}{\sum R}$$



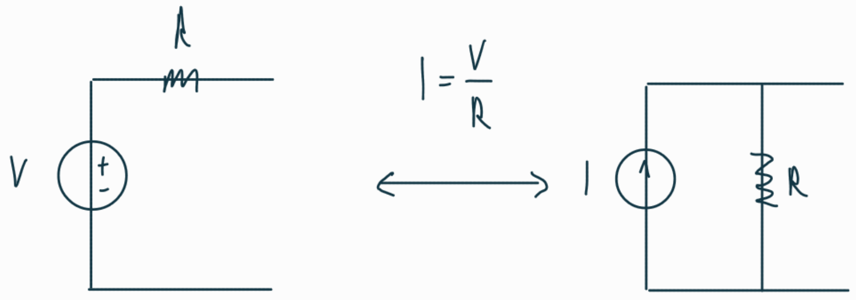
$$i_1 = i_s \frac{R_2}{R_1 + R_2}$$

$$i_2 = i_s \frac{R_1}{R_1 + R_2}$$

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c}, \\ R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c}, \\ R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}. \end{cases} \qquad \begin{cases} R_a = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_b = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \\ R_c = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \end{cases}$$

(Δ-to-Y) (Y-to-Δ)

NVM, MCM



Method 2 to only get R_{Th} alone

For circuits with or without dependent sources:

Step 1: Deactivate all independent sources;

Step 2: Apply a test voltage v_T or test current i_T source to the designated terminals;

Step 3: Calculate the terminal current i_T if a test voltage v_T source is used and vice versa;

Step 4: Get the Thévenin resistance by $R_{Th} = \frac{v_T}{i_T}$

$\max P \Rightarrow R_o = R_{TH}$

superposition

$W = \frac{1}{2} I^2 L$ $\qquad Z_L = j\omega L$

$W = \frac{1}{2} V^2 C$ $\qquad Z_c = \frac{1}{j\omega C}$

inductors $\Leftrightarrow$ resistors

capacitor $\Leftrightarrow$ resistors^{-1}

$k = \frac{M}{\sqrt{L_1 L_2}}$

$I = \frac{V}{Z}$

9.13 for Thévenin in AC circuits

$Z_{Th} = (100 + j1600) + Z_r$, where Z_r is the reflected impedance of Z_{11} due to the transformer:

$Z_{11} = (500 + j100) + (200 + j3600) = (700 + j3700)\ \Omega$

$$Z_r = \left(\frac{\omega M}{|Z_{11}|}\right)^2 Z_{11}^* = \left(\frac{1200}{|700+j3700|}\right)^2 (700 - j3700)$$

$Z_{Th} = (100 + j1600) + Z_r = (171.09 + j1224.26)\ \Omega$

! $j = i$ from complex numbers $j^2 = -1$ etc.

for +/-

rectangular form

for ·/:

polar form

$$a + bj = Ae^{j\theta} = A(\cos\theta + j\sin\theta) = A \angle \theta = A\cos(wt + \theta)$$

$$A = \sqrt{a^2 + b^2} \qquad \theta = \arctan\frac{b}{a}$$

j /

$$\frac{1}{j} = \frac{j}{-1} = -j$$

Assignment 1

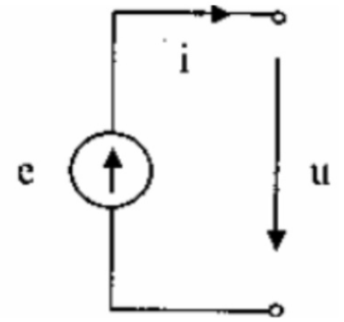
1. Given ideal voltage source as shown in figure. Find the wrong answer?

A. $u \in i$

B. $u = e$

C. $u \in e$

Assignment 4 starts on page 13, this is recap for me!



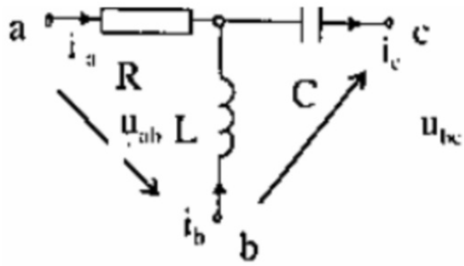
2. Given AC circuit as shown in figure. Which formula is wrong

A. $i_a + i_b - i_c = 0$

B. $i_a + i_b + i_c = 0$

C. $u_{ab} = Ri_a - Ldi_b/dt$

D. $u_{bc} = Ldi_b/dt + 1/C\int i_c dt$



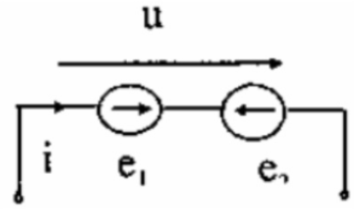
3. Given DC source as shown in figure. Find the correct answer?

A. $u = e_1 + e_2$

B. $u = e_2 - e_1$

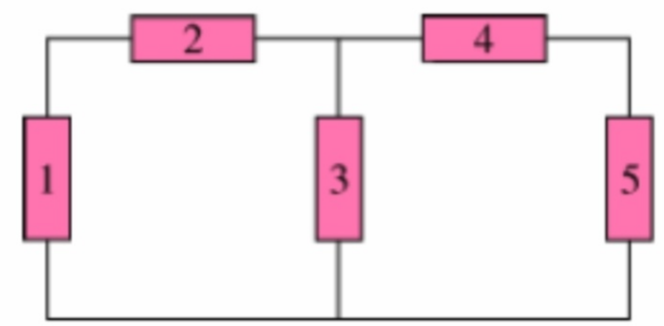
C. $u = e_1 - e_2$

D. $u = e_1 - e_2 - i$



womp

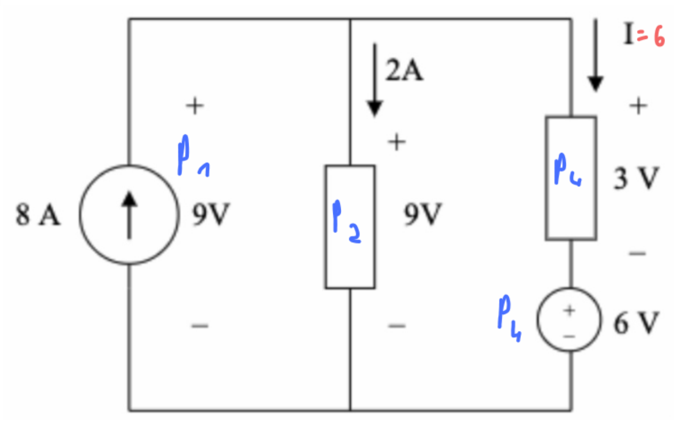
4. The figure shows a circuit with five elements. If p_1 = -200 W, p_2 = 60W, p_4 = 40W, p_5 = 30W. Calculate the power received or delivered by element 3 p_3.



$-P_3 = P_1 + P_2 + P_4 + P_5 \Rightarrow -P_3 = -200 + 60 + 40 + 30 \Rightarrow -P_3 = -70 \Rightarrow$

$\Rightarrow P_3 = 70$

5. Find I and P power of each element in the circuit below



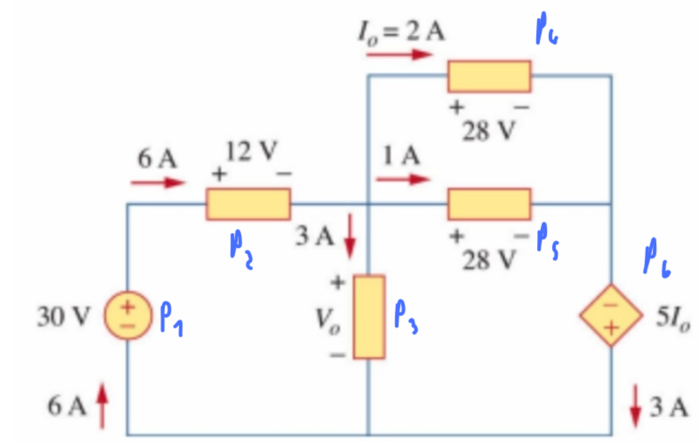
$P_1 = -72$ W

$P_2 = 18$ W

$P_3 = 18$ W

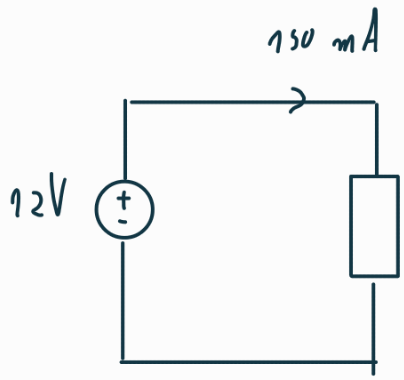
$P_4 = 36$ W

6. Find V_0 and the power absorbed by each element in the circuit below



$P_1 = -180$ W
$P_2 = 72$ W
$P_4 = 56$ W
$P_5 = 28$ W
$P_6 = -30$ W
$P_3 = 54$ W

7. A 12V car battery supplies a current of 150mA to a lightbulb. Find
a. The power absorbed by the bulb
b. The energy absorbed by the bulb over an interval of 20 minutes



$E = 1800 \cdot 20 \cdot 60 = 1800 \cdot 1200 = \ldots$

$P = 1800$ W

8. A flashlight battery has a rating of 0.8 Ah and a lifetime of 10 hours. It has two batteries 3V in series. Find
a. The current
b. Power P
c. Energy stored

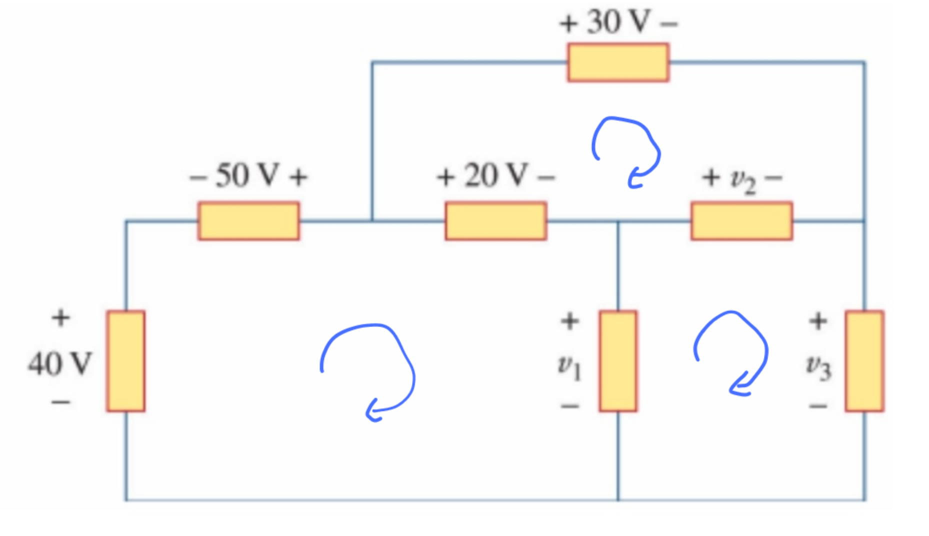
$\Rightarrow I = 0.08$ A $\quad P = 0.08 \cdot 6 = 0.48$ W $\quad E = 10 \cdot 60 \cdot 60 \cdot 0.48 =$

9. A 30W light bulb is connected to a 120V source and is left on continuously in an otherwise dark staircase. Determine:
a. The current through the lightbulb
b. The cost of operating the light for one (non-leap) year if electricity costs 12 cents per kWh.

$I = \frac{P}{U} \Rightarrow I = \frac{30}{120} = \frac{1}{4} = 0.25$ A $\quad P = 30$ W $= 30 \frac{1000}{3600}$ kWh

b) ...

10. Find v_1, v_2 and v_3 in the circuit below



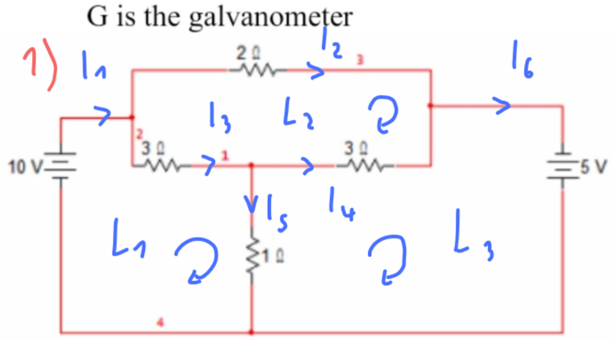
$v_1 = -40 - 50 + 20 = 70$ V

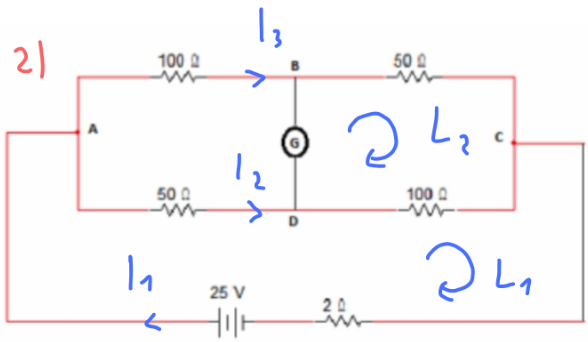
$v_2 = 30 - 20 = 10$ V

$v_3 = 70 - 10 = 60$ V

Assignment 2

1. Write KCL and KVL equations at node 1 (3 loops) and node A (2 loops) for the given circuits G is the galvanometer





1) $I_1 = I_2 + I_3$

$I_3 = I_5 + I_4$

$I_2 + I_4 = I_6$

$I_1 = I_5 + I_6$

$L_1: -10 + 3I_3 + I_5 = 0$

$L_2: 2I_2 - 3I_4 - 3I_3 = 0$

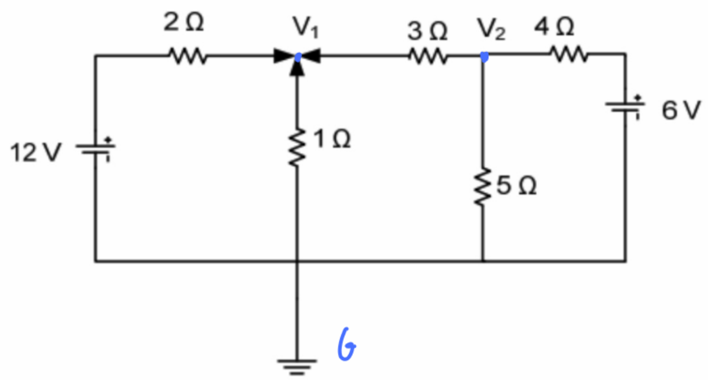
$L_3: -5 - I_5 + 3I_4 = 0$

2) $I_1 = I_2 + I_3$

$L_1: -25 + 2I_1 + 50I_2 + 100I_2 = 0$

$L_2: 100I_3 + 50I_3 - 100I_2 - 50I_2 = 0$

2. Given circuit below

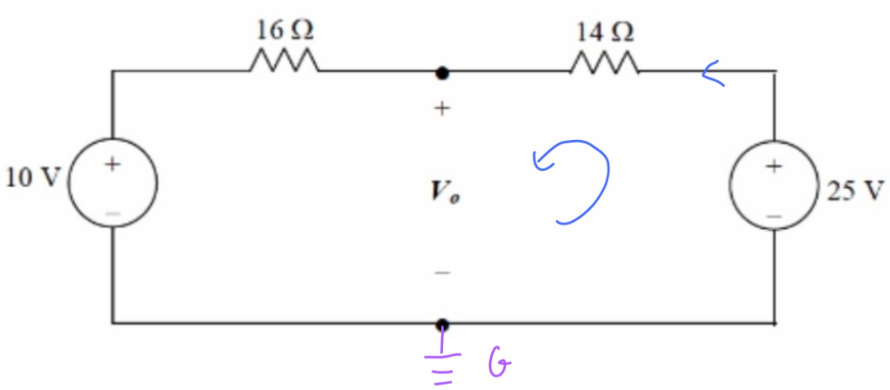


$$1: \frac{12 - V_1}{2} + \frac{V_2 - V_1}{3} - \frac{V_1}{1} = 0$$

$$2: \frac{6 - V_2}{4} - \frac{V_2 - V_1}{3} - \frac{V_2}{5} = 0$$

Find V_1, V_2 and current passing through each resistor. 1 is a node among element 2, 3, and 1 Ω and 2 are a node among element 3, 4 and 5Ω

3. Given circuit below. Use KVL to determine V_0 in the circuit



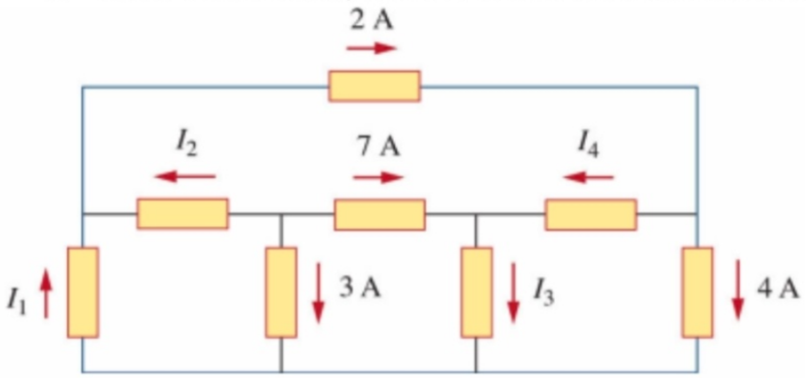
$-25 + 10 + 14I + 16I = 0 \Rightarrow$

$\Rightarrow -15 + 30I \Rightarrow I = 0.5\ A$

$\Rightarrow V_0 =$

$V_0 = 25 + 0.5 \cdot 14 = 25 + 7 = 32$

4. For the circuit below, use KLC to find the branch current I_1 to I_4:



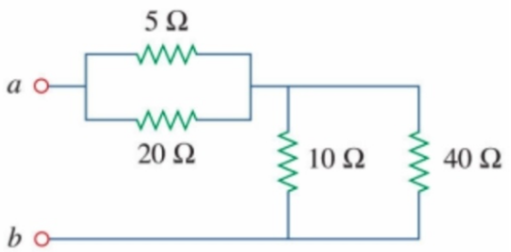
$2 - I_4 - 4 = 0 \Rightarrow I_2 = -2\ A$

$7 + I_4 - I_3 \Rightarrow I_3 = 5\ A$

$-I_2 - 7 - 3 = 0 \Rightarrow I_2 = -10\ A$

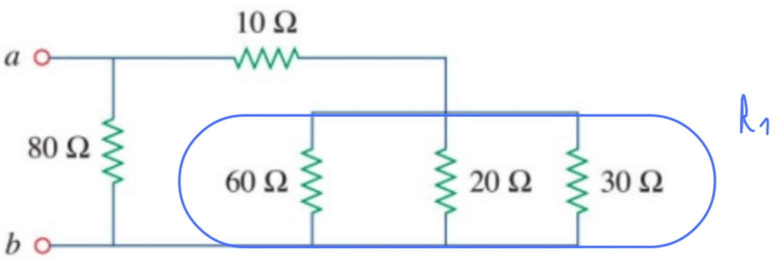
$I_1 + I_2 = 2 \Rightarrow I_1 = 12$

5. Calculate the equivalent resistance, R_{ab}, at terminals a-b for the circuit below



$R_{ab} = \frac{100}{25} + \frac{400}{50} = 4 + 8 = 12\ \Omega$

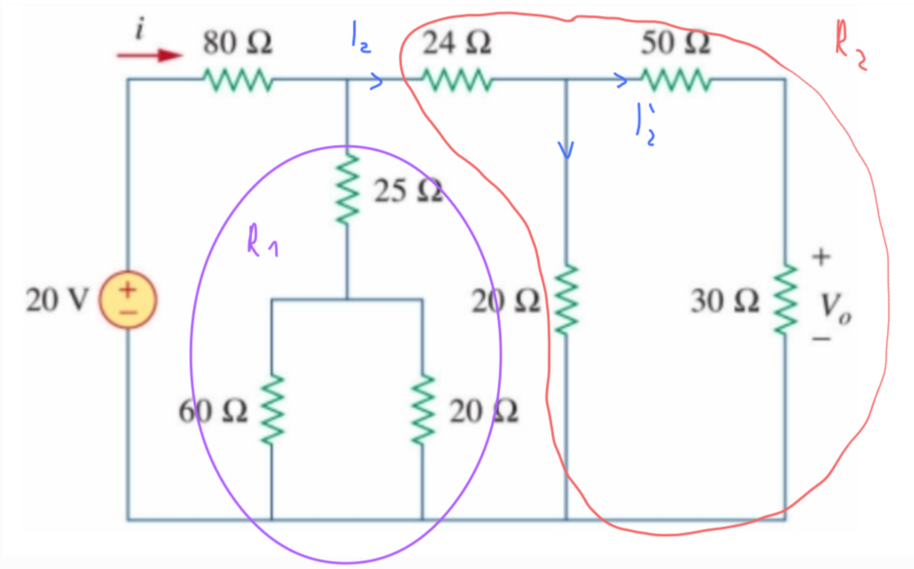
6. Calculate the equivalent resistance, R_{ab}, at terminals a-b for the circuit below



$R_1 = \frac{1}{\frac{1}{60} + {}^{3/}\frac{1}{20} + {}^{2/}\frac{1}{30}} = \frac{1}{\frac{1+3+2}{60}} = 10\ \Omega$

$R_{ab} = \frac{1600}{100} = 16\ \Omega$

7. Find i and V_0 in the circuit below. This is to practice calculating R_{eq} for different purposes – first collapsing the circuit to a single R_{eq} value and then expanding it back out gradually to find V_0

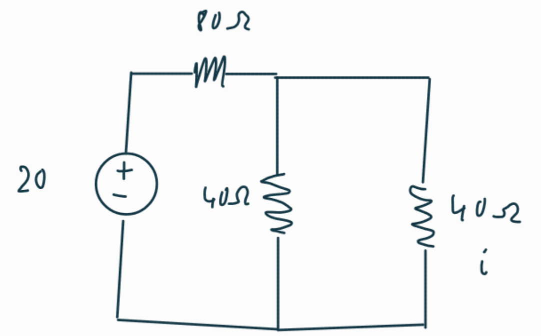


$$R_1 = 25 + \frac{1200}{80} = 25 + 15 = 40\,\Omega$$

$$R_2 = 24 + \frac{1600}{100} = 24 + 16 = 40\,\Omega$$

$$R_{eq} = 80 + \frac{1600}{80} = 80 + 20 = 100\,\Omega$$

$$\Rightarrow I = \frac{20}{100} = 0.2\,A$$

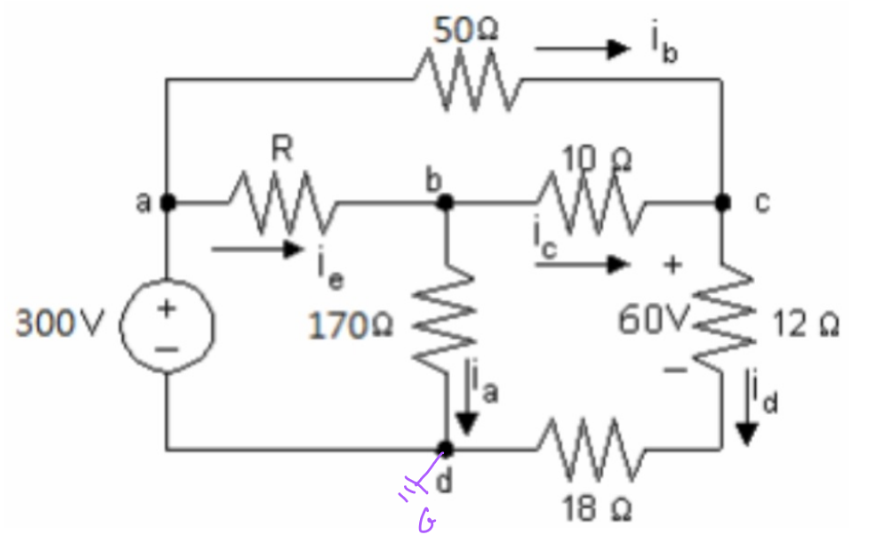


$$I_2 = \frac{40}{40+40} I = 0.5 \cdot 0.2 = 0.01\,A$$

$$I_2' = \frac{20}{100} I_2 = \frac{0.01}{5} = 0.002$$

$$\Rightarrow V_0 = 0.002 \cdot 30 = 0.06\,V$$

8. Given circuit below. Find the value of R?



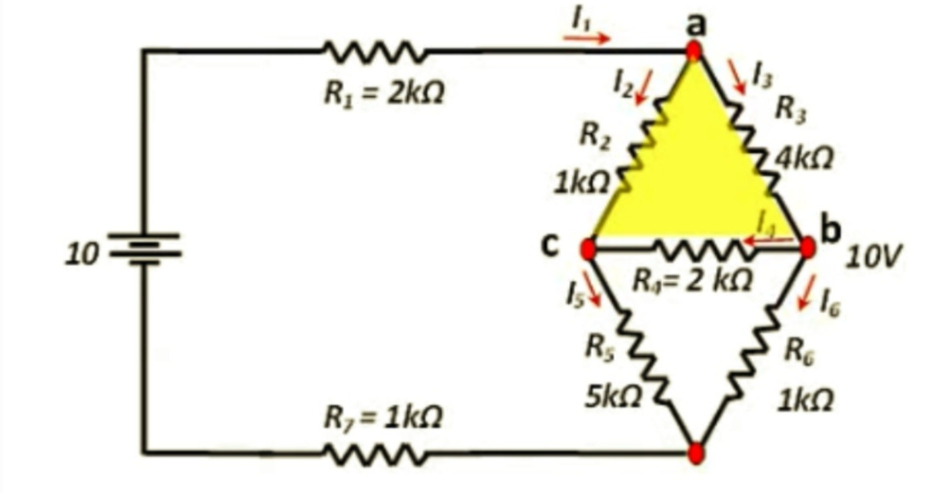
$$i_d = \frac{60}{12} = 5\,A \quad \Rightarrow \quad V_c = i_d \cdot (12 + 18) = 5 \cdot 30 = 150\,V$$

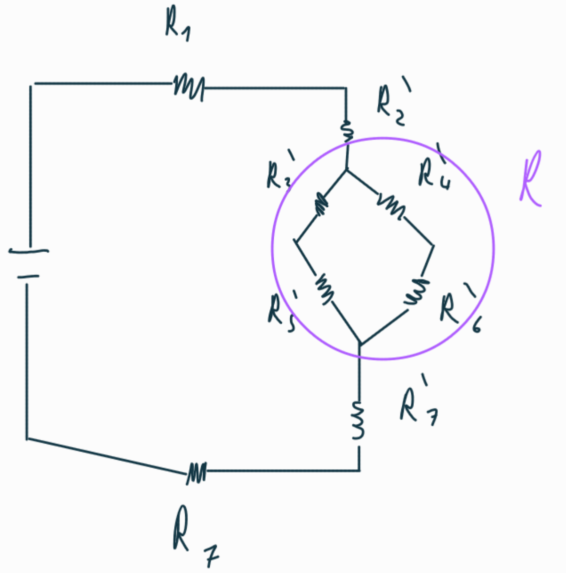
$$i_b = \frac{300 - V_c}{50} = \frac{150}{50} = 3\,A \quad \Rightarrow \quad i_c = 2\,A$$

$$i_c = \frac{V_b - V_c}{10} \Rightarrow \quad 20 = V_b - 150 \quad \Rightarrow \quad V_b = 170\,V$$

$$i_a = \frac{V_b}{170} = 1\,A \quad \Rightarrow \quad i_e = i_a + i_c = 3\,A \quad \Rightarrow \quad 3 = \frac{300 - V_b}{R} \Rightarrow \quad 3R = 130 \quad \Rightarrow \quad R = \frac{130}{3}\,\Omega$$

9. Given circuit below. Find each current in the circuit





$$R_2' = \frac{1 \cdot 4}{1+4+2} = \frac{4}{7} \qquad R_4' = \frac{2 \cdot 4}{7} = \frac{8}{7} \qquad R_3' = \frac{2}{7}$$

$$R_5' = \frac{10}{8} \qquad R_6' = \frac{2}{8} \qquad R_7' = \frac{5}{8}$$

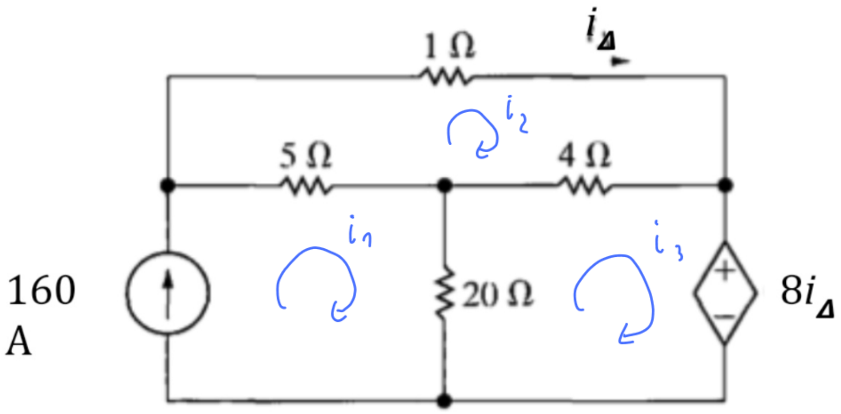
$$R_3' + R_5' = \frac{2}{7} + \frac{5}{4} = \frac{8+35}{28} = \frac{43}{28}$$

$$R_4' + R_6' = \frac{8}{7} + \frac{1}{4} = \frac{32+7}{28} = \frac{39}{28}$$

$$R = \frac{\frac{43}{28} \cdot \frac{39}{28}}{\frac{82}{28}} =$$

Assignment 3

1. The current source is 160 A and the current-controlled voltage source is $8i_\Delta$. Use the **mesh-current method** to determine power on each element in the circuit



$i_1 = 160$ A

$i_2 = i_\Delta$

$i_3 = ?$

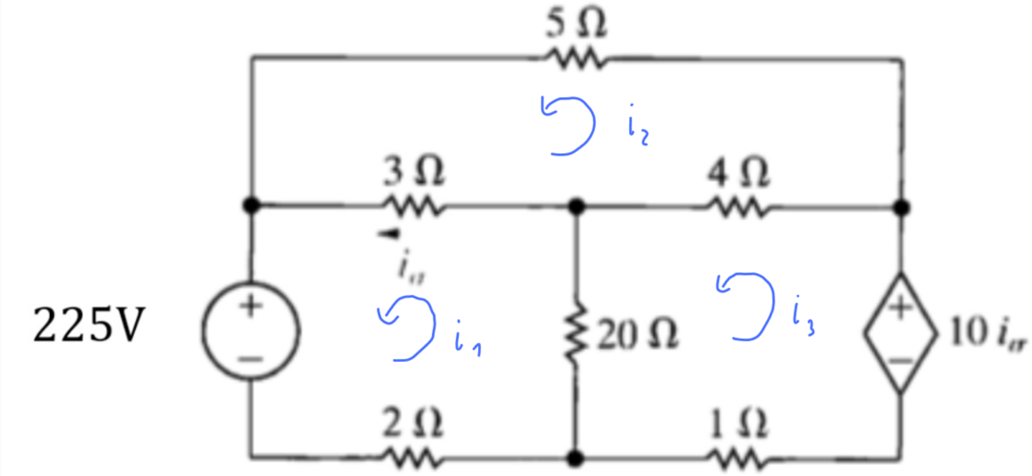
$M_1: \quad 160 + (i_1 - i_2)5 + (i_1 - i_3)20 = 0$

$M_2: \quad 1 \cdot i_2 + (i_2 - i_3)4 + (i_2 - i_1)5 = 0$

$$\Rightarrow \begin{cases} 25 \cdot 160 - 5i_2 - 20i_3 = 0 \\ 10i_2 - 4i_3 - 5 \cdot 160 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -5i_2 - 20i_3 = -160 \cdot 25 \\ 10i_2 - 4i_3 = 5 \cdot 160 \end{cases} \Rightarrow -44i_3 = -45 \cdot 160$$

2. The circuit is shown below. Use **the mesh-current method** to find the power extracted or dissipated by the current controlled voltage source.



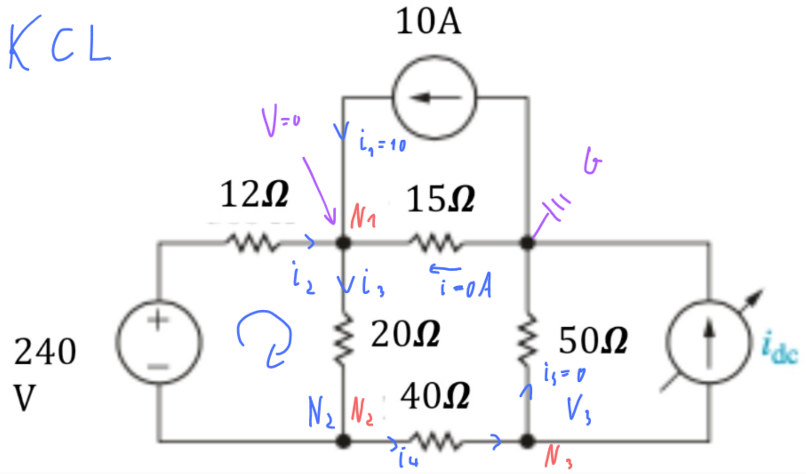
$M_1: \quad 25 i_1 - 3 i_2 - 20 i_3 + 225 = 0$

$M_2: \quad -3 i_1 + 12 i_2 - 4 i_3 - 10(i_1 - i_2) = 0$

$M_3: \quad -20 i_1 - 4 i_2 + 25 i_3 = 0$

3. The variable dc current source in the circuit below is adjusted so that the power developed by the 10A current source is zero. Find the value of i_{dc}.

KCL



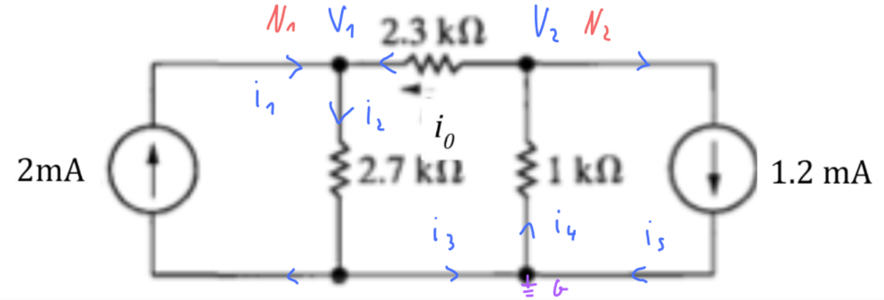
$N_1: \quad 10 + \frac{V_2 + 240 - 0}{12} - \frac{0 - V_2}{20} + 0 = 0$

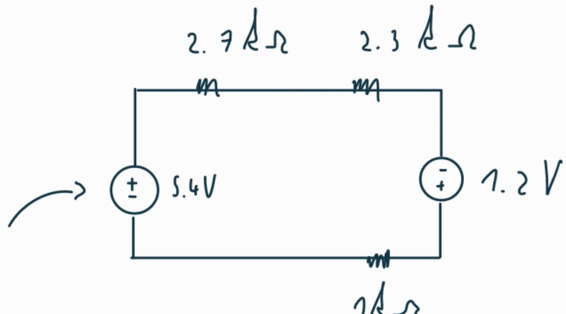
$N_2: \quad \frac{V_2}{20} - \frac{V_2 + 240 - 0}{12} - \frac{V_2 - V_3}{40} = 0$

$N_3: \quad \frac{V_2 - V_3}{40} - \frac{V_3 - 0}{50} - i_{dc} = 0$

$N_1 \Rightarrow V_2 = -225 \Rightarrow V_3 = -625 \Rightarrow i_{dc} = \frac{45}{2}$

4. The circuit is below
a. Use a series of source transformations to find the current i_0 in the circuit.
b. Verify your solution by using the node-voltage method to find i_0.





$\Rightarrow i_0 = \frac{6.6}{6} = 1.1 A$

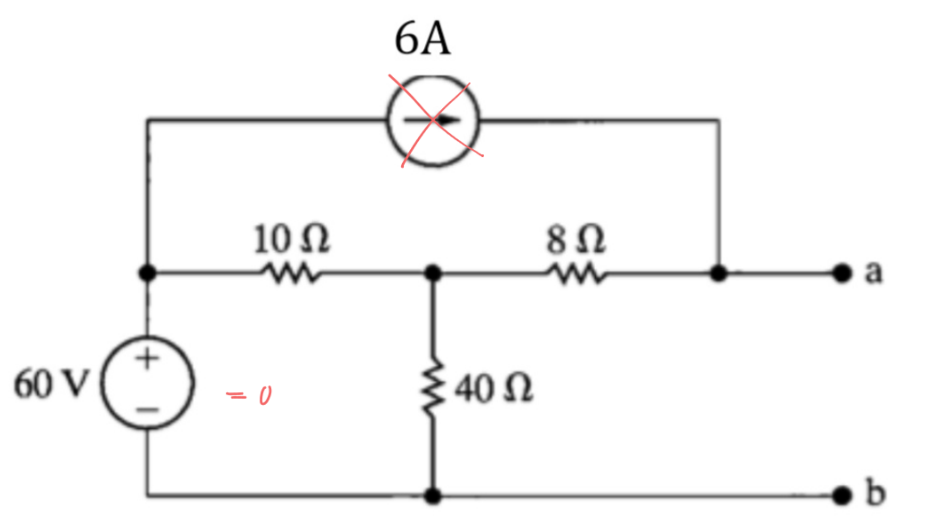
b) $i_1 = 2\,mA \quad i_s = 1.2\,mA$

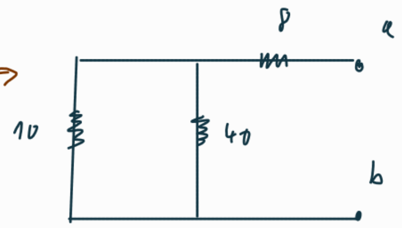
$$N_1: \quad 2 + \frac{V_2 - V_1}{2.3} - \frac{V_1}{2.7} = 0$$

etc.

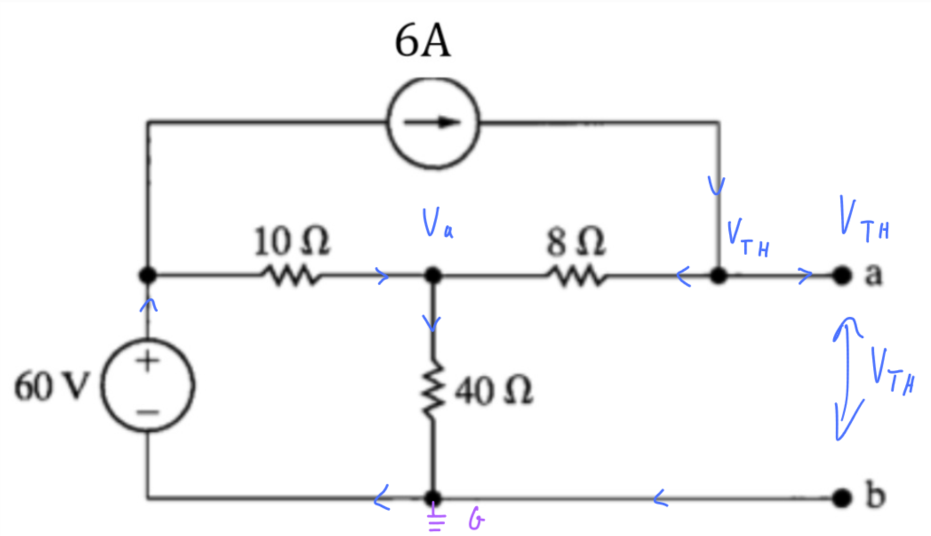
$$N_2: \quad \frac{0 - V_2}{1} - 1.2 - \frac{V_2 - V_1}{2.3} = 0$$

5. The **current source is 6 A** is shown below. Find the Thévenin equivalent with respect to the terminals a, b for the circuit



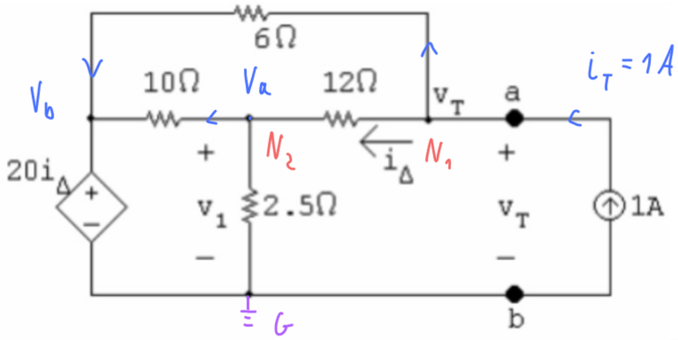


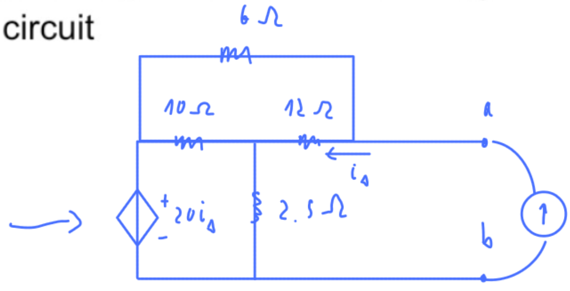
$$R_{TH} = \frac{400}{50} + 8 = 16\,\Omega$$



$$\left.\begin{array}{l} -\dfrac{V_{TH} - V_a}{8} + 6 = 0 \\[2ex] \dfrac{V_{TH} - V_a}{8} + \dfrac{60 - V_a}{10} - \dfrac{V_a}{40} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} V_a = \dots \\ V_{TH} = 144\,V \end{array}$$

6. The current controlled voltage source is $20i_\Delta$. Find the Thévenin equivalent with respect to the terminals a, b for the circuit





$$R_{TH} = \frac{V_{TH}}{i_T}$$

$$i_\Delta = \frac{V_{TH} - V_a}{12}$$

$$N_1: \quad 1 - \frac{V_T - 20 i_\Delta}{6} - \frac{V_T - V_a}{12} = 0$$

$$N_2: \quad \frac{V_T - V_a}{12} - \frac{V_a}{2.5} - \frac{V_a - 20 i_\Delta}{10} = 0$$

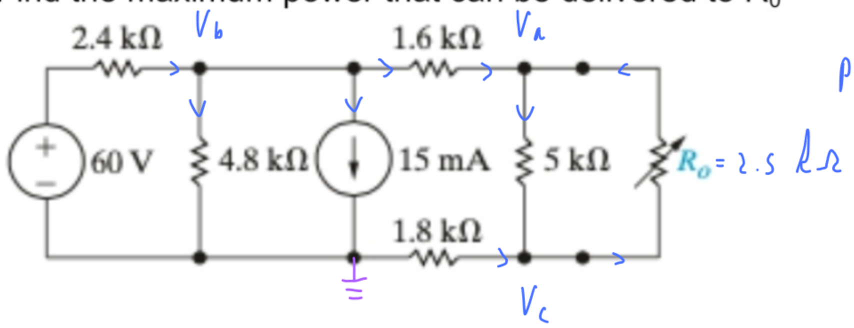
we know i_Δ =)

$$\Rightarrow V_T = 27\,V \rightarrow R_{TH} = 27\,V$$

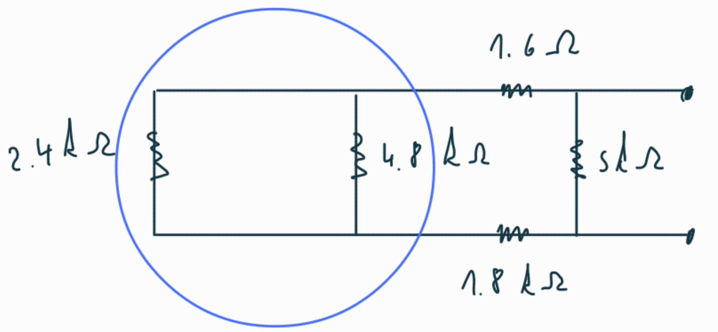
7. The given circuit is below. The variable resistor in the circuit is adjusted for maximum power transfer to R_0

a. Find the value of R_0

b. Find the maximum power that can be delivered to R_0



$P_{max} \Rightarrow R_0 = R_{TH}$



$R = 1.6\,k\Omega$

$R_{TH} = 2.5\,k\Omega = R_0$

$$N_a: \quad \frac{V_c - V_a}{2.5} + \frac{V_b - V_a}{1.6} - \frac{V_a - V_c}{5} = 0$$

$$N_b: \quad \frac{60 - V_b}{2.4} - \frac{V_b}{4.8} - 15 - \frac{V_b - V_c}{1.6} = 0$$

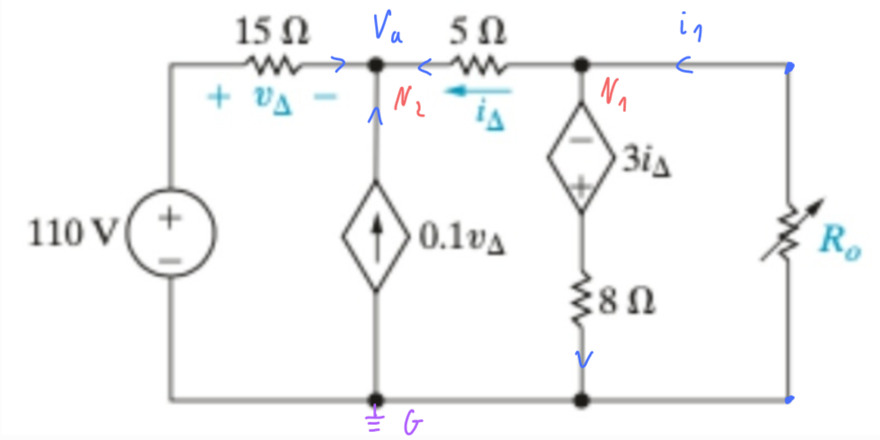
$$N_c: \quad \frac{0 - V_c}{1.8} + \frac{V_a - V_c}{5} - \frac{V_c - V_a}{2.5} = 0$$

$$\Rightarrow \left.\begin{aligned} V_c &= 4.32 \\ V_a &= 8.32 \end{aligned}\right\} \Rightarrow P = \frac{(V_c - V_a)^2}{R_0} = \frac{4^2}{2500} =$$

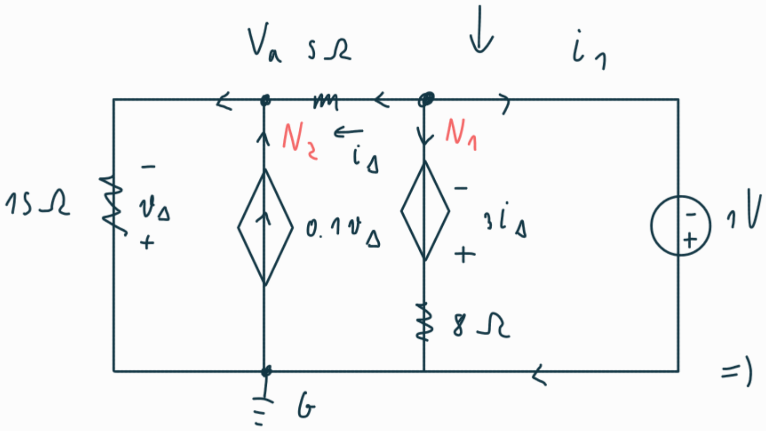
$$= \frac{16}{2500} = 0.064\,W$$

8. The variable resistor (R_0) in the circuit is adjusted until it absorbs maximum power from the circuit.

1. Find the value of R_0.
2. Find the maximum power.
3. Find the percentage of the total power developed in the circuit that is delivered to R_0.



$$i_\Delta = \frac{1-V_a}{5} = \frac{-0.2}{5} = -0.04$$



$$N_1: i_1 + \frac{1+3i_\Delta}{8} + i_\Delta = 0$$

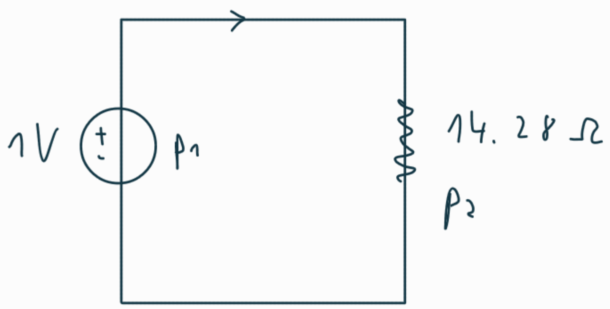
$$N_2: \frac{-V_a}{15} + \frac{V_a}{10} + \frac{1-V_a}{5} = 0 \quad / \cdot 30$$

$$\Rightarrow -2V_a + 3V_a + 6 - 6V_a = 0 \Rightarrow$$

$$\Rightarrow 5V_a = 6 \Rightarrow V_a = 1.2\,V$$

$$i_1 = 0.04 - \frac{1-0.12}{8} = 0.04 - \frac{0.88}{8} = 0.04 - 0.11 = -0.07\,A$$

$$\frac{V_T}{i_T} = R_{TH} = R_0 = \frac{1}{0.07} = 14.28\,\Omega$$

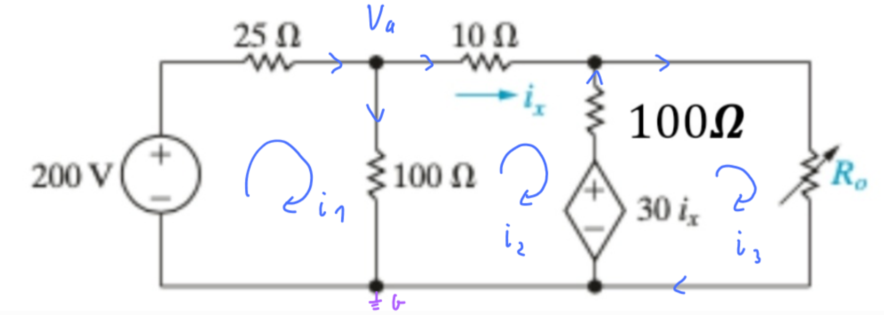


$$p_1 = U \cdot I = 0.07$$

$$p_2 = I^2 \cdot R = 1^2 \cdot \frac{1}{1} = 1$$

2, 3 → back to the original ...

9. The variable resistor (R_0) in the circuit below is adjusted until the power dissipated in the resistor is 225 W. Find the values of R_0 that satisfy this condition.



$I^2 R = 225 \Rightarrow$

$\Rightarrow R = \frac{225}{I^2}$ *

$\Rightarrow I = \frac{\pm 15}{\sqrt{R}}$

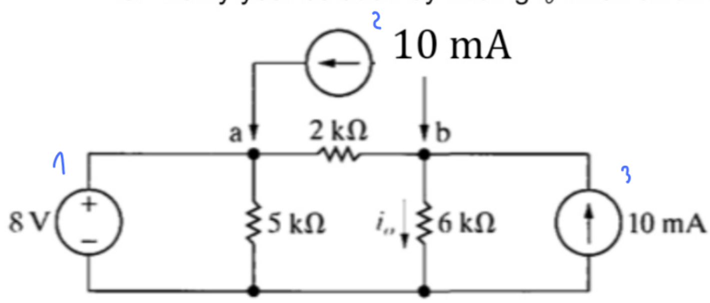
$$\begin{cases} -200 + 25 \cdot i_1 + 100 i_1 - 100 i_2 = 0 \\ 10 i_2 + 100(i_2 - i_3) + 30 i_2 + 100(i_2 - i_1) = 0 \\ 100(i_3 - i_2) + i_3 R_0 - 30 i_2 = 0 \iff 100(i_3 - i_2) + 225 i_3 - 30 i_2 = 0 \end{cases}$$

- $i_1 = \frac{8}{3}$ $\quad i_2 = \frac{4}{3}$ $\quad i_3 = \frac{8}{15}$ $\quad \Rightarrow R = \frac{225}{\frac{8}{225}} = \frac{1}{8} = 0.125\ \Omega$

10. In the circuit below, before the 5 m A current source is attached to the terminals a, b, the current i_0 is calculated and found to be 3.5 mA.

a. Use superposition to find the value of i_0 after the current source is attached.

b. Verify your solution by finding i_0 when all three sources are acting simultaneously.



1) $R_{eq} = \frac{5 \cdot 8}{13} = \frac{40}{13} \Rightarrow V = \frac{6}{8} \cdot 8 = 6 \qquad i_0' = \frac{6}{6} = 1$

2) $R_{eq} = \frac{2 \cdot 11}{13} = \frac{22}{13} \Rightarrow i_0'' = 10 \cdot \frac{2}{13} = \frac{20}{13} \qquad i_0 = i_0' + i_0'' + i_0''' = \frac{90}{13} + 1 = \frac{270}{13}$

3) $i_0''' = 10 \cdot \frac{7}{13} = \frac{70}{13}$

Assignment 4

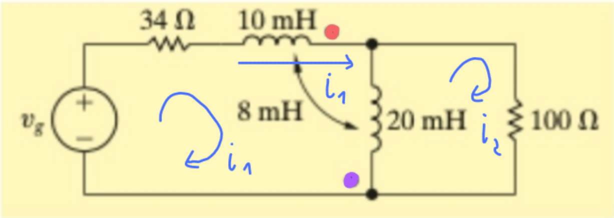
$\frac{1}{j\omega C}$ $\qquad \frac{1}{j} = -j$

in which dot

induced by

1. The two circuits are below

a. Find the average power delivered to the 100 Ω resistor in the circuit shown if v_g = 660cos(5000t) V.



$10\ mH \longrightarrow j\omega L = 5\cdot 10^3 \cdot 10 \cdot 10^{-3} = 50j\ \Omega$

$20\ mH \longrightarrow 100j\ \Omega$

$8\ mH \longrightarrow 40j\ \Omega$

$$\begin{cases} -v_g + 34 i_1 + 50j\, i_1 + 100j\,(i_1 - i_2) + 40j\,(i_1 - i_2) + 40j\, i_1 = 0 \\ 100j\,(i_2 - i_1) + 100\, i_2 - 40j\, i_1 = 0 \end{cases} \quad (=)$$

$$v_g = A\cos(\omega t + \theta) = A\cdot(\cos\theta + j\sin\theta) = 660\cdot(1 + 0\cdot j) = 660$$

$$A e^{j\theta} = A\cos(\omega t + \theta)$$

$$a + bj = A e^{j\theta} = A \angle \theta$$

$$A = \sqrt{a^2 + b^2}$$

$$\theta = \arctan\frac{b}{a}$$

i_1 pleacă $\longrightarrow p\ +$

$\longrightarrow i\ -$

i_1 intră $\longrightarrow p\ -$

$\longrightarrow i\ +$

$$\Rightarrow \begin{cases} 34 i_1 + 230j\, i_1 - 140j\, i_2 = 660 \\ -140j\, i_1 + 100\, i_2 + 100j\, i_2 \end{cases} \Rightarrow i_1 = \frac{100(1+j)}{140}\, i_2$$

$$\Rightarrow \frac{34\cdot 100(1+j)}{140}\, i_2 + \frac{230j\ 100\cdot(1+j)}{140}\, i_2 - 140j\, i_2 = 660$$

$$\Rightarrow \frac{3400(1+j)}{140}\, i_2 + \frac{23000j\,(1+j)}{140}\, i_2 - 140j\, i_2 = 660 \ /\cdot 140$$

$$\Rightarrow 3400 i_2 + 3400j\, i_2 + 23000j\, i_2 - 23000\, i_2 - 19600j\, i_2 = 660 \Rightarrow$$

$$\Rightarrow -19600\, i_2 + 6800j\, i_2 = 660 \Rightarrow \ /\frac{1}{i_2} \Rightarrow$$

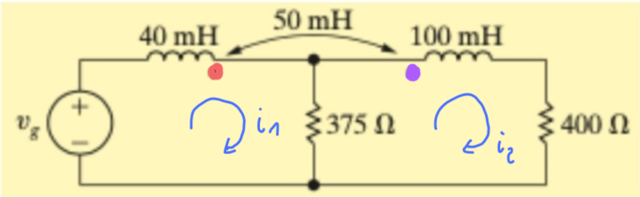
$$\Rightarrow -19600 + 6800j = \frac{660}{i_2} \Rightarrow i_2 = \frac{660}{-19600 + 6800j} = \frac{165}{-4900 + 1700j} = \frac{808500 + 280500j}{4900^2 - 1700^2} =$$

$4900 + 1700j$

$$= \frac{808500 + 280500j}{21120000} = \frac{8085 + 2805j}{211200} = \frac{1617 + 561j}{42240}$$

$$P = \frac{1}{2}\cdot i_2^2 \cdot R$$

b. Find the average power delivered to the 400 Ω resistor in the circuit shown if v_g = 248cos(10,000t) V. Find the average power delivered to the 375 Ω resistor. Find the power developed by the ideal voltage source.



$j\omega L$

$40\ mH \to 40 \cdot 10^{-3} \cdot 10^{4} = 400j$

$50\ mH \to 50 \cdot 10^{-3} \cdot 10^{4} = 500j$

$100\ mH \to 1000j$

$$v_g = 248\cos(10.000t) = 248 \cdot (\cos 0 + j\sin 0) = 248$$

$$\begin{cases} 400j\, i_1 + 375(i_1 - i_2) - 500j\, i_2 = 248 \\ 1000j\, i_2 + 400 i_2 + 375(i_2 - i_1) - 500j\, i_1 = 0 \end{cases} \Rightarrow \begin{cases} (375 + 400j)\, i_1 + (-375 - 500j)\, i_2 = 248 \\ (375 + 1400j)\, i_2 + (-375 - 500j)\, i_1 = 0 \end{cases}$$

$$\Rightarrow i_1 = \ldots \qquad i_2 = \ldots \qquad P_{375} = \frac{1}{2}(i_1 - i_2)^{2?} \cdot 375 =$$

$$P_{v_g} = V \cdot I\,?$$

$$Ae^{j\theta} = A\angle\theta = A\cos(\omega t + \theta) = A(\cos\theta + j\sin\theta)$$

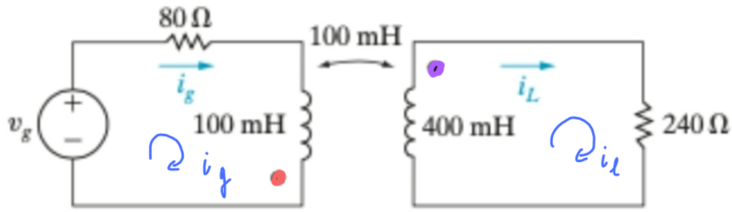
2. The circuit is below

a) Find the steady-state expression for the currents i_g and i_L in the circuit when v_g = 168cos(800t) V

b) Find the coefficient of coupling k.

c) Find the energy stored in the magnetically coupled coils at t = 625π μs and t = 1250π μs.

$$k = \frac{M}{\sqrt{L_1 \cdot L_2}} = \frac{100\ mH}{\sqrt{40000}\ mH} = \frac{1}{2}$$



$j\omega L$

$100\ mH \to 100 \cdot 10^{-3} \cdot 8 \cdot 10^{2} = 80j$

$400\ mH \to 320j$

$$v_g = 168\cos(800t + 0) = 168(\cos 0 + j\sin 0) = 168$$

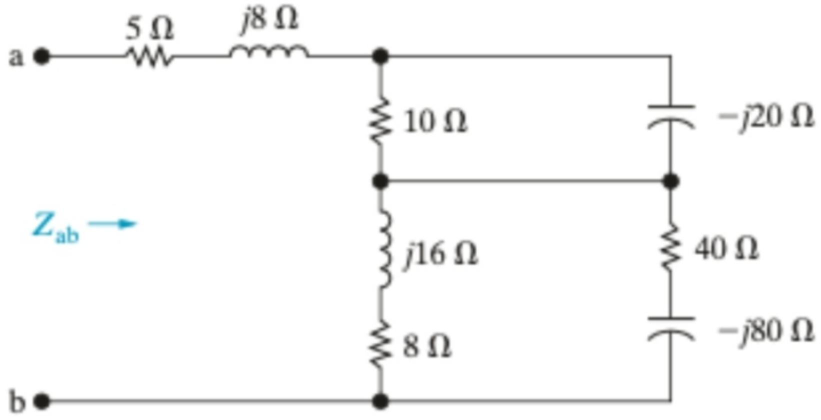
$10(1+j)\, i_g + 10j\, i_L = 21 \Rightarrow -10(1+j)(4+3j)\, i_L + 10j\, i_L = 21$

$\Rightarrow i_L = -0.3(1+6j) \Rightarrow i_g = 0.3(22-3j)$

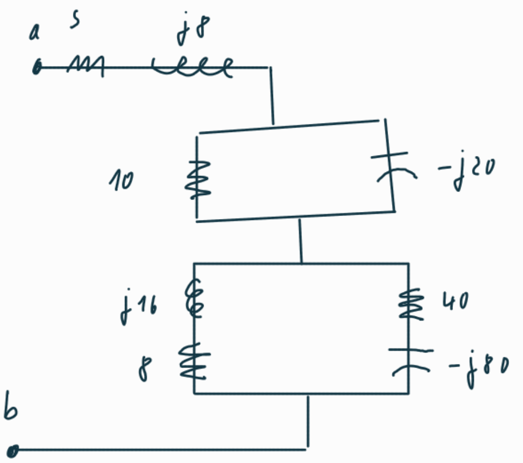
$$\begin{cases} 80 i_g + 80j\, i_g + 80j\, i_L = 168 \\ 240 i_L + 320j\, i_L + 80j\, i_g = 0 \end{cases} \Rightarrow \begin{cases} 20 i_g + 20j\, i_g + 20j\, i_L = 42 \Rightarrow 10 i_g + 10j\, i_g + 10j\, i_L = 21 \Rightarrow \\ 3 i_L + 4j\, i_L + j\, i_g = 0 \Rightarrow /\frac{1}{j} \Rightarrow \frac{3}{j} i_L + 4 i_L + i_g = 0 \Rightarrow \end{cases}$$

$$\Rightarrow 3j\, i_L + 4 i_L = -i_g \Rightarrow i_g = -(4+3j)\, i_L$$

4. Find the impedance Zab in the circuit



$\longrightarrow$



$$Z_{ab} = 5 + 8j + \frac{(-200j)}{10(1+2j)} + \frac{8(1+2j)\cdot 40\cdot(1-2j)}{48-64j} = 5 + 8j - \frac{\overset{1-2j\,/}{20j}}{1+2j} + \frac{\overset{3+4j\,/}{20(1+4)}}{3-4j} =$$

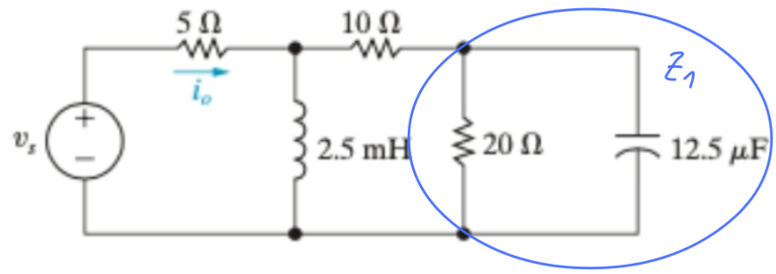
$$= 5 + 8j - \frac{20j-40}{5} + \frac{300+400j}{25} = \frac{625+200j-100j+200+300+400j}{25} = \frac{1125+500j}{25} = 45+20j$$

5. The circuit is operating in the sinusoidal steady state. Find $i_0(t)$ if $v_s(t) = 25$ ~~sin~~ cos 4000t V.

$\frac{1}{j\omega C}$

$\parallel$

$A\cos(\omega t + \theta)$



$$2.5\,mH \longrightarrow 2.5\cdot 10^{-3}\cdot 4\cdot 10^{3} = 10j$$

$$12.5\,\mu F \longrightarrow \frac{1}{j\cdot 4\cdot 10^{3}\cdot 12.5\cdot 10^{-6}} = \frac{1}{50j\cdot 10^{-3}} = \frac{1}{5}\cdot 10^{2}\cdot\frac{1}{j} = -20j$$

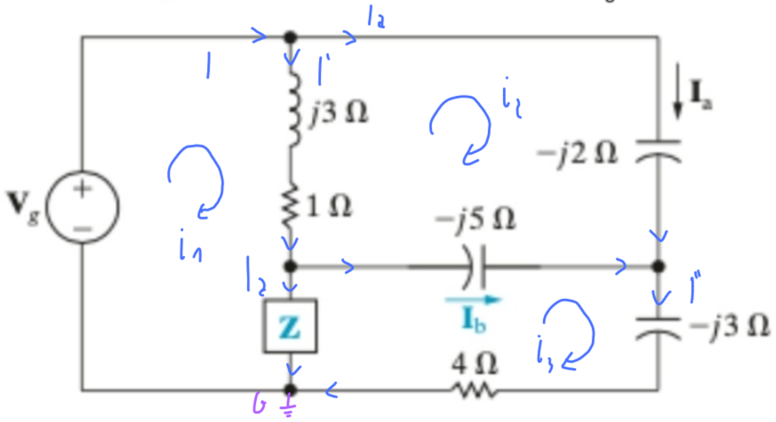
$$Z_1 = \frac{20\cdot(-20j)}{20(1-j)} = \frac{\overset{1+j\,/}{-20j}}{1-j} = \frac{-20j+20}{2} = 10-10j$$

$$Z_{eq} = \frac{(10-10j)\cdot 10j}{20-10j+10j} \overset{+}{=} \frac{200j-100+5}{20} = -5+10j+5 = 10j$$

$$I = \frac{V}{Z} = \frac{25\cos(4000t)}{10j} = \frac{5}{2}(-j)\cos(4000t) = (-j)\cdot\frac{5}{2}(\cos 0 + j\sin 0) = (-j)\,\frac{5}{2}\cdot 1 = \frac{-5}{2}j =$$

$$= -\frac{5}{2}(\cos 90 + j\sin 90^\circ) = -\frac{5}{2}\cos\left(4000t + \frac{\pi}{2}\right) = I$$

6. Find I_b and Z in the circuit below if $V_g = 25\angle 0^0$ V and Ia $= 5\angle 90^0$ A.



$V_g = 25\angle 0^\circ = 25\cos(t+0^\circ) = 25(\cos 0^\circ + j\sin 0^\circ) = 25$

$i_2 = I_a = 5\angle 90^\circ = 5\cos(t+90^\circ) = 5(\cos 90 + j\sin 90) = 5j$

$M_1:\ 3j(i_1-i_2) + i_1 - i_2 + Z(i_1-i_3) = V_g$

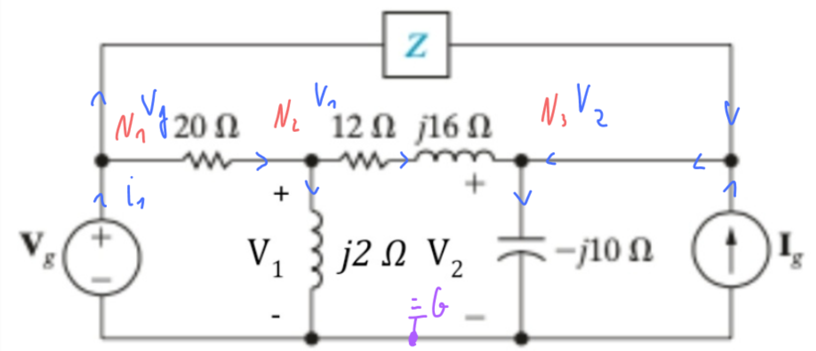
$M_2:\ 3j(i_2-i_1) - 2j\,i_2 - 5j(i_2-i_3) + i_2 - i_1 = 0$

$M_3:\ -5j(i_3-i_2) - 3j\,i_3 + 4i_3 + Z(i_3-i_1) = 0$

$$\Rightarrow \begin{cases} 3j i_1 + i_1 - 5j + Z i_1 - Z i_3 = 10 \\ -15 - 3j i_1 + 10 + 25 - 5j i_3 + 5j - i_1 = 0 \\ -5j i_3 - 25 - 3j i_3 + 4 i_3 + Z i_3 - Z i_1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (1+3j+Z) i_1 + (-Z) i_3 = 5(2+j) \\ (-1-3j) i_1 + (-5j) i_3 = -5(4+j) \\ (-Z) i_1 + (4-8j+Z) i_3 = 25 \end{cases}$$

7. Find the value of Z in the circuit, if V_g = 100 - j50 V, I_g= 30 + j20 A, and V_1 =140+j30V.



$$N_1: \quad i_1 - \frac{V_g - V_1}{20} - \frac{V_g - V_2}{Z} = 0$$

$$i_1 - \frac{-40-80j}{20} + \frac{100-50j - V_2}{Z} = 0$$

$$N_2: \quad \frac{V_g - V_1}{20} - \frac{V_1 - V_2}{4(3+4j)} - \frac{V_1}{2j} = 0 \quad \Rightarrow$$

$$(1) \quad N_3: \quad \frac{V_1 - V_2}{4(3+4j)} - \frac{V_2}{(-j10)} + \frac{V_g - V_2}{Z} + I_g = 0$$

$$\frac{-40-80j}{20} - \frac{140+30j - V_2}{4(3+4j)} - \frac{140+30j}{2j} = 0 \Rightarrow -2-4j-70-15j = \frac{140+30j - V_2}{4(3+4j)} \Rightarrow$$

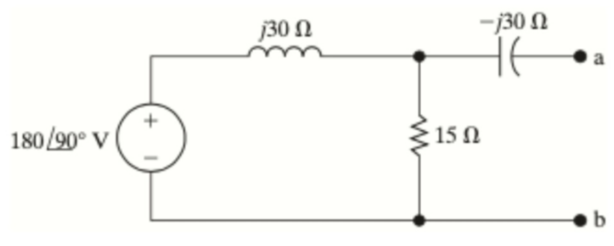
$$\Rightarrow (-72-19j)\,4(3+4j) = 140+30j - V_2 \Rightarrow$$

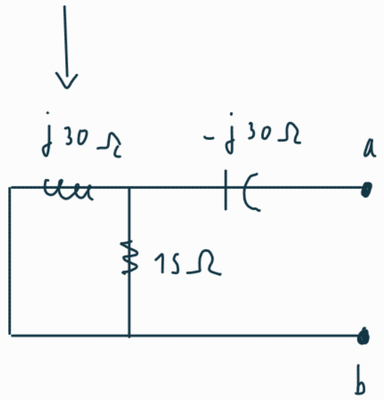
$$\Rightarrow -4(-72-19j)(3+4j) + 140 + 30j = V_2 \Rightarrow V_2 = 700 + 1410j$$

$$N_3: \quad \frac{140+30j - 700 - 1410j}{4(3+4j)} + \frac{700+1410j}{10j} + \frac{100-50j - 700 - 1410j}{Z} + 30 + 20j = 0$$

$$\Rightarrow Z = -90 - 576j$$

8. Use source transformations to find the Norton equivalent circuit with respect to the terminals a,b for the circuit





$$Z_{eq} = -30j + \frac{2 \cdot 15^2 \cdot j}{15 + 30j} =$$

$$= -30j + \frac{450j}{15(1+2j)} = -30j + \frac{30j}{1+2j} =$$

$1-2j$

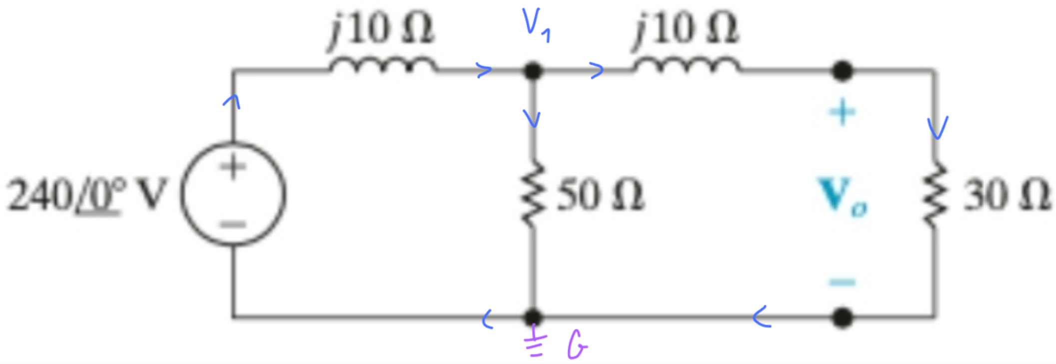
$$= -30j + \frac{30j + 60}{5} = -30j + 6j + 12 =$$

$$= 12 - 24j = 12(1-2j)$$

best bet: $i_N = \frac{V}{R_N} = \frac{180 \angle 90°}{12(1-2j)} = \frac{15(\cos 90° + j \sin 90°)}{1-2j} = \frac{15j - 30}{5} = -6 + 3j = -3(2-j)$

$1+2j$

9. Use the node-voltage method to find V_0 in the circuit:



$240 \angle 0° = 240$

$$\frac{240 - V_1}{10j} - \frac{V_1}{30 + 10j} - \frac{V_1}{50} = 0 \Rightarrow \frac{240 - V_1}{10}(-j) - \frac{V_1}{10(3+j)} - \frac{V_1}{50} = 0 \Rightarrow$$

$3-j$

$$\Rightarrow \frac{-240j + V_1 j}{10} - \frac{(3-j)V_1}{10 \cdot 4} - \frac{V_1}{50} = 0 \quad / \cdot 200 \Rightarrow$$

nimród

$$\Rightarrow -4800j + 20j V_1 - 50(3-j)V_1 - 40V_1 = 0 \Rightarrow$$

$$\Rightarrow 20j V_1 - 150V_1 + 50jV_1 - 40V_1 = 4800j \Rightarrow$$

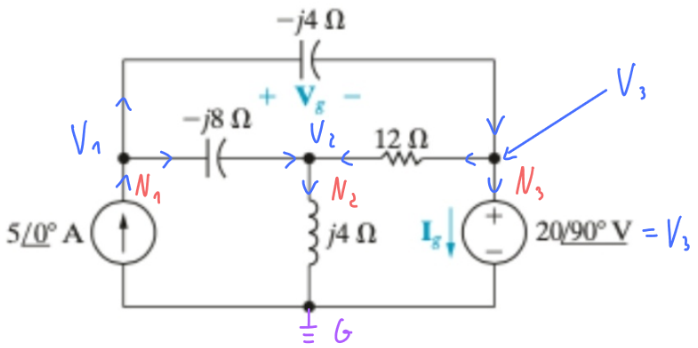
$$\Rightarrow V_1(-190+70j) = 4800j \Rightarrow V_1 = \frac{4800j}{-190+70j} = \frac{480j}{-19+7j} = \frac{336}{41} - \frac{912}{41}j$$

$7j+19$

$3-j$

$$V_0 = V_1 \cdot \frac{30}{30+10j} = V_1 \cdot \frac{3}{3+j} = \frac{9-3j}{4} \cdot V_1 = \frac{29}{41} - \frac{922}{41}j$$

10. Use the node-voltage method to find the phasor voltage V_g in the circuit



$$5\angle 0° A = 5(\cos 0° + j \sin 0°) = 5 \cdot (1 + j \cdot 0) = 5$$

$$20\angle 90° V = 20(\cos 90° + j \sin 90°) = 20(0 + j \cdot 1) = 20j = V_3$$

$$N_1: \quad 5 - \frac{V_1 - V_3}{-4j} - \frac{V_1 - V_2}{-8j} = 0$$

$$N_2: \quad \frac{V_1 - V_2}{-8j} + \frac{V_3 - V_2}{12} - \frac{V_2}{4j} = 0$$

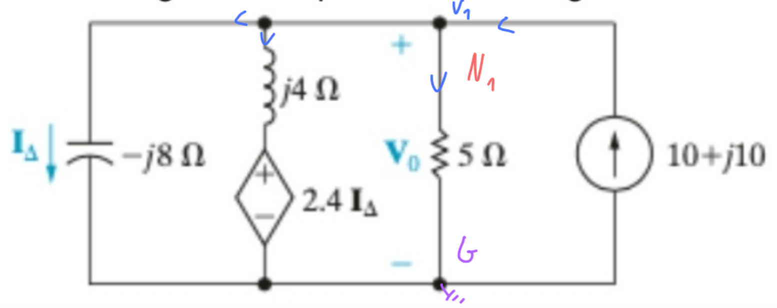
$\Rightarrow V_1 = \frac{-20}{3}$ $\Rightarrow V_g = V_1 - V_3 = \frac{-20}{3} + 20j =$

$V_2 = \dots$ not needed

$$= \frac{-20+60j}{3} = \frac{-20(1-3j)}{3}$$

$$\left(N_3: \quad \frac{V_1 - V_3}{-4j} - I_g - \frac{V_3 - V_2}{12} = 0 \right)$$

11. Use the node-voltage method to find the phasor voltage V_o in the circuit. Express the voltage in both polar and rectangular form.



$$I_\Delta = \frac{V_1}{-8j}$$

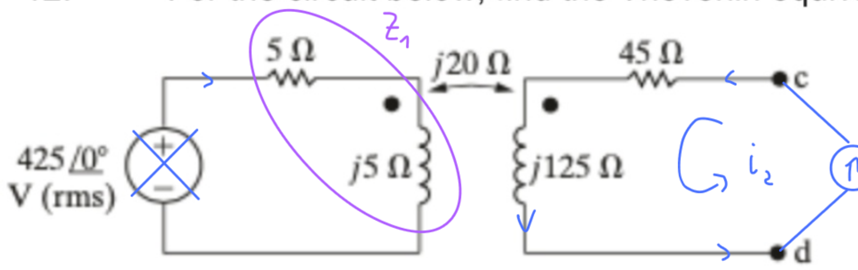
$$\frac{1}{j} = -j$$

$$N_1: \quad 10 + j10 - \frac{V_1}{5} - 2.4 I_\Delta - \frac{V_1}{-8j} = 0 \Rightarrow 10(1+j) - \frac{V_1}{5} + 2.4\frac{V_1}{8j} + \frac{V_1}{8j} = 0 \Rightarrow$$

$$\Rightarrow 10(1+j) - \frac{V_1}{5} + \frac{3V_1}{10} + \frac{V_1}{8j} = 0 \;/ 40 \Rightarrow 400(1+j) - 8V_1 + 12V_1 - 5V_1 = 0$$

$\Rightarrow \quad V_1 = -400(1+j) = -400\cdot\sqrt{2}\, e^{j45^\circ}$

12. For the circuit below, find the Thévenin equivalent with respect to the terminals c,d



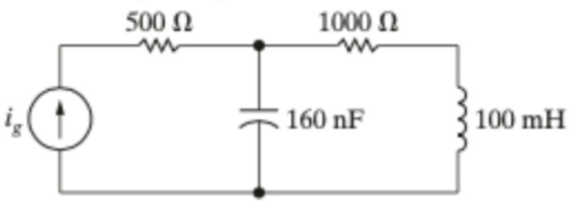
$425\angle 0^\circ = 425$

$1\angle 0\ A$

$$Z_{th} = (45+125j) + Z_r \quad ; \quad Z_r = \left(\frac{\omega M}{|Z_1|}\right)^2 \cdot Z_1^* = \left(\frac{w\cdot 20j}{5\sqrt{2}}\right)^2 \cdot (5-5j) = \ ???$$

w not known

13. Find the average power delivered by the ideal current source in the circuit below if i_g = 4cos(5000t) mA.



$4\cos(5000t+0) = 4(\cos 0^\circ + j\sin 0^\circ) = 4\ mA$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j\cdot 5\cdot 10^{3}\cdot 160\cdot 10^{-9}} = \frac{1}{j\,800\cdot 10^{-6}} = \frac{-j}{8\cdot 10^{-4}} = -j\cdot 0.125\cdot 10000 = -1250j$$

$$Z_L = j\omega L = j\cdot 5\cdot 10^{3}\cdot 100\cdot 10^{-3} = 500j$$

$4+3j$

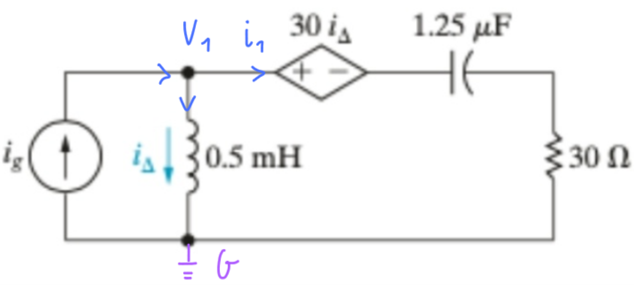
$$Z_{eq} = \frac{(1000+500j)\cdot(-1250j)}{1000-750j} + 500 = \frac{-500(2+j)\cdot 1250j}{250(4-3j)} + 500 = \frac{-2(2j-1)\cdot 1250}{4-3j} + 500 =$$

$$= \frac{-2(8j-4-6-3j)\cdot 1250}{25} + 500 = \frac{-2(-10+5j)\cdot 10\cdot 25\cdot 5}{25} + 500 =$$

$$= -100(-10+5j) + 500 = 1000 - 500j + 500 = 500(3-j)$$

$V = i_g\cdot Z_{eq} = 4\cdot 10^{-3}\cdot 5\cdot 10^{2}\cdot(3-j) = 2(3-j) \qquad P = \frac{1}{2} V\cdot I = \frac{1}{2}\cdot 2\cdot(3-j)\cdot 4\cdot 10^{-3} = 4(3-j)\,10^{-3}$

14. Find the average power dissipated in the 30 Ω resistor in the circuit if i_g = 6cos(20,000t) A. $= 6\ V$



$$0.5\ mH \rightarrow \frac{1}{2}\cdot 10^{-3}\cdot 2\cdot 10^{4} = 10j\ \Omega$$

$$1.25\ \mu F \rightarrow \frac{1}{j\cdot 1.25\cdot 10^{-6}\cdot 2\cdot 10^{4}} = \frac{-j}{2.5\cdot 10^{-2}} =$$

$$= -j\cdot 100\cdot 0.4 = -40j$$

$$6 - \frac{V_1}{10j} - \frac{V_1 + 30 i_\Delta}{-40j+30} = 0 \iff 6 + \frac{V_1 j}{10} - \frac{V_1 - 3jV_1}{-40j+30} = 0 \iff$$

$$i_\Delta = \frac{V_1}{10j} = \frac{-V_1 j}{10}$$

$$30\frac{V_1}{10j} = \frac{3V_1}{j} = -3jV_1$$

$$\iff 6 + \frac{V_1 j}{10} - \frac{V_1(1-3j)}{10(3-4j)} = 0 \quad /\cdot 10 \implies$$

$$3+4j$$

$$\iff 60 + V_1 j - \frac{V_1(3-9j+4j+12)}{25} = 0 \implies$$

$$\implies 60 + V_1 j - \frac{V_1(15-5j)}{25} = 0 \implies$$

$$\implies 60 + V_1 j - \frac{V_1(3-j)}{5} = 0 \quad /\cdot 5 \implies$$

$$\implies 5jV_1 - V_1(3-j) = -300 \implies$$

$$\implies V_1(5j - 3 + j) = -300 \implies V_1 = \frac{-300}{-3+4j} = \frac{300}{3-4j} =$$

$$3+4j$$

$$= \frac{300(3+4j)}{25} = 12(3+4j)$$

$$i_\Delta = \frac{-j\,12(3+4j)}{10} = \frac{6(-3j+4)}{5} \implies i_1 = i_g - i_\Delta = 6 - \frac{6(4-3j)}{5} = \frac{6+18j}{5}$$

$$P = \frac{1}{2}|I|^2\cdot R = \frac{1}{2}\cdot 30\cdot\left(\frac{6(1+3j)}{5}\right)^2 = 3\cdot\frac{36\cdot(1+6j-9)}{5} = \frac{108(-8+6j)}{5} = \frac{-216(4-3j)}{5}$$

3. A 400 Hz sinusoidal voltage with a maximum amplitude of 100 V at t = 0 is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 25 A.

a) What is the frequency of the inductor current? 400 Hz

b) If the phase angle of the voltage is zero, what is the phase angle of the current? 90°

c) What is the inductive reactance of the inductor? 4 Ω

d) What is the inductance of the inductor in millihenry (mH)? $\frac{4}{2\pi\cdot 400} = \frac{1}{2\pi\cdot 100} = \frac{1}{2\pi}\cdot 10^{-2} = \frac{10}{2\pi}\cdot 10^{-3} = \frac{5}{\pi}\ mH$

e) What is the impedance of the inductor? 4 Ω

$400\ Hz \implies \nu = 400\ Hz$