SEMINAR 10

1) Let V, V' be K-vector spaces, $f : V \to V'$ a linear map, $A \leq_K V$ and $A' \leq_K V'$. Show that:

a) $f(A) = \{f(a) \in V' \mid a \in A\} \leq_K V'$;

b) $\overset{-1}{f}(A') = \{x \in V \mid f(x) \in A'\} \leq_K V$.

2) In the $\mathbb{R}$-vector space $\mathbb{R}^{\mathbb{R}} = \{f \mid f : \mathbb{R} \to \mathbb{R}\}$ we consider

$$\mathbb{R}^{\mathbb{R}}_o = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is odd}\},\ \mathbb{R}^{\mathbb{R}}_e = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is even}\}.$$

Show that $\mathbb{R}^{\mathbb{R}}_o$ şi $\mathbb{R}^{\mathbb{R}}_e$ are subspaces of $\mathbb{R}^{\mathbb{R}}$ and $\mathbb{R}^{\mathbb{R}} = \mathbb{R}^{\mathbb{R}}_o \oplus \mathbb{R}^{\mathbb{R}}_e$.

3) Let us consider:

a) $f_1 : \mathbb{R}^2 \to \mathbb{R}^2$, $f_1(x, y) = (-x, y)$ (the symmetry with respect to Oy);

b) $f_2 : \mathbb{R}^2 \to \mathbb{R}^2$, $f_2(x, y) = (x, -y)$ (the symmetry with respect to Ox);

c) $f_3 : \mathbb{R}^2 \to \mathbb{R}^2$, $f_3(x, y) = (x \cos\varphi - y \sin\varphi, x \sin\varphi + y \cos\varphi)$, $\varphi \in \mathbb{R}$, (the plane rotation of angle φ);

d) $f_4 : \mathbb{R}^2 \to \mathbb{R}^3$, $f_4(x, y) = (x + y, 2x - y, 3x + 2y)$.

Show that $f_1,\ f_2,\ f_3,\ f_4$ are $\mathbb{R}$-linear maps. Are they isomorphisms? Are they endomorphisms? Are they automorphisms?

4) Can you find an $\mathbb{R}$-linear map $f : \mathbb{R}^3 \to \mathbb{R}^2$ such that

$$f(1, 0, 3) = (1, 1) \text{ şi } f(-2, 0, -6) = (2, 1)\,?$$