# Temă

**Problem 1.** Given the non-collinear vectors $\mathbf{a}$ and $\mathbf{b}$, prove that the system of vectors $\mathbf{m} = 3\mathbf{a} - \mathbf{b}$, $\mathbf{n} = 2\mathbf{a} + \mathbf{b}$, $\mathbf{p} = \mathbf{a} + 3\mathbf{b}$ is linearly dependent, and that the vectors $\mathbf{n}$ and $\mathbf{p}$ are non-collinear. Express the vector $\mathbf{m}$ in terms of the vectors $\mathbf{n}$ and $\mathbf{p}$.

**Problem 2.** Check whether the points $A(1, 2, -1)$, $B(0, 1, 5)$, $C(-1, 2, 1)$ and $D(2, 1, 3)$ are coplanar.

**Problem 3.** Find the equation of the line passing through the point $A(8, 9)$, for which the segment on the line between the lines $x - 2y + 5 = 0$ and $x - 2y = 0$ has a length of 5.

**Problem 4.** Determine the equation of the plane passing through the origin and the line $x = 1 + 3t, y = -2 + 4t, z = 5 - 2t$.

**Problem 5.** From the point $A(5, 9)$ tangents are drawn to the parabola $y^2 = 5x$. Determine the equation of the chord joining the points of tangency.

**Problem 6.** Find the equation of the conoidal surface generated by a line that remains parallel to the plane $x + z = 0$, rests on the $Ox$ axis and on the circle $x^2 + y^2 = 1,\ z = 0$.

**Problem 7.** Find the points of the skew curve

$$\mathbf{r}(t) = \left(\frac{1}{t}, t, 2t^2 - 1\right)$$

at which the binormals are perpendicular on the line $D$ of equations

$$\begin{cases} x + y = 0 \\ 4x - z = 0 \end{cases} .$$

**Problem 8.** Find the evolute of the curve

$$\begin{cases} x = a\left(\cos t + \ln \operatorname{tg} \dfrac{t}{2}\right), \\ y = a \sin t, \end{cases}$$

with $a > 0$ (the tractrix).

**Problem 9.** Write the equation of the tangent plane at the torus

$$\mathbf{r}(u, v) = ((7 + 5\cos u)\cos v, (7 + 5\cos u)\sin v, 5\sin u)$$

at the point $M(u_0, v_0)$ for which $\cos u_0 = 3/5$ and $\cos v_0 = 4/5$, where $0 < u, v < \pi/2$.

**Problem 10.** Find the asymptotic lines of the surface

$$\mathbf{r}(u, v) = \left(3(u + v), 3(u^2 + v^2), 2(u^3 + v^3)\right). \tag{0.0.1}$$