# Seminar – Eigenvectors and eigenvalues

1. a) Show that $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x_1, x_2, x_3) = (2x_1 - 2x_3, 3x_2, 3x_3)$ is an endomorphism of $_{\mathbb{R}}\mathbb{R}^3$, then determine the eigenvectors and the eigenvalues of $f$.

b) Determine the eigenvalues and the eigenvectors of the matrix:

$$A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \in M_3(\mathbb{R}).$$

2. Which of the following endomorphism is diagonalizable:

a) $f \in End_{\mathbb{R}}(\mathbb{R}^3)$, $f(x, y, z) = (-z, -x, -y)$;

b) $f \in End_{\mathbb{C}}(\mathbb{C}^3)$, $f(x, y, z) = (-z, -x, -y)$;

c) $f \in End_{\mathbb{R}}(\mathbb{R}^3)$, $f(x, y, z) = (-2y - 3z, x + 3y + 3z, z)$;

d) $f \in End_{\mathbb{R}}(\mathbb{R}^4)$, $f(x_1, x_2, x_3, x_4) = (-2x_1, -2x_2, 3x_3, x_3 + 3x_4)$?

3. Let

$$A = \begin{pmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{pmatrix} \in M_3(\mathbb{R}).$$

a) Show that $A$ is diagonalizable.

b) Find a matrix $S \in GL_3(\mathbb{R})$ such that $S^{-1}AS$ is diagonal.

c) Compute $A^n$, $n \in \mathbb{N}^*$.

4. Compute $A^4 - 11A^2 + 22A$ for

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \\ -1 & -1 & 3 \end{pmatrix} \in M_3(\mathbb{R}).$$