$$(u(x)^r)' = u'(x)\cdot r\cdot u(x)^{r-1} \qquad \forall r\in\mathbb{R}^*$$

$$(\ln(u(x)))' = \frac{u'(x)}{u(x)}$$

$$\left(a^{u(x)}\right)' = \ln a\cdot u'(x)\cdot a^{u(x)}$$

$$\sin(u(x)) = u'(x)\cdot\cos(u(x))$$

$$\cos(u(x)) = -u'(x)\cdot\sin(u(x))$$

$$\operatorname{tg}(u(x)) = u'(x)\cdot\frac{1}{\cos^2(u(x))}$$

$$\operatorname{arctg}(u(x)) = \frac{u'(x)}{u(x)^2+1}$$

$$\arcsin(u(x)) = \frac{u'(x)}{\sqrt{1-u^2(x)}}$$

$$(f(x)\cdot g(x)) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$$

$$\int u(x)^r\cdot u'(x)\,dx = \frac{u(x)^{r+1}}{r+1}+c \qquad \forall r\in\mathbb{R}\setminus\{-1\}$$

$$\int\frac{u'(x)}{u(x)}\,dx = \ln|u(x)|+c$$

$$\int u'(x)\cdot a^{u(x)}\,dx = \frac{a^{u(x)}}{\ln a}+c$$

$$\int u'(x)\cdot\sin(u(x))\,dx = -\cos(u(x))+c$$

$$\int u'(x)\cdot\cos(u(x))\,dx = \sin(u(x))+c$$

$$\int\frac{u'(x)}{u(x)^2+a^2}\,dx = \frac{1}{a}\operatorname{arctg}\frac{u(x)}{a}+c$$

$$\int\frac{u'(x)}{u(x)^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{u(x)-a}{u(x)+a}\right|+c$$

$$\int\frac{u'(x)}{\sqrt{u(x)^2\pm a^2}}\,dx = \ln\left|u(x)+\sqrt{u(x)^2\pm a^2}\right|+c$$

$$\int\frac{u'(x)}{\sqrt{a^2-u(x)^2}}\,dx = \arcsin\frac{u(x)}{a}+c$$

$$\int\frac{1}{\cos^2 x}\,dx = \int(\operatorname{tg}^2+1)\,dx = \operatorname{tg}x+c$$

$$\int f(x)\cdot g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\sin^2 x = \frac{\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} \qquad \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x}$$

$$\sin^2 t = \frac{1-\cos 2x}{2} \qquad \cos^2 t = \frac{1+\cos 2x}{2}$$

$$\sin 2t = 2\sin t\cos t \qquad \cos 2t = \cos^2 t - \sin^2 t \qquad \operatorname{tg}^2 x+1 = \frac{1}{\cos^2 x}$$

$$\sin(a\pm b) = \sin a\cos b \pm \sin b\cos a \qquad \cos(a\pm b) = \cos a\cos b \mp \sin a\sin b$$

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for integrals containing $\sin x/\cos x$:

if $R(-\sin x, \cos x) = -R(\sin x, \cos x) \longrightarrow \cos x = t$

if $R(\sin x, -\cos x) = -R(\sin x, \cos x) \longrightarrow \sin x = t$

if $R(-\sin x, -\cos x) = R(\sin x, \cos x) \longrightarrow \operatorname{tg} x = t$

otherwise: $\operatorname{tg}\frac{x}{2} = t \qquad \sin x = \frac{2t}{1+t^2} \qquad \operatorname{tg} x = \frac{2t}{1-t^2}$

$$\cos x = \frac{t^2}{1+t^2} \qquad \operatorname{ctg} x = \frac{1}{\operatorname{tg} x}$$

$$\sqrt{ax^2+bx+c} = \pm\sqrt{a}\,x \pm t, \quad a>0$$

$$\sqrt{ax^2+bx+c} = \pm x\cdot t \pm \sqrt{x}, \quad c>0$$

$\sqrt{ax^2+bx+c} = t(x-x_0)$, x_0 is a solution of the system ax^2+bx+c

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$$\sqrt{x^2+a^2} \longrightarrow x = a\cdot\operatorname{tg} x / a\cdot\operatorname{ctg} x$$

$$\sqrt{x^2-a^2} \longrightarrow x = \frac{a}{\sin t} \Big/ \frac{a}{\cos t}$$

$$\sqrt{a^2-x^2} \longrightarrow x = a\sin x / a\cos x$$

1) $(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$

2) $(\sin x \,\mathrm{tg}\, x)' = \cos x \cdot \frac{\sin x}{\cos x} + \sin x \cdot \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x} = \sin x \cdot \left(\frac{1}{\cos^2 x} + 1\right)$

3) $(\ln a^x)' = (x \ln a)' = \ln a$

4) $\int \ln x \, dx = \int (x)' \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + c$

5) $\int \frac{6x(x^2+1)^2}{(x^2+1)^6+1} dx = \int \frac{2x \cdot 3 \cdot (x^2+1)^2}{(x^2+1)^6+1} dx = \int \frac{((x^2+1)^3)'}{((x^2+1)^3)^2+1} dx = \mathrm{arctg}((x^2+1)^3) dx$

6) $\int \frac{1}{\sqrt{-x^2-5x-6}} dx = \int \frac{1}{\sqrt{\frac{1}{4}-(x+\frac{5}{2})^2}} dx = \arcsin \frac{x+\frac{5}{2}}{\frac{1}{2}} + c = \arcsin(2x+5)$

1) $\int \frac{1}{x^2+3x+4} dx = \int \frac{1}{x^2+2\frac{3}{2}x+\frac{9}{4}-\frac{9}{4}+4} dx = \int \frac{1}{(x+\frac{3}{2})^2+\frac{7}{4}} dx = \frac{2}{\sqrt{7}} \mathrm{arctg} \frac{2x+3}{\sqrt{7}} + c$

2) $\int \sqrt{9-x^2} \, dx = \int \sqrt{9-9\sin^2 t} \cdot 3\cos t \, dt = \int 9 \cos^2 t \, dt$

$x = 3 \sin t$

$dx = 3 \cos t$

3) $\int \frac{e^x \cdot \sin 3x + 3e^x \cos 3x}{(e^x \cdot \sin 3x)^3} dx = \frac{(e^x \cdot \sin 3x)^{-2}}{-2} + c$

4) $(\mathrm{arcctg}\, x)' = \frac{-1}{x^2+1}$

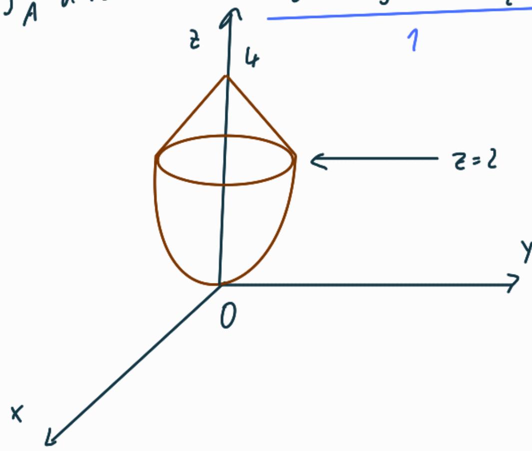
5) $\left(\frac{2^{3x}}{\cos x}\right)' = \frac{3\ln 2 \cdot 2^{3x} \cos x + 2^{3x} \sin x}{\cos^2 x} = \frac{2^{3x}(3 \ln 2 \cos x + \sin x)}{\cos^2 x}$

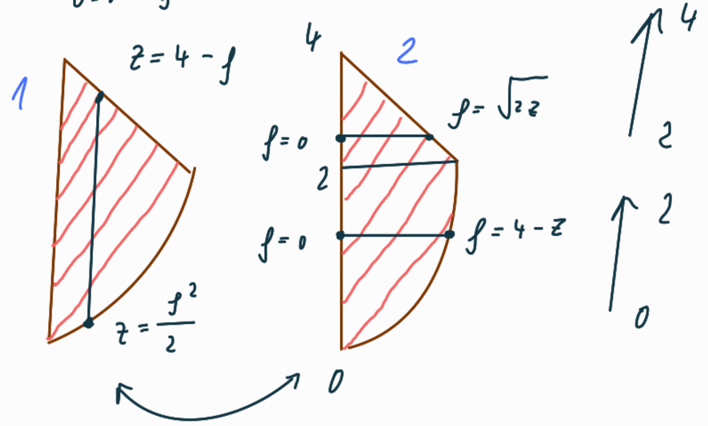
6) $\left(\sqrt{\ln(x^2+1)}\right)' = \frac{1}{2} \cdot \frac{\frac{2x}{x^2+1}}{\sqrt{\ln(x^2+1)}} = \frac{x}{(x^2+1)\sqrt{\ln(x^2+1)}}$

$2z = x^2 + y^2$ $\qquad$ $2z = \rho^2 \Rightarrow \rho = \sqrt{2z}$

$z = 4 - \sqrt{x^2 + y^2}$ $\qquad$ $z = 4 - \rho \Rightarrow \rho = 4 - z$

$$\iiint_A dx\,dy\,dz = \underbrace{\int_{\theta=0}^{\theta=2\pi}\int_{\rho=0}^{\rho=2}\int_{z=\frac{\rho^2}{2}}^{z=4-\rho} \rho\,dz\,d\rho\,d\theta}_{1} = \int_{\theta=0}^{\theta=2\pi}\int_{\rho=0}^{\rho=2} 4 - \rho - \frac{\rho^2}{2}\,d\rho\,d\theta =$$





CALCULE GRESITE AICI

$$= 2\pi\left(8 - 2 - \frac{4}{3}\right) = \frac{28\pi}{3}$$

$$\iiint_A dx\,dy\,dz = \underbrace{\int_{\theta=0}^{\theta=2\pi}\int_{z=0}^{z=2}\int_{\rho=0}^{\rho=\sqrt{2z}} \rho\,d\rho\,dz\,d\theta + \int_{\theta=0}^{\theta=2\pi}\int_{z=2}^{z=4}\int_{\rho=0}^{\rho=4-z} \rho\,d\rho\,dz\,d\theta}_{2} =$$

$$= 2\pi\left(\int_{z=0}^{z=2}\sqrt{2z}\,dz + \int_{z=2}^{z=4} 4 - z\,dz\right) = 2\pi\left(\sqrt{2}\cdot\frac{2}{3}\cdot z^{\frac{3}{2}}\Big|_0^2 + 4z - \frac{z^2}{2}\Big|_2^4\right) =$$

$$= 2\pi\cdot\left(\frac{8}{3} + 16 - 8 - 8 + 2\right) = 2\pi\left(\frac{8}{3} + 2\right) = 2\pi\,\frac{14}{3} = \frac{28\pi}{3}$$

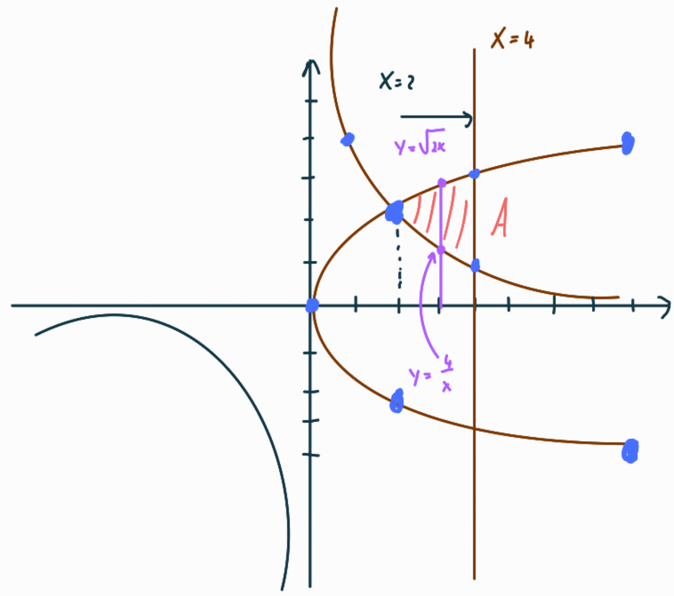
1) $f: \mathbb{R}^n \to \mathbb{R}^m$ $\quad f = (f_1, f_2 \dots f_m)$

$a \in \mathbb{R}^n$, $a = (a_1, \dots a_n)$

$$J(f)(a) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(a) & \frac{\partial f_1}{\partial x_2}(a) & \cdots & \frac{\partial f_1}{\partial x_n}(a) \\ \frac{\partial f_2}{\partial x_1}(a) & \frac{\partial f_2}{\partial x_2}(a) & \cdots & \frac{\partial f_2}{\partial x_n}(a) \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1}(a) & \frac{\partial f_m}{\partial x_2}(a) & \cdots & \frac{\partial f_m}{\partial x_n}(a) \end{pmatrix}$$

$$[df(a)(h)] = J(f)(a)\cdot\begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix} \; m \text{ lines}$$

2)



$$y^2 = 2x \qquad y = \pm\sqrt{2x}$$

$$xy = 4$$

$$x = 4$$

$$\begin{cases} y^2 = 2x \\ y = \frac{4}{x} \end{cases} \Rightarrow \frac{16}{x^2} = 2x \Rightarrow$$

$$\Rightarrow x^3 = 8 \Rightarrow x = 2 \in \mathbb{R}$$

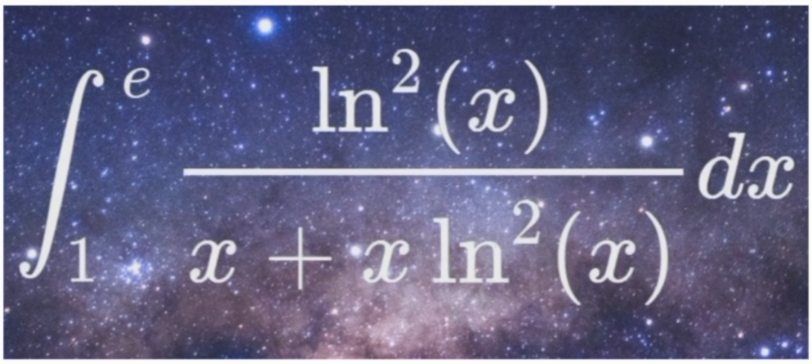
$$\iint_A \frac{1}{xy}\,dx\,dy = \int_{x=2}^{x=4}\int_{y=\frac{4}{x}}^{y=\sqrt{2x}} \frac{1}{xy}\,dy\,dx = \int_{x=2}^{x=4} \frac{1}{x}\left(\ln y \Big|_{y=\frac{4}{x}}^{y=\sqrt{2x}}\right)dx =$$

$$= \int_{x=2}^{x=4} \frac{1}{x}\left(\ln\sqrt{2x} - \ln\frac{4}{x}\right)dx = \int_2^4 \frac{1}{x}\left(\frac{1}{2}\ln 2 + \frac{1}{2}\ln x - 2\ln 2 + \ln x\right)dx =$$

$$= \int_2^4 \frac{1}{x}\left(-\frac{3}{2}\ln 2 + \frac{3}{2}\ln x\right)dx = -\frac{3}{2}\ln 2\left(\ln x\Big|_2^4\right) + \frac{3}{2}\left(\frac{\ln^2 x}{2}\Big|_2^4\right) = \frac{3\ln^2 2}{4}$$

$$\int_\gamma = \int \|\gamma'(t)\|\,dt$$

$$\int_\gamma f(x,y,z)\,ds = \int f(t,t,t)\cdot\|\gamma'(t)\| =$$



$$= \int \frac{\frac{1}{x}\ln^2 x}{1+\ln^2 x}\,dx = \int \ln^2 x\cdot\left(\operatorname{arctg}(\ln x)\right)'dx =$$

$$= \ln^2 x\,\operatorname{arctg}(\ln x) - \int 2\frac{1}{x}\ln x\cdot\operatorname{arctg}(\ln x)\,dx =$$

$$= \ln^2 x\,\operatorname{arctg}(\ln x) - \int \ln x\cdot\left(\operatorname{arctg}^2(\ln x)\right)'dx =$$

$$= \ln^2 x\,\operatorname{arctg}(\ln x) - \ln x\,\operatorname{arctg}^2(\ln x) + \frac{1}{3}\int\frac{3}{x}\operatorname{arctg}^2(\ln x)\,dx$$