

# Signals and Systems

### 3 | KPVTQFWEVKQP

As described in the Foreword, the intuitive notions of signals and systems arise in a rich variety of contexts. Moreover, as we will see in this book, there is an analytical framework - that is, a language for describing signals and systems and an extremely powerful set of tools for analyzing them - that applies equally well to problems in many fields. In this chapter, we begin our development of the analytical framework for signals and systems by introducing their mathematical description and representations. In the chapters that follow, we build on this foundation in order to develop and describe additional concepts and methods that add considerably both to our understanding of signals and systems and to our ability to analyze and solve problems involving signals and systems that arise in a broad array of applications.

## 4 | EQPVKPWQWU-VKOGBCPFBFKUETGVG-VKOGBUKIPCNU

### BB4:3 GzcornguBcpfBOcvjgocvkecnBTgrtgugpvcvkqp

Signals may describe a wide variety of physical phenomena. Although signals can be represented in many ways, in all cases the information in a signal is contained in a pattern of variations of some form. For example, consider the simple circuit in Figure 2.1. In this case, the patterns of variation over time in the source and capacitor voltages,  $v_s$  and  $v_e$  are examples of signals. Similarly, as depicted in Figure 2.2, the variations over time of the applied force  $f$  and the resulting automobile velocity  $v$  are signals. As another example, consider the human vocal mechanism, which produces speech by creating fluctuations in acoustic pressure. Figure 2.3 is an illustration of a recording of such a speech signal, obtained by using a microphone to sense variations in acoustic pressure, which are then converted into an electrical signal. As can be seen in the figure, different sounds correspond to different patterns in the variations of acoustic pressure, and the human vocal system produces intelligible speech by generating particular sequences of these patterns. Alternatively, for the monochromatic picture, shown in Figure 2.4, it is the pattern of variations in brightness across the image that is important.

Text-to-speech

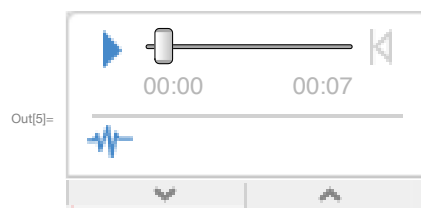
```
In[6]:= Urgcm['Ujqwnf yg ejcug']!
```

Set up of the current directory

```
In[4]:= UgvFktgevtA[PqvgdqmqFktgevtA[]]!
```

Import audio files (.wav and .mp3)

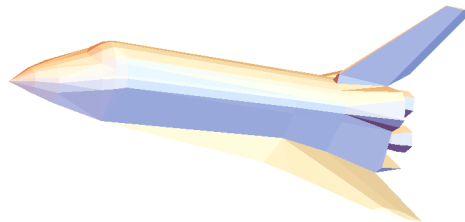
```
In[5]:= Korqtv['GzcorngFcvc/ect:ycx']
```



```
Korqtv['GzcorngFcvc/UjqwnfYgEjcug:or5'; 'UcorngTcvg']
```

66322

**GzcorngFcvc[{'IlgqogvtÅ5F'; 'UrcegUjwvvng'}]**

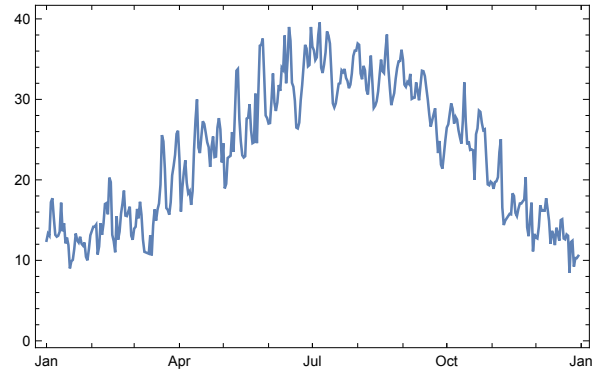


**GzcorngFcvc[{'VguvKocig'; 'Jqwug'}]**

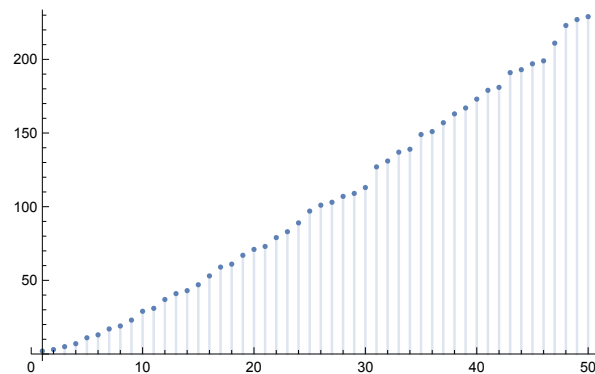


Signals are represented mathematically as functions of one or more independent variables. For example, a speech signal can be represented mathematically by acoustic pressure as a function of time, and a picture can be represented by brightness as a function of two spatial variables. In this book, we focus our attention on signals involving a single independent variable. For convenience, we will generally refer to the independent variable as time, although it may not in fact represent time in specific applications. For example, in geophysics, signals representing variations with depth of physical quantities such as density, porosity, and electrical resistivity are used to study the structure of the earth. Also, knowledge of the variations of air pressure, temperature, and wind speed with altitude are extremely important in meteorological investigations. Figure 2.5 depicts a typical example of annual average vertical wind profile as a function of height. The measured variations of wind speed with height are used in examining weather patterns, as well as wind conditions that may affect an aircraft during final approach and landing.

**FcvgNkuvRnqv[YgcvjgtFcvc[{55:1938; 8:0610};  
'OgcpVgorgtcvwtg'; {{4238; 3; 3}; {4238; 34; 53}; 'FcÀ'}]; Lqkpgf -> Vtwg]**

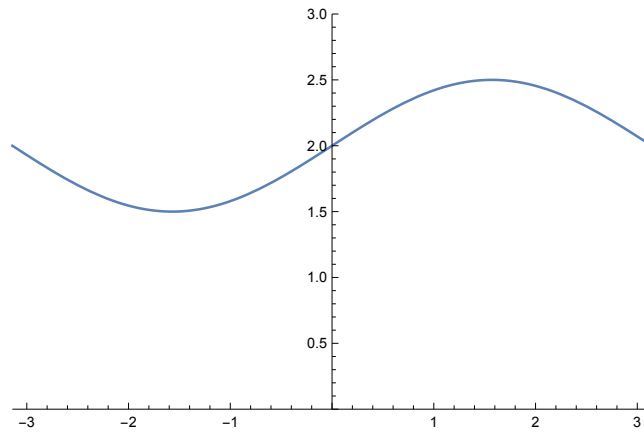


**FkuetgvgRnqv[Rtkog[p]; {p; 72}]**

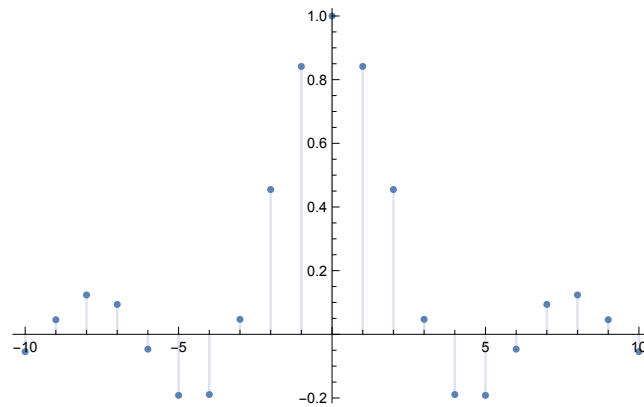


Throughout this book we will be considering two basic types of signals: continuous-time signals and discrete-time signals. In the case of continuous-time signals the independent variable is continuous, and thus these signals are defined for a continuum of values of the independent variable. On the other hand, discrete-time signals are defined only at discrete times, and consequently, for these signals, the independent variable takes on only a discrete set of values. A speech signal as a function of time and atmospheric pressure as a function of altitude are examples of continuous-time signals. The weekly Dow-Jones stock market index, as illustrated in Figure 2.6, is an example of a discrete-time signal. Other examples of discrete-time signals can be found in demographic studies in which various attributes, such as average budget, crime rate, or pounds of fish caught, are tabulated against such discrete variables as family size, total population, or type of fishing vessel, respectively.

**Rnqv[2:7\*Ukp[z]+4; {z; -Rk; Rk}; RnqvTcpiq → {{-Rk; Rk}; {2; 5}}]**



**FkuetgvGRnqv[Ukpe[z]; {z; -32; 32; 3}]**



To distinguish between continuous-time and discrete-time signals, we will use the symbol  $t$  to denote the continuous-time independent variable and  $n$  to denote the discrete-time independent variable. In addition, for continuous-time signals we will enclose the independent variable in parentheses  $(\cdot)$ , whereas for discrete-time signals we will use brackets  $[\cdot]$  to enclose the independent variable. We will also have frequent occasions when it will be useful to represent signals graphically. Illustrations of a continuous-time signal  $x(t)$  and a discrete-time signal  $x[n]$  are shown in Figure 2.7. It is important to note that the discrete-time signal  $x[n]$  is defined only for integer values of the independent variable. Our choice of graphical representation for  $x[n]$  emphasizes this fact, and for further emphasis we will on occasion refer to  $x[n]$  as a discrete-time sequence.

A discrete-time signal  $x[n]$  may represent a phenomenon for which the independent variable is inherently discrete. Signals such as demographic data are examples of this. On the other hand, a very important class of discrete-time signals arises from the sampling of continuous-time signals. In this case, the discrete-time signal  $x[n]$  represents successive samples of an underlying phenomenon for which the independent variable is continuous. Because of their speed, computational power, and flexibility, modern digital processors are used to implement

many practical systems, ranging from digital autopilots to digital audio systems. Such systems require the use of discrete-time sequences representing sampled versions of continuous-time signal - e.g., aircraft position, velocity, and heading for an autopilot or speech and music for an audio system. Also, pictures in newspapers - or in this book, for that matter- actually consist of a very fine grid of points, and each of these points represents a sample of the brightness of the corresponding point in the original image. No matter what the source of the data, however, the  $x[n]$  is defined only for integer values of  $n$ . It makes no more sense to refer to the  $3\frac{1}{2}$  sample of a digital speech signal than it does to refer to the average budget for a family with  $2\frac{1}{2}$  family members.

Throughout most of this book we will treat discrete-time signals and continuous-time signals separately but in parallel, so that we can draw on insights developed in one setting to aid our understanding of another. In Chapter 7 we will return to the question of sampling, and in that context we will bring continuous-time and discrete-time concepts together in order to examine the relationship between a continuous-time signal and a discrete-time signal obtained from it by sampling.

#### BB4:4 UkipcnBGpgtiABcpfBRqygt

From the range of examples provided so far, we see that signals may represent a broad variety of phenomena. In many, but not all, applications, the signals we consider are directly related to physical quantities capturing power and energy in a physical system. For example, if  $v(t)$  and  $i(t)$  are, respectively, the voltage and current across a resistor with resistance  $R$ , then the instantaneous power is

$$p(t) = v(t) i(t) = \frac{1}{R} v^2(t) \quad (1.1)$$

The total energy expended over the time interval  $t_1 \leq t \leq t_2$  is

$$\int_{t_1}^{t_2} p(t) dt = \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt, \quad (1.2)$$

and the average power over this time interval is

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t) dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{1}{R} v^2(t) dt \quad (1.3)$$

Similarly, for the automobile depicted in Figure 2.2, the instantaneous power dissipated through friction is  $p(t) = b v^2(t)$ , and we can then define the total energy and average power over a time interval in the same way as in eqs. (2.2) and (2.3).

With simple physical examples such as these as motivation, it is a common and worthwhile convention to use similar terminology for power and energy for any continuous-time signal  $x(t)$  or any discrete-time signal  $x[n]$ . Moreover, as we will see shortly, we will frequently find it convenient to consider signals that take on complex values. In this case, the total energy over the time interval  $t_1 \leq t \leq t_2$  in a continuous-time signal  $x(t)$  is defined as

$$\int_{t_1}^{t_2} |x(t)|^2 dt, \quad (1.4)$$

where  $|x|$  denotes the magnitude of the (possibly complex) number  $x$ . The time-averaged power is obtained by dividing eq. (2.4) by the length,  $t_1 - t_2$ , of the time interval. Similarly, the total energy in a discrete-time signal  $x[n]$  over the time interval  $n_1 \leq n \leq n_2$  is defined as

$$\sum_{n=n_1}^{n_2} |x[n]|^2, \quad (1.5)$$

and dividing by the number of points in the interval,  $n_2 - n_1 + 1$ , yields the average power over the interval. It is important to remember that the terms “power” and “energy” are used here independently of whether the quantities in eqs. (2.4) and (2.5) actually are related to physical energy.<sup>1</sup> Nevertheless, we will find it convenient to use these terms in a general fashion.

Furthermore, in many systems we will be interested in examining power and energy in signals over an infinite time interval, i.e., for  $-\infty < t < +\infty$  or for  $-\infty < n < +\infty$ . In these cases, we define the total energy as limits of eqs. (2.4) and (2.5) as the time interval increases without bound. That is, in continuous time,

$$E_\infty \triangleq \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{+\infty} |x(t)|^2 dt, \quad (1.6)$$

and in discrete time,

$$E_\infty \triangleq \lim_{N \rightarrow \infty} \sum_{n=-N}^{+N} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |x[n]|^2, \quad (1.7)$$

Note that for some signals the integral in eq. (2.6) or sum in eq. (2.7) might not converge- e.g., if  $x(t)$  or  $x[n]$  equals a nonzero constant value for all time. Such signals have infinite energy, while signals with  $E_\infty < \infty$  have finite energy.

In an analogous fashion, we can define the time-averaged power over an infinite interval as

$$P_\infty \triangleq \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad (1.8)$$

and

$$P_\infty \triangleq \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 \quad (1.9)$$

in continuous time and discrete time, respectively. With these definitions, we can identify three important classes of signals. The first of these is the class of signals with finite total energy, i.e., those signals for which  $E_\infty < \infty$ . Such a signal must have zero average power, since in the continuous time case, for example, we see from eq. (2.8) that

$$P_\infty = \lim_{T \rightarrow \infty} \frac{E_\infty}{2T} = 0. \quad (1.10)$$



An example of a finite-energy signal is a signal that takes on the value 1 for  $0 \leq t \leq 1$  and 0 otherwise. In this case,  $E_\infty = 1$  and  $P_\infty = 0$ .

A second class of signals are those with finite average power  $P_\infty$ . From what we have just seen, if  $P_\infty > 0$ , then, of necessity,  $E_\infty = \infty$ . This, of course, makes sense, since if there is a nonzero average energy per unit time (i.e., nonzero power), then integrating or summing this over an infinite time interval yields an infinite amount of energy. For example, the constant signal  $x[n] = 4$  has infinite energy, but average power  $P_\infty = 16$ . There are also signals for which neither  $P_\infty$  nor  $E_\infty$  are finite. A simple example is the signal  $x(t) = t$ . We will encounter other examples of signals in each of these classes in the remainder of this and the following chapters.

## 5 | PaSWKUV-UJCPPQPBUCORNKPIBVJGQTGO

The Nyquist-Shannon Sampling theorem is a fundamental one providing the condition on the sampling frequency of a band-width limited continuous-time signal in order to be able to reconstruct it perfectly from its discrete-time (sampled) version. It stated that the sampling frequency must be at least two times the highest frequency of the continuous-time signal spectrum.

### BB5:3 UcornkpiBVjgqtgo

Sampling a continuous-time signal is getting one sample each sampling period  $T_s[s]$ , which means a sampling frequency equivalent to  $f_s = \frac{1}{T_s}[\text{Hz}]$ .

The mean important decision to make is the sampling frequency value of  $f_s$ : Indeed, if its value is too big, we'll get  $f_s$  samples per second, so large storage or large band width in order to transmit the data. If on the other hand, the value is too small, we won't be able to reconstruct the signal from the samples.

Hence, there is a tradeoff to make in order to minimise the needed resources (storage or frequency band-width) while being able to reproduce the original signal from the samples.

One theorem provides an answer to this question, it's the *Nyquist-Shannon Sampling Theorem* which states:

*A continuous-time signal  $x(t)$  can be sampled at a frequency  $f_s$  in order to get a discrete-time copy of it  $x[n]$ , and afterwards be reconstructed perfectly to its original form  $x(t)$  if  $f_s > 2 f_{\max}$  with  $f_{\max}$  is the maximum frequency value of the  $x(t)$  signal spectrum*

Next, we'll consider a Sinc<sup>2</sup> function and see what happen to the sampled signal on the frequency domain using Fourier Transform

### BB5:4 UcornkpiBRtqeguu

#### VkogBFqockpBCpcnÅuku

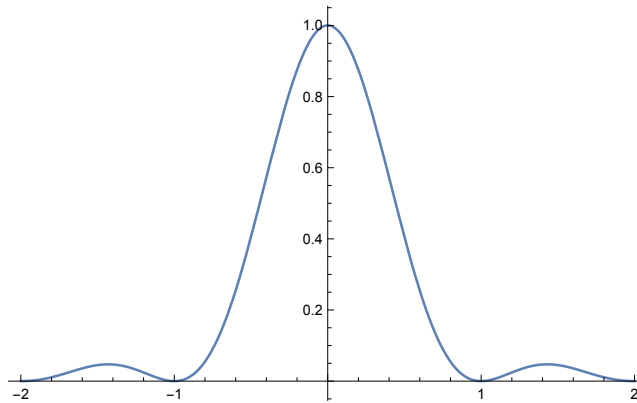
Consider the function  $x(t) = \text{Sinc}^2(t)$  which represent a continuous-time signal.

RnqvBqhBvjgBukipcnBz(v) = Ukpe<sup>4</sup>(v)...

In[40]:= **z[v\_] ...= (Ukpe[Rk\*v])~4!**

**Rnqv[z[v]; {v; -4; 4}]**

Out[41]=



Consider a sampling frequency  $f_s$  and let see what happen for different values of  $f_s$ .

RnqvBqhBvjgBeqpvkpqwv-wkogBcpfBfkuetvgv-wkogB[ucorngf]Bukipcnu...

In[42]:= **Ocpkrwncvg[**

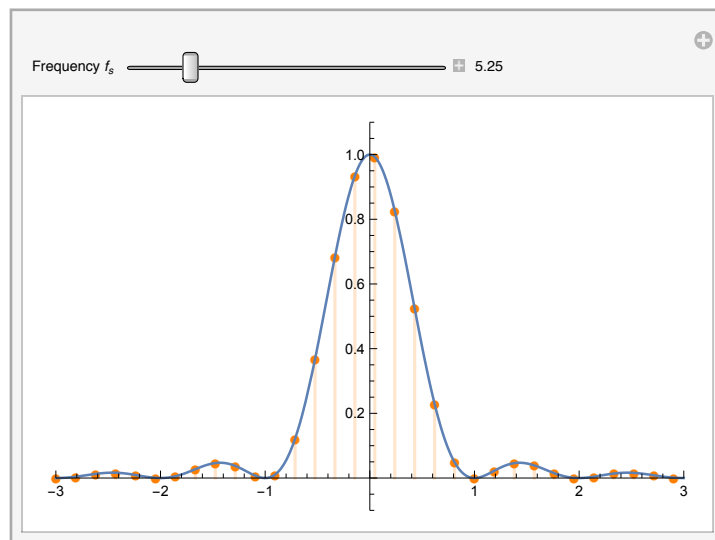
**Ujqy[FkuetvgvRnqv[z[v]; {v; -5; 5; 3/hu}; RnqvTcpiq → {{-5; 5}; {-2;3; 3;3}};**

**Rgthqtocpeglqcn → 'SwcnkvÀ'; RnqvUvÀng → Fktgevkxg[Qtcpiq; Vjkem];**

**RnqvOctmgtu → Cwvqocvke]; Rnqv[z[v]; {v; -5; 5}]];**

**{{hu; 37:47; 'HtgswwpeÀ h\_u '}; 2:7; 52; Crrgctcpeg → 'Ncdgngf'; UcxgFghkpvkqpu → Vtwg]**

Out[42]=



As we can see, for high values of  $f_s$  we can at least recognize visually the original continuous-time form on the  $x(t)$  signal. However, for small values of  $f_s$  such as  $f_s = 0.1$  hz for instance, we can no more visually retrieve the form of the original Sinc<sup>2</sup> function, with only four sample over the 20 interval duration it's quite impossible to go back from samples to continu-

ous signal. Indeed, in such case, we are sure not satisfying the Nyquist-Shannon sampling condition.

In the time domain, we can somehow visually understand what's happening, however, the theorem is more understandable when analyzed on the frequency domain.

HtgswgpeÀBFqockp

The spectrum analysis of signal  $x(t)$  allows us to understand easily and even prove the theorem ourselves.

$$\text{TF}[\text{Sinc}^2(a\,t)] = \frac{1}{|a|} \text{tri}\left(\frac{f}{a}\right)$$

(1.11)

HqwtkggtBVtcpuhtqtoBqhBvjgBukipcnBz(v)...

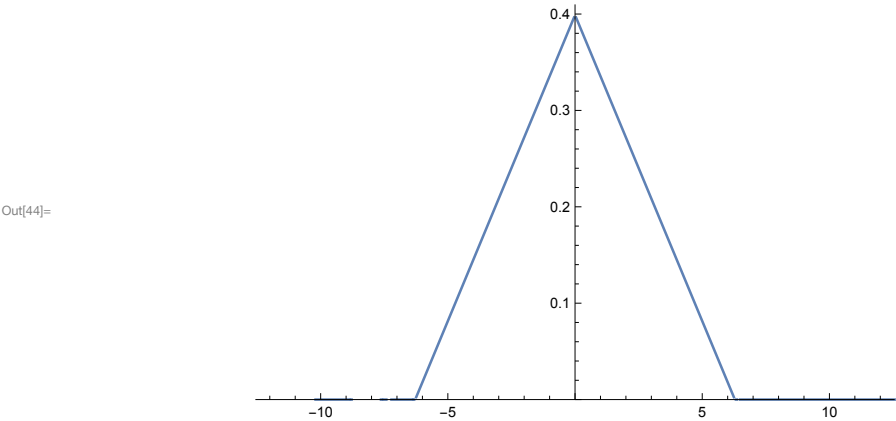
In[43]:= UkornkhÀ[HqwtkggtVtcpuhtqto[z[v]; v; ω]

Out[43]=

$$\frac{3}{6\sqrt{4}\pi^{5/4}}(-4\omega\text{Ukip}[\omega]+(-4\pi+\omega)\text{Ukip}[-4\pi+\omega]+(4\pi+\omega)\text{Ukip}[4\pi+\omega])$$

UrgevtwoBqhBvjgBukipcnBz(v)...

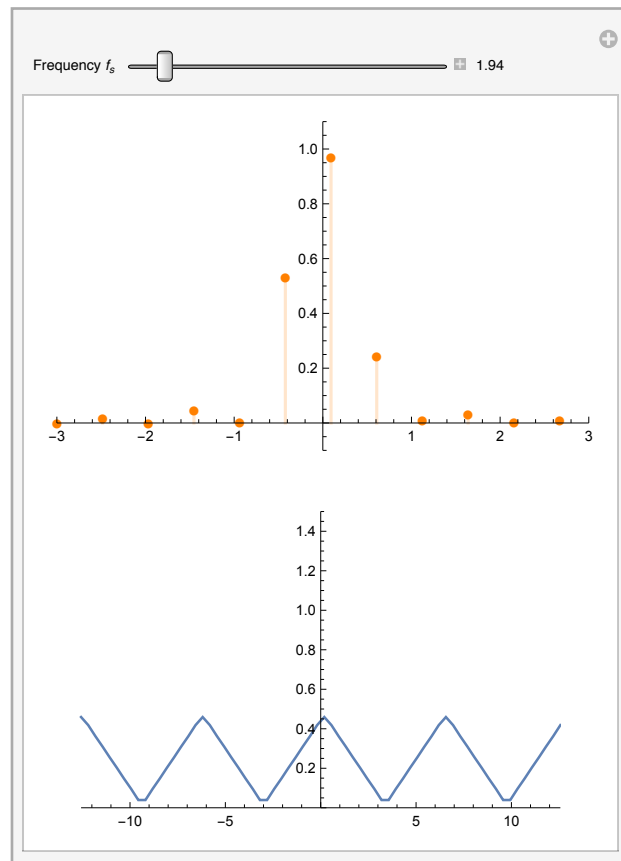
In[44]:= Rnqv[HqwtkggtVtcpuhtqto[z[v]; v; ω];  
{ω; -32\*4 Rk; 32\*4 Rk}; RnqvTcpi g → {{-4\*4 Rk; 4\*4 Rk}; {2; 2:63}}]



UcorngfBukipcnBz[p]B[nghv]BcpfBkvbBHqwtkgdBvtcpuhqtoBurgevtwo...

In[45]:= **Ocpkrwncvg[Tqy[{FkuetgvgRnqv[z[v]; {v; -5; 5; 3/hu}; RnqvTcpiq → {{-5; 5}; {-2;3; 3:3}};**  
**Rgthqtocpeglqcn → 'SwcnkvÀ'; RnqvUvÀng → Fktgevkxg[Qtcpiq; Vjkem];**  
**RnqvOctmgtu → Cwvqocvke; KocigUkÁg → 562]; NkuvNkpgRnqv[**  
**Lqkp@@Vcdng[Cdu[Hqwtkg[Bcdng[P[z[p/hu]; {p; -6\*hu; 6\*hu+3}]]]; 6];**  
**FcvvTcpiq → {-6 Rk; 6 Rk}; RnqvTcpiq → {{-6 Rk; 6 Rk}; {2; 3:7}}; KocigUkÁg → 522]]];**  
**{{hu; 37:47; 'HtgswgpeÀ h<sub>u</sub> '}; 2:7; 42; Crrgctcpeg → 'Ncdngf'}}**

Out[45]=



The spectrum of the sampled signal is a periodic form where its period is  $2\pi$ , and as long as we respect the Nyquist-Shannon condition, its form over one period is equivalent to the spectrum of the original continuous-time signal  $x(t)$ .

Indeed, while the sampling frequency  $f_s$  is large enough, there is no overlapping between the periods of the spectrum, and then, the original signal can be retrieved by means of a low passband filter. However, if the sampling frequency is too low, such as  $f_s = 1.4$  in the above example, there will be overlapping of the spectrum, which we call an aliasing effect. In such a case, we can no more retrieve the original signal using a simple low-passband filter, and we need then a more powerful signal processing by means of equalization. The condition to avoid the overlapping is that  $f_s > 2f_{\max}$ , which represents perfectly the Nyquist-Shannon sampling theorem.

\* The signal looks continuous on the plot but any signal processed (transmitted, stored or displayed) is always a sampled version of the original continuous time parameter

### BB5:5 TgeqpuvtwekqpBRtqeguu

The reconstruction process consists in the generation of the original continuous-time signal  $x(t)$  from the sampled version  $x[n]$ . As long as there is no aliasing effect, a simple low pass-band filter is enough to retrieve the original signal

HknavgtkpiBvjgBUrgevtwoBqhBvjgBucorngfBukipcnBz[p]...

In[85]:= **Ocpkrwncvg[**

**Ujqy[NkuvNkpgRnqv[Lqkp@@Vcdng[Cdu[HqwtkgT[Vcdng[P[z[p/hu]]; {p; -6\*hu; 6\*hu}]]]; 4];**

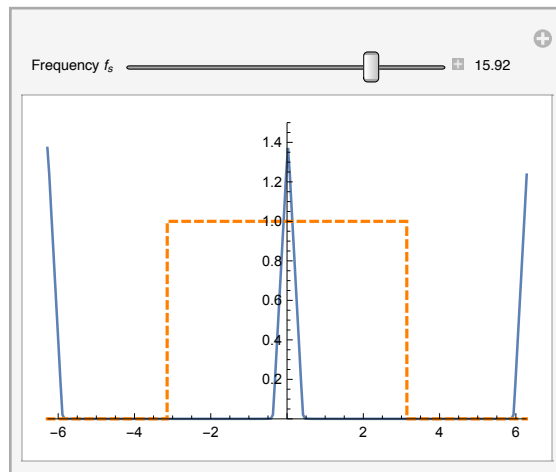
**FcvcTcpig → {-4 Rk; 4 Rk}; RnqvTcpig → {{-4 Rk; 4 Rk}; {2; 3:7}}; KocigUkÁg → 522];**

**Rnqv[JgcxkukfgRk[z/(4\*Rk)]; {z; -4 Rk; 4 Rk}; Gzenwukqpu → Pqpg;**

**RnqvUvÀng → Fktgevkxg[Qtcpig; Vjkem; Fcujgf]]];**

**{{hu; 0; 'HtgswgpeÀ h\_u'}; 2:7; 42; Crrgctcpeg → 'Ncdngf']}]**

Out[85]=



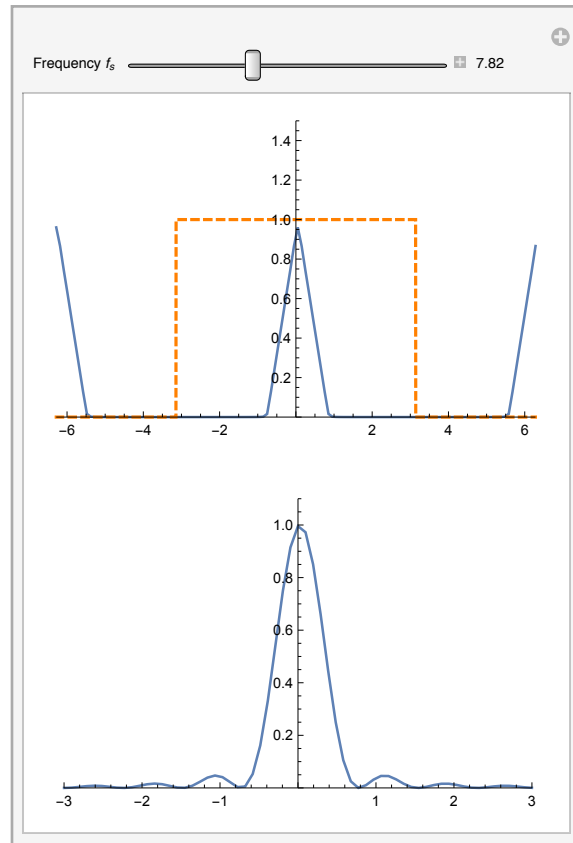
Depending on the existence of aliasing or not, the retrieved continuous-time signal could be the perfect copy of the original or a distorted one.

UrgevtwoBqhBvjgBucorngBukipcnBcpfBvjgBtgeqpvtwevgfBxgtukqpBqhBvjgBukipcnBz[p]...

In[121]:= Ocpkrwncvg[Tqy[

```
{Ujqy[NkuvNkpgRnqv[Lqkp@@Vcdng[Cdu[Hqwtkg[Vcdng[P[z[p/hu]]; {p, -6 hu, 6 hu}]]]; 4];
  Rgthqtocpeglqcn → 'SwcnkvÀ'; FcvcTcpiq → {-4 Rk; 4 Rk};
  RnqvTcpiq → {{-4 Rk; 4 Rk}; {2; 3:7}}; KocigUkÁg → 522;
  Rnqv[JgcxkukfgRk[z/(4 * Rk)]; {z, -4 Rk; 4 Rk}; Gzenwukqpu → Pqpg;
  RnqvUvÀng → Fktgevkxg[Qtcpig; Vjkem; Fcuigf] ]];
  NkuvNkpgRnqv[Tg[KpxgtugHqwtkg[Hqwtkg[Vcdng[P[z[p/hu]]; {p, -6 hu, 6 hu}]]]];
  FcvcTcpiq → {-5; 5}; RnqvTcpiq → {{-5; 5}; {2; 3:3}};
  KocigUkÁg → 522; Rgthqtocpeglqcn → 'SwcnkvÀ'] ]];
  {{hu; 0; 'HtgswgpeÀ h_u'; 2:7; 42; Crrgctcpeg → 'Ncdgngf'}}
```

Out[121]=



As long as the sampling frequency verifies the Nyquist-Shannon condition, we can reconstruct the original signal without distortion.

## 6 | VTCPUHQTOCVKQPUBQHBVJGBKPFGRGPFPGPVBXCTKCDNG

A central concept in signal and system analysis is that of the transformation of a signal. For example, in an aircraft control system, signals corresponding to the actions of the pilot are transformed by electrical and mechanical systems into changes in aircraft thrust or the positions of aircraft control surfaces such as the rudder or ailerons, which in turn are transformed through the dynamics and kinematics of the vehicle into changes in aircraft velocity and heading. Also, in a high-fidelity audio system, an input signal representing music as recorded on a cassette or compact disc is modified in order to enhance desirable characteristics, to remove recording noise, or to balance the several components of the signal (e.g., treble and bass). In this section, we focus on a very limited but important class of elementary signal transformations that involve simple modification of the independent variable, i.e., the time axis. As we will see in this and subsequent sections of this chapter, these elementary transformations allow us to introduce several basic properties of signals and systems. In later chapters, we will find that they also play an important role in defining and characterizing far richer and important classes of systems.

### BB6:3 GzcornguBqhBVtcpuhtocvkqpuBqhBvjgBKpfrgrpgpvBXctkcdng

A simple and very important example of transforming the independent variable of a signal is a time shift. A time shift in discrete time is illustrated in Figure 2.8, in which we have two signals  $x[n]$  and  $x[n - n_0]$  that are identical in shape, but that are displaced or shifted relative to each other. We will also encounter time shifts in continuous time, as illustrated in Figure 2.9, in which  $x(t - t_0)$  represents a delayed (if  $t_0$  is positive) or advanced (if  $t_0$  is negative) version of  $x(t)$ . Signals that are related in this fashion arise in applications such as radar, sonar, and seismic signal processing, in which several receivers at different locations observe a signal being transmitted through a medium (water, rock, air, etc.). In this case, the difference in propagation time from the point of origin of the transmitted signal to any two receivers results in a time shift between the signals at the two receivers.

A second basic transformation of the time axis is that of time reversal. For example, as illustrated in Figure 2.10, the signal  $x[-n]$  is obtained from the signal  $x[n]$  by a reflection about  $n = 0$  (i.e., by reversing the signal). Similarly, as depicted in Figure 2.11, the signal  $x(-t)$  is obtained from the signal  $x(t)$  by a reflection about  $t = 0$ . Thus, if  $x(t)$  represents an audio tape recording, then  $x(-t)$  is the same tape recording played backward. Another transformation is that of time scaling. In Figure 2.12 we have illustrated three signals,  $x(t)$ ,  $x(2t)$ , and  $x(t/2)$ , that are related by linear scale changes in the independent variable. If we again think of the example of  $x(t)$  as a tape recording, then  $x(2t)$  is that recording played at twice the speed, and  $x(t/2)$  is the recording played at half-speed.

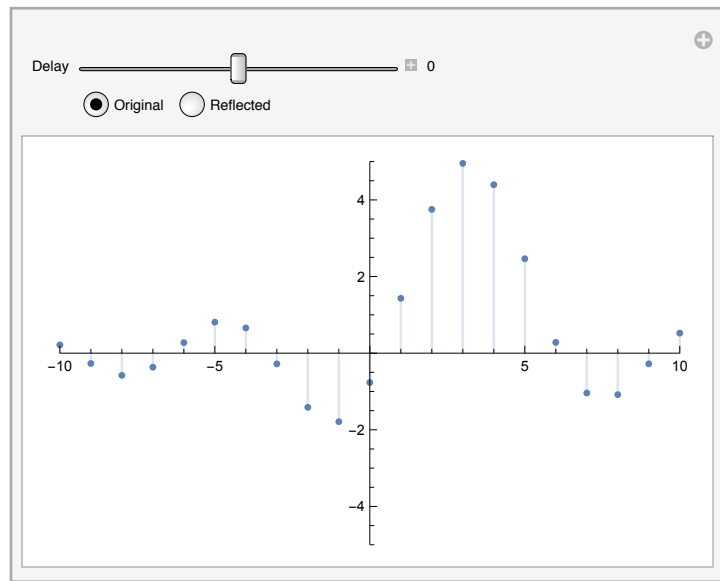
It is often of interest to determine the effect of transforming the independent variable of a given signal  $x(t)$  to obtain a signal of the form  $x(\alpha t + \beta)$ , where  $\alpha$  and  $\beta$  are given numbers. Such a transformation of the independent variable preserves the shape of  $x(t)$ , except that the



resulting signal may be linearly stretched if  $|\alpha| < 1$ , linearly compressed if  $|\alpha| > 1$ , reversed in time if  $\alpha < 0$ , and shifted in time if  $\beta$  is nonzero. This is illustrated in the following set of examples.

```
In[70]:= Qtkikpcn[d_ ; p_ ; f_] ... := -Ukpe[d*(p-f)] + 7*Ukpe[d*(p-f)-5]!
Ocpkrwncvg[FkuetgvgRnqv[Qtkikpcn[d ; p ; f]; {p ; -32+f ; 32+f ; 3};
  RnqvTcpiq → {{-32 ; 32}; {-7 ; 7}}; Rgthqtocpeglqcn → 'SwcnkvÀ';
  {{f ; 2 ; 'FgncÀ'} ; -7 ; 7 ; 3 ; Crrgctcpeg → 'Ncdgngf'};
  {{d ; 3 ; ''} ; {3 → 'Qtkikpcn' ; -3 → 'Tghngevgf'} ; EqpvtqnVÀrg → TcfkqDwvvqpDct};
  UcxgFghkpkvkqpu → Vtwg]
```

Out[71]=

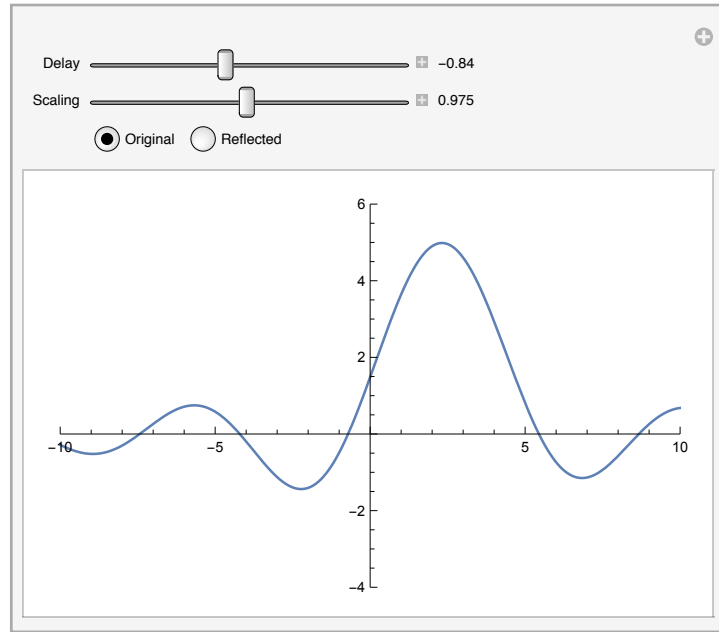


```

In[72]:= Qtkikpcn[d_ ; z_ ; u_ ; f_] ...= -2:7*Ukpe[d*(u*z-f)]+7*Ukpe[d*(u*z-f)-5]!
Ocpkrwncvg[Rnqv[Qtkikpcn[d; z; u; f]; {z; -32; 32}; RnqvTcpiq → {{-32; 32}; {-6; 8}};
Rgthqtocpeglqcn → 'SwcnkvÀ'; {{f; 2; 'FgncÀ'; -7; 7; Crrgctcpeg → 'Ncdgngf'};
{{u; 3; 'Uecnkpi'}; 2; 4; Crrgctcpeg → 'Ncdgngf'}; {{d; 3; ''}; {3 → 'Qtkikpcn'; -3 → 'Tghngevgf'};
EqpvtqnVÀrg → TcfkqDwvvqpDct}; UcxgFghkpkvkqpu → Vtwg]

```

Out[73]=



#### BB6:4 RgtkqfkeBUkipcnu

An important class of signals that we will encounter frequently throughout this book is the class of *periodic signals*. A periodic continuous-time signal  $x(t)$  has the property that there is a positive value of  $T$  for which

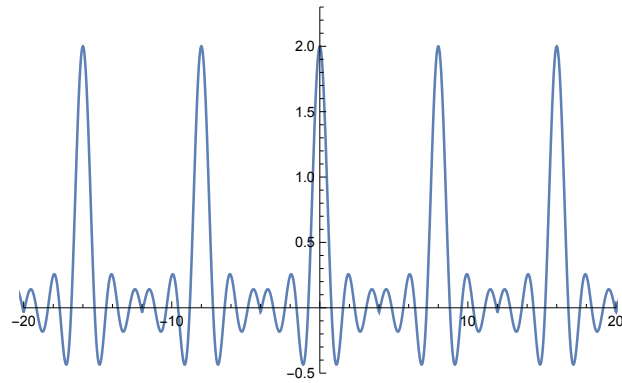
$$x(t) = x(t + T) \quad (1.12)$$

for all values of  $t$ . In other words, a periodic signal has the property that it is unchanged by a time shift of  $T$ . In this case, we say that  $x(t)$  is *periodic with period  $T$* . Periodic continuous-time signals arise in a variety of contexts. For example, as illustrated in Problem 2.61, the natural response of systems in which energy is conserved, such as ideal *LC* circuits without resistive energy dissipation and ideal mechanical systems without frictional losses, are periodic and, in fact, are composed of some of the basic periodic signals that we will introduce in Section 2.3.

```

In[16]:= V = 6!
i[z_ /! -V ≤ z ≤ V] ...= 4 * Ukpe[V * z]!
i[z_] ...= i[Oqf[z + V; 4 * V] - V]!
Rnqv[i[z]; {z; -1 * V; 1 * V};
RnqvTcpiq → {{-7 * V; 7 * V}; {-2:7; 4:5}}; Rgthqtocpeglqcn → 'SwcnkvÀ'
    
```

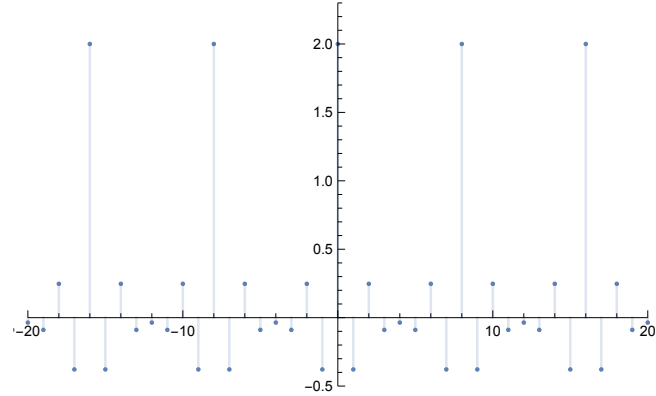
Out[18]=



```

In[29]:= O = 6!
i[p_ /! -O ≤ p ≤ O] ...= 4 * Ukpe[O * p]!
i[p_] ...= i[Oqf[p + O; 4 * O] - O]!
FkuetvgvRnqv[i[p]; {p; -1 * O; 1 * O};
RnqvTcpiq → {{-7 * O; 7 * O}; {-2:7; 4:5}}; Rgthqtocpeglqcn → 'SwcnkvÀ'
    
```

Out[32]=



An example of a periodic continuous-time signal is given in Figure 3.3. From the figure or from eq. (3.1), we can readily deduce that if  $x(t)$  is periodic with period  $T$ , then  $x(t) = x(t + mT)$  for all  $t$  and for any integer  $m$ . Thus,  $x(t)$  is also periodic with period  $2T$ ,  $3T$ ,  $4T$ , .... The *fundamental* period  $T_0$  of  $x(t)$  is the smallest positive value of  $T$  for which eq. (2.11) holds. This definition of the fundamental period works, except if  $x(t)$  is a constant. In this case the fundamental period is undefined, since  $x(t)$  is periodic for any choice of  $T$  (so there is no smallest positive value). A signal  $x(t)$  that is not periodic will be referred to as an aperiodic signal.

Periodic signals are defined analogously in discrete time. Specifically, a discrete-time signal

$x[n]$  is periodic with period  $N$ , where  $N$  is a positive integer, if it is unchanged by a time shift of  $N$ , i.e., if

$$x[n] = x[n + N] \quad (1.13)$$

for all values of  $n$ . If eq. (2.12) holds, then  $x[n]$  is also periodic with period  $2N, 3N, \dots$ . The fundamental period  $N_0$  is the smallest positive value of  $N$  for which eq. (1.12) holds. An example of a discrete-time periodic signal with fundamental period  $N_0 = 3$  is shown in Figure 2.15.

### BB6:5 GxgpBcpfBQffBUkipcnu

Another set of useful properties of signals relates to their symmetry under time reversal. A signal  $x(t)$  or  $x[n]$  is referred to as an even signal if it is identical to its time-reversed counterpart, i.e., with its reflection about the origin. In continuous time a signal is even if

$$x(-t) = x(t) \quad (1.14)$$

while a discrete-time signal is even if

$$x[-n] = x[n] \quad (1.15)$$

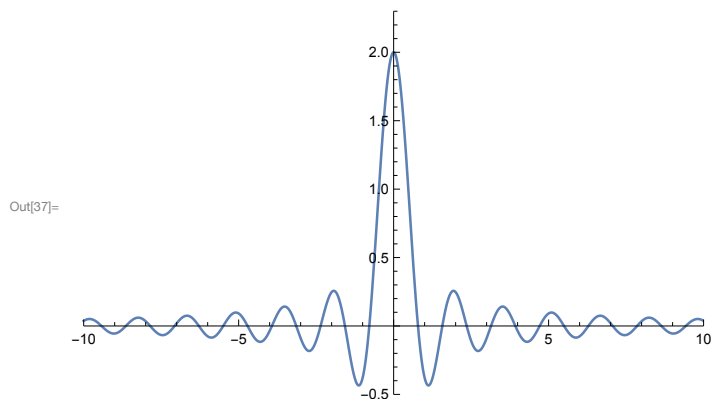
A signal is referred to as odd if

$$x(-t) = -x(t) \quad (1.16)$$

$$x[-n] = -x[n] \quad (1.17)$$

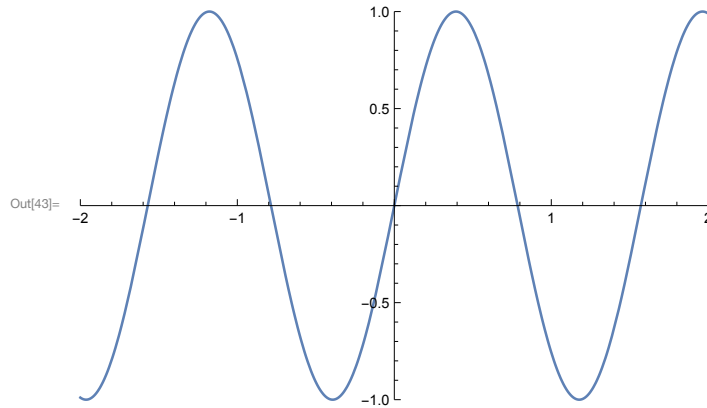
In[37]:= **i[z\_] ...= 4 \* Ukpe[V \* z]!**

**Rnqv[i[z]; {z, -32; 32}; RnqvTcpiq → {{-32; 32}; {-2;7; 4;5}}; Rgthqtocpeglqcn → 'SwcnkvA']**



In[43]:= **i4[z\_] ...= Ukp[V\*z]!**

**Rnqv[i4[z]; {z, -4; 4}; RnqvTcpiq → {{-4; 4}; {-3; 3}}; Rgthqtoceplqcn → 'SwcnkvÀ']**



An odd signal must necessarily be 0 at  $t = 0$  or  $n = 0$ , since eqs. (2.16) and (2.17) require that  $x(0) = -x(0)$  and  $x[0] = -x[0]$ . Examples of even and odd continuous-time signals are shown in Figure 3.17.

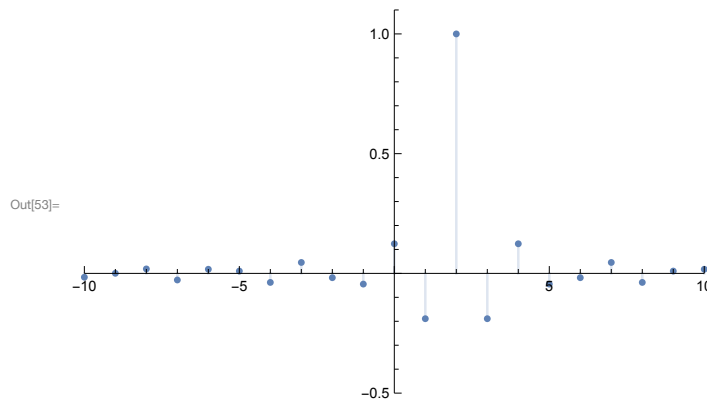
An important fact is that any signal can be broken into a sum of two signals, one of which is even and one of which is odd. To see this, consider the signal

$$\mathcal{E}_v \{x(t)\} = \frac{1}{2}[x(t) + x(-t)] \quad (1.18)$$

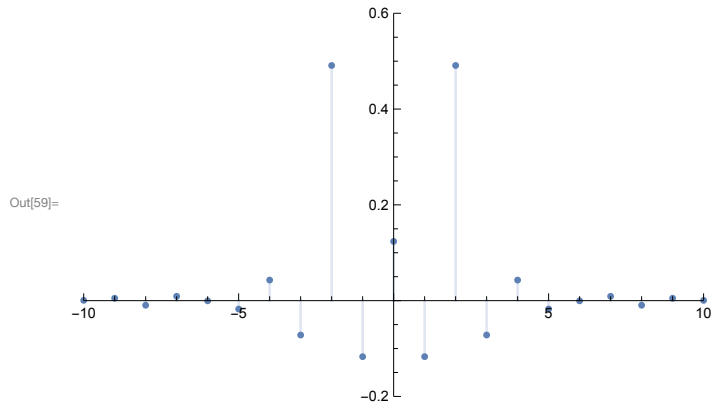
In[53]:= **i5[p\_] ...= Ukpe[V\*(p-4)]!**

**FkuetgvgRnqv[i5[p]; {p, -32; 32};**

**RnqvTcpiq → {{-32; 32}; {-2; 7; 3; 3}}; Rgthqtoceplqcn → 'SwcnkvÀ']**



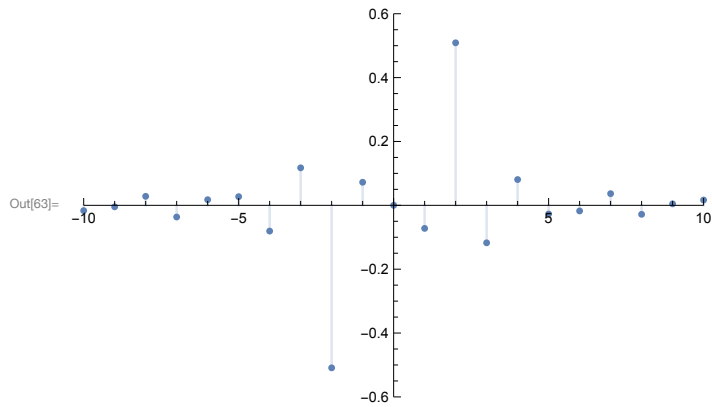
```
In[58]:= i6[p_] ...= (i5[p]+i5[-p])/4!
FkuetvgvRnqv[i6[p]; {p; -32; 32};
RnqvTcpiq → {{-32; 32}; {-2;4; 2;8}}; Rgthqtoceplqcn → 'SwcnkvÀ']
```



which is referred to as the even part of  $x(t)$ . Similarly, the odd part of  $x(t)$  is given by

$$Od \{x(t)\} = \frac{1}{2}[x(t) - x(-t)] \quad (1.19)$$

```
In[62]:= i6[p_] ...= (i5[p]-i5[-p])/4!
FkuetvgvRnqv[i6[p]; {p; -32; 32};
RnqvTcpiq → {{-32; 32}; {-2;8; 2;8}}; Rgthqtoceplqcn → 'SwcnkvÀ']
```



It is a simple exercise to check that the even part is in fact even, that the odd part is odd, and that  $x(t)$  is the sum of the two. Exactly analogous definitions hold in the discrete-time case. An example of the even-odd decomposition of a discrete-time signal is given in Figure 1.18.

## 7 | GZRQPGPVKCNBCPFBUKPWUQKFCNBUKIPCNU

In this section and the next, we introduce several basic continuous-time and discrete-time signals. Not only do these signals occur frequently, but they also serve as basic building blocks from which we can construct many other signals.

### BB7:3 Eqpvkpwqwu–VkogBEqorngzBGzrqpgpvkcnBcpfBUkpwuqkfcnBUkipcnu

The continuous-time complex exponential signal is of the form

$$x(t) = A e^{at} \quad (1.20)$$

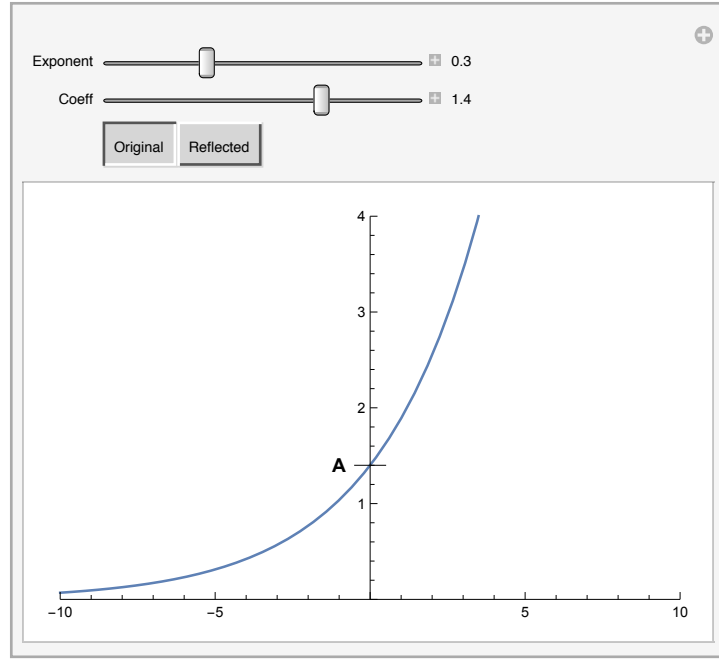
where  $C$  and  $a$  are, in general, complex numbers. Depending upon the values of these parameters, the complex exponential can exhibit several different characteristics.

#### ■ Real Exponential Signals

As illustrated in Figure 1.19, if  $C$  and  $a$  are real [in which case  $x(t)$  is called a real exponential], there are basically two types of behavior. If  $a$  is positive, then as  $t$  increases  $x(t)$  is a growing exponential, a form that is used in describing many different physical processes, including chain reactions in atomic explosions and complex chemical reactions. If  $a$  is negative, then  $x(t)$  is a decaying exponential, a signal that is also used to describe a wide variety of phenomena, including the process of radioactive decay and the responses of  $RC$  circuits and damped mechanical systems. In particular, as shown in Problems 1.61 and 1.62, the natural responses of the circuit in Figure 1.1 and the automobile in Figure 1.2 are decaying exponentials. Also, we note that for  $a = 0$ ,  $x(t)$  is constant.

```
In[10]:= Qtkikpcn3[d_ ; C_ ; z_] ...= C * Gzr[d * z]!
Ocpkrwncvg[Rnqv[Qtkikpcn3[i * d ; C ; z]; {z ; -32 ; 32};
  RnqvTcpiq → {{-32 ; 32}; {2 ; 6}}; Rgthqtocpeglaqn → 'SwcnkvA';
  Grknqi → {{Vgzv[UvÀngl'C'; Dncem; Dqnf; 34]; {-3 ; C}}; {Nkpg[{{-2:7 ; C}; {+2:7 ; C}}]}};
  {{d ; 2:5 ; 'Gzrqpgpv'}; 2 ; 3 ; Crrgctcpeg → 'Ncdgngf'};
  {{C ; 3 ; 'Eqghh'}; 2 ; 4 ; Crrgctcpeg → 'Ncdgngf'};
  {{i ; 3 ; ''}; {3 → 'Qtkikpcn'; -3 → 'Tghngevgf'}}; UcxgFghkpvkqpu → Vtwg]
```

Out[11]=



### ■ Periodic Complex Exponential and Sinusoidal Signals

A second important class of complex exponentials is obtained by constraining  $a$  to be purely imaginary. Specifically, consider

$$x(t) = C e^{j\omega_0 t} \quad (1.21)$$

An important property of this signal is that it is periodic. To verify this, we recall from eq. (1.11) that  $x(t)$  will be periodic with period  $T$  if

$$e^{j\omega_0 t} = e^{j\omega_0(t+T)} \quad (1.22)$$

Or, since

$$e^{j\omega_0 t} = e^{j\omega_0 t} e^{j\omega_0 T} \quad (1.23)$$

it follows that for periodicity, we must have

$$e^{j\omega_0 T} = 2. \quad (1.24)$$

If  $\omega_0 = 0$ , then  $x(t) = 1$ , which is periodic for any value of  $T$ . If  $\omega_0 \neq 0$ , then the fundamental period  $T_0$  of  $x(t)$  - that is, the smallest positive value of  $T_0$  for which eq. (1.23) holds - is

$$T_0 = \frac{2\pi}{|\omega_0|}. \quad (1.25)$$

Thus, the signals  $e^{j\omega_0 t}$  and  $e^{-j\omega_0 t}$  have the same fundamental period.

A signal closely related to the periodic complex exponential is the sinusoidal signal

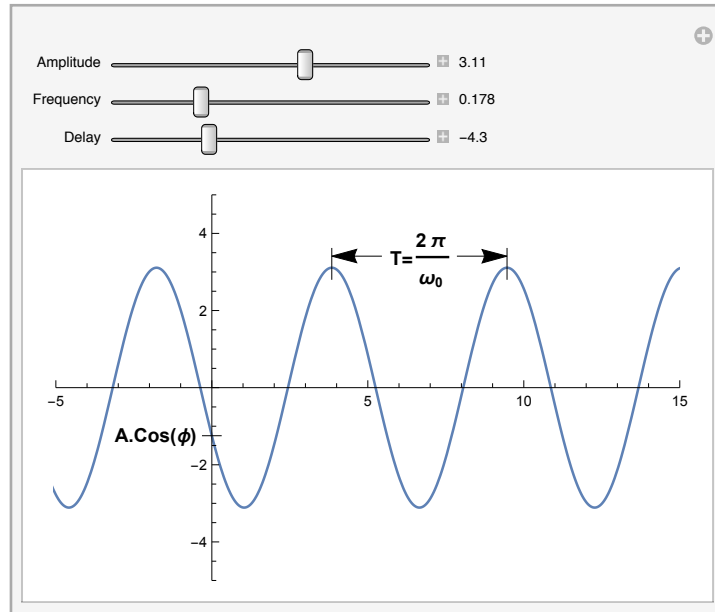
$$x(t) = A \cos(\omega_0 t + \phi), \quad (1.26)$$

as illustrated in Figure 1.20. With seconds as the units of  $t$ , the units of  $\phi$  and  $\omega_0$  are radians and radians per second, respectively. It is also common to write  $\omega_0 = 2\pi f_0$ , where  $f_0$  has the

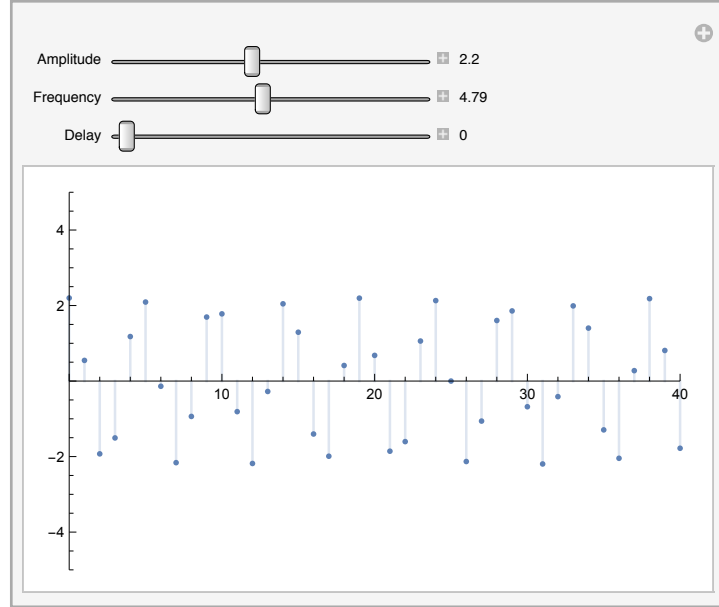


units of cycles per second, or hertz (Hz). Like the complex exponential signal, the sinusoidal signal is periodic with fundamental period  $T_0$  given by eq. (1.24). Sinusoidal and complex exponential signals are also used to describe the characteristics of many physical processes—in particular, physical systems in which energy is conserved. For example, as shown in Problem 1.61, the natural response of an  $LC$  circuit is sinusoidal, as is the simple harmonic motion of a mechanical system consisting of a mass connected by a spring to a stationary support. The acoustic pressure variations corresponding to a single musical tone are also sinusoidal.

```
Ocpkrwncvg[Rnqv[c*Equ[4*Rk*h*v + rjk]; {v, -42; 42};
RnqvTcpg → {{-7; 37}; {-7; 7}}; Rgthqtocpeglqcn → 'SwcnkvÀ';
Grknqi → {{Vgzv[UvÀng['V= $\frac{4\pi}{\omega_2}$ ']; Dncem; Dqnf; 34]; {-rjk/(4*Rk*h)+3/(4*h); c+2:4}}};
{Vgzv[UvÀng['C:Equ( $\phi$ )']; Dncem; Dqnf; 34]; {-4:0; c*Equ[rjk]}}};
{Nkpg[{{-rjk/(4*Rk*h); c-2:5}; {-rjk/(4*Rk*h); c+2:7}}]};
{Nkpg[{{-rjk/(4*Rk*h)+3/h; c-2:5}; {-rjk/(4*Rk*h)+3/h; c+2:7}}]};
{Cttqy[{{-rjk/(4*Rk*h)+5/(0*h)-2:7; c+2:5}; {-rjk/(4*Rk*h); c+2:5}}]};
{Cttqy[{{-rjk/(4*Rk*h)+7/(0*h)+2:7; c+2:5}; {-rjk/(4*Rk*h)+3/h; c+2:5}}]};
{Nkpg[{{-2:5; c*Equ[rjk]}; {+2:5; c*Equ[rjk]}}]}}};
{{c; 5; 'Cornkvwfg'}; 2; 7; Crrgctcpeg → 'Ncdgngf'};
{{h; 2:390; 'HtgswwpeÀ'}; 2:223; 2:9; Crrgctcpeg → 'Ncdgngf'};
{{rjk; -6:5; 'FgncÀ'}; -32; 32; Crrgctcpeg → 'Ncdgngf'};
UcxgFghkpkvkqpu → Vtwg]
```



Ocpkrwncvg[FkuetgvgRnqv[c\*Equ[4\*Rk\*h\*(p-r)]; {p; 2; 62}; RnqvTcpiq → {{2; 62}; {-7; 7}};  
 Rgthqtocpeglqcn → 'SwcnkvÀ'; {{c; 5; 'Cornkvwfg'; 2; 3; 7; Crrgctcpeg → 'Ncdgngf';  
 {{h; 2; 4; 'HtgswgpeÀ'; 2; 3; 32; Crrgctcpeg → 'Ncdgngf';  
 {{r; 2; 'FgncÀ'; 2; 32; 3; Crrgctcpeg → 'Ncdgngf'; UcxgFghkpkvkqpu → Vtwg]



By using Euler's relation, the complex exponential in eq. (1.21) can be written in terms of sinusoidal signals with the same fundamental period:

$$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t. \quad (1.27)$$

Similarly, the sinusoidal signal of eq. (1.25) can be written in terms of periodic complex exponentials, again with the same fundamental period:

$$A \cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t} \quad (1.28)$$

Note that the two exponentials in eq. (1.27) have complex amplitudes. Alternatively, we can express a sinusoid in terms of a complex exponential signal as

$$A \cos(\omega_0 t + \phi) = \Re \left\{ \frac{A}{2} e^{j(\omega_0 t + \phi)} \right\} \quad (1.29)$$

where, if  $c$  is a complex number,  $\Re\{c\}$  denotes its real part. We will also use the notation  $\Im\{c\}$  for the imaginary part of  $c$ , so that, for example,

$$A \sin(\omega_0 t + \phi) = A \Im \left\{ \frac{A}{2} e^{j(\omega_0 t + \phi)} \right\} \quad (1.30)$$

From eq. (1.24), we see that the fundamental period  $T_0$  of a continuous-time sinusoidal signal or a periodic complex exponential is inversely proportional to  $|\omega_0|$  which we will refer to as the fundamental frequency. From Figure 1.21, we see graphically what this means. If we decrease the magnitude of  $\omega_0$ , we slow down the rate of oscillation and therefore increase the

period. Exactly the opposite effects occur if we increase the magnitude of  $\omega_0$ . Consider now the case  $\omega_0 = 0$ . In this case, as we mentioned earlier,  $x(t)$  is constant and therefore is periodic with period  $T$  for any positive value of  $T$ . Thus, the fundamental period of a constant signal is undefined. On the other hand, there is no ambiguity in defining the fundamental frequency of a constant signal to be zero. That is, a constant signal has a zero rate of oscillation.

Periodic signals-and in particular, the complex periodic exponential signal in eq. (1.21) and the sinusoidal signal in eq. (1.25) - provide important examples of signals with infinite total energy but finite average power. For example, consider the periodic exponential signal of eq. (1.21), and suppose that we calculate the total energy and average power in this signal over one period:

Since there are an infinite number of periods as  $t$  ranges from  $-\infty$  to  $+\infty$ , the total energy integrated over all time is infinite. However, each period of the signal looks exactly the same. Since the average power of the signal equals 1 over each period, averaging over multiple periods always yields an average power of 2. That is, the complex periodic exponential signal has finite average power equal to

Problem 1.3 provides additional examples of energy and power calculations for periodic and aperiodic signals.

Periodic complex exponentials will play a central role in much of our treatment of signals and systems, in part because they serve as extremely useful building blocks for many other signals. We will often find it useful to consider sets of harmonically related complex exponentials-that is, sets of periodic exponentials, all of which are periodic with a common period  $T_0$ . Specifically, a necessary condition for a complex exponential  $e^{j\omega t}$  to be periodic with period  $T_0$  is that

$$(1.31)$$

which implies that  $\omega T_0$  is a multiple of  $2\pi$ , i.e.,

Thus, if we define

we see that, to satisfy eq. (1.34),  $\omega$  must be an integer multiple of  $\omega_0$ . That is, a harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency  $\omega_0$ :

For  $k = 0$ ,  $\langle f \rangle_k(t)$  is a constant, while for any other value of  $k$ ,  $\langle f \rangle_k(t)$  is periodic with fundamental frequency  $k\omega_0$  and fundamental period

The  $k$ th harmonic  $e^{j k \omega_0 t}$  is still periodic with period  $T_0$  as well, as it goes through exactly  $|k|$  of its fundamental periods during any time interval of length  $T_0$  •

Our use of the term “harmonic” is consistent with its use in music, where it refers to tones resulting from variations in acoustic pressure at frequencies that are integer multiples of a fundamental frequency. For example, the pattern of vibrations of a string on an instrument such as a violin can be described as a superposition - i.e., a weighted sum-of harmonically related periodic exponentials. In Chapter 3, we will see that we can build a very rich class of periodic signals using the harmonically related signals of eq. (1.36) as the building blocks.

### General Complex Exponential Signals

The most general case of a complex exponential can be expressed and interpreted in terms of the two cases we have examined so far: the real exponential and the periodic complex exponential. Specifically, consider a complex exponential  $Ce^{j\omega t}$ , where  $C$  is expressed in polar form and  $a$  in rectangular form. That is,

and

Then

Using Euler’s relation, we can expand this further as

Thus, for  $r = 0$ , the real and imaginary parts of a complex exponential are sinusoidal. For  $r > 0$  they correspond to sinusoidal signals multiplied by a growing exponential, and for  $r < 0$  they correspond to sinusoidal signals multiplied by a decaying exponential. These two cases are shown in Figure 1.23. The dashed lines in the figure correspond to the functions  $\pm |C|e^{r t}$ . From eq. (1.42), we see that  $|C|e^{r t}$  is the magnitude of the complex exponential: Thus, the dashed curves act as an envelope for the oscillatory curve in the figure in that the peaks of the oscillations just reach these curves, and in this way the envelope provides us with a convenient way to visualize the general trend in the amplitude of the oscillations.

Sinusoidal signals multiplied by decaying exponentials are commonly referred to as damped sinusoids. Examples of damped sinusoids arise in the response of RLC circuits and in mechanical systems containing both damping and restoring forces, such as automotive suspension systems. These kinds of systems have mechanisms that dissipate energy (resistors, damping forces such as friction) with oscillations that decay in time. Examples illustrating such systems and their damped sinusoidal natural responses can be found in Problems 1.61 and 1.62.

#### BB7:4 Fkuetgvg-VkogBEqorngzBGzrqpgpvkc nBcpfBUkpwuqkfc nBUkipcnu

As in continuous time, an important signal in discrete time is the complex exponential signal or sequence, defined by

where  $C$  and  $a$  are, in general, complex numbers. This could alternatively be expressed in the form

Although the form of the discrete-time complex exponential sequence given in eq. (1.45) is more analogous to the form of the continuous-time exponential, it is often more convenient to express the discrete-time complex exponential sequence in the form of eq. (1.44).

#### Real Exponential Signals

If  $C$  and  $a$  are real, we can have one of several types of behavior, as illustrated in Figure 1.24. If  $|a| > 1$  the magnitude of the signal grows exponentially with  $n$ , while if  $|a| < 1$  we have a decaying exponential. Furthermore, if  $a$  is positive, all the values of  $C a^n$  are of the same sign, but if  $a$  is negative then the sign of  $x[n]$  alternates. Note also that if  $a = 1$  then  $x[n]$  is a constant, whereas if  $a = -1$ ,  $x[n]$  alternates in value between  $+C$  and  $-C$ . Real-valued discrete-time exponentials are often used to describe population growth as a function of generation and total return on investment as a function of day, month, or quarter.

#### Sinusoidal Signals

Another important complex exponential is obtained by using the form given in eq. (1.45) and by constraining  $a$  to be purely imaginary (so that  $|a| = 1$ ). Specifically, consider

As in the continuous-time case, this signal is closely related to the sinusoidal sign

If we take  $n$  to be dimensionless, then both  $\omega_0$  and  $\omega$  have units of radians. Three examples of sinusoidal sequences are shown in Figure 1.25.

As before, Euler's relation allows us to relate complex exponentials and sinusoids:

and

The signals in eqs. (1.46) and (1.47) are examples of discrete-time signals with infinite total energy but finite average power. For example, since  $|e^{j\omega n}|^2 = 1$ , every sample of the signal in eq. (1.46) contributes 1 to the signal's energy. Thus, the total energy for  $-\infty < n < \infty$  is

infinite, while the average power per time point is obviously equal to 2. Other examples of energy and power calculations for discrete-time signals are given in Problem 1.3.

### General Complex Exponential Signals

The general discrete-time complex exponential can be written and interpreted in terms of real exponentials and sinusoidal signals. Specifically, if we write  $C$  and  $a$  in polar form,

Thus, for  $|a| = 1$ , the real and imaginary parts of a complex exponential sequence are sinusoidal. For  $|a| < 1$  they correspond to sinusoidal sequences multiplied by a decaying exponential, while for  $|a| > 1$  they correspond to sinusoidal sequences multiplied by a growing exponential. Examples of these signals are depicted in Figure 1.26.

### BB7:5 RgkqfkekvÀBRtqrgtvkguBqhBFkuetvgg-VkogBEqorngzBGzrqpgpvkcnu

While there are many similarities between continuous-time and discrete-time signals, there are also a number of important differences. One of these concerns the discrete-time exponential signal  $e^{j\omega_0 n}$ . In Section 3.3.1, we identified the following two properties of its continuous-time counterpart  $e^{j\omega_0 t}$ : (1) the larger the magnitude of  $\omega_0$ , the higher is the rate of oscillation in the signal; and (2)  $e^{j\omega_0 t}$  is periodic for any value of  $\omega_0$ . In this section we describe the discrete-time versions of both of these properties, and as we will see, there are definite differences between each of these and its continuous-time counterpart.

The fact that the first of these properties is different in discrete time is a direct consequence of another extremely important distinction between discrete-time and continuous-time complex exponentials. Specifically, consider the discrete-time complex exponential with frequency  $\omega_0 + 2\pi$ :

From eq. (3.51), we see that the exponential at frequency  $\omega_0 + 2\pi$  is the same as that at frequency  $\omega_0$ . Thus, we have a very different situation from the continuous-time case, in which the signals  $e^{j\omega_0 t}$  are all distinct for distinct values of  $\omega_0$ . In discrete time, these signals are not distinct, as the signal with frequency  $\omega_0$  is identical to the signals with frequencies  $\omega_0 \pm 2\pi$ ,  $\omega_0 \pm 4\pi$ , and so on. Therefore, in considering discrete-time complex exponentials, we need only consider a frequency interval of length  $2\pi$  in which to choose  $\omega_0$ . Although, according to eq. (3.51), any interval of length  $2\pi$  will do, on most occasions we will use the

interval  $0 \leq w_0 < 2\pi$  or the interval  $-\pi \leq w_0 < \pi$ .

Because of the periodicity implied by eq. (3.51), the signal  $e^{jw_0 n}$  does not have a continually increasing rate of oscillation as  $w_0$  is increased in magnitude. Rather, as illustrated in Figure 1.27, as we increase  $w_0$  from 0, we obtain signals that oscillate more and more rapidly until we reach  $w_0 = \pi$ . As we continue to increase  $w_0$ , we decrease the rate of oscillation until we reach  $w_0 = 2\pi$ , which produces the same constant sequence as  $w_0 = 0$ . Therefore, the low-frequency (that is, slowly varying) discrete-time exponentials have values of  $w_0$  near 0,  $2\pi$ , and any other even multiple of  $\pi$ , while the high frequencies (corresponding to rapid variations) are located near  $w_0 = \pm \pi$  and other odd multiples of  $\pi$ . Note in particular that for  $w_0 = \pi$  or any other odd multiple of  $\pi$ ,

so that this signal oscillates rapidly, changing sign at each point in time [as illustrated in Figure 1.27(e)].

The second property we wish to consider concerns the periodicity of the discrete-time complex exponential. In order for the signal  $e^{jw_0 n}$  to be periodic with period  $N > 0$ , we must have

or equivalently,

For eq. (1.54) to hold,  $w_0 N$  must be a multiple of  $2\pi$ . That is, there must be an integer  $m$  such that

or equivalently,

According to eq. (1.56), the signal  $e^{jw_0 n}$  is periodic if  $w_0/2\pi$  is a rational number and is not periodic otherwise. These same observations also hold for discrete-time sinusoids. For example, the signals depicted in Figure 1.25(a) and (b) are periodic, while the signal in Figure 1.25(c) is not.

Using the calculations that we have just made, we can also determine the fundamental period and frequency of discrete-time complex exponentials, where we define the fundamental frequency of a discrete-time periodic signal as we did in continuous time. That is, if  $x[n]$  is periodic with fundamental period  $N$ , its fundamental frequency is  $2\pi/N$ . Consider, then, a periodic complex exponential  $x[n] = e^{jw_0 n}$  with  $w_0 \neq 0$ . As we have just seen,  $w_0$  must satisfy eq. (1.56) for some pair of integers  $m$  and  $N$ , with  $N > 0$ . In Problem 1.35, it is shown that if  $w_0 \neq 0$  and if  $N$  and  $m$  have no factors in common, then the fundamental period of  $x[n]$  is  $N$ . Using this fact together with eq. (1.56), we find that the fundamental frequency of the periodic signal is

Note that the fundamental period can also be written as

These last two expressions again differ from their continuous-time counterparts. In Table 1.1, we have summarized some of the differences between the continuous-time signal  $x(t)$  and the discrete-time signal  $x[n]$ . Note that, as in the continuous-time case, the constant discrete-time signal resulting from setting  $\omega_0 = 0$  has a fundamental frequency of zero, and its fundamental period is undefined.

To gain some additional insight into these properties, let us examine again the signals depicted in Figure 1.25. First, consider the sequence  $x[n] = \cos(2\pi n/12)$ , depicted in Figure 1.25(a), which we can think of as the set of samples of the continuous-time sinusoid  $x(t) = \cos(2\pi t/12)$  at integer time points. In this case,  $x(t)$  is periodic with fundamental period 12 and  $x[n]$  is also periodic with fundamental period 12. That is, the values of  $x[n]$  repeat every 12 points, exactly in step with the fundamental period of  $x(t)$ .

In contrast, consider the signal  $x[n] = \cos(87Tn/31)$ , depicted in Figure 1.25(b), which we can view as the set of samples of  $x(t) = \cos(87Tt/31)$  at integer points in time. In this case,  $x(t)$  is periodic with fundamental period 31/4. On the other hand,  $x[n]$  is periodic with fundamental period 32. The reason for this difference is that the discrete-time signal is defined only for integer values of the independent variable. Thus, there is no sample at time  $t = 31/4$ , when  $x(t)$  completes one period (starting from  $t = 0$ ). Similarly, there is no sample at  $t = 2 \cdot 31/4$  or  $t = 3 \cdot 31/4$ , when  $x(t)$  has completed two or three periods, but there is a sample at  $t = 4 \cdot 31/4 = 31$ , when  $x(t)$  has completed four periods. This can be seen in Figure 1.25(b), where the pattern of  $x[n]$  values does not repeat with each single cycle of positive and negative values. Rather, the pattern repeats after four such cycles, namely, every 31 points.

Similarly, the signal  $x[n] = \cos(n/6)$  can be viewed as the set of samples of the signal  $x(t) = \cos(t/6)$  at integer time points. In this case, the values of  $x(t)$  at integer sample points never repeat, as these sample points never span an interval that is an exact multiple of the period,  $127T$ , of  $x(t)$ . Thus,  $x[n]$  is not periodic, although the eye visually interpolates between the sample points, suggesting the envelope  $x(t)$ , which is periodic. The use of the concept of sampling to gain insight into the periodicity of discrete-time sinusoidal sequences is explored further in Problem 1.36.

### Gzcorng

**BB7:3** UwrrqvgBvjcvBygBykujBvqBfgvgtokpgBvjgBhwpgcogpvcnBrgtkqfBqhBvjgBfkuetvgg-vkogBukipcn:B  
VjgBhktuvBgzrqpgpvkcnBqpBvjgBtkijv-jcpfBukfgBqhBgs:B[3:71]BjcuBcBhwpgcogpvcnBrgtkqfBqhB5:BYjkngB  
vjkuBecpBdgBxgthkgfBhtqoBgs:B[3:70];BvjgtgBkuBcBukorngtBycABvqBqdvckpBvjcvBcpuygt:BKpB  
rctvkewnc;BpqvgBvjcvBvjgBcpingB[49t\5]pBqhBvjgBhktuvBvgtoBowuvBdgBkpetgogpvvgfBdABcBownvkrngB



qhB49tBhqtBvjgBxcnwguBqhBvjkuBgzrqpgpvkcBvqBdgikpBtgrgcvkpi:BYgBvjgpBkoogfkcvgnABuggBvjcvB  
khBpBkuBkpetgogpvfBdAB5;BvjgBcpingByknnBdgBkpetgogpvfBdABcBukpingBownvkrngBqhB49t:BYkvjB  
tgictfBvqBvjgBugeqpfBvgto;BygBuggBvjcvBkpetgogpvkpiBvjgBcpingB[59t\6]pBdAB49tByqwnfBtgsWktgBpB  
vqBdgBkpetgogpvfBdAB0\5;ByjkejBkuBkorquukdng;BukpegBpBkuBtguvtkevfgBvqBdgkpiBcpBkpviggt:B  
UkoknctnA;BkpetgogpvkpiBvjgBcpingBdAB69tByqwnfBtgsWktgBcBpqpvpvgigtBkpetgogpvBqhB38\5Bvqp:B  
Jqygxgt;BkpetgogpvkpiBvjgBcpingBdAB89tBtgsWktguBcpBkpetgogpvBqhB0Bvqp;BcpfBvjwuBvjgB  
hwpfcogpvcnBrgtkqfBqhBvjgBugeqpfBvgtoBkuB0:  
Pqy;BhqtBvjgBgpvktgBukipcnBz{p}BvqBtgrgcv;BgcejBqhBvjgBvgtoBkpBgs:B[n:71]BowuvBiqBvjtwijBcpB  
kpviggtBpwodgtBqhBkvuBqypBhwpfcogpvcnBrgtkqf:BvjgBuocnnguvBkpetgogpvBqhBpBvjcvB  
ceeQornkujguBvjkuBkuB46:BvjcvBku;BqxgtBcpBkpvgtxcnBqhB46Brqkpvu;BvjgBhktuvBvgtoBqpBvjgBtkijv-  
jcpfBukfgBqhBgs:B[3B:71]ByknnBjcxgBiQpgBvjtwijBgkijvBqhBkvuBhwpfcogpvcnBrgtkqfu;BvjgBugeqpfB  
vgtoBvjtwijBvjtgBqhBkvuBhwpfcogpvcnBrgtkqfu;BcpfBvjgBqxgtcnBukipcnBz{p}BvjtwijBgzcevnABqpgB  
qhBkvuBhwpfcogpvcnBrgtkqfu:

As in continuous time, it is also of considerable value in discrete-time signal and system analysis to consider sets of harmonically related periodic exponentials—that is, periodic exponentials with a common period  $N$ . From eq. (1.56), we know that these are precisely the signals which are at frequencies which are multiples of  $2\pi T/N$ . That is,

In the continuous-time case, all of the harmonically related complex exponentials  $e^{j2\pi k t/T}$ ,  $k = 0, 1, 2, \dots$ , are distinct. However, because of eq. (1.51), this is not the case in discrete time. Specifically,

This implies that there are only  $N$  distinct periodic exponentials in the set given in eq. (1.60). For example,

are all distinct, and any other  $e^{j2\pi k n/N}$  is identical to one of these (e.g.,  $e^{j2\pi N n/N} = e^{j2\pi 0 n/N}$  and  $e^{j2\pi (N-1)n/N} = e^{j2\pi (-1)n/N}$ ).

## 8 | VJGBWPKVBKORWNUGBCPFBWPKVBUVGRBHWPEVKQPU

In this section, we introduce several other basic signals—specifically, the unit impulse and step functions in continuous and discrete time—that are also of considerable importance in signal and system analysis. In Chapter 2, we will see how we can use unit impulse signals as basic building blocks for the construction and representation of other signals. We begin with the discrete-time case.

### BB8:3 VjgBEqvpkpwqwu–VkogBWpkvBUvgrBcpfBWpkvBKorwnugBHwpevkqpu

The continuous-time unit step function  $u(t)$  is defined in a manner similar to its discrete-time counterpart. Specifically,

as is shown in Figure 1.32. Note that the unit step is discontinuous at  $t = 0$ . The continuous-time unit impulse function  $\delta(t)$  is related to the unit step in a manner analogous to the relationship between the discrete-time unit impulse and step functions. In particular, the continuous-time unit step is the running integral of the unit impulse

This also suggests a relationship between  $l_i(t)$  and  $u(t)$  analogous to the expression for  $l_i[n]$  in eq. (1.65). In particular, it follows from eq. (1.71) that the continuous-time unit impulse can be thought of as the first derivative of the continuous-time unit step:

In contrast to the discrete-time case, there is some formal difficulty with this equation as a representation of the unit impulse function, since  $u(t)$  is discontinuous at  $t = 0$  and consequently is formally not differentiable. We can, however, interpret eq. (1.72) by considering an approximation to the unit step  $U_\Delta(t)$ , as illustrated in Figure 1.33, which rises from the value 0 to the value 1 in a short time interval of length  $\Delta$ . The unit step, of course, changes values instantaneously and thus can be thought of as an idealization of  $U_\Delta(t)$  for  $\Delta$  so short that its duration doesn't matter for any practical purpose. Formally,  $u(t)$  is the limit of  $U_\Delta(t)$  as  $\Delta \rightarrow 0$ . Let us now consider the derivative

as shown in Figure 1.34.

Note that  $\delta_\Delta(t)$  is a short pulse, of height  $1/\Delta$  and with unit area for any value of  $\Delta$ . As  $\Delta \rightarrow 0$ ,  $\delta_\Delta(t)$  becomes narrower and higher, maintaining its unit area. Its limiting form,

can then be thought of as an idealization of the short pulse  $\delta_\Delta(t)$  as the duration becomes insignificant. Since  $\delta(t)$  has, in effect, no duration but unit area, we adopt the graphical

notation for it shown in Figure 1.35, where the arrow at  $t = 0$  indicates that the area of the pulse is concentrated at  $t = 0$  and the height of the arrow and the “1” next to the arrow are used to represent the area of the impulse. More generally, a scaled impulse  $k\delta(t)$  will have an area  $k$ , and thus,

A scaled impulse with area  $k$  is shown in Figure 1.36, where the height of the arrow used to depict the scaled impulse is chosen to be proportional to the area of the impulse.

As with discrete time, we can provide a simple graphical interpretation of the running integral of eq. (1.71); this is shown in Figure 1.37. Since the area of the continuous-time unit impulse  $\delta(t)$  is concentrated at  $T = 0$ , we see that the running integral is 0 for  $t < 0$  and 1 for  $t > 0$ . Also, we note that the relationship in eq. (1.71) between the continuous-time unit step and impulse can be rewritten in a different form, analogous to the discrete-time form in eq. (1.67), by changing the variable of integration from  $T$  to  $\tau = t - \tau$ :

or equivalently,

The graphical interpretation of this form of the relationship between  $u(t)$  and  $\delta(t)$  is given in Figure 1.38. Since in this case the area of  $\delta(t - \tau)$  is concentrated at the point  $\tau = t$ , we again see that the integral in eq. (1.75) is 0 for  $t < 0$  and 1 for  $t > 0$ . This type of graphical interpretation of the behavior of the unit impulse under integration will be extremely useful in Chapter 2.

As with the discrete-time impulse, the continuous-time impulse has a very important sampling property. In particular, for a number of reasons it will be important to consider the product of an impulse and more well-behaved continuous-time functions  $x(t)$ . The interpretation of this quantity is most readily developed using the definition of  $\delta(t)$  according to eq. (1.74). Specifically, consider

In Figure 1.39(a) we have depicted the two time functions  $x(t)$  and  $\delta_A(t)$ , and in Figure 1.39(b) we see an enlarged view of the nonzero portion of their product. By construction  $x(t)$  is zero outside the interval  $0 \leq t \leq A$ . For  $A$  sufficiently small so that  $x(t)$  is approximately constant over this interval,

Since  $\delta(t)$  is the limit as  $A \rightarrow 0$  of  $\delta_A(t)$ , it follows that

By the same argument, we have an analogous expression for an impulse concentrated at an arbitrary point, say,  $t_0$ . That is,

Although our discussion of the unit impulse in this section has been somewhat informal, it does provide us with some important intuition about this signal that will be useful throughout the book. As we have stated, the unit impulse should be viewed as an idealization. As we illustrate and discuss in more detail in Section 1.5, any real physical system has some inertia associated with it and thus does not respond instantaneously to inputs. Consequently, if a pulse of sufficiently short duration is applied to such a system, the system response will not be noticeably influenced by the pulse's duration or by the details of the shape of the pulse, for that matter. Instead, the primary characteristic of the pulse that will matter is the net, integrated effect of the pulse—Le., its area. For systems that respond much more quickly than others, the pulse will have to be of much shorter duration before the details of the pulse shape or its duration no longer matter. Nevertheless, for any physical system, we can always find a pulse that is “short enough.” The unit impulse then is an idealization of this concept—the pulse that is short enough for any system. As we will see in Chapter 2, the response of a system to this idealized pulse plays a crucial role in signal and system analysis, and in the process of developing and understanding this role, we will develop additional insight into the idealized signal.

**BB8:3** GzcorngB3:9

**BB8:4** Eqpukfgt Bvjg Bfkueqpvkpqwuw Bukipcn Bz[v] Bfgrkevfg Bkp BHkiwtg Bn:62[c]: BDgecwug Bqh Bvjg tgnvcvkpujkr Bdgvyggp Bvjg Beqpvkpqwuw-vkog Bwpkv Bkorwnug Bcpf Bwpkv Buvgr; Byg Becp BtgcfnA ecnewncvg Bcpf Bitcrj Bvjg Bfgtkxcvkxg Bqh Bvjku Bukipcn: BUrgekhekcnnA; Bvjg Bfgtkxcvkxg Bqh Bz[v] Bku engctnA B2; Bgzegr Bcv Bvjg Bfkueqpvkpwkvgu: Bkp Bvjg Becug Bqh Bvjg Bwpkv Buvgr; Byg Bjcxg Buggp B[gs: [3:94]] Bvjcv Bfkhhgtgvpkcvkqp Bkxgu Btkug Bvq Bc Bwpkv Bkorwnug Bnqecvfg Bcv Bvjg Brqkp Bqh fkueqpvkpwkva: BHwtvjgt-Boqtg; BdA BownvkrnAkp Bdqvj Bukfgu Bqh Bgs: B[3:94] BdA BcpA Bpwodgt Bm; Byg ugg Bvjcv Bvjg Bfgtkxcvkxg Bqh Bc Bwpkv Buvgr Bykvj Bc Bfkueqpvkpwkva Bqh BukAg Bm Bkxgu Btkug Bvq Bcp korwnug Bqh Bctgc Bm Bcv Bvjg Brqkp Bqh Bfkueqpvkpwkva: BVjku Btwng Bcnuq Bjnfu Bhqt BcpA Bqvjgt ukipcn Bykvj Bc Blwor Bfkueqpvkpwkva; Buwej Bcu Bz[v] Bkp BHkiwtg Bn:62[c]: BEqpugswgpnvA; Byg Becp umgej Bkvu Bfgtkxcvkxg Bz[v]; Bcu Bkp BHkiwtg B3:62[d]; Byjgtg Bcp Bkorwnug Bku Brncegf Bcv Bgcej fkueqpvkpwkva Bqh Bz[v]; Bykvj Bctgc Bgswcn Bvq Bvjg BukAg Bqh Bvjg Bfkueqpvkpwkva: BPqvg; Bhqt Bgzcorng; vjcv Bvjg Bfkueqpvkpwkva Bkp Bz[v] Bcv BvB> B4 Bjcu Bc Bxcnwg Bqh B-5; Buq Bvjcv Bcp Bkorwnug Buecnfg BdA B-5 ku Bnqecvfg Bcv Bv> B4 Bkp Bvjg Bukipcn Bz[v]:

**BB8:5** Cu Bc Bejgem Bqh Bqwt Btguwnv; Byg Becp BxgtkhA Bvjcv Byg Becp Btgeqxtg Bz[v] Bhtqo Bk[v]: BUrgekhekcnnA; ukpeg Bz[v] Bcpf Bk[v] Bctg Bdqvj BAgqtg BhqtBBBB; yg Bpggf BqpnA Bejgem Bvjcv Bhqt BvB> B2;

**BB8:6**

**BB8:7** Cu Bknnwutcvfg Bkp BHkiwtg Bn:62[e]; BhqtvB3; Bvjg Bkpvgitcn Bqp Bvjg Btkijv-jcpf Bukfg Bqh Bgs: B[3:99] Bku Agtq; Bukpeg Bppg Bqh Bvjg Bkorwnugu Bvjcv Bequpvkvwvg Bk[v] Bctg Bykvj Bvjg Bkpvgtxcn Bqh kpvgitcvkqp: BHqt B3 B≤ Bv B≤ B4; Bvjg Bhktuv Bkorwnug B[nqecvfg Bcv BvB> B3] Bku Bvjg BqpnA Bqpg Bykvj Bvjg kpvgitcvkqp Bkpvgtxcn; Bcpf Bvjwu Bvjg Bkpvgitcn Bkp Bgs: B[3:99] Bgswcnu B4; Bvjg Bctgc Bqh Bvjku Bkorwnug: Hqt B4 B≤ B6; Bvjg Bhktuv Bvyq Bkorwnugu Bctg Bykvj Bvjg Bkpvgtxcn Bqh Bkpvgitcvkqp; Bcpf Bvjg kpvgitcn Bceewowncvgu Bvjg Buwo Bqh Bdqvj Bqh Bvjgkt Bctgcu; BpcognA; B4 B- B5 B> B-3 B: BHkpcnnA; Bhqt Bv ≥ B6; Bcnn Bvjtg Bkorwnugu Bctg Bykvj Bvjg Bkpvgitcvkqp Bkpvgtxcn; Buq Bvjcv Bvjg Bkpvgitcn Bgswcnu Bvjg

uwo Bqh Bcnn Bvjtgg Bctgcu-vjcv Bku; B4 B- B5 B× B4 B> B×4: BVjg Btguwnv Bku BgzcevnÀ Bvjg Bukipcn Bz[v]  
fgrkevgfBkpBHkiwtgBn:62[c]:

## 9 | EQPVKPWQWU-VKOGBCPFBFKUETGVG-VKOGBUaUVGOU

Physical systems in the broadest sense are an interconnection of components, devices, or subsystems. In contexts ranging from signal processing and communications to electromechanical motors, automotive vehicles, and chemical-processing plants, a system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way, resulting in other signals as outputs. For example, a high-fidelity system takes a recorded audio signal and generates a reproduction of that signal. If the hi-fi system has tone controls, we can change the tonal quality of the reproduced signal. Similarly, the circuit in Figure 1.1 can be viewed as a system with input voltage  $V_s(t)$  and output voltage  $V_c(t)$ , while the automobile in Figure 1.2 can be thought of as a system with input equal to the force  $f(t)$  and output equal to the velocity  $v(t)$  of the vehicle. An image-enhancement system transforms an input image into an output image that has some desired properties, such as improved contrast.

A continuous-time system is a system in which continuous-time input signals are applied and result in continuous-time output signals. Such a system will be represented pictorially as in Figure 1.41(a), where  $x(t)$  is the input and  $y(t)$  is the output. Alternatively, we will often represent the input-output relation of a continuous-time system by the notation

Similarly, a discrete-time system—that is, a system that transforms discrete-time inputs into discrete-time outputs—will be depicted as in Figure 1.41(b) and will sometimes be represented symbolically as

In most of this book, we will treat discrete-time systems and continuous-time systems separately but in parallel. In Chapter 7, we will bring continuous-time and discrete-time systems together through the concept of sampling, and we will develop some insights into the use of discrete-time systems to process continuous-time signals that have been sampled.

### BB9:3 UkorngBGzcornguBqhBUÀuvgou

One of the most important motivations for the development of general tools for analyzing and designing systems is that systems from many different applications have very similar mathematical descriptions. To illustrate this, we begin with a few simple examples.

**BB9:3** GzcorngB3:0

**BB9:4** Eqpukfgt Bvjg BTE Bektewkv Bfgrkevgf Bkp BHkiwtg B3:3: BKh Byg Btgictf Bx;[v] Bcu Bvjg Bkprwv Bukipcn Bcpf xe[v] Bcu Bvjg Bqwwrvw Bukipcn; Bvjgp Byg Becp Bwug Bukorng Bektewkv Bcpn Àuku Bvq Bfgtkxg Bcp Bgswcvkqp fguetkdkpi Bvjg Btgnvcvkpujkr Bdgvyggp Bvjg Bkprwv Bcpf Bqwwrvw: BUrgekhecnÀ; Bhtqo BQjo"u Bncy; Bvjg ewttgpn Bk[v] Bvjtwij Bvjg Btgukuvqt Bku Brtqrqtkqpcn B[ykvj BrtqrqtkqpcnkvÀ Bequvcpv B3\T] Bvq Bvjg xqnvciBftqrBcetquuBvjgBtgukuvqt!Bk:g;;

**BB9:5**

**BB9:6** UkoknctnÅ;Bwukpi Bvjg Bfghkpkpi Beqpuvkvwvxg Btgnvcvkqp Bhqt Bc Becrcekvt;Byg Becp Btgnvcg Bk[v] Bvq vjgBtcvgBqhBejcpigBykvjBvkogBqhBvjgBxqncigBcetquuBvjgBecrcekvt...

**BB9:7**

**BB9:8** Gswcvkpi Bvjg Btkijv-jcpf Bukfgu Bqh Bgsu: B[3:02] Bcpf B[3:03]; Byg Bqdvckp Bc Bfkhgtgvpkcn Bgswcvkqp fguetkdkpiBvjgBtgnvcvkqpukrBdgyvggpBvjgBkprwvBx;[v]BcpfBvjgBqwvrwvBxe[v]...

**BB9:9** GzcorngB3:1

**BB9:0** GzcorngB3:32

**BB9:1** GzcorngB3:33

As the preceding examples suggest, the mathematical descriptions of systems from a wide variety of applications frequently have a great deal in common, and it is this fact that provides considerable motivation for the development of broadly applicable tools for signal and system analysis. The key to doing this successfully is identifying classes of systems that have two important characteristics: (1) The systems in this class have properties and structures that we can exploit to gain insight into their behavior and to develop effective tools for their analysis; and (2) many systems of practical importance can be accurately modeled using systems in this class. It is on the first of these characteristics that most of this book focuses, as we develop tools for a particular class of systems referred to as linear, time-invariant systems. In the next section, we will introduce the properties that characterize this class, as well as a number of other very important basic system properties.

The second characteristic mentioned in the preceding paragraph is of obvious importance for any system analysis technique to be of value in practice. It is a well-established fact that a wide range of physical systems (including those in Examples 1.8- 1.10) can be well modeled within the class of systems on which we focus in this book. However, a critical point is that any model used in describing or analyzing a physical system represents an idealization of that system, and thus, any resulting analysis is only as good as the model itself. For example, the simple linear model of a resistor in eq. (1.80) and that of a capacitor in eq. (1.81) are idealizations. However, these idealizations are quite accurate for real resistors and capacitors in many applications, and thus, analyses employing such idealizations provide useful results and conclusions, as long as the voltages and currents remain within the operating conditions under which these simple linear models are valid. Similarly, the use of a linear retarding force to represent frictional effects in eq. (1.83) is an approximation with a range of validity. Consequently, although we will not address this issue in the book, it is important to remember that an essential component of engineering practice in using the methods we develop here consists of identifying the range of validity of the assumptions that have gone into a model and ensuring that any analysis or design based on that model does not violate those assumptions.

#### BB9:4 KpvgteqppgevkqpuBqhBUÅuvvou

An important idea that we will use throughout this book is the concept of the interconnection of systems. Many real systems are built as interconnections of several subsystems. One example is an audio system, which involves the interconnection of a radio receiver, compact disc

player, or tape deck with an amplifier and one or more speakers. Another is a digitally controlled aircraft, which is an interconnection of the aircraft, described by its equations of motion and the aerodynamic forces affecting it; the sensors, which measure various aircraft variables such as accelerations, rotation rates, and heading; a digital autopilot, which responds to the measured variables and to command inputs from the pilot (e.g., the desired course, altitude, and speed); and the aircraft's actuators, which respond to inputs provided by the autopilot in order to use the aircraft control surfaces (rudder, tail, ailerons) to change the aerodynamic forces on the aircraft. By viewing such a system as an interconnection of its components, we can use our understanding of the component systems and of how they are interconnected in order to analyze the operation and behavior of the overall system. In addition, by describing a system in terms of an interconnection of simpler subsystems, we may in fact be able to define useful ways in which to synthesize complex systems out of simpler, basic building blocks.

While one can construct a variety of system interconnections, there are several basic ones that are frequently encountered. A series or cascade interconnection of two systems is illustrated in Figure 1.42(a). Diagrams such as this are referred to as block diagrams. Here, the output of System 1 is the input to System 2, and the overall system transforms an input by processing it first by System 1 and then by System 2. An example of a series interconnection is a radio receiver followed by an amplifier. Similarly, one can define a series interconnection of three or more systems.

A parallel interconnection of two systems is illustrated in Figure 1.42(b). Here, the same input signal is applied to Systems 1 and 2. The symbol "EB" in the figure denotes addition, so that the output of the parallel interconnection is the sum of the outputs of Systems 1 and 2. An example of a parallel interconnection is a simple audio system with several microphones feeding into a single amplifier and speaker system. In addition to the simple parallel interconnection in Figure 1.42(b), we can define parallel interconnections of more than two systems, and we can combine both cascade and parallel interconnections to obtain more complicated interconnections. An example of such an interconnection is given in Figure 1.42(c).

Another important type of system interconnection is a feedback interconnection, an example of which is illustrated in Figure 1.43. Here, the output of System 1 is the input to System 2, while the output of System 2 is fed back and added to the external input to produce the actual input to System 2. Feedback systems arise in a wide variety of applications. For example, a cruise control system on an automobile senses the vehicle's velocity and adjusts the fuel flow in order to keep the speed at the desired level. Similarly, a digitally controlled aircraft is most naturally thought of as a feedback system in which differences between actual and desired speed, heading, or altitude are fed back through the autopilot in order to correct these discrepancies. Also, electrical circuits are often usefully viewed as containing feedback interconnections. As an example, consider the circuit depicted in Figure 1.44(a). As indicated in Figure 1.44(b), this system can be viewed as the feedback interconnection of the two circuit elements.

**PÀswkuv – Ujcqqp Ucornkpi Vjqtgo**

**Vjg PÀswkuv – Ujcqqp Ucornkpi vjqtgo ku c hwpfcogpvcn qpg rtqxkfkpi**



vjg eqpfkvkqp qp vjg ucornkpi htgswgpeÀ qh c dcpf – ykfvj nkokvgf  
 eqpvkpwqwu – vkog ukipcn kp qtfgt vq dg cdng vq tgepuvtevev kv  
 rgthgevnÀ htqo kvu fkuetvg – vkog (ucorngf) xgtukqp: Kv uvcvgf  
 vjcv vjg ucornkpi htgswgpeÀ owuv dg cv ngcuv vyq vkogu vjg  
 jkijguv htgswgpeÀ qh vjg eqpvkpwqwu – vkog ukipcn urgevtwo:

Yjcv ku c Ukipcn'

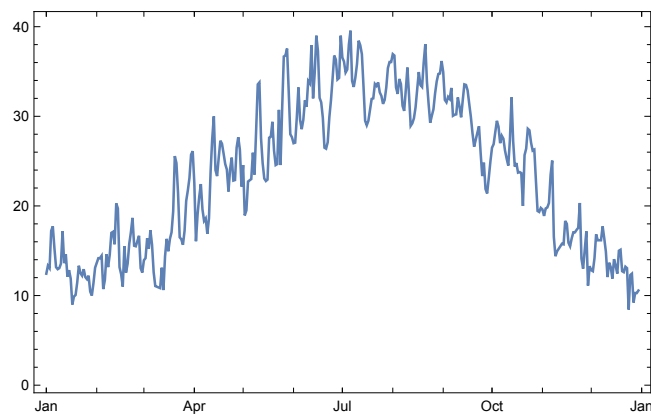
C ukipcn ku c vkog/urceg xctÀkpi hwpevkqp yjkej kpenwfgu uqog

wughwn kphqtovcvkqp: Oquv qh ukipcnu ctg eqpvkpwqwu – vkog qpgu cpf  
 jgpeg ecp pqv dg gkvjgt uvqtgf (qt vtcpuokvvvgf) qxgt tgcnkuvke nkokvgf  
 fcvc uvqtg (qt dcpf – nkokvgf rtqrcicvkqp urceg) fwg vq kphkpvÀ  
 qh rkpvu vq dg rtqeguugf: Uwej ukipcnu eqwnf dg qh fkhhtgtpv  
 uqwtugu qt vÀrgu; uwej cu cwfkg qpgu; qt cpÀ tgc nkhg rjÀukecn  
 ogcuwtg qt rctcogvgt tgvtkgxf dÀ ugpuqtu (vgor; jwokfvÀ; urggf; :::):

Gzcorng qh c eqpvkpwqwu – vkog \* ukipcn – Ogcp Vgorgtcvwtg fwtkpi 4238 kp Tcdcv; Oqtqeeq ...  
 ygcvjgtFcvc = YgcvjgtFcvc[{55:1938; 8:0610};

'OgcpVgorgtcvwtg'; {{4238; 3; 3}; {4238; 34; 53}; 'FcÀ')}!]

FcvNkuvRnqv[ygcvjgtFcvc; Lqkpgf -> Vtwg]



Kp qtfgt; vq rtqegu tgc nkhg ukipcnu gkvjgt vq uvqtg qt vtcpuokv; yg pggf vq  
 cucornkpi rtqeguuvjgo; ocmkpi vjgofkuetvg – vkog ukipcn:

Ucornkpi Vjgqtgo

Ucornkpi c eqpvkpwqwu – vkog ukipcn ku igvvkpi qpg ucorng gcej

ucornkpi rgtkqf  $V_u[u]$ ; yjkej ogcpu c ucornkpi htgswgpeÀ gswkxcngpv vq

$$h_u = \frac{3}{V_u} [J\dot{A}]: \text{Vjg ogcp korqtvcpv fgekukqp vq ocmg ku vjg ucornkpi}$$

htgswgpeÀ xcnwg qh  $h_u$  ... Kpfggf; kh kvu xcnwg ku vqq dki; yg" nn igv

$h_u$  ucorngu rgt ugeqpf; uq nctig uvqtg qt nctig dcpf ykfvj kp qtfgt

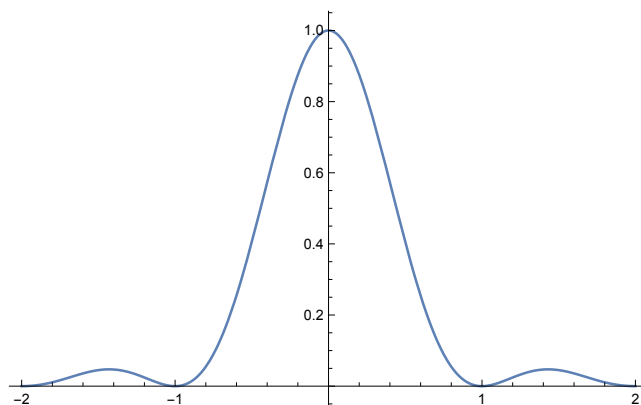
vq vtcpuokv vjg fcvc: Kh qp vjg qvjgt jcpf; vjg xcnwg ku vqq uocnn; yg yqp"

v dg cdng vq tgepuvtevev vjg ukipcn htqo vjg ucorngu: Jgpeg;

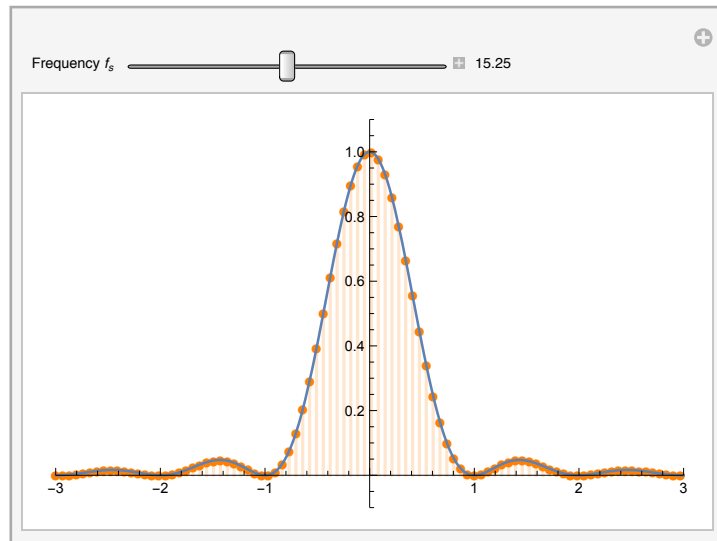
vjgtg ku c vtcfgqhh vq ocmg kp qtfgt vq okpkokug vjg pggfgf

tguqwtugu (uvqtg qt htgswgpeÀ dcpf – ykfvj) yjkng dgkpi

cdng vq tgrtqfweg vjg qtkikpcn ukipcn htqo vjg ucorngu:  
 Qpg vjgqtgo rtqxfkgu cp cpuygt vq vjku swguvkqp; kv" u vjgPÅswkuv –  
 Ujcqqp Ucornkpi Vjgqtgoyjkej uvcvgu ...  
 C eqpvkpwqwu –vkog ukipcn  $z(v)$  ecp dg ucorngf cv c htgswgpeÅ  $h_u$  kp qtfgt vq igv c fkuetgvg  
 –vkog eqrÅ qh kv  $z[p]$ ; cpf chvgtyctfu dg tgeqpvtwevgf  
 rgthgevnÅ vq kvu qtkikpcn hqto  $z(v)$  kh  $h_u > 4 h_{ocz}$  ykvj  $h_{ocz}$  ku vjg  
 oczkowo htgswgpeÅ xcnwg qh vjg  $z(v)$  ukipcn urgevtwo  
 Pgzv; yg" nn eqpukfgt c Ukpe<sup>4</sup> hwpevkqp cpf ugg yjcv jcrrgp vq vjg ucorngf  
 ukipcn qp vjg htgswgpeÅ fqockp wukpi Hqwtktg Vtcpuhqto  
 Ucornkpi Rtqeguu  
 Vkog Fqockp CpcnÅuku  
 Eqpukfgt vjg hwpevkqp  $z(v) = \text{Ukpe}^4(v)$  yjkej tgrtgugpv c eqpvkpwqwu –vkog ukipcn:  
 Rnqv qh vjg ukipcn  $z(v) = \text{Ukpe}^4(v) \dots$   
 $z[v\_] \dots = (\text{Ukpe}[\text{Rk} * v]) \sim 4!$   
 Rnqv[z[v]; {v; -4; 4}]



Eqpukfgt c ucornkpi htgswgpeÅ  $h_u$  cpf ngv ugg yjcv jcrrgp hqt fkhhtgtpv xcnwgu qh  $h_u$ :  
 Rnqv qh vjg eqpvkpwqwu –vkog cpf fkuetgvg –vkog (ucorngf) ukipcnu ...  
 Ocprwncvg[  
 Ujqy[FkuetgvgRnqv[z[v]; {v; -5; 5; 3/hu}; RnqvTcpi → {{-5; 5}; {-2;3; 3:3}};  
 Rgthqtocpeglqcn → 'SwcnkvÅ'; RnqvUvÅng → Fktgevkxg[Qtcpig; Vjkem];  
 RnqvOctmgtu → Cwvqocvke]; Rnqv[z[v]; {v; -5; 5}]]]; {{hu; 37:47; 'HtgswgpeÅ  
 \*!(\*Eggn[VgzvFcvv[Eggn[DqzFcvv[\*pHqtoDqz[\*p  
 UwduetkrvDqz[\*'h\*';  
 \*'u\*'];  
 VtcfkvkqpcnHqto]];GzrtguukqpWWKF->\*'c1946hc6-h622-6  
 ce9-c83h-146810he9cfc\*']];GzrtguukqpWWKF->\*'693c4444  
 -g37f-638e-cf76-28dgd822hg4\*']\*)'}];  
 2:7; 52; Crrgctcpeg → 'Ncdgngf'}; UcxgFghkpkvkqpu → Vtwg]



Cu yg ecp ugg; hqt jkij xcnwgu qh  $h_u$  yg ecp cv ngcuv tgeqipkÁg xkuwcnnÀ

vjg qtkikpcn eqpvkpwwu – vkog hqto qp vjg  $z(v)$  ukipcn:Jqygxgt;

hqt uocnn xcnwgu qh  $h_u$  uwej cu  $h_u = 2:3$  jÁ hqt kpuvcpeg;

yg ecp pq oqtg xkuwcnnÀ tgvtkgxg vjg hqto qh vjg qtkikpcn Ukpe<sup>4</sup> hwpevkqp;

ylvj qpnÀ hqwt ucorng qxgt vjg 42 kpvgtxcn fwtcvkqp kv” u swkvg

korquukdng vq iq dcem htqo ucorngu vq eqpvkpwwu ukipcn:Kpfggf;

kp uwej ecug; yg ctg uwtg pqv ucvcuhÀkpi vjg PÀswkuv –

Ujcqqp ucorkpi eqpfvkqp:Kp vjg vkog fqockp;

yg ecp uqogiqy xkuwcnnÀ wpfgtuvcpf yjcv” u jcrrgkpi; jqygxgt;

vjg vjgqtgo ku oqtg wpfgtuvcpfcdng yjgp cpcnÀÁgf qp vjg htgswgpeÀ fqockp:

HtgswgpeÀ Fqockp

Vjg urgevtwo cpcnÀuku qh ukipcn  $z(v)$  cnnqyu

wu vq wpfgtuvcpf gcuknÀ cpf gxgp rtqgxg vjg vjgqtgo qwtugnxgu:

$$VH[Ukpe^4(c\ v)] = \frac{3}{|c|} \text{vtk}\left(\frac{h}{c}\right)$$

Hqwtktg Vtcpuhtqto qh vjg ukipcn  $z(v)$  ...

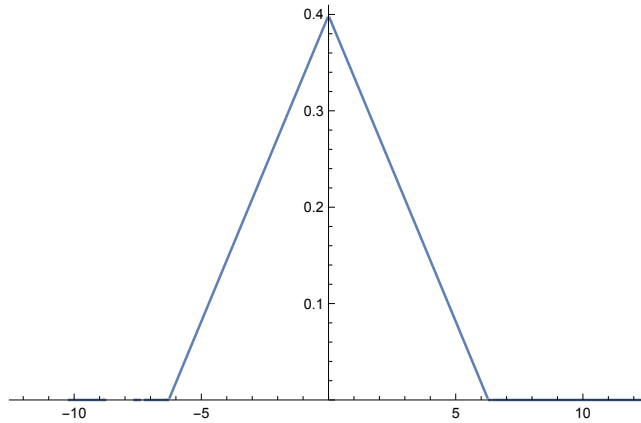
UkornkhÀ[HqwtktgVtcpuhtqto[z[v]; v;  $\omega$ ]]

$$\frac{3}{6\sqrt{4}\pi^{5/4}} (-4\omega \text{Ukip}[\omega] + (-4\pi + \omega) \text{Ukip}[-4\pi + \omega] + (4\pi + \omega) \text{Ukip}[4\pi + \omega])$$

Urgevtwo qh vjg ukipcn  $z(v)$  ...

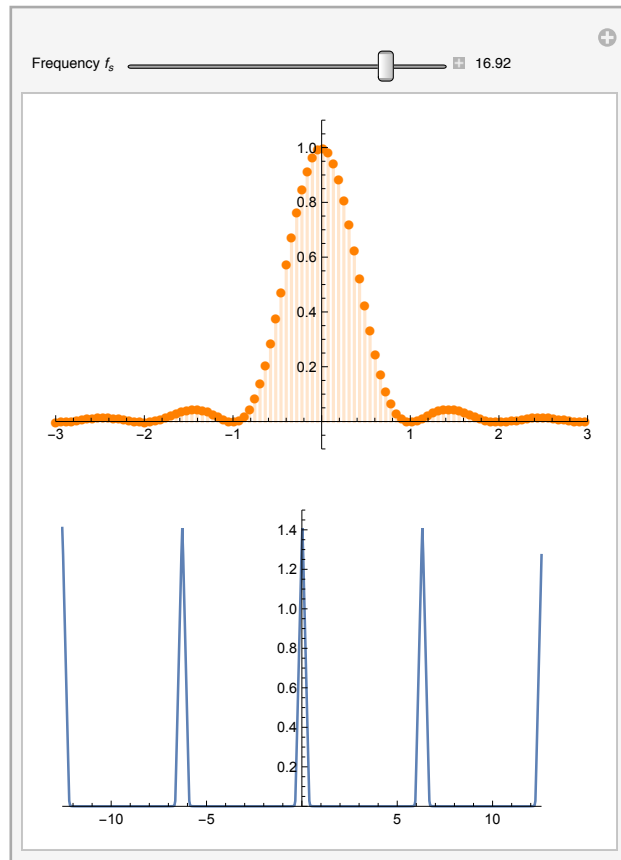
Rnqv[HqwtktgVtcpuhtqto[z[v]; v;  $\omega$ ];

{ $\omega$ ; -32 \* 4 Rk; 32 \* 4 Rk}; RnqvTcpiig → {{-4 \* 4 Rk; 4 \* 4 Rk}; {2; 2:63}}}



Ucorngfukipcn  $z[p]$  (nghv) cpf kvu Hqwtkgv vtcpuhqto urgevtwo ...

Ocpkrwncvg[Tqy[{Fkuegvgrnqv[z[v]; {v; -5; 5; 3/hu}; RnqvTcpiq → {{-5; 5}; {-2:3; 3:3}}];  
 RgthqtocpegIqcn → 'SwcnkvÀ'; RnqvUvÀng → Fktgevkxg[Qtcpiq; Vjkem];  
 RnqvOcmgtu → Cwvqocvke; KocigUkÁg → 562]; NkuvNkpgRnqv[  
 Lqkp@@Vcdng[Cdu[Hqwtkgv[Vcdng[P[z[p/hu]]; {p; -6\*hu; 6\*hu+3}]]]; 6];  
 FcvcTcpiq → {-6 Rk; 6 Rk}; RnqvTcpiq → {{-6 Rk; 6 Rk}; {2; 3:7}};  
 KocigUkÁg → 522}}]; {{hu; 37:47;  
 'HtgswgpeÀ \*!(\*Egnn[VgzvFcvc[Egnn[DqzFcvc[\*pHqtoDqz[\*pUwduetkrvDqz[\*'h\*';  
 \*'u\*'];  
 VtcfkvkqpcnHqto]];GzrtguukqpWWKF->\*c4ed2263-8cgh-6h15-cg5g-  
 e6f2443d0hg3\*'];GzrtguukqpWWKF->\*3ffgdc5c-4368-636e-d67d-  
 fh82634d9gc3\*']\*)'}];  
 2:7; 42; Crrgctcpeg → 'Ncdgngf'}}]



Vjg urgevtwo qh vjg uconrgf ukipcn ku c rgtkqfke hqto yjgtg kv" u rgtkqf ku  $4\pi$ ;  
 cpf cu nqpi cu yg turgerv vjg PÅswkuv – Ujcppqp eqpfkvkqp;  
 kvu hqto qxgt qpg rgtkqf ku gswkxcngpv vq vjg urgevtwo qh vjg qtkikpcn eqpvkpwqwu –  
 vkog ukipcn  $z(v)$ :Kpfggf; yjknv vjg uconrkpi htgswgpeÅ  $h_u$  ku nctig gpqwij;  
 vjgtg ku pq qxgtncrrkpi dgvyggp vjg rgtkqfu qh vjg urgevtwo; cpf vjgp;  
 vjg qtkikpcn ukipcn ecp dg tgvtkgxgf dÅ ogcp qh c nqy rcuudcpf hknvgt:Jqyngxgt;  
 kh vjg uconrkpi htgswgpeÅ ku vqq nqy; uwej cu  $h_u = < 3:6$  kp vjg cdqyg gzcornv;  
 vjgtg yknn dg qxgtncrrkpi qh vjg urgevtwo; yjkej yg ecnn cpcnkukpi ghheg:Kp uwej c ecug;  
 yg ecp pq oqtg tgvtkgxg vjg qtkikpcn ukipcn wukpi c ukornv nqy – rcuudcpf hknvgt;  
 cpf yg pggf vjgp c oqtg rvygthwn ukipcn rtqeguukpi dÅ ogcpu qh  
 gswcnkÅcvkqp:Vjg eqpfkvkqp vq cxqkf vjg qxgtncrrkpi ku vjcv  $h_u > 4 h_{ocz}$ ;  
 yjkej tgrtgugpvu rgthgevnÅ vjg PÅswkuv – Ujcppqp uconrkpi vjgtgo:  
 \* Vjg ukipcn nqpmu eqpvkpwqwu qp vjg rnqv dwv cpÅ  
 ukipcn rtqeguugf (vtcpuokvvvgf; uvqtgf qt fkurncÅgf) ku cnycÅu  
 c uconrgf xgtukqp qh vjg qtkikpcn eqpvkpwqwu vkog rctcogvgt  
 Tgeqpvtwevkqp Rtqegu  
 Vjg tgeqpvtwevkqp rtqegu eqpukuvu kp vjg igpgtcvkqp qh vjg qtkikpcn eqpvkpwqwu –  
 vkog ukipcn  $z(v)$  htqo vjg uconrgf xgtukqp  $z[p]$ :Cu nqpi cu vjgtg ku pq cnkcukpi ghheg;  
 c ukornv nqy rcuudcpf hknvgt ku gpqwij vq tgvtkgxg vjg qtkikpcn ukipcn

Hknvgtkpi vjg Urgevtwo qh vjg ucorngf ukipcn z[p] ...

Ocpkrwncvg[Ujqy[

NkuvNkpgRnqv[Lqkp@@Vcdng[Cdu[Hqwtkg[Vcdng[P[z[p/hu]]; {p; -6\*hu; 6\*hu}]]]; 4];

FcvcTcpi → {-4 Rk; 4 Rk}; RnqvTcpi → {{-4 Rk; 4 Rk}; {2; 3:7}}; KocigUkÁg → 522];

Rnqv[JgcxkukfgRk[z/(4\*Rk)]; {z; -4 Rk; 4 Rk}; Gzenwukqpu → Pqpg;

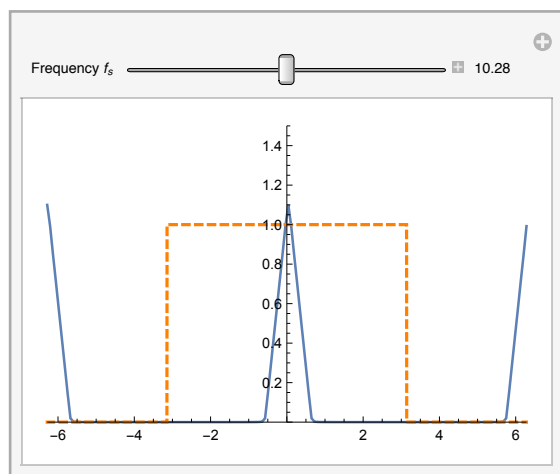
RnqvUvÀng → Fktgevkxg[Qtcpi; Vjkem; Fcujgf]]]; {{hu; 0; 'HtgswgpeÀ

\*!(\*Eggn[VgzvFcvc[Eggn[DqzFcvc[\*pHqtoDqz[\*pUwduetkrvDqz[\*'h'; \*'u\*'];

VtcfkvkqpcnHqto]];GzrtguukqpWWKF->\*7hd06291-e239-69gf-d4dd-312

h065g3dd3\*']);GzrtguukqpWWKF->\*4523fd70-92h0-6egf-c2h6-f59eg417hhd2\*

]\*)'); 2:7; 42; Crrgctcpeg → 'Ncdgngf']}]



Fgrgpfkpi qp vjg gzkuvpeg qh cnkcukpi qt pqv; vjg tgvtkgxgf eqpvkpwqwu -

vkog ukipcn eqwnf vjg rgthgev eqrÀ qh vjg qtkikpcn qt c fkuvtvgf qpg;

Urgevtwo qh vjg ucorng ukipcn cpf vjg tgequvtwevgf xgtukap qh vjg ukipcn z[p] ...

Ocpkrwncvg[Tqy[{Ujqy[

NkuvNkpgRnqv[Lqkp@@Vcdng[Cdu[Hqwtkg[Vcdng[P[z[p/hu]]; {p; -6 hu; 6 hu}]]]; 4];

Rgthqtocpeglqcn → 'SwcnkvÀ'; FcvcTcpi → {-4 Rk; 4 Rk};

RnqvTcpi → {{-4 Rk; 4 Rk}; {2; 3:7}}; KocigUkÁg → 522];

Rnqv[JgcxkukfgRk[z/(4\*Rk)]; {z; -4 Rk; 4 Rk}; Gzenwukqpu → Pqpg;

RnqvUvÀng → Fktgevkxg[Qtcpi; Vjkem; Fcujgf]]];

NkuvNkpgRnqv[Tg[KpxgtugHqwtkg[Hqwtkg[Vcdng[P[z[p/hu]]; {p; -6 hu; 6 hu}]]]]];

FcvcTcpi → {-5; 5}; RnqvTcpi → {{-5; 5}; {2; 3:3}}; KocigUkÁg → 522];

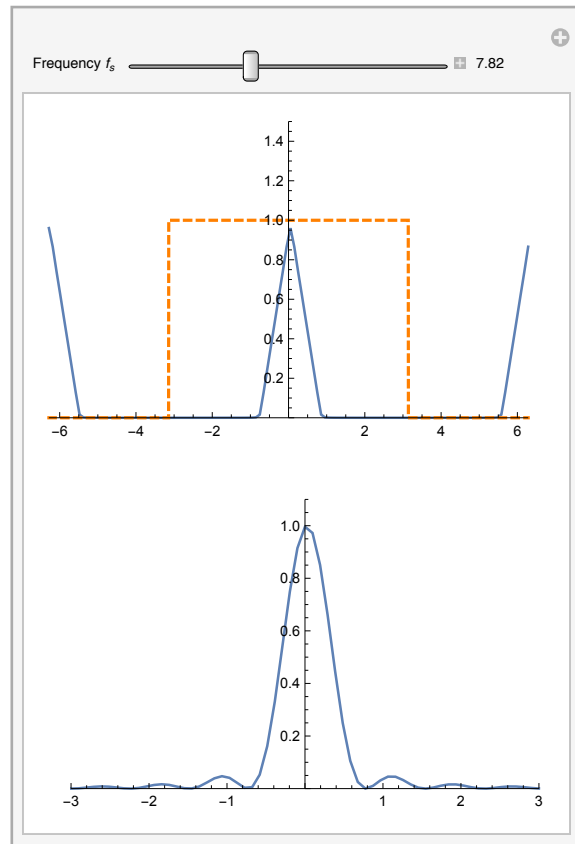
Rgthqtocpeglqcn → 'SwcnkvÀ']]); {{hu; 0; 'HtgswgpeÀ

\*!(\*Eggn[VgzvFcvc[Eggn[DqzFcvc[\*pHqtoDqz[\*pUwduetkrvDqz[\*'h'; \*'u\*'];

VtcfkvkqpcnHqto]];GzrtguukqpWWKF->\*543f15d2-4ch4-692g-0117-

dc9gc4029c0f\*']);GzrtguukqpWWKF->\*13fffd50-2237-6hhh-cg6d-f22dc85ef4e5

\*)'); 2:7; 42; Crrgctcpeg → 'Ncdgngf']}]



Cu nqpi cu vjg ucornkpi htgswgpeÀ xgtkhkgu vjg PÀswkuv – Ujcqqp eqpfkvkqp;  
 yg ecp tgeqpvtwev vjg qtkikpcn ukipcn ykvjqwv fkuvqtvkqp:  
 Hwtvjgt Gzrnqtcvkqpu  
 Fkikvcn Ukipcn Rtqeguukpi  
 Cpcnqi cpf Fkikvcn Oqfwncvkqp Vgejpkswgu  
 Cwvjqtujuv kphqtocvkqp  
 Ijcuucpg Cpkdc  
 45 Lwpg 4239  
 ijcuucpg@gok:ce:oc

## 0 | DCUKEBUaUVGOBRTQRGTVKGU

In this section we introduce and discuss a number of basic properties of continuous-time and discrete-time systems. These properties have important physical interpretations and relatively simple mathematical descriptions using the signals and systems language that we have begun to develop.

### BB0:3 UÀuvgouBykvjBcpfBykvjqwvBOgoqtÀ

A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the input at only that same time. For example, the system specified by the relationship

is memoryless, as the value of  $y[n]$  at any particular time  $n_0$  depends only on the value of  $x[n]$  at that time. Similarly, a resistor is a memoryless system; with the input  $x(t)$  taken as the current and with the voltage taken as the output  $y(t)$ , the input-output relationship of a resistor is

where  $R$  is the resistance. One particularly simple memoryless system is the identity system, whose output is identical to its input. That is, the input-output relationship for the continuous-time identity system is

and the corresponding relationship in discrete time is

An example of a discrete-time system with memory is an accumulator or summer

and a second example is a delay

A capacitor is an example of a continuous-time system with memory, since if the input is taken to be the current and the output is the voltage, then

where  $C$  is the capacitance.

Roughly speaking, the concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time. For example, the delay in eq. (1.93) must retain or store the



preceding value of the input. Similarly, the accumulator in eq. (1.92) must “remember” or store information about past inputs. In particular, the accumulator computes the running sum of all inputs up to the current time, and thus, at each instant of time, the accumulator must add the current input value to the preceding value of the running sum. In other words, the relationship between the input and output of an accumulator can be described as

or equivalently,

Represented in the latter way, to obtain the output at the current time  $n$ , the accumulator must remember the running sum of previous input values, which is exactly the preceding value of the accumulator output.

In many physical systems, memory is directly associated with the storage of energy. For example, the capacitor in eq. (1.94) stores energy by accumulating electrical charge, represented as the integral of the current. Thus, the simple RC circuit in Example 1.8 and Figure 1.1 has memory physically stored in the capacitor. Similarly, the automobile in Figure 1.2 has memory stored in its kinetic energy. In discrete-time systems implemented with computers or digital microprocessors, memory is typically directly associated with storage registers that retain values between clock pulses.

While the concept of memory in a system would typically suggest storing past input and output values, our formal definition also leads to our referring to a system as having memory if the current output is dependent on future values of the input and output. While systems having this dependence on future values might at first seem unnatural, they in fact form an important class of systems, as we discuss further in Section 1.6.3.

#### BB0:4 Kpxgtvkd knkvÀBcpfBKpxgtugBUÀuv gou

A system is said to be invertible if distinct inputs lead to distinct outputs. As illustrated in Figure 1.45(a) for the discrete-time case, if a system is invertible, then an inverse system exists that, when cascaded with the original system, yields an output  $w[n]$  equal to the input  $x[n]$  to the first system. Thus, the series interconnection in Figure 1.45(a) has an overall input-output relationship which is the same as that for the identity system.

An example of an invertible continuous-time system is

for which the inverse system is

This example is illustrated in Figure 1.45(b). Another example of an invertible system is the accumulator of eq. (1.92). For this system, the difference between two successive values of the output is precisely the last input value. Therefore, in this case, the inverse system is

as illustrated in Figure 1.45(c). Examples of noninvertible systems are

that is, the system that produces the zero output sequence for any input sequence, and

in which case we cannot determine the sign of the input from knowledge of the output.

The concept of invertibility is important in many contexts. One example arises in systems for encoding used in a wide variety of communications applications. In such a system, a signal that we wish to transmit is first applied as the input to a system known as an encoder. There are many reasons for doing this, ranging from the desire to encrypt the original message for secure or private communication to the objective of providing some redundancy in the signal (for example, by adding what are known as parity bits) so that any errors that occur in transmission can be detected and, possibly, corrected. For lossless coding, the input to the encoder must be exactly recoverable from the output; i.e., the encoder must be invertible.

#### BB0:5 EcwucnkVÀ

A system is causal if the output at any time depends on values of the input at only the present and past times. Such a system is often referred to as being nonanticipative, as the system output does not anticipate future values of the input. Consequently, if two inputs to a causal system are identical up to some point in time to or  $n_0$ , the corresponding outputs must also be equal up to this same time. The RC circuit of Figure 1.1 is causal, since the capacitor voltage responds only to the present and past values of the source voltage. Similarly, the motion of an automobile is causal, since it does not anticipate future actions of the driver. The systems described in eqs. (1.92)-(1.94) are also causal, but the systems defined by

and

are not. All memoryless systems are causal, since the output responds only to the current value of the input.

Although causal systems are of great importance, they do not by any means constitute the only systems that are of practical significance. For example, causality is not often an essential constraint in applications in which the independent variable is not time, such as in image processing. Furthermore, in processing data that have been recorded previously, as often happens with speech, geophysical, or meteorological signals, to name a few, we are by no means constrained to causal processing. As another example, in many applications, including

historical stock market analysis and demographic studies, we may be interested in determining a slowly varying trend in data that also contain high-frequency fluctuations about that trend. In this case, a commonly used approach is to average data over an interval in order to smooth out the fluctuations and keep only the trend. An example of a non causal averaging system is

**BB0:3** GzcornG3:34

**BB0:4** Yjgp Bejgemkpi Bvjg Becwucnk vA Bqh Bc BuAuvgo; Bkv Bku Bkorqtcv pV Bvq Bnqqm BectghwnnA Bcv Bvjg Bkprwv-qwvrvw Btgncvkq: BVq Bknnwuvtcvg Buqog Bqh Bvjg Bkuuwgu Bkpxqnxgf Bkp Bfqkpi Bvjku; Byg Byknn Bejgem vjg Becwucnk vA Bqh Bvyq Brctvkewnct BuAuvgo:

**BB0:5** Vjg Bhktuv BuAuvgo Bku Bfghkpgf BdA

**BB0:6**

**BB0:7** Pqvg Bvjcv Bvjg Bqwrvwv B{p2} Bcv Bc Brqukvkxg Bvkog Bpq Bfgrgpfu BqpnA Bqp Bvjg Bxcnwg Bqh Bvjg Bkprwv ukipcn Bz{-p2} Bcv Bvkog B{-p2}; Byjkej Bku Bpgicvkxg Bcpf Bvjgtghqtg Bkp Bvjg Brcuv Bqh Bp2: BYg BocA Bdg vgorvgf Bvq Beqpenwfg Bcv Bvjku Brqkpv Bvjcv Bvjg Bikxgp BuAuvgo Bku Becwucn: BJqygxgt; Byg Bujqwnf cnycAu Bdg Bectghwn Bvq Bejgem Bvjg Bkprwv-qwvrvw Btgncvkq Bhq Bcnn Bvkogu: BKp Brctvkewnct; Bhq Bp ≤ B2; Bg:i: Bp B> B- B6; Byg Bugg Bvjcv B{-6} B> Bz{6}; Buq Bvjcv Bvjg Bqwrvwv Bcv Bvjku Bvkog Bfgrgpfu Bqp Bc hwwtg Bxcnwg Bqh Bvjg Bkprwv: BJgpeg; Bvjg BuAuvgo Bku Bpqv Becwucn:

**BB0:8** Kv Bku Bcnuq Bkorqtcv pV Bvq Bfkuvkpiwkuj BectghwnnA Bvjg Bghhgevu Bqh Bvjg Bkprwv Bhtqo Bvjqug Bqh BcpA qvjgt Bhwpevkqpu Bwugf Bkp Bvjg Bfghkpvkq Bqh Bvjg BuAuvgo: BHqt BgzcornG; Beqpukfgt Bvjg BuAuvgo

**BB0:9**

**BB0:0** Kp Bvjku BuAuvgo; Bvjg Bqwrvwv Bcv BcpA Bvkog Bv Bgswcnu Bvjg Bkprwv Bcv Bvjcv Bucog Bvkog BownvkrnkG dABcBpwodgt Bvjcv Bxctkgu Bykvj Bvkog: BUrgekhekcennA; Byg Becp Btgytkvg Bgs: B[n:328] Bcu

**BB0:1**

**BB0:32** yjgtg Bi[v] Bku Bc Bvkog-xctA kpi Bhwpevkq; BpcognA Bi[v] B> Bequ[vB×B3]: BVjwu; BqpnA Bvjg Bewttgpn Bxcnwg qh Bvjg Bkprwv Bz[v] Bkphnwgegu Bvjg Bewttgpn Bxcnwg Bqh B B B B Bqwrvwv B{v}; Bcpf Byg Beqpenwfg Bvjcv vjku BuAuvgo Bku Becwucn B[cpf; Bkp Bhcev; BogoqtAnguu]:

## BB0:6 Uvcdknk vA

Stability is another important system property. Informally, a stable system is one in which small inputs lead to responses that do not diverge. For example, consider the pendulum in Figure 1.46(a), in which the input is the applied force  $x(t)$  and the output is the angular deviation  $y(t)$  from the vertical. In this case, gravity applies a restoring force that tends to return the pendulum to the vertical position, and frictional losses due to drag tend to slow it down. Consequently, if a small force  $x(t)$  is applied, the resulting deflection from vertical will also be small. In contrast, for the inverted pendulum in Figure 1.46(b), the effect of gravity is to apply a force that tends to increase the deviation from vertical. Thus, a small applied force leads to a large vertical deflection causing the pendulum to topple over, despite any retarding forces due to friction.

The system in Figure 1.46(a) is an example of a stable system, while that in Figure 1.46(b) is unstable. Models for chain reactions or for population growth with unlimited food supplies

and no predators are examples of unstable systems, since the system response grows without bound in response to small inputs. Another example of an unstable system is the model for a bank account balance in eq. (1.86), since if an initial deposit is made (i.e.,  $x[0]$  = a positive amount} and there are no subsequent withdrawals, then that deposit will grow each month without bound, because of the compounding effect of interest payments.

There are also numerous examples of stable systems. Stability of physical systems generally results from the presence of mechanisms that dissipate energy. For example, assuming positive component values in the simple RC circuit of Example 1.8, the resistor dissipates energy and this circuit is a stable system. The system in Example 1.9 is also stable because of the dissipation of energy through friction.

The preceding examples provide us with an intuitive understanding of the concept of stability. More formally, if the input to a stable system is bounded (i.e., if its magnitude does not grow without bound), then the output must also be bounded and therefore cannot diverge. This is the definition of stability that we will use throughout this book. For example, consider applying a constant force  $f(t) = F$  to the automobile in Figure 1.2, with the vehicle initially at rest. In this case the velocity of the car will increase, but not without bound, since the retarding frictional force also increases with velocity. In fact, the velocity will continue to increase until the frictional force exactly balances the applied force; so, from eq. (1.84), we see that this terminal velocity value  $V$  must satisfy

i.e.,

As another example, consider the discrete-time system defined by eq. (1.104), and suppose that the input  $x[n]$  is bounded in magnitude by some number, say,  $B$ , for all values of  $n$ . Then the largest possible magnitude for  $y[n]$  is also  $B$ , because  $y[n]$  is the average of a finite set of values of the input. Therefore,  $y[n]$  is bounded and the system is stable. On the other hand, consider the accumulator described by eq. (1.92). Unlike the system in eq. (1.104), this system sums all of the past values of the input rather than just a finite set of values, and the system is unstable, since the sum can grow continually even if  $x[n]$  is bounded. For example, if the input to the accumulator is a unit step  $u[n]$ , the output will be

That is,  $y[0] = 1$ ,  $y[1] = 2$ ,  $y[2] = 3$ , and so on, and  $y[n]$  grows without bound.

**BB0:33** GzcorngB3:35

**BB0:34** Kh Byg Buwurgev Bvjcv Bc BuÀuvgo Bku Bwpuvcdng; Bvjgp Bc Bwughwn BuvtcvgiÀ Bvq BxgthkÀ Bvjku Bku Bvq nqqm Bhqt Bc Burgekhke Bdqwpfgf Bkprwv Bvjcv Bngcfu Bvq Bcp Bwpdqwpfgf Bqwwrvv: BHkpfkpi Bqpg Buwej gzcornng Bgpcdngu Bwu Bvq Beqpenwfg Bvjcv Bvjg Bkxgp BuÀuvgo Bku Bwpuvcdng: BKh Buwej Bcp Bgzcornng fqgu Bpqv Bgzkuv Bqt Bku Bfkhkewnv Bvq Bhkpf; Byg Bowuv Bejgem Bhqt BuvcdknkvÀ BdÀ Bwukpi Bc Bogvjqf vjcv Bfqgu Bpqv BwvknkÀg Burgekhke Bgzcornng Bqh Bkprwv Bukipcnu: BVq Bknnwuvtcvg Bvjku Bcrrtqcej; Bngv wuBejgemBvjgBuvcdknkvÀBqhBvyqBuÀuvgou;

**BB0:35**

**BB0:36** cpf

**BB0:37**

**BB0:38** Kp Buggmkpi Bc Burgekhke Beqwpvgtgzcornng Bkp Bqtfgt Bvq Bfkurtqyg BuvcdknkvÀ; Byg Bokijv BvtÀ Bukornng dqwpfgf Bkprwv Buwej Bcu Bc Bequvcpv Bqt Bc Bwpkv Buvgr: BHqt BuÀuvgo BU3 Bkp Bgs: B[3:321]; Bc eqpuvcpv Bkprwv Bz[v] B> B3 BÀkgnfu BÀ[v] B> Bv; Byjkej Bku Bwpdqwpfgf; Bukpeg Bpq Bocvgt Bvjcv Bhkpvkq eqpuvcpv Byg Brkem; BÀ[v] Byknn Bgzeggf Bvjcv Bequvcpv Bhqt Buqog Bv: BYg Beqpenwfg Bvjcv BuÀuvgo BU3 kuBwpuvcdng:

**BB0:39** Hqt BuÀuvgo BU4; Byjkej Bjcrpgu Bvq Bdg Buvcdng; Byg Byqwnf Bdg Bwpcdng Bvq Bhkpf Bc Bdqwpfgf Bkprwv vjcv Btguwnv Bkp Bcp Bwpdqwpfgf Bqwwrvv: BUq Byg Brtqeggf Bvq BxgthkÀ Bvjcv Bcnn Bdqwpfgf Bkprwv tguwnv Bkp Bdqwpfgf Bqwwrvv: BUrgekhkecnÀ; Bngv BD Bdg Bcp BctdkvtctÀ Brqukvkxg Bpwodgt; Bcpf Bngv z[v] Bdg Bcp BctdkvtctÀ Bukipcn Bdqwpfgf BdÀ BD! Bvjcv Bku; Byg Bctg Bocmkpi Bpq Bcuuworvkq Bcdqww Bz[v]; gzevrvBvjcv

**BB0:30**

**BB0:31** qt

**BB0:42**

**BB0:43** hqtBcnnBv:BWukpiBvjgBfghkpvkqpBqhBU4BkpBgs:B[3:332];BygBvjgpBuggBvjcvBkhBz[v]BucvkuhkguBgs:[3:333];BvjgpBÀ[v]BowuvBucvkuhÀ

**BB0:44** Yg Beqpenwfg Bvjcv Bkh BcpÀ Bkprwv Bvq BU4 Bku Bdqwpfgf BdÀ Bcp BctdkvtctÀ Brqukvkxg Bpwodgt BD; Bvjg eqttgurqpfkpiB-qwwrvvBkuBiwctcpvgfBvqBdgBdqwpgfBdÀBg0B•BVjuw;BU4BkuBuvcdng;

## BB0:7 VkogBKpxctkcepg

Conceptually, a system is time invariant if the behavior and characteristics of the system are fixed over time. For example, the RC circuit of Figure 1.1 is time invariant if the resistance and capacitance values  $R$  and  $C$  are constant over time: We would expect to get the same results from an experiment with this circuit today as we would if we ran the identical experiment tomorrow. On the other hand, if the values of  $R$  and  $C$  are changed or fluctuate over time, then we would expect the results of our experiment to depend on the time at which we run it. Similarly, if the frictional coefficient  $b$  and mass  $m$  of the automobile in Figure 1.2 are constant, we would expect the vehicle to respond identically independently of when we drive it. On the other hand, if we load the auto's trunk with heavy suitcases one day, thus increasing  $m$ , we would expect the car to behave differently than at other times when it is not so heavily loaded.

The property of time invariance can be described very simply in terms of the signals and systems language that we have introduced. Specifically, a system is time invariant if a time shift in the input signal results in an identical time shift in the output signal. That is, if  $y[n]$  is

the output of a discrete-time, time-invariant system when  $x[n]$  is the input, then  $y[n - n_0]$  is the output when  $x[n - n_0]$  is applied. In continuous time with  $y(t)$  the output corresponding to the input  $x(t)$ , a time-invariant system will have  $y(t - t_0)$  as the output when  $x(t - t_0)$  is the input.

To see how to determine whether a system is time invariant or not, and to gain some insight into this property, consider the following examples:

**BB0:45** GzcorngB3:36

**BB0:46** EqpukfgtBvjgBeqpvkpwqwu-vkogBuAuvgoBfghkpgfBdA

**BB0:47**

**BB0:48** VqBejgemBvjcvBvjkuBuAuvgoBkuBvkogBkpxctkcpv;BygBowuvBfgvgtokpgByjgvjgtBvjgBvkog-Bkpxctkcpv  
rtqrgtvABjqnfuBhqtBcpABkprwvBcpfBcpABvkogBujkhvBv2:BVjwu;BngvBz3[v]BdgBcpBctdktctABkprwvBvq  
vjkuBuAuvgo;BcpfBngv

**BB0:49**

**BB0:40** dgBvjgBeqttgurqpfkpiBqwwrvv:BVjgpBeqpvkfgtBcBugeqpfBkprwvBqdvckpgfBdABujkhvkpiBz3[v]Bkp  
vkog...

**BB0:41**

**BB0:52** VjgBqwwrvvBeqttgurqpfkpiBvqBvjkuBkprwvBku

**BB0:53**

**BB0:54** UkoknctnA:BhtqoBgs:B[n:337];

**BB0:55**

**BB0:56** EqorctkpiBgsu:B[3:339]BcpfB[3:330];BygBuggBvjcvBA4[v]B>BA3B[vB-Bv2];BcpfBvjgtghqgt;BvjkuBuAuvgo  
kuBvkogBkpxctkcpv:

**BB0:57** GzcorngB3:37

**BB0:58** CuBcBugeqpfBgzcorg;BeqpvkfgtBvjgBfkuetgvg-vkogBuAuvgo

**BB0:59**

**BB0:50** VjkuBkuBcBvkog-xctAkpBuAuvgo;BcBhcevBvjcvBecpBdgBxgtkhkgfBwukpiBvjgBucogBhqtocnBrtqegfwtg  
cuBvjcvBwugfBkpBvjgBrtgegfkiBgzcorgB[uggBRTqdnogB3:40]:BJqygxgt;ByjgpBcBuAuvgoBkuBuwurgevgf  
qhBdgkpiBvkogBxctAkp;BcpBcrrtqejBvqBujqykpiBvjkuBvjcvBkuBqhvgpBxgtABwughwnBkuBvqBuggmBc  
eqwpvgtgzcorg-k:g;;BvqBwugBqwtBkpvwkvkqpBvqBhkpBcpBkprwvBukipcnBhqtByjkejBvjgBeqpfkvkqp  
qhBvkogBkpxctkcpvBkuBxqncvgf:BKpBrctvkewnt;BvjgBuAuvgoBkpBvjkuBgzcorgBtgrtgugpvuBc  
uAuvgoBykvjBcBvkog-xctAkpBickp:BHqtBgzcorg;BkhBygBmpqyBvjcvBvjgBewttgvpBkprwvBxcnwgBkuB3;  
ygBecppqvBfgvgtokpgBvjgBewttgvpBqwwrvvBxcnwgBykvjqwvBmpqykpiBvjgBewttgvpBvkog:

**BB0:51** EqpugswgpvnA;BeqpvkfgtBvjgBkprwvBukipcnBz3{p}B>Bn>{p};ByjkejBAkgnfuBcpBqwwrvvBA3{p}BvjcvBku  
kfgpvkecnA B2B[ukpegBpn>{p}B>B2]:BJqygxgt;BvjgBkprwvBz4{p}B>Bn>{p-3}BAkgnfuBvjgBqwwrvvBa4{p}  
>Bpn>{pB-B3}B>Bn>{pB-B3}:BVjwu;ByjkgBz4{p}BkuBcBujkhvgfBxgtukqpBqhBzB3{p};Ba4{p}BkuBpqvBc  
ujkhvgfBxgtukqpBqhBakB{p}:

**BB0:62** YjkgBvjgBuAuvgoBkpBvjgBrtgegfkiBgzcorgBjcuBcBvkog-xctAkpBickpBcpfBcuBcBtguwnvBkuBcBvkog-  
xctAkpBuAuvgo:BvjgBuAuvgoBkpBgs:B[3:19]BjcuBcBequvcpvBBBBBcpf;BkpBhcev;BkuBvkogBkpxctkcpv:  
QvjgtBgzcorguBqhBvkog-kpxctkcpvBuAuvgoBctgBikxgpBdABgsu:B[3:13]-B[3:326]:BVjgBhqnnqykpi  
gzcorgBknnwvutcvguBcBvkog-xctAkpBuAuvgo:

**BB0:63** GzcorngB3:38

**BB0:64** EqpukfgtBvjgBuAuvgo

**BB0:65**

**BB0:66** VjkuBuAuvgoBtgrtgugpvuBcBvkogBuecnkpi:BVjcvBku;BÅ[v]BkuBcBvkog-eqortguugfB[dÅBcBhcevqtBqhB4]xgtukqpBqhBz[v]:BKpwwkvkxgnÅ;Bvjgp;BcpÅBvkogBujkhvBkpBvjgBkprwvByknnBcnuqBdgBeqortguugfBdÅcBhcevqtBqhB4;BcpfBkvBkuBhqtBvjkuBtgcupBvjcvBvjgBuAuvgoBkuBpqvBvkogBkpxctkcpv:BVqfgoqpuvctvgBvjkuBdÅBeqwpvgtgzcorng;BeqpukfgtBvjgBkprwvBzB3[v]BujqypBkpBHkiwtgBn:69[c]BcpfBvjgtguwnvkpiBqwwrvwBÅ3[v]BfgrkevgfBkpBHkiwtgBn:69[d]:BKHBygBvjgpBujkhvBvjgBkprwvBdÅB4-k:g;;eqpukfgtBz4[v]B>Bz3[vB-B4];BcuBujqypBkpBHkiwtgBn:69[e]-ygBqdvckpBvjgBtguwnvkpiBqwwrvwBÅ4[v]B>z4[4v]BujqypBkpBHkiwtgBn:69[f]:BEqorctkpiBHkiwtguBn:69[f]BcpfB[g];BygBuggBvjcvBa4[v]BqHBÅB3[vB-4];BuqBvjcvBvjgBuAuvgoBkuBpqvBvkogBkpxctkcpv:B[KpBhcev;BÅ4[v]B>BÅB3[vB-B3];BuqBvjcvBvjgqwwrvwBvkogBujkhvBkuBqpnÅBjcnhBcuBdkiBcuBkvBujqwnfBdgBhqtBvkogBkpxctkcpv;BfwgBvqBvjgBvkogBeqortguukqpBkorctvgfBdÅBvjgBuAuvgo:]

## BB0:8 NkpgctkvÅ

A linear system, in continuous time or discrete time, is a system that possesses the important property of superposition: If an input consists of the weighted sum of several signals, then the output is the superposition- That is, the weighted sum - of the responses of the system to each of those signals. More precisely, let  $y_1(t)$  be the response of a continuous-time system to an input  $x_1(t)$ , and let  $y_2(t)$  be the output corresponding to the input  $x_2(t)$ . Then the system is linear if:

2. response to  $x_1(t) + x_2(t)$  is  $Y_1(t) + Y_2(t)$ .

2. The response to  $ax_1(t)$  is  $ay_1(t)$ , where  $a$  is any complex constant.

The first of these two properties is known as the additivity property; the second is known as the scaling or homogeneity property. Although we have written this description using continuous-time signals, the same definition holds in discrete time. The systems specified by eqs. (1.91)-(1.100), (1.102)-(1.104), and (1.119) are linear, while those defined by eqs. (1.101) and (1.114) are nonlinear. Note that a system can be linear without being time invariant, as in eq. (1.119), and it can be time invariant without being linear, as in eqs. (1.101) and (1.114).

The two properties defining a linear system can be combined into a single statement:

continuous time:  $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$ , (2.121)

discrete time:  $ax_1[n] + bx_2[n] \rightarrow ay_1[n] + by_2[n]$ . (2.122)

Here,  $a$  and  $b$  are any complex constants. Furthermore, it is straightforward to show from the definition of linearity that if  $x_k[n]$ ,  $k = 1, 2, 3, \dots$ , are a set of inputs to a discrete-time linear system with corresponding outputs  $y_k[n]$ ,  $k = 1, 2, 3, \dots$ , then the response to a linear combination of these inputs given by

is

This very important fact is known as the superposition property, which holds for linear systems in both continuous and discrete time.

A direct consequence of the superposition property is that, for linear systems, an input which is zero for all time results in an output which is zero for all time. For example, if  $x[n] = y[n]$ , then the homogeneity property tells us that

In the following examples we illustrate how the linearity of a given system can be checked by directly applying the definition of linearity.

**BB0:67** GzcorngB4:39

**BB0:68** EqpukfgtBcBuÀuvgoBUByjqugBkprwvBz[v]BcpfBqwwrvwBÀ[v]BctgBtgnvcgfBdÀ

**BB0:69**

**BB0:60** VqBfgvgtokpgByjgvjgtBqtBpqvBUkuBnkpugt;BygBeqpukfgtBvyqBctdkvtctÀBkprwvuBz3[v]BcpfBz4[v]:

**BB0:61**

**BB0:72** NgvBz5[v]BdgBcBnkpugtBeqodkpcvkqpBqhBz3[v]BcpfBz4[v]:BVjcvBku;

**BB0:73**

**BB0:74** yjgtgBcBcpfBdBctgBctdkvtctÀBuecnctu:BKhBz5[v]BkuBvjgBkprwvBvqBU;BvjgpBvjgBeqttgurqpfkpiBqwwrvwocÀBdgBgzrtguugfBcu

**BB0:75**

**BB0:76** YgBeqpenwfgBvjcvBvjgBuÀuvgoBUBkuBnkpugt:

**BB0:77** GzcorngB4:30

**BB0:78** NgvBwuBcrrnÀBvjgBnkpugtÀ-ejgemkpiBrtqegfwtgBqhBvjgBrtgkxquwBgzcornngBvqBcpqvjgtBuÀuvgoBUEqpukfgtBkprwvBz[v]BcpfBqwwrvwBÀ[v]BctgBtgnvcgfBdÀ

**BB0:79**

**BB0:70** FghkpkpiBz3[v];Bz4[v];BcpfBz5[v]BcuBkpBvjgBrtgkxquwBgzcornng;BygBjcxg

**BB0:71**

**BB0:82** cpf

**BB0:83**

**BB0:84** EngctnÀ; BygBecpBurgekhÀBz3[v];Bz4[v];Bc;BcpfBdBuwejBvjcvBÀ5[v]BkuBpqvBvjgBucogBcuBcÀ3[v]B×dÀ4[v];B BHqtBgzcornngB;khz3[v]B>Bn;z4[v]B>BQ;cB>B4;BcpfBdB>B2;vjgpÀ5[v]B>B[4z3[v]]4B>B6;Bdww4À3[v]B>B4[z3[v]]4B>B4:BYgBeqpenwfgBvjcvBvjgBuÀuvgoBUkuBpqvBnkpugt:

**BB0:85** GzcorngB4:31

**BB0:86** KpBejgemkpiBvjgBnkpugtÀBqhBcBuÀuvgo;BkvBkuBkorqtvcpvBvqBtgogodgtBvjcvBvjgBuÀuvgoBowuvucvkuhÀBdqvjBvjgBcflkvkxkvÀBcpfBjqoqigpgkvÀBrtgrgtvkguBcpfBvjcvBvjgBukipcnu;BcuBygnnBcuBcpÀuecnkpiBequvcpvu;BctgBcnnqygfBvqBdgBeqorngz:BVqBgorjcukÀgBvjgBkorqtvcpvBqhBvjgugBrqkpvu;eqpukfgtBvjgBuÀuvgoBurgekhkfgBdÀ



**BB0:87****BB0:88** Cu Bujqyp Bkp BRTqdnGo B4:41; Bvjku BuÀuvgo Bku Bcfffkxg! Bjyqyxt; Bkv Bfqgu Bpqv BucvkuhÀ Bvjg Bjq-oqigpgkvÀBrtqrgtvÀ;BcuBygBpqyBfgoqpvtcvg:BNgv**BB0:89****BB0:80** dgBcpBctdkvtctÀBeqornGzBkprwvBykvjBtgcBcpfBkocikpctÀBrctvuBt{p}BcpfBu{p};BtgurgevknÀ;uqBvjcvBvjgBeqttgurqpfkpiBqwwrvBku**BB0:81****BB0:92** Pqy;BeqpukfgtBuecnkpiBz3{p}BdÀBcBeqornGzBpwodgt;BhqtBgzcornG;BcB>B!Bk:g;;Beqpukfgt vjgBkprwv**BB0:93****BB0:94** VjgBqwwrvBqttgurqpfkpiBvqBz4{p}Bku**BB0:95****BB0:96** yjkejBkuBpqvBgswcnBvqBvjgBuecnGfBxgtukqpBqhBÀ3{p};**BB0:97** GzcornG B4:42**BB0:98** EqpukfgtBvjgBuÀuvgo**BB0:99****BB0:90** Vjku BuÀuvgo Bku Bpqv BnkpGct; Bcu Becp Bdg Bxgtkhkgf Bkp Bugxgtcn BycÀu: BHqt BgzcornG; Bvjg BuÀuvgo xkqncvGuBvjgBcfffkxkvÀBrtqrgtvÀ...BKHBz3{p}B>B4BcpfBz4B{p}B>B5;Bvjg**BB0:91****BB0:02** Jqyqyxt;BvjgBtGurqpugBvqBz5B{p}B>BzB3{p}B×Bz4B{p}Bku**BB0:03****BB0:04** yjkejBfqgu BpqvBgswcn BÀ3{p} B×À4{p} B>B38:BCnvgtpcvknÀ;Bukpeg BÀ{p} B>B5Bkh Bz{p} B>B2;Byg Bugg vjcv Bvjg BuÀuvgo BxkqncvGu Bvjg B,Ágtq-kp\Ágtq-qww,, BrtqrgtvÀ Bqh BnkpGct BuÀuvGou BixGp Bkp Bgs: [n:347]:**BB0:05** KvBocÀBuggoBuwtrtkukpiBvjcvBvjgBuÀuvgoBkpBvjgBcdqXgBgzcornG BkuBqpnpkGct;BukpegBgs:B[n:354] ku Bc BnkpGct Bgswcnkq: BQp Bvjg Bqvjgt Bjcpf; Bcu BfgrkevGf Bkp BHkiwtg B4:60; Bvjg Bqwwrv Bqh Bvjku uÀuvgoBecpBdgBtgrtgugpvGfBcuBvjgBuwoBqhBvjgBqwwrvBqhBcBnkpGctBuÀuvgoBcpfBcpqvjgtBukipcn gswcn Bvq Bvjg BÁgtq-kprwv BtGurqpug Bqh Bvjg BuÀuvgo: BHqt Bvjg BuÀuvgo Bkp Bgs: B[4:354]; Bvjg BnkpGct uÀuvgoBku**BB0:06****BB0:07** cpfBvjgBÁgtq-kprwvBtGurqpugBku**BB0:08****BB0:09**

VjgtgBctg;BkpBhcev;BnctigBencuGuBqhBuÀuvGouBkpBdqvjBeqpvkpqwuBcpfBfkuetgvgBvkogBvjcvBecp dg BtgrtgugpvGf Bcu Bkp BHkiwtg B4:60-k:g;; Bhqt Byjkej Bvjg Bqxgtcnn BuÀuvgo Bqwwrv Beqpukuvu Bqh Bvjg uwrgrtqukvkq Bqh Bvjg BtGurqpug Bqh Bc BnkpGct BuÀuvgo Bykvj Bc BÁgtq-kprwv BtGurqpug: BCu Bujqyp Bkp RtqdnGo B4:69; Buwej BuÀuvGou Beqttgurqpf Bvq Bvjg Bencu Bqh BkpetgogpvcnnÀ BnkpGct BuÀuvGou-Bk B:g;; uÀuvGou Bkp Beqpvkpqwu Bqt Bfkuetgvg Bvkog Bvjcv BtGurqpf BnkpGctnÀ Bvq Bejcpigu Bkp Bvjg Bkprwv: BKp qvjgt Byqtfu; Bvjg Bfkhhgtgpeg BdgvygGp Bvjg BtGurqpugu Bvq BcpÀ Bvyq Bkprwv Bvq Bcp BkpetgogpvcnnÀ nkpGct BuÀuvgo Bku Bc BnkpGct B[k:g;; Bcfffkxg Bcpf Bjqoqigpgqu] Bhwpevkq Bqh Bvjg Bfkhhgtgpeg dgvygGp:vjg Bvyq Bkprwv: BHqt BgzcornG; Bkh Bz3{p} Bcpf Bz4{p} Bctg Bvyq Bkprwv Bvq Bvjg BuÀuvgo

urgekhhkgfBdÀBgs:B[4:354];BcpfBkhBÀ3{p}BcpfBÀ”k{p}BctgBvjgBeqttgurqpfpkiBqwvrwvu;Bvjgp

## 1 | UWOOCta

In this chapter, we have developed a number of basic concepts related to continuous-time and discrete-time signals and systems. We have presented both an intuitive picture of what signals and systems are through several examples and a mathematical representation for signals and systems that we will use throughout the book. Specifically, we introduced graphical and mathematical representations of signals and used these representations in performing transformations of the independent variable. We also defined and examined several basic signals, both in continuous time and in discrete time. These included complex exponential signals, sinusoidal signals, and unit impulse and step functions. In addition, we investigated the concept of periodicity for continuous-time and discrete-time signals.

In developing some of the elementary ideas related to systems, we introduced block diagrams to facilitate our discussions concerning the interconnection of systems, and we defined a number of important properties of systems, including causality, stability, time invariance, and linearity.

The primary focus in most of this book will be on the class of linear, time-invariant (LTI) systems, both in continuous time and in discrete time. These systems play a particularly important role in system analysis and design, in part due to the fact that many systems encountered in nature can be successfully modeled as linear and time invariant. Furthermore, as we shall see in the following chapters, the properties of linearity and time invariance allow us to analyze in detail the behavior of LTI systems.