Tarea II Error Cuadratico Medio

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\left(\frac{2\pi n}{T} t\right) + \sum_{n=1}^{\infty} b_n \cdot \sin\left(\frac{2\pi n}{T} t\right)$$

$$\omega = \frac{2\pi}{T}$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos\left(\frac{2\pi n}{T} t\right) \partial t$$

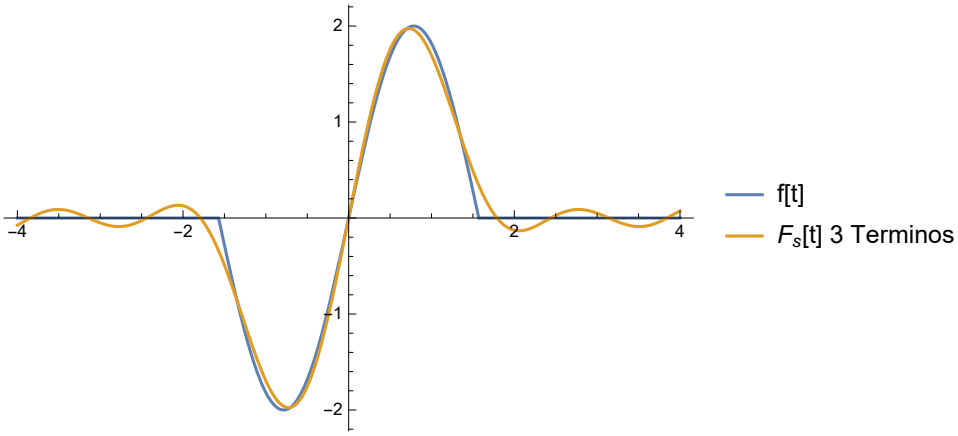
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin\left(\frac{2\pi n}{T} t\right) \partial t$$

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \partial t$$

$$E_k = \frac{1}{T} \int_{-T/2}^{T/2} [\epsilon_k(t)]^2 dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} [f(t) - S_k(t)]^2 dt$$

```
Clear["Global`*"]
T = 2 Pi;
f = (A * Sin[2 t] - 0) * HeavisideTheta[t - (-Pi / 2)] +
    (0 - A * Sin[2 t]) * HeavisideTheta[t - (Pi / 2)];
a0 = (2 / T) * Integrate[f, {t, -T / 2, T / 2}];
aN = (2 / T) * Integrate[f * Cos[(2 * Pi * n * t) / T], {t, (-T / 2), (T / 2)}];
bN = (2 / T) * Integrate[f * Sin[(2 * Pi * n * t) / T], {t, (-T / 2), (T / 2)}];
Fs = (a0 / 2) + Sum[Limit[aN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
  Sum[Limit[(bN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}]
EK = (1 / T) * Integrate[(f - Fs) ^ 2, {t, (-T / 2), (T / 2)}] // N;
Plot[{f /. A → 2, Fs /. A → 2}, {t, -4, 4},
 PlotRange → All, PlotLegends → {"f[t]", "Fs[t] 3 Terminos"}]
Print["f(t)= ", f, "\na0=", a0, "\nan= ", aN, "\nb0= ",
 bN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", EK]
```



$$f(t) = -A\,\text{HeavisideTheta}\left[-\frac{\pi}{2}+t\right]\text{Sin}[2t] + A\,\text{HeavisideTheta}\left[\frac{\pi}{2}+t\right]\text{Sin}[2t]$$

$$a_0 = 0$$

$$a_n = 0$$

$$b_0 = -\frac{4A\,\text{Sin}\left[\frac{n\pi}{2}\right]}{\left(-4+n^2\right)\pi}$$

$$F_s(t) = \frac{4A\,\text{Sin}[t]}{3\pi} + A\,\text{Cos}[t]\,\text{Sin}[t] + \frac{4A\,\text{Sin}[3t]}{5\pi} \quad ; \text{ 3 terminos}$$

$$E_k = 0.00251395\,A^2$$

Ejercicio 1

In[204]:=
```
Clear["Global`*"]
f = (1 - 0) HeavisideTheta[t - 0] +
    (-1 - (1)) HeavisideTheta[t - (T / 2)] + (0 - (-1)) HeavisideTheta[t - T];
A0 = (2 / T) (Integrate[(1), {t, 0, T / 2}] + Integrate[(-1), {t, (T / 2), T}]);
AN = Simplify[(2 / T) (Integrate[(1) * Cos[(2 * Pi * n * t) / T], {t, 0, T / 2}] +
      Integrate[(-1) Cos[(2 * Pi * n * t) / T], {t, (T / 2), T}]), n ∈ Integers];
BN = Simplify[(2 / T) (Integrate[(1) * Sin[(2 * Pi * n * t) / T], {t, 0, T / 2}] +
      Integrate[(-1) Sin[(2 * Pi * n * t) / T], {t, (T / 2), T}]), n ∈ Integers && n ≥ 1];
Fs = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 5}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
Ek = Simplify[(1 / T) * Integrate[((f - Fs)^2), {t, 0, T}], T ∈ Reals && T > 0] // N;
Print["f(t)= ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", Ek]
Plot[{f /. T → 2, Fs /. T → 2, Fs5 /. T → 2}, {t, -1, 3}, PlotRange → All,
 PlotLegends → {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"}]
```

$$f(t) = \text{HeavisideTheta}[t] + \text{HeavisideTheta}[t - T] - 2\,\text{HeavisideTheta}\left[t - \frac{T}{2}\right]$$

$$a_0 = 0$$

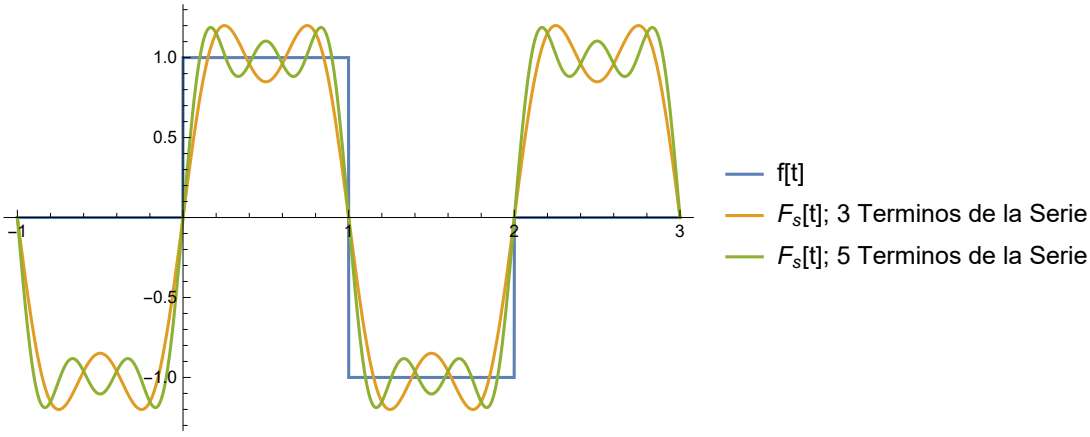
$$a_n = \frac{2\,\text{Sin}[n\pi] - \text{Sin}[2\,n\pi]}{n\,\pi}$$

$$b_0 = -\frac{4\,\text{Cos}[n\pi]\,\text{Sin}\left[\frac{n\pi}{2}\right]^2}{n\,\pi}$$

$$F_s(t) = \frac{4\left(3\,\text{Sin}\left[\frac{2\pi t}{T}\right] + \text{Sin}\left[\frac{6\pi t}{T}\right]\right)}{3\,\pi} \quad ;\ 3\ \text{terminos}$$

$$E_k = 0.0993673$$

Out[213]=



Ejercicio 2

In[599]:=
```
Clear["Global`*"]
f = (1 - 0) HeavisideTheta[t - (-Pi / 2)] +
    (-1 - (1)) HeavisideTheta[t - (Pi / 2)] + (0 - (-1)) HeavisideTheta[t - (3 Pi / 2)];
T = 2 Pi;
A0 =
  (2 / T) (Integrate[(1), {t, -Pi / 2, Pi / 2}] + Integrate[(-1), {t, (Pi / 2), (3 Pi / 2)}]);
AN = Simplify[(2 / T) (Integrate[(1) * Cos[(2 * Pi * n * t) / T], {t, -Pi / 2, Pi / 2}] +
      Integrate[(-1) Cos[(2 * Pi * n * t) / T], {t, (Pi / 2), (3 Pi / 2)}]), n ∈ Integers];
BN = Simplify[(2 / T) (Integrate[(1) * Sin[(2 * Pi * n * t) / T], {t, -Pi / 2, Pi / 2}] + Integrate[
       (-1) Sin[(2 * Pi * n * t) / T], {t, (Pi / 2), (3 Pi / 2)}]), n ∈ Integers && n ≥ 1];
Fs = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 5}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
EK = Simplify[(1 / T) * Integrate[((f - Fs)^2), {t, -Pi / 2, 3 Pi / 2}], T ∈ Reals && T > 0] // N;
Print["f(t)= ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", EK]
Plot[{f, Fs, Fs5}, {t, -3 Pi / 2, 5 Pi / 2}, PlotRange → All,
 PlotLegends → {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"},
 ExclusionsStyle → Blue]
```

$$f(t) = \text{HeavisideTheta}\left[-\frac{3\pi}{2}+t\right] - 2\,\text{HeavisideTheta}\left[-\frac{\pi}{2}+t\right] + \text{HeavisideTheta}\left[\frac{\pi}{2}+t\right]$$

$a_0=0$

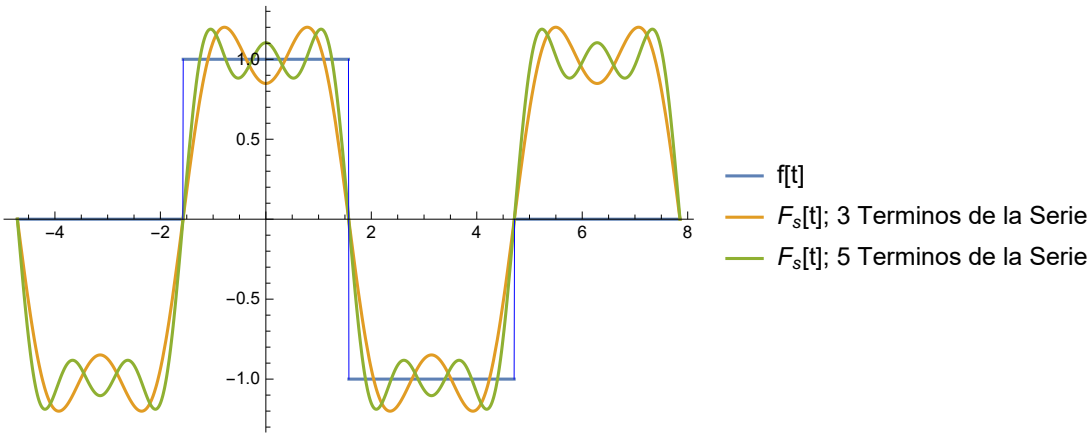
$$a_n = \frac{4\,\text{Sin}\left[\frac{n\pi}{2}\right]^3}{n\pi}$$

$$b_0 = -\frac{2\,\text{Sin}\left[\frac{n\pi}{2}\right]\text{Sin}[n\pi]}{n\pi}$$

$$F_s(t) = -\frac{4\,(-3\,\text{Cos}[t]+\text{Cos}[3\,t])}{3\pi} \quad ; \text{ 3 terminos}$$

$E_k = 0.0993673$

Out[609]=



Ejercicio 3

In[610]:=
```
Clear["Global`*"]
f = (A - 0) HeavisideTheta[t - (0)] + (0 - (A)) HeavisideTheta[t - (T/2)];
T = 1 * 10^-6;
A0 = (2/T) (Integrate[(A), {t, 0, T/2}] + Integrate[(0), {t, (T/2), (T)}]);
AN = Simplify[(2/T) (Integrate[(A) * Cos[(2 * Pi * n * t)/T], {t, 0, T/2}] +
      Integrate[(0) Cos[(2 * Pi * n * t)/T], {t, (T/2), (T)}]), n ∈ Integers];
BN = Simplify[(2/T) (Integrate[(A) * Sin[(2 * Pi * n * t)/T], {t, 0, T/2}] +
      Integrate[(0) Sin[(2 * Pi * n * t)/T], {t, (T/2), (T)}]), n ∈ Integers && n ≥ 1];
Fs = Simplify[(A0/2) + Sum[Limit[AN * Cos[(2 * Pi * n * t)/T], n → k], {k, 1, 3}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t)/T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0/2) + Sum[Limit[AN * Cos[(2 * Pi * n * t)/T], n → k], {k, 1, 5}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t)/T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
EK = Simplify[(1/T) * Integrate[((f - Fs)^2), {t, 0, T}], T ∈ Reals && T > 0] // N;
Print["f(t)= ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", EK]
Plot[{f /. A → 2, Fs /. A → 2, Fs5 /. A → 2}, {t, -T/2, 4 T/2}, PlotRange → All,
 PlotLegends → {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"}]
```

$$f(t) = -A\,\text{HeavisideTheta}\left[-\frac{1}{2\,000\,000}+t\right] + A\,\text{HeavisideTheta}[t]$$

$$a_0 = A$$

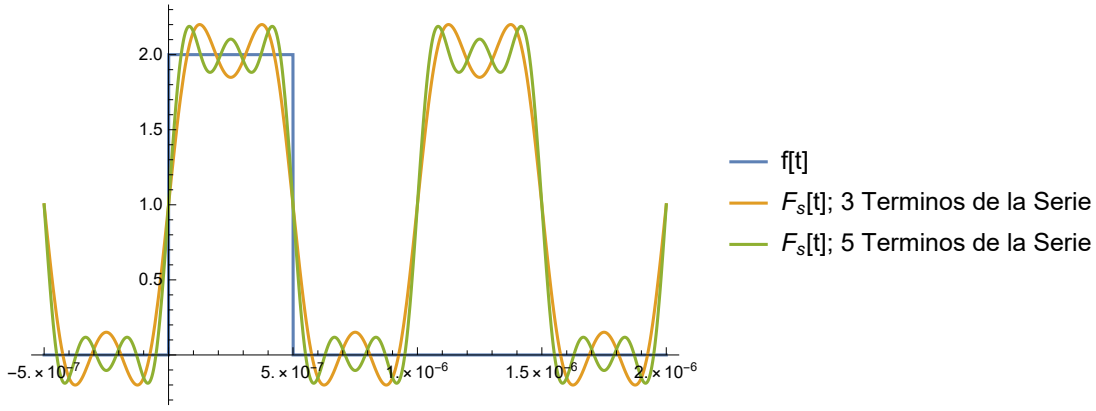
$$a_n = \frac{A\,\text{Sin}[n\pi]}{n\pi}$$

$$b_0 = \frac{2A\,\text{Sin}\left[\frac{n\pi}{2}\right]^2}{n\pi}$$

$$F_s(t) = \frac{A\left(3\pi + 12\,\text{Sin}[2\,000\,000\,\pi t] + 4\,\text{Sin}[6\,000\,000\,\pi t]\right)}{6\pi} \quad ; \text{ 3 terminos}$$

$$E_k = 0.0248418\,A^2$$

Out[620]=



Ejercicio 4

In[621]:=
```
Clear["Global`*"]
f = ((A * t / Pi) - 0) HeavisideTheta[t - (0)] + ((-A * (t - 2 Pi) / Pi) - (A * t / Pi))
    HeavisideTheta[t - (Pi)] + (0 - (-A * (t - 2 Pi) / Pi)) HeavisideTheta[t - (2 Pi)];
T = 2 Pi;
A0 = (2 / T)
   (Integrate[(A * t / Pi), {t, 0, Pi}] + Integrate[(-A * (t - 2 Pi) / Pi), {t, Pi, (2 Pi)}]);
AN = Simplify[(2 / T) (Integrate[(A * t / Pi) * Cos[(2 * Pi * n * t) / T], {t, 0, Pi}] + Integrate[
        (-A * (t - 2 Pi) / Pi) Cos[(2 * Pi * n * t) / T], {t, (Pi), (2 Pi)}]), n ∈ Integers];
BN = Simplify[(2 / T) (Integrate[(A * t / Pi) * Sin[(2 * Pi * n * t) / T], {t, 0, Pi}] + Integrate[
        (-A * (t - 2 Pi) / Pi) Sin[(2 * Pi * n * t) / T], {t, (Pi), (2 Pi)}]), n ∈ Integers];
Fs = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 5}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
EK = Simplify[(1 / T) * Integrate[((f - Fs)^2), {t, 0, T}], T ∈ Reals && T > 0] // N;
Print["f(t)= ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", EK]
Plot[{f /. A → 2, Fs /. A → 2, Fs5 /. A → 2}, {t, -2 Pi, 2 Pi}, PlotRange → All, PlotLegends →
  {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"}]
```

$$f(t) = \frac{A\,t\,\text{HeavisideTheta}[t]}{\pi} + \frac{A\,(-2\pi+t)\,\text{HeavisideTheta}[-2\pi+t]}{\pi} + \left(-\frac{A\,t}{\pi} - \frac{A\,(-2\pi+t)}{\pi}\right)\text{HeavisideTheta}[-\pi+t]$$

$$a_0 = A$$

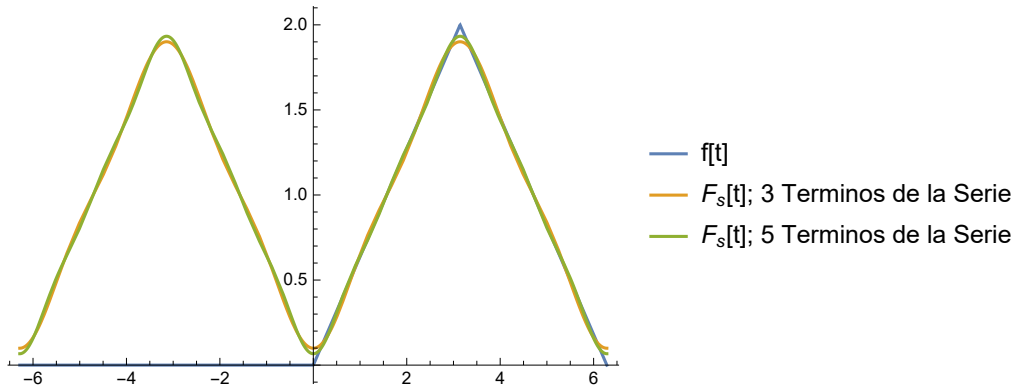
$$a_n = \frac{4\,A\,\text{Cos}[n\pi]\,\text{Sin}\left[\frac{n\pi}{2}\right]^2}{n^2\pi^2}$$

$$b_0 = \frac{A\left(2\,\text{Sin}[n\pi] - \text{Sin}[2\,n\pi]\right)}{n^2\pi^2}$$

$$F_s(t) = \frac{A\left(9\pi^2 - 72\,\text{Cos}[t] - 8\,\text{Cos}[3\,t]\right)}{18\pi^2} \quad ; \text{ 3 terminos}$$

$$E_k = 0.000191551\,A^2$$

Out[631]=



Ejercicio 5

In[632]:=
```
Clear["Global`*"]
f = ((A * t / Pi) - 0) HeavisideTheta[t - (0)] + ((0) - (A * t / Pi)) HeavisideTheta[t - (2 Pi)];
T = 2 Pi;
A0 = (2 / T) (Integrate[(A * t / Pi), {t, 0, 2 Pi}]);
AN = Simplify[
    (2 / T) (Integrate[(A * t / Pi) * Cos[(2 * Pi * n * t) / T], {t, 0, 2 Pi}]), n ∈ Integers];
BN = Simplify[(2 / T) (Integrate[(A * t / Pi) * Sin[(2 * Pi * n * t) / T], {t, 0, 2 Pi}]),
   n ∈ Integers];
Fs = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
     Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 5}] +
     Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
Ek = Simplify[(1 / T) * Integrate[((f - Fs)^2), {t, 0, 2 Pi}], T ∈ Reals && T > 0] // N;
Print["f(t)= ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t)= ", Fs, "  ; 3 terminos\nEk= ", Ek]
Plot[{f /. A → 1, Fs /. A → 1, Fs5 /. A → 1}, {t, -2 Pi, 5 Pi}, PlotRange → All,
 PlotLegends → {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"}]
```

$$f(t) = \frac{A\, t\, \text{HeavisideTheta}[t]}{\pi} - \frac{A\, t\, \text{HeavisideTheta}[-2\pi + t]}{\pi}$$

$$a_0 = 2\, A$$

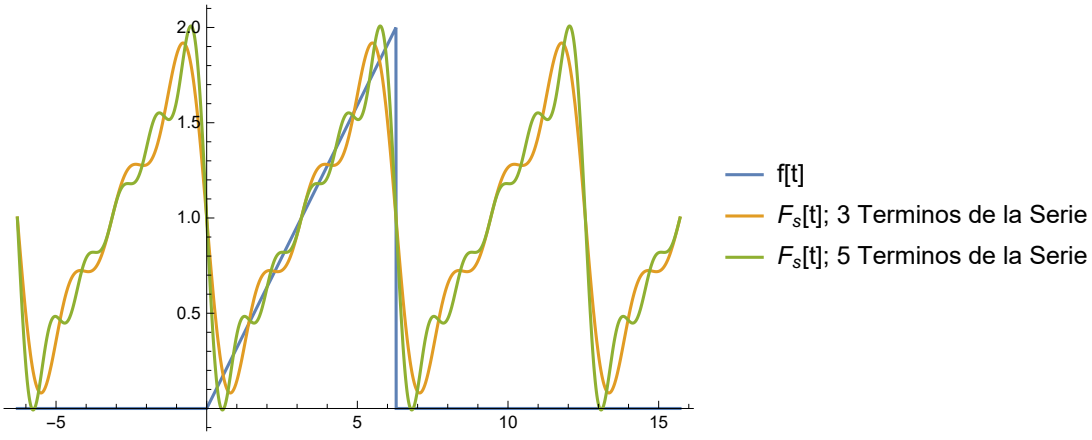
$$a_n = \frac{A\left(-1 + \text{Cos}[2\, n\, \pi] + 2\, n\, \pi\, \text{Sin}[2\, n\, \pi]\right)}{n^2\, \pi^2}$$

$$b_0 = \frac{A\left(-2\, n\, \pi\, \text{Cos}[2\, n\, \pi] + \text{Sin}[2\, n\, \pi]\right)}{n^2\, \pi^2}$$

$$F_s(t) = -\frac{A\left(-3\, \pi + 6\, \text{Sin}[t] + 3\, \text{Sin}[2\, t] + 2\, \text{Sin}[3\, t]\right)}{3\, \pi} \quad ;\ 3\ \text{terminos}$$

$$E_k = 0.0575146\, A^2$$

Out[642]=



Ejercicio 6

In[578]:=
```
Clear["Global`*"]
T = 2 Pi;
f = (A - 0) HeavisideTheta[t - 0] +
    (-A - (A)) HeavisideTheta[t - (T / 2)] + (0 - (-A)) HeavisideTheta[t - T];
A0 = (2 / T) (Integrate[(A), {t, 0, T / 2}] + Integrate[(-A), {t, (T / 2), T}]);
AN = Simplify[(2 / T) (Integrate[(A) * Cos[(2 * Pi * n * t) / T], {t, 0, T / 2}] +
      Integrate[(-A) Cos[(2 * Pi * n * t) / T], {t, (T / 2), T}]), n ∈ Integers];
BN = Simplify[(2 / T) (Integrate[(A) * Sin[(2 * Pi * n * t) / T], {t, 0, T / 2}] +
      Integrate[(-A) Sin[(2 * Pi * n * t) / T], {t, (T / 2), T}]), n ∈ Integers && n ≥ 1];
Fs = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 3}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 3}], n ∈ Integers && n ≥ 0];
Fs5 = Simplify[(A0 / 2) + Sum[Limit[AN * Cos[(2 * Pi * n * t) / T], n → k], {k, 1, 5}] +
    Sum[Limit[(BN * Sin[(2 * Pi * n * t) / T]), n → k], {k, 1, 5}], n ∈ Integers && n ≥ 0];
EK = Simplify[(1 / T) * Integrate[((f - Fs)^2), {t, 0, T}], T ∈ Reals && T > 0] // N;
Print["f(t) = ", f, "\na0=", A0, "\nan= ", AN,
 "\nb0= ", BN, "\nFs(t) = ", Fs, "  ; 3 terminos\nEk= ", EK]
Plot[{f /. A → 2, Fs /. A → 2, Fs5 /. A → 2}, {t, -Pi, 3 Pi}, PlotRange → All,
 PlotLegends → {"f[t]", "Fs[t]; 3 Terminos de la Serie", "Fs[t]; 5 Terminos de la Serie"}]
```

$f(t) = A\,\text{HeavisideTheta}[t] + A\,\text{HeavisideTheta}[-2\pi + t] - 2A\,\text{HeavisideTheta}[-\pi + t]$

$a_0 = 0$

$$a_n = \frac{A\left(2\sin[n\pi] - \sin[2n\pi]\right)}{n\pi}$$

$$b_0 = \frac{A\,(1 - 2\cos[n\pi] + \cos[2n\pi])}{n\pi}$$

$$F_s(t) = \frac{4A\left(3\sin[t] + \sin[3t]\right)}{3\pi} \quad ; \text{ 3 terminos}$$

$E_k = 0.0993673\,A^2$

Out[588]=