$$\int [x^n]\, dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \left[\frac{x^n}{a}\right] dx = \frac{1}{a} \cdot \frac{x^{n+1}}{n+1} + C$$

$$\int [a]\, dx = ax + C$$

$$\int \csc^2 x\, dx = -\cot x$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

$$\int f(x)\, dx = F(x) + C$$

$$\int e^u\, du = \frac{e^u}{u'} + C$$

has to be constant

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

---

u substitution:

$$\int 4x e^{x^2}\, dx =$$

$U = x^2$

$du = 2x\, dx$ (that's $u'dx$)

$dx = \dfrac{du}{2x}$

$$= \int \cancel{4}^{2} \cancel{x}\, e^u \frac{du}{\cancel{2x}} =$$

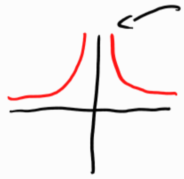
$$= \int 2e^u\, du = 2e^u + c = \underline{\underline{2e^{x^2} + C}}$$

Improper Integrals

$$\int_{-2}^{3} \frac{1}{x^2}\, dx$$

improper integral, bcs we should evaluate it on the interval $(-2;3)$ but $x \neq 0$

asymptote



$$\int_{-2}^{3} \frac{1}{x^2}\, dx = \lim_{a \to 0^-} \int_{-2}^{a} \frac{1}{x^2}\, dx + \lim_{b \to 0^+} \int_{b}^{3} \frac{1}{x^2}\, dx =$$

$$\left(-\frac{1}{x}\right)\Big|^{a} \qquad \left(-\frac{1}{x}\right)\Big|^{3}$$

$$= \lim_{a\to 0^-} \left[-\frac{1}{x}\right]\Bigg|_{-2} + \lim_{b\to 0^+}\left[-\frac{1}{x}\right]\Bigg|_{b}$$

$$= \lim_{a\to 0^-}\left[\frac{-1}{a}-\frac{1}{2}\right] + \lim_{b\to 0^+}\left[-\frac{1}{3}\overset{+}{-}\left(-\frac{1}{b}\right)\right] =$$

$$= \left(\infty - \frac{1}{2}\right) + \left(-\frac{1}{3}+\infty\right) = \underline{\underline{\infty}}$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

$$\int |x|\,dx = \begin{cases} \int x\,dx & x \geq 0 \\ \int -x\,dx & x < 0 \end{cases}$$

---

u-substitution hard:

$$\int 2x\sqrt{4x-5}\,dx$$

$$u = 4x-5 \longrightarrow u+5 = 4x$$
$$du = 4 \qquad x = \frac{1}{4}[u+5]$$
$$dx = \frac{du}{4}$$

$$\int 2\cdot\frac{1}{4}(u+5)\,u^{1/2}\,\frac{du}{4} =$$

$$= \frac{1}{8}\int u^{1/2}(u+5)\,du = \frac{1}{8}\int\left(u^{3/2}+5u^{1/2}\right)du =$$

$$= \frac{1}{8}\left(u^{5/2}\cdot\frac{2}{5} + 5u^{3/2}\cdot\frac{2}{3}\right)+C = \frac{1}{20}u^{5/2} + \frac{5}{12}u^{3/2}+C =$$

$$= \underline{\underline{\frac{1}{20}(4x-5)^{5/2} + \frac{5}{12}(4x-5)^{3/2}+C}}$$

---

u-substitution definite integrals:

$$\int_0^2 2x(x^2+4)^2\,dx$$

$$u = x^2+4$$
$$du = 2x\,dx$$
$$dx = \frac{du}{2x}$$

$$u = 0^2+4 = 4$$
$$u = 2^2+4 = 8$$

$$\int \cancel{2x}(u^2)\frac{du}{\cancel{2x}} = \int_4^8 u^2\,du =$$

$$= \frac{u^3}{3}\Bigg|_4^8 = \frac{8^3}{3} - \frac{4^3}{3} = \underline{\underline{\frac{448}{3}}}$$

Integration by parts:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$\int x e^x dx =$$

$u = x$
$du = 1dx$
$v = e^x$ (integral of dv)
$dv = e^x dx$

$$= xe^x - \int e^x dx$$

$$= \underline{xe^x - e^x + C}$$

$$\int x \sin x \, dx$$

$u = x$
$du = dx$
$v = -\cos x$
$dv = \sin x \, dx$

$$= uv - \int v\,du =$$

$$= x(-\cos x) - \int (-\cos x)\,dx =$$

$$= \underline{-x \cdot \cos x + \sin x + C}$$

$$\int x^2 \ln x \, dx$$

$u = \ln x$
$du = \frac{1}{x} dx$
$V = \frac{x^3}{3}$
$dv = x^2 dx$

$$= uv - \int v\,du =$$

$$= \frac{\ln x \cdot x^3}{3} - \int \frac{x^{\cancel{3}2}}{3} \cdot \frac{1}{\cancel{x}} dx$$

$$= \frac{x^3 \cdot \ln x}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + C = \underline{\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C}$$

$$\int x^2 \sin x \, dx$$

$u = x^2$
$du = 2x \, dx$
$V = -\cos x$
$dv = \sin x \, dx$

$$= x^2(-\cos x) - \int (-\cos x) 2x \, dx$$

$$= -x^2 \cos x + 2 \int x \cos x \, dx \longrightarrow \int x \cos x \, dx$$

$u = x$
$du = 1dx$
$V = \sin x$
$dv = \cos x \, dx$

$$= x \sin x - \int \sin x \, dx$$

$$= -x \sin x + \cos x + C$$

$$= \underline{-x^2 \cos x + 2x \sin x + 2\cos x + C}$$

$= x\sin x + \cos x + C$

$$\int (\ln x)^2 \, dx$$

$u = (\ln(x))^2$

$du = 2\ln(x) \cdot \frac{1}{x}$

$dv = dx$

$v = x$

$$= x(\ln x)^2 - \int \cancel{x} \cdot 2(\ln x) \cdot \frac{1}{\cancel{x}} \cdot dx$$

$$= x(\ln x)^2 - 2\int \ln(x)\, dx$$

$$= x(\ln x)^2 - 2(x\ln x - x) = x(\ln x)^2 - 2x\ln x + 2x + C$$

$$\int e^x \sin x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$dv = e^x dx$

$v = e^x$

$$= e^x \sin x - \int e^x \cos x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$dv = e^x dx$

$v = e^x$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx) \quad \Big| + \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

Integration by partial fractions:

$$\int \frac{x}{(x-1)(x-2)^2} dx = \int \frac{A}{(x-1)} dx + \int \frac{B}{(x-2)} dx + \int \frac{C}{(x-2)^2} dx$$

$$\frac{x}{\ldots} = \frac{A}{\ldots} + \frac{B}{\ldots} + \frac{C}{\ldots} \quad \Big| \cdot (x-1)(x-2)^2$$

$$\frac{}{(x-1)(x-2)^2} = \frac{}{x-1} + \frac{}{x-2} + \frac{}{(x-2)^2} \qquad / \cdot (x-1)(x-2)$$

$x = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$ $x=2$ $\boxed{C=2}$

$2 = A(0) + B(1)(0) + C(1)$ $x=1$ $\boxed{A=1}$

$\underline{\underline{2 = C}}$

$1 = A(1-2)^2 + B(0) + C(0)$ $x=3$ $\boxed{B=-1}$

$\underline{\underline{1 = A}}$

$3 = A(1) + B(2)(1) + C(2)$

$3 = 1 + 2B + 4$

$\underline{\underline{B = -1}}$

$$\int \frac{x}{(x-1)(x-2)^2}\,dx = \int \frac{1}{(x-1)}\,dx + \int \frac{-1}{(x-2)}\,dx + \int \frac{2}{(x-2)^2}\,dx$$

$$= \ln|x-1| - \ln|x-2| + 2\int \frac{1}{(x-2)^2}\,dx = \qquad u = x-2, \quad du = dx$$

$$= \ln|x-1| - \ln|x-2| + 2\int u^{-2}\,du =$$

$$= \ln|x-1| - \ln|x-2| + 2\,\frac{-1}{u} =$$

$$= \underline{\underline{\ln\left|\frac{x-1}{x-2}\right| - \frac{2}{x-2} + C}}$$

Improper Integrals (Convergence / Divergence)

$$\int_1^\infty \frac{1}{x}\,dx = L \rightarrow \text{converges}$$

$$= \infty \rightarrow \text{diverges}$$

$$= \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln x \Big|_1^t = \lim_{t\to\infty} \left[\ln t - \cancel{\ln 1}_{\,0}\right] = \lim_{t\to\infty} \ln t = \underline{\underline{\infty}}$$

diverges

$$\int^\infty \frac{1}{\ } \qquad \int^\infty \qquad \frac{1}{\ }\Big|^t$$

$$\int_1 \frac{1}{x^2}\,dx = \int_1 x^{-2}\,dx = \lim_{t\to\infty} \left.\frac{-1}{x}\right|_1 = \lim_{t\to\infty}\left(\frac{-1}{t} - \frac{-1}{1}\right) =$$

$$= \lim_{t\to\infty}\left(\underset{\downarrow \atop 0}{-\frac{1}{t}} + \underset{\downarrow \atop 1}{1}\right) = 0 + 1 = \underline{\underline{1}} \to \text{converges}$$

Integral of log

$$\int \log_a u\,du = \frac{u\cdot\log_a\left(\frac{u}{e}\right)}{u'} + C$$

(u has to be linear)

$$\int \log_5(x+7)\,dx = \frac{(x+7)\cdot\log_5\left(\frac{x+7}{e}\right)}{1} + C$$