

$$\frac{d}{dx} [c] = 0$$

$$\frac{d}{dx} [x^m] = m \cdot x^{m-1}$$

$$\frac{d}{dx} [\pi] = 0$$

$$\frac{d}{dx} \left[\frac{1}{x^m} \right] = - \frac{1}{x^{m+1}}$$

$$\frac{d}{dx} [\pi^e] = 0$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} [\sin u] = \cos u \cdot u'$$

$$\frac{d}{dx} [\cos u] = -\sin u \cdot u'$$

$$\frac{d}{dx} [\tan u] = \sec^2(u) \cdot u'$$

$$\frac{d}{dx} [\cot u] = -\csc^2(u) \cdot u'$$

$$\frac{d}{dx} [\sec u] = \sec u \cdot \tan u \cdot u' \quad \left(\sec = \frac{1}{\cos} \right)$$

$$\frac{d}{dx} [\csc u] = -\csc u \cdot \cot u \cdot u'$$

$$\frac{d}{dx} [\ln u] = \frac{u'}{u}$$

$$\frac{d}{dx} [\log_a u] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} [e^u] = e^u \cdot u'$$

$$\frac{d}{dx} [a^u] = a^u \cdot u' \cdot \ln(a)$$

$$\frac{d}{dx} [u \cdot v] = u' \cdot v + u \cdot v'$$

$$\frac{d}{dx} \left[\frac{u}{v} \right] = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$$\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx} [x^x] =$$

$$\begin{aligned} 1. \quad y &= x^x \\ 2. \quad \ln y &= \ln(x^x) \\ \ln y &= x \ln x \end{aligned}$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} (x \ln x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \left(\frac{1}{x} \right) \Big|_y$$

$$\frac{dy}{dx} = y [1 + \ln x]$$

$$\frac{dy}{dx} = x^x [1 + \ln x]$$

Find tangent line equation

① $f(x) = 2x^2 - 5x + 3$ at $x_1 = 2$

$$y - y_1 = m(x - x_1)$$

$$y_1 = 2(2)^2 - 5 \cdot 2 + 3 \Rightarrow y_1 = 1$$

$$m = f'(x) \Rightarrow m = 4x - 5 \text{ at } x = 2$$

$$m = 3$$

$$\boxed{y - 1 = 3(x - 2)}$$

or

$$\boxed{y = 3x - 5}$$

Find the points where the graph has a horizontal tangent line:

$$f(x) = x^3 - 6x^2 + 15$$

$$f'(x) = 0$$

$$f'(x) = 3x^2 - 12x$$

$$3x^2 - 12x = 0$$

$$3x(x - 4) = 0$$

$$\underline{x_1 = 0}$$

$$\underline{x_2 = 4}$$

finding y :

$$f(0) = 0^3 - 6 \cdot 0^2 + 15 = 15$$

$$P_1 = (0, 15)$$

$$f(4) = 4^3 - 6 \cdot 4^2 + 15 = 79$$

$$P_2 = (4, 79)$$

Find the equation of the normal line

$$y = x^3 - 4x^2 + 5 \text{ at } x = 2$$

1) find y coordinate at $x = 2$:

$$y = 2^3 - 4(2)^2 + 5 \Rightarrow \underline{y = -3}$$

2) slope of the tangent line:

$$y' = 3x^2 - 8x$$

$$\text{at } x=2$$

$$\underline{m = -4}$$

3) normal line :

$$P = (2, -3) \quad m_{\text{tan}} = -4$$

$$m_n = +\frac{1}{4}$$

$$y - (-3) = \frac{1}{4}(x - 2)$$

$$\underline{y + 3 = \frac{1}{4}(x - 2)}$$

or

$$\underline{y = \frac{1}{4}x - \frac{7}{2}}$$

Inverse trig. functions

$$\frac{d}{dx} (\sin^{-1}(u)) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\tan^{-1}(u)] = \frac{u'}{1+u^2}$$

$$\frac{d}{dx} [\cos^{-1}(u)] = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} [\sec^{-1}(u)] = \frac{u'}{|u|\sqrt{u^2-1}}$$

critical points : when • $f'(x) = 0$

• $f'(x) = \text{DNE}$

Find local min/max of the function:

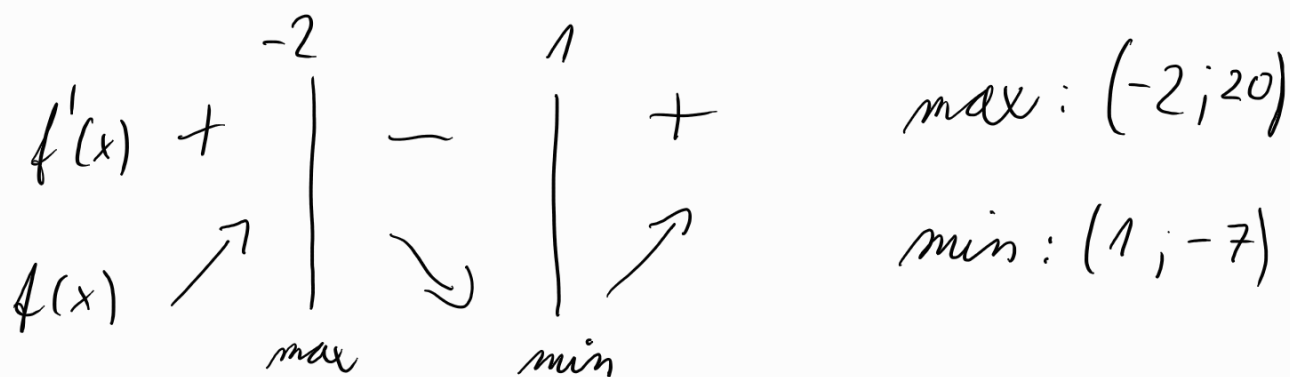
$$f(x) = 2x^3 + 3x^2 - 12x$$

$$f'(x) = 6x^2 + 6x - 12$$

critical points: $f'(x) = 0$

$$6x^2 + 6x - 12 = 0$$

$$(x-1)(x+2) = 0 \quad \begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$$



increasing $f(x) \Rightarrow f'(x)$ positive

decreasing $f(x) \Rightarrow f'(x)$ negative

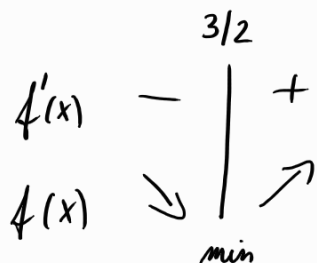
Find the intervals where the function is increasing/decreasing

$$f(x) = x^2 - 3x + 1$$

$$f'(x) = 2x - 3$$

$$2x - 3 = 0$$

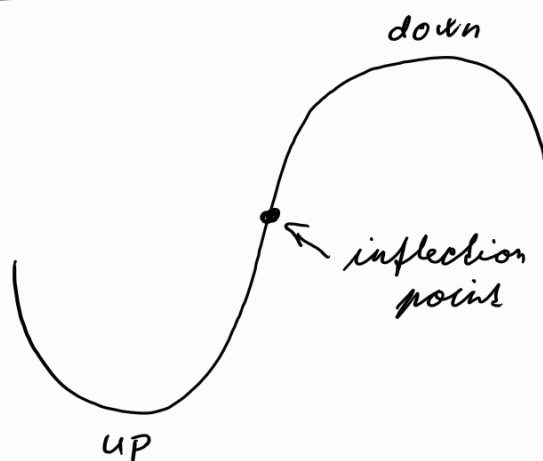
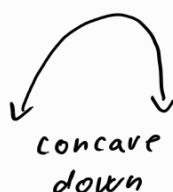
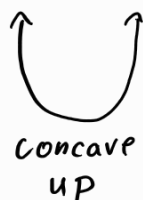
$$x = 3/2$$



increasing: $(3/2; \infty)$

decreasing $(-\infty; 3/2)$

concavity:



Find inflection points and where the fun. is concave up/down

$$f(x) = x^3 - 9x^2 + 7x$$

$$f'(x) = 3x^2 - 18x + 7$$

$$f''(x) = 6x - 18$$

$$0 = 6x - 18$$

$$0 = 6(x - 3) \Rightarrow x = 3$$

inflection
point

$$f''(x) \quad - \quad | \quad +$$

down up

↓

infl.
point

concave down: $(-\infty; 3)$

concave up: $(3; \infty)$
