$$y = \lim_{n\to\infty} \left[1+\frac{1}{n}\right]^n$$

$$\ln y = \lim_{n\to\infty} \ln\left[1+\frac{1}{n}\right]^n \qquad a\cdot b = \frac{b}{1/a}$$

$$\ln y = \lim_{n\to\infty} (n)\ln\left(1+\frac{1}{n}\right) \qquad \boxed{\frac{d}{dx}\ln(u) = \frac{u'}{u}}$$

$$\ln y = \lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{1/n} \overset{H}{=} \lim_{n\to\infty} \frac{\frac{-1/n^2}{1+1/n}}{-1/n^2} \begin{matrix}\cdot n^2\\ \cdot n^2\end{matrix} = \qquad \boxed{\begin{matrix} u = 1+\frac{1}{n} \\ u'\ 0 - \frac{1}{n^2}\end{matrix}}$$

$$\ln y = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \frac{1}{1+0} = 1 \qquad \ln y = 1$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \underline{\underline{e}}$$

$$\lim_{x\to 4} \frac{\frac{1}{x}-\frac{1}{4}}{x-4} \begin{matrix}\cdot 4x\\ \cdot 4x\end{matrix} = \lim_{x\to 4}\frac{4-x}{(x-4)\,4x} = \lim_{x\to 4}\frac{-\cancel{(x-4)}}{4x\cancel{(x-4)}} =$$

$$= \lim_{x\to 4}\frac{-1}{4x} = \underline{\underline{-\frac{1}{16}}}$$

$$\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16} \begin{matrix}\cdot(\sqrt{x}+4)\\ \cdot(\sqrt{x}+4)\end{matrix} = \lim_{x\to 16}\frac{\cancel{x-16}}{\cancel{(x-16)}(\sqrt{x}+4)} =$$

$$= \lim_{x\to 16}\frac{1}{\sqrt{x}+4} = \frac{1}{\sqrt{16}+4} = \underline{\underline{\frac{1}{8}}}$$

$$\lim_{x\to 7} \frac{|x-7|}{x-7} = DNE \qquad \begin{array}{ll} x>7 & \oplus \\ x<7 & \ominus \end{array}$$

$$\lim_{x\to 7^+} \frac{x-7}{x-7} = \underline{\underline{1}}$$

$$\lim_{x\to 7^-} \frac{-(x-7)}{x-7} = \underline{\underline{-1}}$$

$$\lim_{x\to 0} \frac{\tan(3x)}{5x} = \frac{3}{5}$$

$$\lim_{x\to 0} \frac{\sin 3x}{\cos 3x} \cdot \frac{1}{5x} \cdot \frac{3}{3} = \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{3}{5\cos(3x)} =$$

$y=3x$

$$= \lim_{y\to 0} \underbrace{\frac{\sin y}{y}}_{=1} \cdot \lim_{x\to 0} \underbrace{\frac{1}{\cos 3x}}_{=1} \cdot \frac{3}{5} =$$

(bec $\cos 0 = 1$)

$$= 1 \cdot 1 \cdot \frac{3}{5} = \underline{\underline{\frac{3}{5}}}$$

Find horizontal asymptotes using limits:

$$f(x) = \frac{5x+8x^2}{3+2x^2} + 5$$

$$\lim_{x\to\infty} \frac{\cancel{5x}+8x^2}{\cancel{3}+2x^2} + 5 = \lim_{x\to\infty} \frac{8\cancel{x^2}}{2\cancel{x^2}} + 5 = 4+5 = \underline{\underline{9}}$$

// asymptote $y=9$

$$\lim_{x \to 0} x \cdot \sin(1/x) = 0$$

squeeze theorem:

$$\left(-1 \leq \sin\left(\frac{1}{x}\right) \leq 1\right) \cdot x$$

$$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$$

$$h(x) \leq f(x) \leq g(x)$$

$$0 \leq f(x) \leq 0$$

$$\| $$

$$0$$

$$\lim_{x \to 0} h(x) = \lim_{x \to 0} -x = 0$$

$$\lim_{x \to 0} g(x) = \lim_{x \to 0} x = 0$$

Find the value of c that will make the function continuous:

$$f(x) \begin{cases} 7x^2 + cx & x < 2 \\ 2x^3 + 5c + 3 & x \geq 2 \end{cases}$$

at $x=2$

$$7x^2 + cx = 2x^3 + 5c + 3$$

$$7(2)^2 + 2c = 2(2)^3 + 5c + 3$$

$$28 + 2c = 16 + 5c + 3$$

$$9 = 3c$$

$$\boxed{c = 3}$$

Is the function continuous and differentiable?

$$f(x) \begin{cases} x & x \leq 1 \\ x^3 & x > 1 \end{cases}$$

continuous ✓

differentiable X

3 step continuity evaluation:

1) $f(x)$ must exist:

$f(1) = 1$ ✓

2) $\lim_{x \to 1} f(x)$ must exist:

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = \underline{\underline{1}}$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^3 = \underline{\underline{1}}$

$\lim_{x \to 1} f(x) = 1$ ✓

3) $f(x) = \lim f(x)$

$f(1) = \lim_{x \to 1} f(x)$

$1 = 1$ ✓

<u>differentiability:</u>

$$f'(x) \begin{cases} 1 & x \leq 1 \\ 3x^2 & x > 1 \end{cases}$$

is $f'(x)$ continuous?

$\lim_{x \to 1^-} f'(x) = \underline{\underline{1}}$

$\lim_{x \to 1^+} f'(x) = \lim_{x \to 1^+} 3x^2 = \underline{\underline{3}}$

$\lim_{x \to 1^-} f'(x) \neq \lim_{x \to 1^+} f(x)$

$f'(x)$ NOT CONTINUOUS

$\Downarrow$

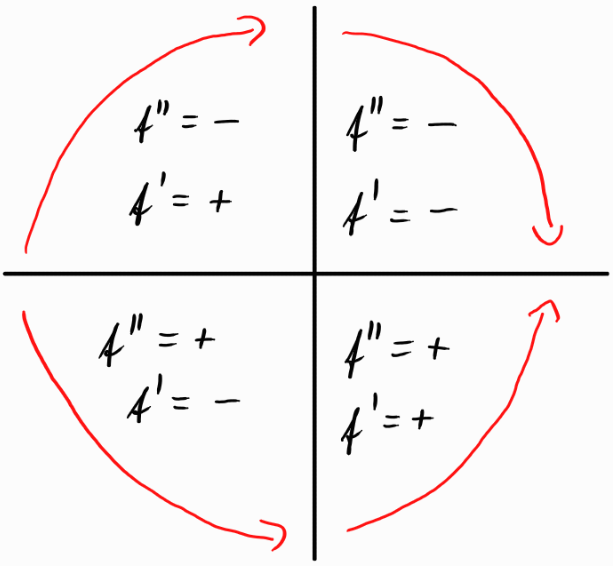
$f(x)$ NOT DIFFERENTIABLE

L'hobital Rule

$\lim \dfrac{f(x)}{\ldots} = \lim \dfrac{f'(x)}{\ldots}$

$\lim\limits_{x \to c} \; g(x) = \lim\limits_{x \to c} \; g'(x)$

Graph sketching:



- $y = 3x^4 + 4x^3$

$0 = x^3(3x+4)$

$\downarrow$ $x = 0$ $\qquad$ $\downarrow$ $x = -\frac{4}{3}$

x	y
$-4/3$	0
0	0
-1	-1
$-2/3$	$-16/27$

- $y' = 12x^3 + 12x^2$

$0 = 12x^2(x+1)$

$\downarrow$ $x = 0$ $\qquad$ $\downarrow$ $x = -1$

for $x = -1$ $\qquad$ $y = ?$

$y = 3(-1)^4 + 4(-1)^3 = 3 - 4 = -1$

$\underbrace{y = -1}$

- $y'' = 36x^2 + 24x$

$0 = 12x(3x+2)$

$\downarrow$ $x = 0$ $\qquad$ $\downarrow$ $x = -2/3$

for $x = -2/3$ $\qquad$ $y = ?$

$y = 3(-2/3)^4 + 4(-2/3)^3 = \frac{16}{27} - \frac{32}{27}$

$y = -\frac{16}{27}$

		-1		$-2/3$		0	
f'	$-$		$+$		$+$		$+$
f''	$+$		$+$		$-$		$+$