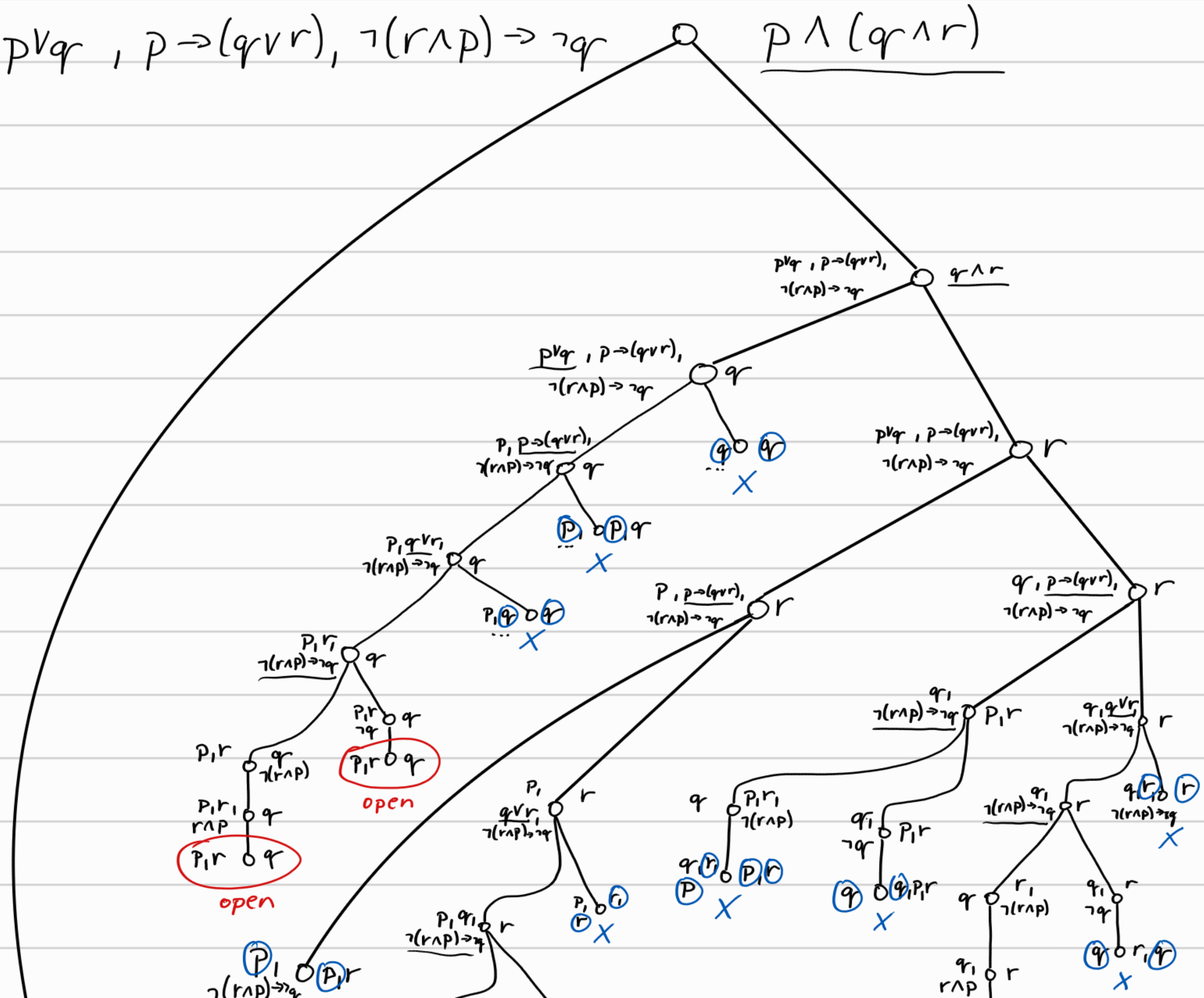
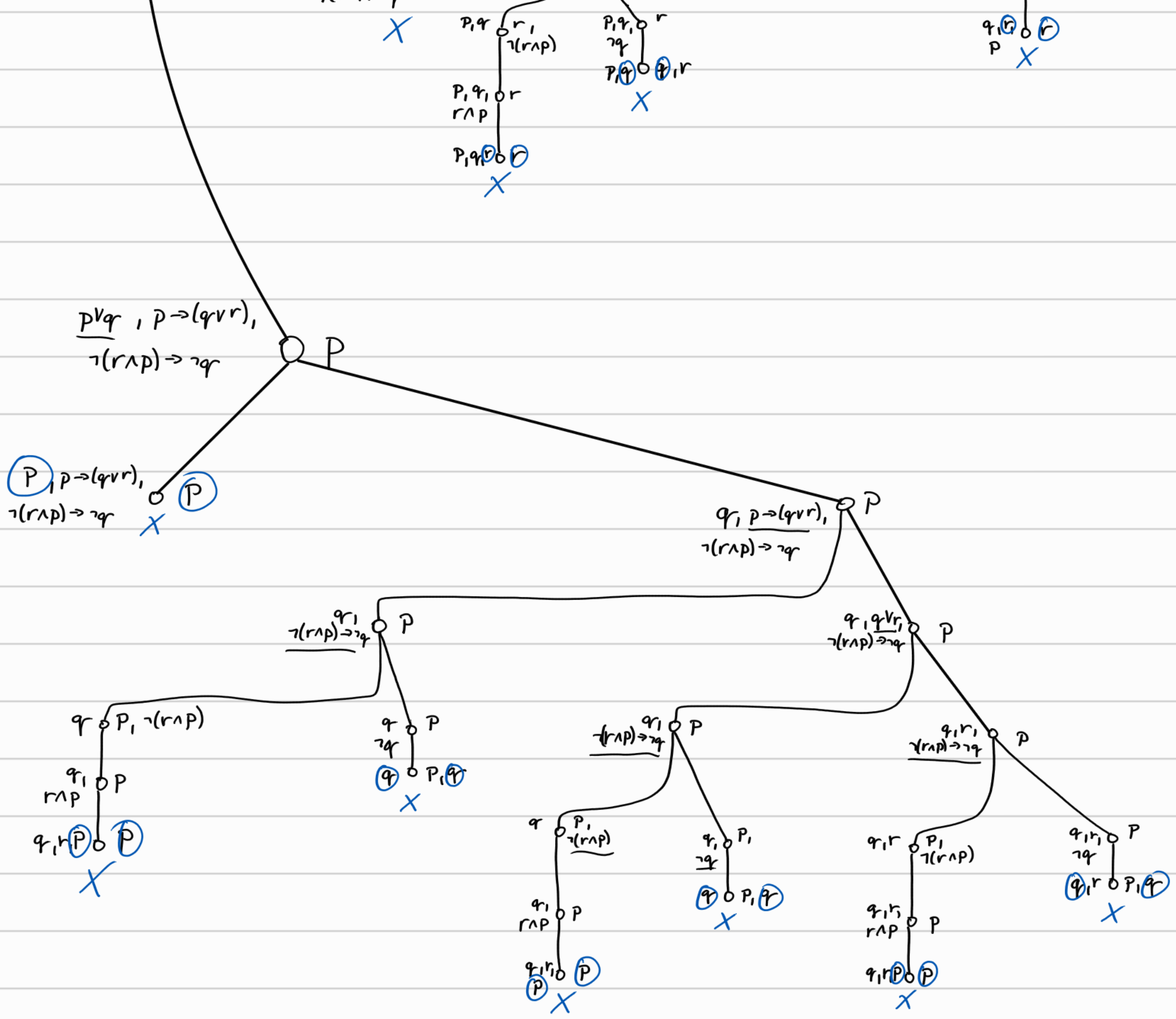
①

| a | b | c | $a \vee b$ | $c \to \neg b$ | $\neg(a \wedge c)$ | $(b \wedge \neg a) \to c$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 |

- Alice likes running.
- Unknown is Bob likes cycling.
- Chris does not like dancing.

② $p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q$ $\quad$ $\underline{p \wedge (q \wedge r)}$






counterexample p, r are T (or 1) and q is F (0)

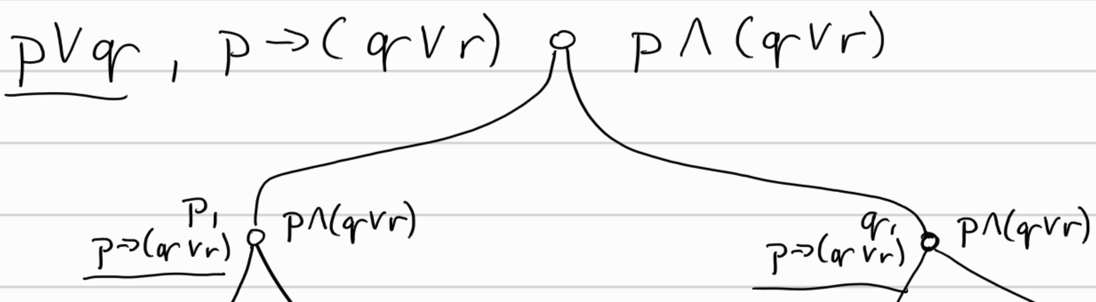
$p = T,\ r = T,\ q = F$

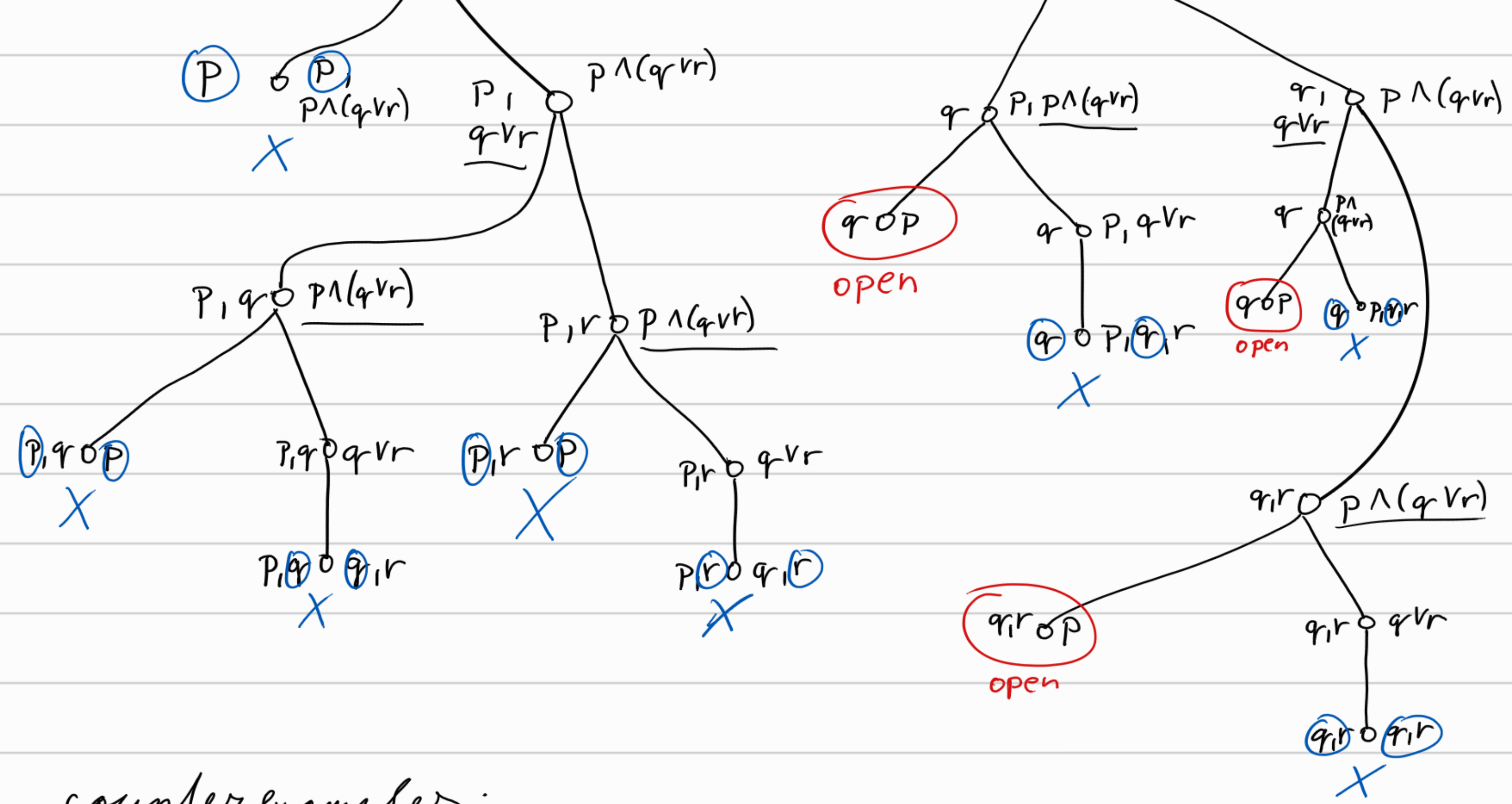
$p \vee q = T \vee F = T$

$p \to (q \vee r) = T \to T = T$ $\qquad T \models F$

$\neg(r \wedge p) \to \neg q = \neg T \to \neg F = F \to T = T$

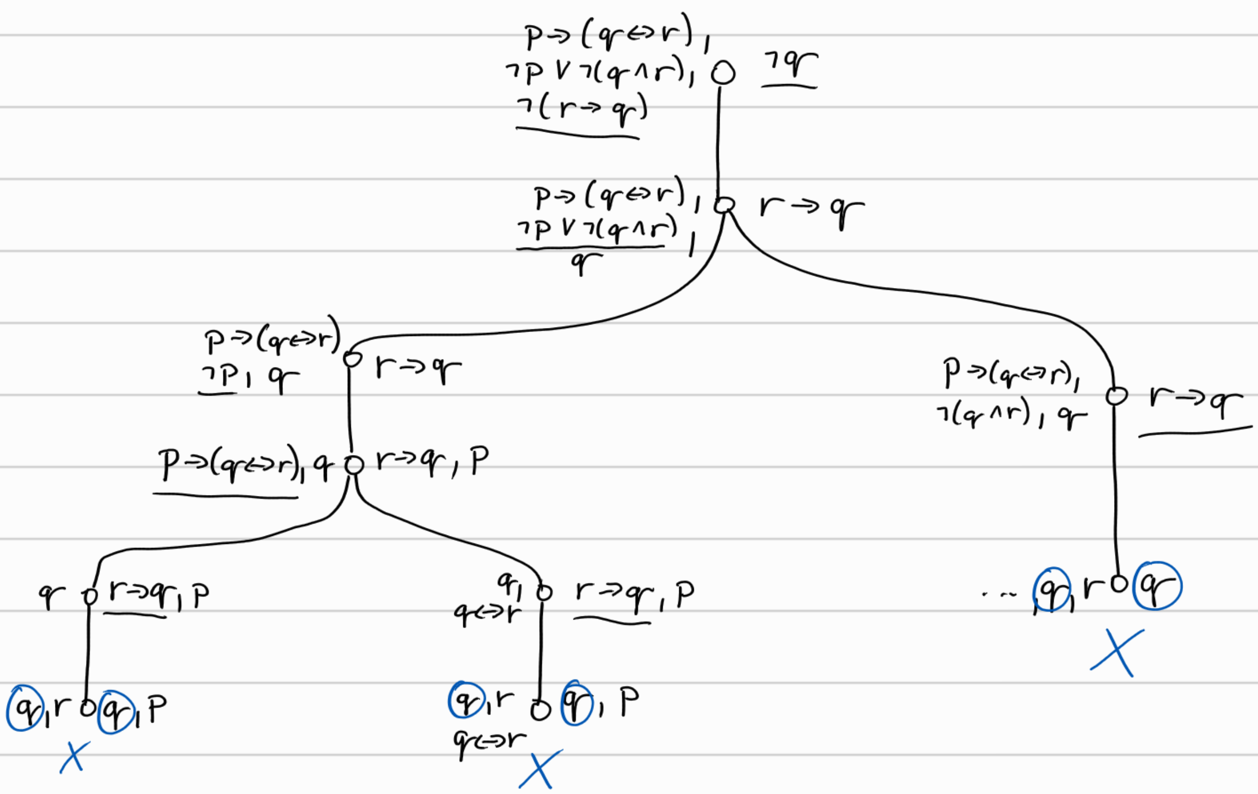
$\dashv\ p \wedge (q \wedge r) = T \wedge (F \wedge T) = T \wedge F = F$






counterexamples:

- $q = 1$, $p = 0$, $r = 0$
- $q = 1$, $p = 0$, $r = 1$




valid

③

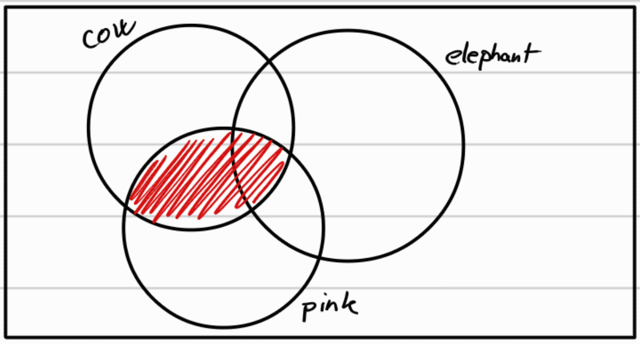
| | |
|---|---|
| 1 | $p \Rightarrow \neg(r \Rightarrow q)$ |
| 2 | $p$      ass |
| 3 | $\neg(r \Rightarrow q)$      E (1,2) |

| | | |
|---|---|---|
| 3 | $\neg(r \to q)$ | $E_{\to}(1,2)$ |
| 4 | $\neg(\neg q \wedge r)$ | ass |
| 5 | $r$ | ass |
| 6 | $\neg q$ | ass |
| 7 | $\neg q \wedge r$ | $I_{\wedge}(5,6)$ |
| 8 | $\perp$ | $\perp(4,7)$ |
| 9 | $q$ | $E_{\neg}(6-8)$ |
| 10 | $r \to q$ | $I_{\to}(5,9)$ |
| 11 | $\perp$ | $\perp(3,10)$ |
| 12 | $(\neg q \wedge r)$ | $E_{\neg}(4,5-11)$ |
| 13 | $p \to (\neg q \wedge r)$ | $I_{\to}(2,12)$ |

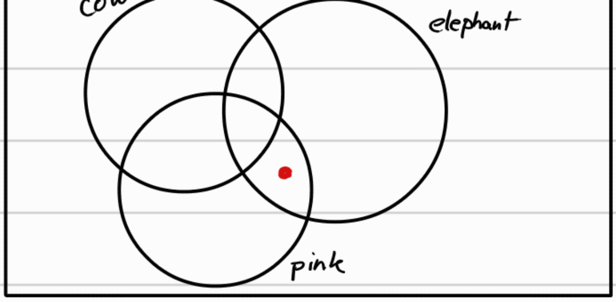
| | | |
|---|---|---|
| 1 | $\neg(\neg p \vee \neg q)$ | $p \wedge q$ |
| 2 | $\neg p$ | ass |
| 3 | $\neg p \vee \neg q$ | $I_{\vee}(2)$ |
| 4 | $\perp$ | $\perp(1,3)$ |
| 5 | $p$ | $E_{\neg}(2,3-4)$ |
| 6 | $\neg q$ | ass |
| 7 | $\neg p \vee \neg q$ | $I_{\vee}(6)$ |
| 8 | $\perp$ | $\perp(1,6)$ |
| 9 | $q$ | $E_{\neg}(6,7-8)$ |
| 10 | $p \wedge q$ | $I_{\wedge}(5,9)$ |

4

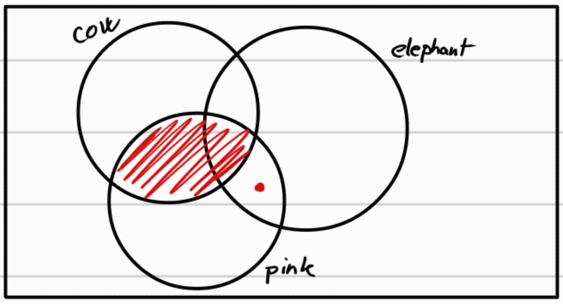
no cow is pink:




some elephants are pink:




together: Some elephants are not cows.




(6)

$\circ\ \forall x \exists y (Ryx \to Rxx)$

$\overset{+}{\circ}\ \exists y (Rya \to Raa)$

$\circ\ Raa \to Raa$

$Raa\ \circ\ Raa$

$\times$

$\circ\ \forall x ((\exists y Ryx) \to Rxx)$

$\overset{+}{\circ}\ (\exists y Rya) \to Raa$

$\exists y Rya\ \circ\ Raa$

$Rba\ \overset{+}{\circ}\ Raa$

open

(7) $\forall x\ (R \dots \to \exists R \dots)$

(7)

| | | |
|---|---|---|
| 1 | $\forall x(Px \to \exists y Rxy)$ | |
| 2 | $\forall x(\exists y Ryx \to \neg Px)$ | |
| 3 | $\exists x Px$ | |
| 4 | a $Pa$ | |
| 5 | $Pa \to \exists y Ray$ | $E_\forall 1$ |
| 6 | $\exists y Ray$ | $E_\to (4,5)$ |
| 7 | b $Rab$ | |
| 8 | $\exists y Ryb$ | $I_\exists (7)$ |
| 9 | $\exists y Ryb \to \neg Pb$ | $E_\forall (2)$ |
| 10 | $\neg Pb$ | $E_\to (8,9)$ |
| 11 | $\exists x \neg Px$ | $I_\exists (10)$ |
| 12 | $\exists x \neg Px$ | $E_\exists (6, 7-11)$ |
| 13 | $\exists x \neg Px$ | $E_\exists (3, 4-12)$ |

$\exists x(Cx \land \neg Ax)$

| | | |
|---|---|---|
| 1 | $\forall x(Ax \to \neg Bx)$ | |
| 2 | $\exists x(Bx \land Cx)$ | |
| 3 | a $Ba \land Ca$ | |
| 4 | $Ca$ | $E_\land (3)$ |
| 5 | $Ba$ | $E_\land (3)$ |
| 6 | $Aa$ | ass |
| 7 | $Aa \to \neg Ba$ | $E_\forall (1)$ |
| 8 | $\neg Ba$ | $E_\to (6,7)$ |
| 9 | $\bot$ | $\bot (5,9)$ |
| 10 | $\neg Aa$ | $I_\neg (6, 7-9)$ |
| 11 | $Ca \land \neg Aa$ | $I_\land (4,10)$ |
| 12 | $\exists x((Cx \land \neg Ax)$ | $I_\exists (11)$ |
| 13 | $\exists x(Cx \land \neg Ax)$ | $E_\exists (2, 3-12)$ |

(10)

$\langle a^* \rangle p \to \top$ everywhere

$[b]\neg p$