①

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ -1 & -2 & -1 & -10 \\ 4 & 0 & a & 4+2a \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & -9 \\ 4 & 0 & a & 4+a \end{array}\right]$$

for any a
no sol

③ $\left[\begin{array}{cc|c} a & 1 & a^2 \\ 1 & a & a+1 \end{array}\right]$

if $a=0$ $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right]$ $x_1 = 1$, $x_2 = 0$ ✓

if $a=1$ $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$ no sol for $a=1$

if $a=-1$: $\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 1 \\ 1 & -1 & 0 \end{array}\right]$ no sol for $a=-1$

other: $\left[\begin{array}{cc|c} a & 1 & a^2 \\ 1 & a & a+1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1/a & a \\ 1 & a & a+1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1/a & a \\ 0 & a-1/a & 1 \end{array}\right]$

$$\frac{1}{a} = a - \frac{1}{a} \qquad \frac{2}{a} = a \qquad 2 = a^2$$

$$\boxed{a = \pm\sqrt{2}}$$

$$\left[\begin{array}{cc|c} \sqrt{2} & 1 & 2 \\ 1 & \sqrt{2} & \sqrt{2}+1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1/\sqrt{2} & 2/\sqrt{2} \\ 1 & \sqrt{2} & \sqrt{2}+1 \end{array}\right]$$

④ $\left[\begin{array}{ccc|c} 1/8 & 1 & -1 & 7 \\ 1/8 & 4 & -3 & 29 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 8 & -8 & 56 \\ 1 & 32 & -24 & 232 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 8 & -8 & 56 \\ 0 & 24 & -16 & 176 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 8 & 0 & 64 \end{array}\right.$ $\quad (x_1 = 64 - 8x_{\dots}$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{cases} x_2 \text{ free} \\ x_3 = 1 \end{cases}$$

$$\underline{x} = \begin{bmatrix} 64 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -8 \\ 1 \\ 0 \end{bmatrix}$$

(5)

$$\begin{array}{cc} & 37 \\ 1 & \begin{bmatrix} \; A \; \end{bmatrix} \end{array} \quad \begin{bmatrix} \\ B \\ \\ \end{bmatrix}$$

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(1) $$c_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$\rightarrow c_1 = 2 \;;\; c_2 = 3 \;;\; c_3 = 0$

$\rightarrow c_1 = -3 \;;\; c_2 = -1 \;;\; c_3 = 1$

(2) $$T\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) - T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right)$$

$T(\underline{0}) \neq \underline{0}$ not a transfor.

(3) not invertible : $\det(A) = 0$

$$p \det \begin{bmatrix} p & 2 \\ -1 & 2 \end{bmatrix} = p(2p+2) = 2p^2 + 2p = 2p(p+1)$$

$p = 0$ or $p = -1$

④ $\begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix}$

row sum $= 3 \rightarrow \lambda = 3$

$\det A = 6 \rightarrow \lambda_1 \cdot \lambda_2 \cdot \lambda_3$

trace $A = 0 \rightarrow \lambda_1 + \lambda_2 + \lambda_3$

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 6$

$-\lambda_1 = \lambda_2 + \lambda_3 \Rightarrow -3 = \lambda_2 + \lambda_3 \Rightarrow \lambda_2 = -3 - \lambda_3$

$3(-3-\lambda_3)\lambda_3 = 6$

$(-3-\lambda_3)\lambda_3 = 2$

$-3\lambda_3 - \lambda_3^2 = 2$

$\lambda^2 + 3\lambda + 2 = 0$

$\lambda_2 = -2 \qquad \lambda_3 = -1$

⑤ $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 9 & -7 \\ 0 & 2 & -5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & 1 & -5/2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{cases} x_1 = x_3 - 5x_4 \\ x_2 = 5/2\,x_3 - 3x_4 \\ x_3, x_4 \text{ free} \end{cases}$

$\underline{x} = x_3 \begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}$

$\text{Nul}\, A = \text{Span}\left(\begin{bmatrix} 1 \\ 5/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right)$

$a + 5/2\, b + c = 0$

$-5a - 3b + d = 0$

$$\left[\begin{array}{cccc|c} 1 & 5/2 & 1 & 0 & 0 \\ -5 & -3 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 2 & 5 & 2 & 0 & 0 \\ 0 & 19 & 10 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 38 & 95 & 38 & 0 & 0 \\ 0 & 95 & 50 & 10 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 38 & 0 & -12 & -10 & 0 \\ 0 & 95 & 50 & 10 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 19 & 0 & -6 & -5 & 0 \\ 0 & 19 & 10 & 2 & 0 \end{array}\right]$$

6

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

every vector in Row A $\alpha \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$

$$\underline{u} - \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix} = \begin{bmatrix} \alpha - 3 \\ -\alpha + 3\beta - 2 \\ \beta - 9 \end{bmatrix}$$ $\rightarrow$ has to be orthogonal to Row A

$$\text{Row } A = \left(\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \right)$$

$$\begin{bmatrix} \alpha - 3 \\ -\alpha + 3\beta - 2 \\ \beta - 9 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} = (\alpha - 3) - (-\alpha + 3\beta - 2)$$

$$\alpha - 3 + \alpha - 3\beta + 2 = 0$$

$$\underline{2\alpha - 3\beta - 1 = 0}$$

$$\begin{bmatrix} \alpha - 3 \\ -\alpha + 3\beta - 2 \\ \beta - 9 \end{bmatrix} \begin{bmatrix} 0 & 3 & 1 \end{bmatrix} = 3(-\alpha + 3\beta - 2) + (\beta - 9)$$

$$-3\alpha + 9\beta - 6 + \beta - 9$$

$$\underline{-3\alpha + 10\beta - 15 = 0}$$

$$\left[\begin{array}{cc|c} 2 & -3 & 1 \\ -3 & 10 & 15 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & -3 & 1 \\ 0 & 11/2 & 33/2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \end{array}\right]$$

$$\underline{u} = 5\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

7a

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0 \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\downarrow \qquad\qquad \downarrow$$

$$x_1 + x_3 = 0 \qquad x_1 + x_2 = 0$$

$$x_1 = -x_3 \qquad x_1 = -x_2$$

$$x_2 = x_3$$

$$\underline{x} = \begin{bmatrix} a \\ -a \\ -a \end{bmatrix} \qquad ||\underline{x}|| = 1$$

$$||\underline{x}||^2 = \underline{x} \cdot \underline{x} = a^2 + a^2 + a^2 = 3a^2$$

$$3a^2 = 1$$

$$a^2 = \frac{1}{3} \qquad a = \pm \frac{1}{\sqrt{3}}$$

$$\underline{x} = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix} \qquad \underline{y} = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

7b

$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 4 \end{bmatrix} = -2 \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ -14 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$4 \quad 2 \quad 3 \qquad 0 \qquad 0 \qquad 0$

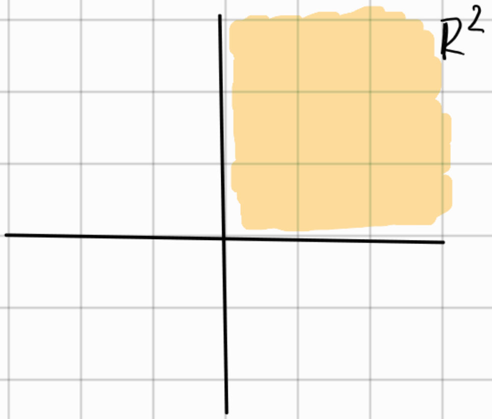
$$\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ 7 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

7c $\quad \underline{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \underline{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{w} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

7d $\quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

7e



$$\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \geq 0 \right\}$$