H2.a $\dot{y} = \cos(t) - \frac{y}{3}$

$w_0 = y(0) = 0$ up to $t = 2$ $h = 1$

euler method:

$$w_{i+1} = w_i + h f(t_i, w_i)$$

$t_0 = 0$ $t_1 = 1$

$$w_1 = w_0 + h f(t_0, w_0)$$
$$= 0 + 1 \cdot f(0,0)$$
$$= 0 + 1 \cdot 1$$

$y(1) = w_1 = 1$

$$w_2 = w_1 + h f(t_1, w_1)$$
$$= 1 + 1 \cdot f(t_1, w_1)$$
$$= 1 + 1 \cdot 0.20697$$

$y(2) = w_2 = 1.20697$

Ralston Method:

$$w_{i+1} = w_i + \frac{1}{4} h \left(f(t_i, w_i) + 3 f\left(t_i + \frac{2}{3}h, w_i + \frac{2}{3}h \cdot f(t_i, w_i)\right)\right)$$

$y(1) = w_1 = 0 + \frac{1}{4} \cdot 1 \left(1 + 3 \cdot 0.5637\right) = 0.6728$

$f(t_0, w_0) = f(0,0) = 1$

$f\left(t_0 + \frac{2}{3}h, w_0 + \frac{2}{3}h \cdot f(t_0, w_0)\right) = f\left(\frac{2}{3}, \frac{2}{3}\right)$

$f\left(\frac{2}{3}, \frac{2}{3}\right) = 0.5637$

$$y(2) = w_2 = 0.6728 + \frac{1}{4} \cdot 1 \left(0.3160 + 3 \cdot (-0.3902)\right) = \underline{0.4592}$$

$$f(t_1, w_1) = f(1, 0.6728) = 0.3160$$
$$f\left(t_1 + \tfrac{2}{3}h,\ w_1 + \tfrac{2}{3}h \cdot f(t_1, w_1)\right) = f\left(\tfrac{5}{3}, 0.8835\right) = -0.3902$$

2stage AB:

$$w_{i+1} = w_i + \frac{h}{2}\left(3f(t_i, w_i) - f(t_{i-1}, w_{i-1})\right)$$

$$w_1 = w_0 + \frac{1}{4}h\left(f(t_0, w_0) + 3f\left(t_0 + \tfrac{2}{3}h,\ w_0 + \tfrac{2}{3}h f(t_0, w_0)\right)\right)$$

$$\underline{w_1 = 0.6728}$$

$$y(2) = w_2 = w_1 + \frac{h}{2}\left(3f(t_1, w_1) - f(t_0, w_0)\right)$$

$$= 0.6728 + \frac{1}{2}\left(3 \cdot 0.3160 - 1\right) = \underline{0.6468}$$

$$f(t_1, w_1) = f(1, 0.6728) = 0.3160$$
$$f(t_0, w_0) = f(0, 0) = 1$$