Tangent lines:

$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$$y = m(x - x_0) + y_0$$

$$\text{slope of normal} = \frac{-1}{\text{slope of tangent}}$$

Derivatives:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

derivative of linear function: $f(x) = ax + b \to f'(x) = a$

derivative of constant function: $f(x) = c \to f'(x) = 0$

General Power Rule: $f(x) = x^r \to f'(x) = r \cdot x^{r-1}$

$f(x)$	$f'(x)$
c (constant)	0
x	1
$1/x$	$-1/x^2 \quad (x \neq 0)$
$\sqrt{x}$	$1/2\sqrt{x} \quad (x \neq 0)$
x^r	$r \cdot x^{r-1}$
$\lvert x \rvert$	$x/\lvert x \rvert = \operatorname{sgn} x$

notation:

$$D_x y = y' = \frac{dy}{dx} = \frac{d}{dx} f(x) = f'(x) = D_x f(x) = D f(x)$$

value of derivative of a function at a particular number x_0 notation:

$$D_x y \Big|_{x=x_0} = y' \Big|_{x=x_0} = \frac{dy}{dx}\Big|_{x=x_0} = \frac{d}{dx} f(x) \Big|_{x=x_0} = f'(x_0) = D_x f(x_0)$$

$$\frac{dx}{dy} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

differentiation rules

$$(f+g)'(x) = f'(x) + g'(x)$$

$$(f-g)'(x) = f'(x) + g'(x)$$

$$(Cf)'(x) = C f'(x) \qquad (C - \text{constant eg. } 5)$$

product rule:

$$(fg)'(x) = f'(x) g(x) + f(x) g'(x)$$

reciprocal rule:

$$\left(\frac{1}{f}\right)'(x) = \frac{-f'(x)}{(f(x))^2} \qquad \frac{d}{dx} x^{-n} = -n x^{-n-1}$$

quotient rule:

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

eg.

$$g = x-1$$

$$\frac{d}{dx}\left(\sqrt{x-1}\right) = \frac{d}{dg}\left(\sqrt{g}\right) \cdot \frac{d}{dx}(x-1)$$

$$= \frac{1}{2\sqrt{g}} \cdot 1 = \boxed{\frac{1}{2\sqrt{x-1}}}$$

Chain Rules:

$$\frac{d}{dx} u^n = n\,u^{n-1}\frac{du}{dx} \qquad \frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{1}{u}\right) = \frac{-1}{u^2} \cdot \frac{du}{dx} \qquad \frac{d}{dx}|u| = \frac{u}{|u|} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\sin(Cx) = \cos(Cx) \cdot C \qquad \frac{d}{dx}\cos(Cx) = -\sin(Cx) \cdot C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\sec x = \sec x \cdot \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x \qquad \frac{d}{dx}\csc x = -\csc x \cdot \cot x$$

$$\frac{d}{dx} a^u = a^u \cdot u' \cdot \ln a$$

$$\left(\frac{d}{dx}\left(4^{3x}\right) = 4^{3x} \cdot 3 \cdot \ln(4)\right)$$

$$\lim_{x \to a^+} \frac{c}{x} = +\infty \qquad \lim_{x \to a^-} \frac{c}{x} = -\infty$$

$$\left(\lim_{x \to a} \frac{c}{0} = \pm\infty\right)$$

Finding asymptotes:

- Horizontal:

  $f(x)$ will have the horizontal asymptote $y = L$ if either

  $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$

- Vertical