1

Horner form: $x(x^2 - 9x + 26) - 24$

$$= x(x(x-9)+26) - 24$$

$$p(3.47) = (3.47-2)(3.47-3)(3.47-4)$$

$$= 1.47 \times 0.47 \times (-0.53) \stackrel{3sf}{=} \underline{-0.366}$$

$$p(3.47) = 3.47^3 - 9 \times (3.47)^2 + 26 \times 3.47 - 24$$

$26 \times 3.47 = 90.22 \stackrel{3sf}{=} 90.2$ $\quad 3.47^2 \stackrel{3sf}{=} 12.0$ $\quad 3.47^3 = 3.47 \times 12.0 \stackrel{3sf}{=} 41.6$

$9 \times (3.47)^2 = 108$

$$= 41.6 - 108 + 90.2 - 24 = \underline{\underline{-0.2}}$$

$$p(3.47) = \underbrace{3.47\underbrace{(3.47\underbrace{(3.47-9)}_{-5.53}+26)}_{6.87}}_{23.6} - 24 = \underline{-0.4}$$

2

$\dot{y} = t - y^2$ $\quad w_0 = y(0.0) = 2$ $\quad$ step $= 0.5$ $\quad$ up to $t = 1.5$

$t_0 = 0$ $\quad t_1 = 0.5$ $\quad t_2 = 1.0$ $\quad t_3 = 1.5$

3rd order to bootstrap:

Heun's 3rd order:

$k_{0,1} = 0.5 \cdot f(t_0, w_0) = 0.5 \cdot f(0, 2) = 0.5 \cdot (-4) = \underline{-2}$

$k_{0,2} = 0.5 \cdot f(1/6, 1.333) = 0.5 \cdot (-1.610) = \underline{-0.805}$

$k_{0,3} = 0.5 \cdot f(1/3, 1.463) = 0.5 \cdot (-1.807) = \underline{-0.904}$

$$w_1 = w_0 + \frac{1}{4}(k_{0,1} + 3k_{0,3})$$

$$= 2 + \frac{1}{4}(-2 + 3 \cdot (-0.904)) = 0.822$$

AB predictor:

$\hat{w}_2 = 0.822 + 0.25\left(3 \cdot f(t_1, w_1) - f(t_0, w_0)\right)$

$$\hat{w}_2 = 0.822 + 0.25\left(3 \cdot f(t_1, w_1) - f(t_0, w_0)\right)$$

$$= 0.822 + 0.25\left(3 \cdot f(0.5, 0.822) - f(0,2)\right)$$

$$= 0.822 + 0.25\left(3 \cdot (-0.176) - (-4)\right)$$

$$\underline{\hat{w}_2 = 1.69}$$

AM corrector:

$$w_2 = 0.822 + \frac{1}{24}\left(5 \cdot f(t_2, w_2) + 8 f(t_1, w_1) - f(t_0, w_0)\right)$$

$$= 0.822 + \frac{1}{24}\left(5 \cdot \underbrace{\cancel{f(1, 1.69)}}_{-1.856} + 8 \cdot (-0.176) - (-4)\right)$$

$$\underline{w_2 = 0.543}$$

AB predictor:

$$\hat{w}_3 = w_2 + 0.25\left(3 f(t_2, w_2) - f(t_1, w_1)\right)$$

$$= 0.543 + 0.25\left(3 \cdot \underbrace{\cancel{f(1, 0.543)}}_{0.705} - (-0.176)\right)$$

$$\underline{\hat{w}_3 = 1.116}$$

AM corrector:

$$w_3 = w_2 + \frac{1}{24}\left(5 f(t_3, w_3) + 8 f(t_2, w_2) - f(t_1, w_1)\right)$$

$$= 0.543 + \frac{1}{24}\left(5 \cdot 0.255 + 8 \cdot 0.705 - (-0.176)\right)$$

$$\underline{\underline{w_3 = 0.838}}$$

3. $g(x) = ax + b$

$$a = \frac{(\overline{XY} - \overline{X} \cdot \overline{Y})}{(\overline{X^2} - \overline{X}^2)}$$

$$b = \overline{Y} - a\overline{X}$$

$$\overline{X} = \frac{1}{5} \cdot \sum_{i=1}^{5} x_i = \frac{1}{5}(0+2+4+7+10) = 4.6$$

$$\overline{Y} = \frac{1}{5} \cdot \sum_{i=1}^{5} y_i = \frac{1}{5}(0.9+0.4+0.1+0.3+1.4) = 0.62$$

$$\overline{XY} = \frac{1}{5} \cdot \sum_{i=1}^{5} x_i y_i = \frac{1}{5}(0+0.8+0.4+2.1+14) = 3.46$$

$$\overline{X^2} = \frac{1}{5} \cdot \sum_{i=1}^{5} x_i^2 = \frac{1}{5}(0+4+16+49+100) = 33.8$$

$$a = \frac{(3.46 - 4.6 \cdot 0.62)}{(33.8 - 4.6^2)} = \underline{0.048}$$

$$b = \overline{Y} - a\overline{X} = 0.62 - 0.048 \cdot 4.6 = \underline{0.399}$$

$$g(x) = y \approx 0.048x + 0.399$$

(6) $p_0 = 1 \qquad p_1 = 2 \qquad f(x) = e^x - x - 5$

interval $[1, 2]$

accuracy of 0.001

secant rule:

$$p_{n+1} = p_n - \frac{p_n - p_{n-1}}{f(p_n) - f(p_{n-1})} f(p_n)$$

$$f(p_0) = f(1) = -3.2817$$
$$f(p_1) = f(2) = 0.3891$$

$$p_2 = 2 - \frac{2 - 1}{0.3891 - (-3.2817)} 0.3891$$

$$\underline{p_2 = 1.8940}$$

$$f(p_2) = f(1.8940) = -0.2481$$

$$p_3 = p_2 - \frac{p_2 - p_1}{f(p_2) - f(p_1)} f(p_2)$$

$$= 1.8940 - \frac{1.8940 - 2}{-0.2481 - 0.3891} \cdot (-0.2481)$$

$$\underline{p_3 = 1.9353}$$

$$f(p_3) = f(1.9353) = -0.000918$$

$$p_4 = p_3 - \frac{p_3 - p_2}{f(p_3) - f(p_2)} f(p_3)$$

$$= 1.9353 - \frac{1.9353 - 1.8940}{-0.000918 - (-0.2481)} \cdot (-0.000918)$$

$$\underline{p_4} = 1.9360 \qquad |p_3 - p_4| < 0.001$$

stop!

(7) Divided differences:

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{0.82 - 1.51}{0.8 - 0.3} = \underline{-1.38}$$

$$f[x_2, x_3] = \frac{0.6 - 0.82}{1.1 - 0.8} = \underline{-0.73333}$$

$$f[x_3, x_4] = \frac{0.3 - 0.6}{2 - 1.1} = \underline{-0.33333}$$

$$f[x_4, x_5] = \frac{0.22 - 0.3}{2.5 - 2} = \underline{-0.16}$$

$$f[x_1, x_2, x_3] = \frac{-0.73333 - (-1.38)}{1.1 - 0.3} = \underline{0.80834}$$

$$f[x_2, x_3, x_4] = \frac{-0.33333 - (-0.73333)}{2 - 0.8} = \underline{0.33333}$$

$$f[x_3, x_4, x_5] = \frac{-0.16 - (-0.33333)}{2.5 - 1.1} = \underline{0.12381}$$

$$f[x_1, x_2, x_3, x_4] = \frac{0.33333 - 0.80834}{2 - 0.3} = \underline{-0.27942}$$

$$f[x_2, x_3, x_4, x_5] = \frac{0.12381 - 0.33333}{2.5 - 0.8} = \underline{-0.12325}$$

$$f[x_1, x_2, x_3, x_4, x_5] = \frac{-0.12325 - (-0.27942)}{2.5 - 0.3} = \underline{0.07099}$$

interpolating polynomial:

$$p(x) = a_1 + (x-x_1)(a_2 + (x-x_2)(a_3 + (x-x_3)(a_4 + (x-x_4)a_5)))$$

$$p(x) = 1.51 + (x-0.3)(-1.38 + (x-0.8)(0.80834 + (x-1.1)(-0.27942 + (x-2)\cdot 0.07099)$$

$$\underline{p(1) = 0.6621}$$