1)

a) $$\left[\begin{array}{ccccc|c} 2 & -3 & -7 & 13 & 1 & -3 \\ 1 & 1 & -1 & 4 & 2 & 4 \\ -1 & 1 & 3 & -6 & -1 & 0 \end{array}\right] \underset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 2 & -3 & -7 & 13 & 1 & -3 \\ -1 & 1 & 3 & -6 & -1 & 0 \end{array}\right] \underset{R_2 : R_2 - 2R_1}{\sim}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ -1 & 1 & 3 & -6 & -1 & 0 \end{array}\right] \underset{R_3 : R_3 + R_1}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ 0 & 2 & 2 & -2 & 1 & 4 \end{array}\right] \underset{R_3 : R_3 + 2/5 R_2}{\sim}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ 0 & 0 & 0 & 0 & -1/5 & -2/5 \end{array}\right] \underset{R_3 : R_3 \cdot (-5)}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \text{ REF } \underset{\substack{R_1 : R_1 - 2R_3 \\ R_2 : R_2 + 3R_3}}{\sim}$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 0 & 0 \\ 0 & -5 & -5 & 5 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \underset{R_2 / (-5)}{\sim} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \underset{R_1 : R_1 - R_2}{\sim}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 5 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad \text{RREF}$$

b) $$\begin{cases} x_1 = -1 + 2x_3 - 5x_4 \\ x_2 = 1 - x_3 + x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \\ x_5 = 2 \end{cases}$$

2) a) If a SLE has a unique solution, then the number of basic variables is larger than the number of free variables:

TRUE

• if a solution is unique, there are no free

If a solution is unique, there are no free variables

Hence, # basic var > # free var

$\downarrow$

0

b) If a SLE of n eq. has a matrix with a pivot in each col, then the SLE has a unique sol.

TRUE

c) A consistent system of lin. eq. with fewer eq. than unknowns can never have a unique sol.

TRUE # basic var $\leq$ # equations

in this case also # basic var $<$ # var

Hence there must be at least one free var.

d) Suppose 3×5 coefficient matrix for a system has 3 pivot cols. Then the system is consistent

TRUE

If the (3×5) matrix has 3 pivot cols then the RREF of the augmented matrix cannot contain a row of the form $[0 \dots 0 \; b]$ with b nonzero