H8.a (ii)

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$$A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & 7 & 3 \\ 3 & 1 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Gauss - Seidel method:

$$x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_i^{(m+1)} = \left(b_i - \sum_{j<i} a_{ij} x_j^{(m+1)} - \sum_{j>i} a_{ij} x_j^{(m)} \right) / a_{ii}$$

$$x_1^{(1)} = \left(b_1 - a_{12} x_2^{(0)} - a_{13} x_3^{(0)} \right) / a_{11} = (2 - (-1)\cdot 0 - 1 \cdot 0) / 2 = 1$$

$$x_2^{(1)} = \left(b_2 - a_{21} x_1^{(1)} - a_{23} x_3^{(0)} \right) / a_{22} = (-1 - 4 \cdot 1 - 3 \cdot 0) / 7 = -0.7143$$

$$x_3^{(1)} = \left(b_3 - a_{31} x_1^{(1)} - a_{32} x_2^{(1)} \right) / a_{33} = (3 - 3 \cdot 1 - 1 \cdot (-0.7143)) / 6 = 0.1190$$

$$x^{(1)} = \begin{bmatrix} 1 \\ -0.7143 \\ 0.1190 \end{bmatrix}$$

$$x_1^{(2)} = \left(b_1 - a_{12} x_2^{(1)} - a_{13} x_3^{(1)} \right) / a_{11} = (2 - (-1) \cdot (-0.7143) - 1 \cdot 0.1190) / 2$$
$$= 0.5833$$

$$x_2^{(2)} = \left(b_2 - a_{21} x_1^{(2)} - a_{23} x_3^{(1)} \right) / a_{22} = (-1 - 4 \cdot 0.5833 - 3 \cdot 0.1190) / 7$$
$$= -0.5272$$

$$x_3^{(2)} = \left(b_3 - a_{31} x_1^{(2)} - a_{32} x_2^{(2)} \right) / a_{33} = (3 - 3 \cdot 0.5833 - 1 \cdot (-0.5272)) / 6$$
$$= 0.2962$$

$$x^{(2)} = \begin{bmatrix} 0.5833 \\ -0.5272 \\ 0.2962 \end{bmatrix}$$

$$\| x^{(2)} - x^{(1)} \| = \left\| \begin{bmatrix} 0.5833 \\ \end{bmatrix} \begin{bmatrix} 1 \\ \end{bmatrix} \right\| = \begin{bmatrix} 0.4167 \end{bmatrix}$$

$$\left\| \begin{bmatrix} -0.5272 \\ 0.2962 \end{bmatrix} - \begin{bmatrix} -0.7143 \\ 0.1190 \end{bmatrix} \right\| \quad \begin{bmatrix} 0.1871 \\ 0.1772 \end{bmatrix}$$

$$= \sqrt{0.4167^2 + 0.1871^2 + 0.1772^2} = \underline{0.49}$$

$$Ax = \begin{bmatrix} 2 & -1 & 1 \\ 4 & 7 & 3 \\ 3 & 1 & 6 \end{bmatrix} \begin{bmatrix} 0.5833 \\ -0.5272 \\ 0.2962 \end{bmatrix} = \begin{bmatrix} 1.9900 \\ -0.4686 \\ 2.9999 \end{bmatrix}$$

$$\|Ax - b\| = \left\| \begin{bmatrix} 1.9900 \\ -0.4686 \\ 2.9999 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\| = \underline{0.5315}$$