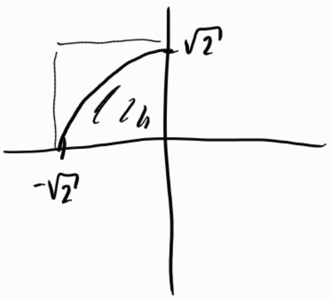
(3) $$\int_{-2}^{2} (x+2)\,dx = \underbrace{\int_{-2}^{2} x\,dx}_{\downarrow\ \frac{x^2}{2}} + \underbrace{\int_{-2}^{2} 2\,dx}_{\downarrow\ 2x}$$

$$\frac{x^2}{2}\Big|_{-2}^{2} + 2x\Big|_{-2}^{2} = \left(\frac{2^2}{2} - \frac{(-2)^2}{2}\right) + \left(2(2) - 2(-2)\right) = \underline{\underline{8}}$$

(8) $$\int_{-\sqrt{2}}^{0} \sqrt{2-x^2}\,dx = \text{quarter disk}$$



$$= \underline{\underline{\frac{\pi}{2}}}$$

(18) $$\int_{0}^{a} x^2\,dx = \frac{a^3}{3}$$

$$\int_{2}^{3} (x^2-4)\,dx = \int_{2}^{3} x^2\,dx - \int_{2}^{3} 4\,dx = \frac{x^3}{3}\Big|_{2}^{3} - 4x\Big|_{2}^{3} =$$

$$= \frac{3^3}{3} - \frac{2^3}{3} - \left(4(3) - 4(2)\right) = \frac{19}{3} - \frac{12}{3} = \underline{\underline{\frac{7}{3}}}$$

(26) $$\int_{1/4}^{3} \frac{1}{s}\,ds = \int_{1}^{3} \frac{1}{s}\,ds - \int_{1}^{1/4} \frac{1}{s}\,ds = \ln(3) - \ln\left(\frac{1}{4}\right) = \ln(3) + \ln(4) =$$

$$= \underline{\underline{\ln(12)}}$$

28 average values

$g(x) = x+2$ over $[a,b]$

$$\frac{1}{b-a}\cdot\int_a^b (x+2)\,dx = \frac{1}{b-a}\cdot\left(\int_a^b x\,dx + \int_a^b 2\,dx\right) = \frac{1}{b-a}\left(\frac{x^2}{2}\Big|_a^b + 2x\Big|_a^b\right)$$

$$= \frac{1}{b-a}\left(\frac{b^2}{2}-\frac{a^2}{2}+2b-2a\right) = \frac{1}{b-a}\left(\frac{b^2-a^2+4b-4a}{2}\right) =$$

$$= \frac{b^2-a^2+4b-4a}{2(b-a)} = \frac{b^2-a^2}{2(b-a)} + \frac{2(b-a)}{b-a} = \underline{\underline{\frac{b+a}{2}+2}}$$

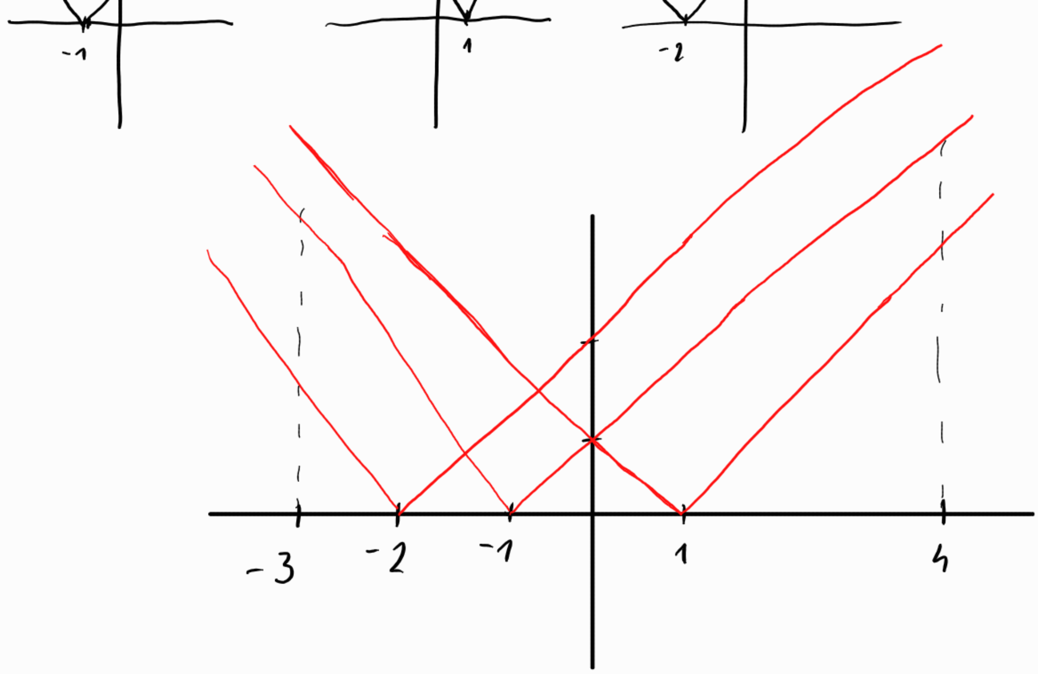
34 $\int_{-3}^{2} f(x)\,dx$ $\qquad f(x)\begin{cases} 1+x & x<0 \\ 2 & x\geq 0 \end{cases}$

$$\int_{-3}^{0}(1+x) + \int_0^2 2 = \left(x+\frac{x^2}{2}\right)\Big|_{-3}^{0} + 2x\Big|_0^2 =$$

$$= 0+\frac{0}{2} - \left(-3+\frac{(-3)^2}{2}\right) + (2\cdot 2 - 2\cdot 0) = -\frac{3}{2}+4 = \underline{\underline{\frac{5}{2}}}$$

39 $\int_{-3}^{4} \left(|x+1| - |x-1| + |x+2|\right) dx$

$$= \int_{-3}^{4} |x+1|\,dx - \int_{-3}^{4} |x-1|\,dx + \int_{-3}^{4} |x+2|\,dx$$



$$\left(2-\frac{25}{2}\right)-\left(8+\frac{9}{2}\right)+\left(\frac{1}{2}+18\right)=\frac{41}{2}$$

(8) $$\int_4^9 \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right) dx = \int_4^9 x^{1/2} dx - \int_4^9 x^{-1/2} dx =$$

$$= \frac{2x\sqrt{x}}{3}\Big|_4^9 - 2\sqrt{x}\Big|_4^9 \qquad x^{3/2}=\sqrt{x^3}=x\sqrt{x}$$

$$=\left(18-\frac{16}{3}\right)-\left(6-4\right)=\frac{32}{3}$$

(9) $$\int_{-\pi/4}^{-\pi/6} \cos x\, dx = \sin x\Big|_{-\pi/4}^{-\pi/6} = -\frac{1}{2}+\frac{1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2\sqrt{2}}$$

(6) $$\int_{-1}^{2} (1-2x)\, dx = \int_{-1}^{2} 1\, dx - \int_{-1}^{2} 2x\, dx = x - x^2\Big|_{-1}^{2} =$$

$$= (2-4) - (-1-1) = -2+2 = \underline{\underline{0}}$$

(13) $$\int_{-\pi}^{\pi} e^x\,dx = e^x\Big|_{-\pi}^{\pi} = \underline{\underline{e^{\pi} - e^{-\pi}}}$$

(14) $$\int_{-2}^{2} (e^x - e^{-x})\,dx = \int_{-2}^{2} e^x\,dx - \int_{-2}^{2} e^{-x}\,dx = e^x\Big|_{-2}^{2} - \left(-\frac{1}{e^x}\right)\Big|_{-2}^{2} =$$

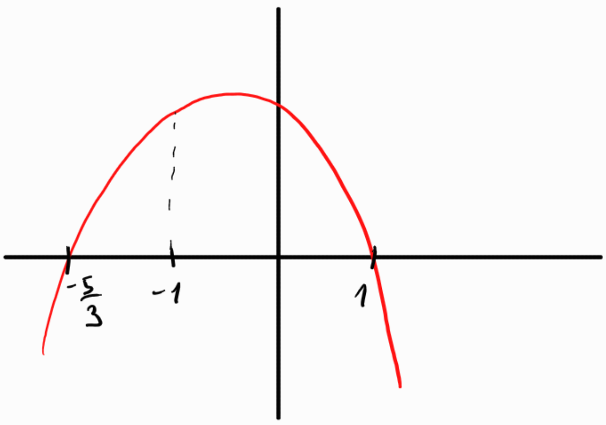
$$= \left(e^x + \frac{1}{e^x}\right)\Big|_{-2}^{2} = \left(e^2 + \frac{1}{e^2}\right) - \left(e^{-2} + \frac{1}{e^{-2}}\right) = e^2 + \frac{1}{e^2} - \frac{1}{e^2} - e^2 = \underline{\underline{0}}$$

(16) $$\int_{-1}^{1} 2^x\,dx = \frac{2^x}{\ln(2)}\Bigg|_{-1}^{1} = \frac{2}{\ln 2} - \frac{1}{2\ln(2)} = \underline{\underline{\frac{3}{2\ln(2)}}}$$

(24) $$y = 5 - 2x - 3x^2 = (5+3x)(1-x)$$

$$\downarrow \qquad \downarrow$$

$$-\frac{5}{3} \qquad 1$$

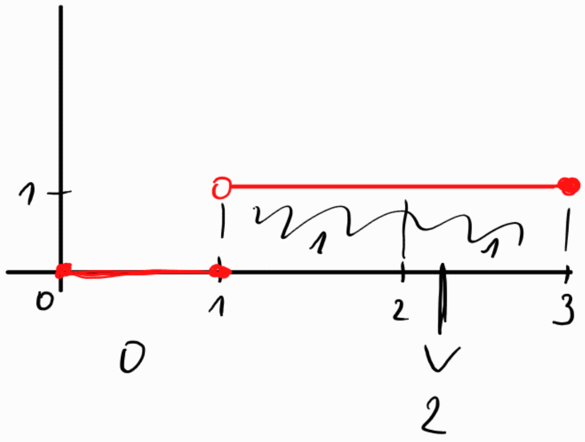


$$\int_{-1}^{1} (5-2x-3x^2)\,dx = \int_{-1}^{1} 5\,dx - \int_{-1}^{1} 2x\,dx - \int_{-1}^{1} 3x^2\,dx =$$

$$= 5x\Big|_{-1}^{1} - x^2\Big|_{-1}^{1} - x^3\Big|_{-1}^{1} = (5x - x^2 - x^3)\Big|_{-1}^{1} =$$

$$= (5-1-1) - (-5-1+1) = 3 + 5 = \underline{\underline{8}}$$

(38)



average value $\quad \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 1 = \underline{\underline{\frac{2}{3}}}$

(12) $$\int_0^{2\pi} (1+\sin x)\,dx = \int_0^{2\pi} 1\,dx + \int_0^{2\pi} \sin x\,dx = (x - \cos x)\Big|_0^{2\pi} =$$

$$= (2\pi - \cos 2\pi) - (0 - \cos 0) = \underline{\underline{2\pi}}$$

(13) $$\int_{-2}^{2} (e^x - e^{-x})\,dx = \int_{-2}^{2} e^x\,dx - \int_{-2}^{2} e^{-x}\,dx =$$

$$= (e^x + e^{-x})\Big|_{-2}^{2} = e^2 + e^{-2} - e^2 - e^{-2} = \underline{\underline{0}}$$

(1) $\int e^{5-2x}\,dx = -\frac{1}{2}\cdot e^{5-2x}$

(6) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\sin u}{\cancel{u}}\cdot 2\cancel{u}\,du = \int \sin u \cdot 2\,du =$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$= \int 2\sin u = 2\cdot(-\cos u) =$

$= -2\cos\sqrt{x} + C$

(12) $\int \frac{\ln t}{t}\,dt = \int \frac{u}{\cancel{t}}\,\cancel{t}\cdot du = \int u\,du = \frac{u^2}{2} + C =$

$u = \ln t$

$du = \frac{1}{t}\cdot dt$

$= \frac{\ln^2 t}{2} + C$

(17) $\int \frac{dx}{e^x+1} = \int \frac{1}{u}\cdot\frac{1}{u-1}\,du = \int \frac{du}{u(u-1)} = \int \frac{du}{u-1} - \int \frac{du}{u} =$

$u = e^x + 1$

$du = e^x\,dx$

$= \ln(|u-1|) - \ln(|u|) = \ln(e^x) - \ln(e^x+1)$

$= x - \ln(e^x+1) + C$

(25) $\int \sin ax\,\cos^2 ax\,dx = \int \sin ax\,(1-\sin^2 ax)\,dx =$

$= \int \sin ax - \sin^3 ax\,dx = \int \sin u - \sin^3 u\,\frac{du}{a} =$

$u = ax$

$du = a\cdot dx$

$= \frac{1}{a}\left(\int \sin u\,du - \int \sin^3 u\,du\right) =$

$$= \frac{1}{a}\left(\cancel{-\cos^3 u} \cancel{+\cos^3 u} - \frac{\cos(u)^3}{3}\right) =$$

$$= -\frac{\cos(ax)^3}{3a} + C$$

(36) $$\int \frac{\sin^3 x}{\cos^4 x}\,dx = \int \frac{\sin^{\cancel{3}\,2} x}{\cos^4 x} \cdot \frac{1}{-\cancel{\sin} x}\,du = \int -\frac{\sin^2 x}{u^4}\,du$$

$u = \cos x$

$du = -\sin x\,dx$

$$= \int -\frac{1-\cos^2 x}{u^4}\,du = -\int \frac{1-u^2}{u^4}\,du$$

$$= -\int \frac{1}{u^4} - \frac{\cancel{u^2}}{u^{\cancel{4}\,2}}\,du = -\int \frac{1}{u^4} - \frac{1}{u^2}\,du$$

$$= -\left(\int \frac{1}{u^4}\,du - \int \frac{1}{u^2}\,du\right) = -\left(-\frac{1}{3u^3} + \frac{1}{u}\right) =$$

$$= \frac{1}{3\cos^3 x} - \frac{1}{\cos x} + C$$

(15) $$\int_0^e a^x\,dx \quad (a>0) = \frac{a^x}{\ln(a)}\Bigg|_0^e = \frac{a^e}{\ln(a)} - \frac{a^0}{\ln(a)} = \frac{a^e - 1}{\ln(a)}$$

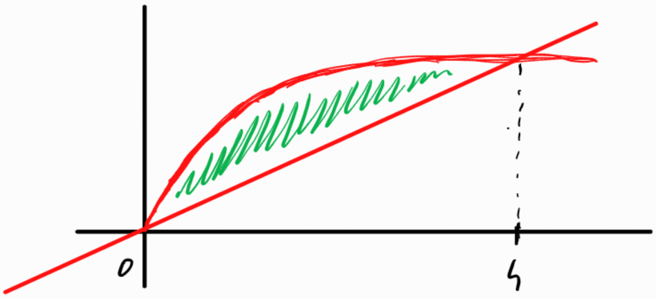
(16) $$\int_{-1}^{1} 2^x\,dx = \frac{2^x}{\ln(2)}\Bigg|_{-1}^{1} = \frac{2}{\ln(2)} - \frac{1}{2\ln(2)} = \frac{3}{2\ln(2)}$$

(18) $$\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx = \int_0^{1/2} \frac{1}{\sqrt{1^2-x^2}} = \arcsin(x)\Bigg|_0^{1/2} =$$

$= \arcsin\left(\frac{1}{2}\right) - \arcsin(0) = \frac{\pi}{6} - 0 = \underline{\underline{\frac{\pi}{6}}}$

(26) area of below $y=\sqrt{x}$ and above $y=\frac{x}{2}$

$\sqrt{x} = \frac{x}{2}$ $\qquad x_1 = 0$, $\quad x_2 = 4$



$$\int_0^4 \sqrt{x}\,dx - \int_0^4 \frac{x}{2}\,dx =$$

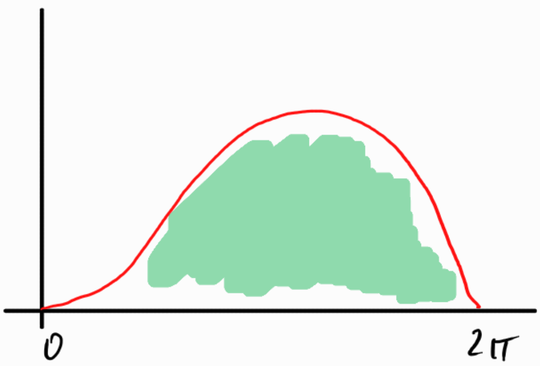
$$= \frac{2x\sqrt{x}}{3}\Bigg|_0^4 - \frac{x^2}{4}\Bigg|_0^4 = \frac{16}{3} - \frac{16}{4} = \underline{\underline{\frac{4}{3}}}$$

(30) area under $y=e^{-x}$ and above $y=0$ from $x=-a$ to $x=0$

$$\int_{-a}^{0} e^{-x}\,dx = -e^{-x}\Big|_{-a}^{0} = -\frac{1}{e^x}\Bigg|_{-a}^{0} =$$

$$= -\frac{1}{e^0} - \left(-e^a\right) = \underline{\underline{-1+e^a}}$$

(31) area below $y=1-\cos x$ and above $y=0$ between two consecutive intersections of these graphs

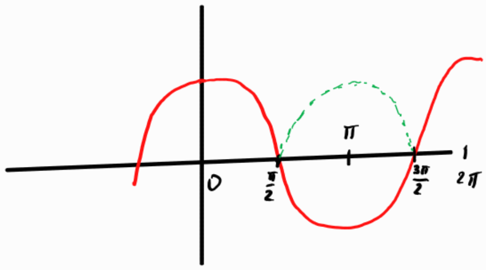


$$\int_0^{2\pi} (1-\cos x)\,dx = (x-\sin x)\Big|_0^{2\pi} =$$

$$= (2\pi - 0) - (0-0) = \underline{\underline{2\pi}}$$

(33)

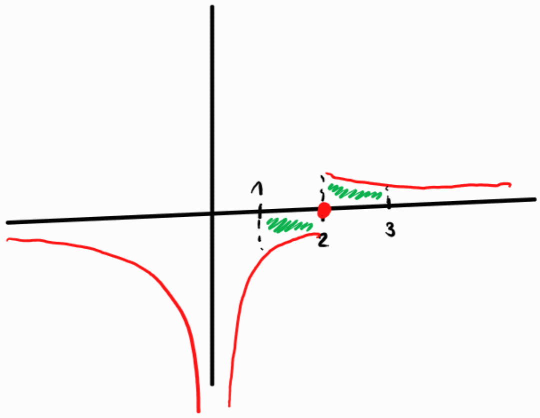
$$\int_0^{3\pi/2} |\cos x|\,dx = \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{3\pi/2} \cos x\,dx =$$



$$= \sin x\Big|_0^{\pi/2} - \sin x\Big|_{\pi/2}^{3\pi/2} = (1-0) - (-1-1) = 1+1+1 = \underline{\underline{3}}$$

(34)

$$\int_1^3 \frac{\operatorname{sgn}(x-2)}{x^2}\,dx =$$



$$= -\int_1^2 \frac{dx}{x^2} + \int_2^3 \frac{dx}{x^2} =$$

$$= -\left(-\frac{1}{x}\right)\Bigg|_1^2 + \left(-\frac{1}{x}\right)\Bigg|_2^3 =$$

$$= \left(\frac{1}{2} - 1\right) + \left(-\frac{1}{3} + \frac{1}{2}\right) = \cancel{\frac{1}{2}} \cancel{-1} - \frac{1}{3} + \cancel{\frac{1}{2}} = \underline{\underline{-\frac{1}{3}}}$$

(37) find average values

$f(x) = 2^x$ over $\left[0, \frac{1}{\ln(2)}\right]$

$$\text{avg} = \frac{1}{1/\ln 2} \int_0^{1/\ln 2} 2^x\,dx = \ln(2)\,\frac{2^x}{\ln(2)}\Bigg|_0^{1/\ln 2} = 2^x\Big|_0^{1/\ln(2)} = \underline{\underline{e-1}}$$

(35) average values:

$f(x) = 1 + x + x^2 + x^3$ over $[0,2]$

$$\boxed{\text{avg} = \frac{1}{b-a}\int_a^b x\,dx}$$

$$= \frac{1}{2}\cdot\int_0^2 (1+x+x^2+x^3) = \frac{1}{2}\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}\right)\Bigg|^2 =$$

$$avg = \frac{1}{2} \int_0^{2} (1+x+x^2+x^3)dx = \frac{1}{2}\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}\right)\Big|_0^{2} =$$

$$= \frac{1}{2}\left(2+2+\frac{8}{3}+4\right) = \underline{\underline{\frac{16}{3}}}$$

(36) average values:

$f(x) = e^{3x}$ over $[-2;2]$

$$\boxed{\int e^{ax}dx = \frac{1}{a}\cdot e^{ax}}$$

$$avg = \frac{1}{2-(-2)} \int_{-2}^{2} e^{3x}\,dx = \frac{1}{4}\cdot\left(\frac{1}{3}e^{3x}\right)\Big|_{-2}^{2} =$$

$$= \underline{\underline{\frac{1}{12}\left(e^{6} - e^{-6}\right)}}$$

(38) average values

$$g(t)\begin{cases} 0 & \text{if } 0\le t\le 1 \\ 1 & \text{if } 1<t<3 \end{cases} \quad \text{over } [0;3]$$

$$avg = \frac{1}{3}\left[\int_0^1 0\,dx + \int_1^3 1\,dx\right] = \frac{1}{3}\left[0 + x\Big|_1^3\right] =$$

$$\frac{1}{3}\cdot x\Big|_1^3 = \frac{1}{3}(3-1) = \underline{\underline{\frac{2}{3}}}$$

(39) $$\frac{d}{dx}\int_2^x \frac{\sin t}{t}\,dt = \underline{\underline{\frac{\sin x}{x}}}$$

(40) $$\frac{d}{dt}\int_t^3 \frac{\sin x}{x}\,dx = \frac{d}{dt}\left(-\int^t \frac{\sin x}{x}\,dx\right) = -\frac{\sin t}{t}$$

(41) $$\frac{d}{dx}\int_{x^2}^{0}\frac{\sin t}{t}\,dt = \frac{d}{dx}\left(-\int_{0}^{x^2}\frac{\sin t}{t}\,dt\right) = -2\cancel{x}\,\frac{\sin x^2}{x^{\cancel{2}}} = \underline{\underline{-2\,\frac{\sin x^2}{x}}}$$

(42) $$\frac{d}{dx}\,x^2\int_{0}^{x^2}\frac{\sin u}{u}\,du = 2x\int_{0}^{x^2}\frac{\sin u}{u}\,du + x^2\,\frac{d}{dx}\int_{0}^{x^2}\frac{\sin u}{u}\,du =$$

$$= 2x\int_{0}^{x^2}\frac{\sin u}{u}\,du + x^2\left[\frac{2x\sin(x^2)}{x^2}\right]$$