(1) $f(x) = x^3 - x - 1 \qquad [1,2] \qquad p_0 = 2 \qquad p_1 = 1$

compute $f(p_3)$

$$p_2 = p_1 - \frac{p_1 - p_0}{f(p_1) - f(p_0)} \cdot f(p_1) = 1 - \frac{1-2}{-1-5} \cdot (-1) = \underline{1.1667}$$

$$p_3 = p_2 - \frac{p_2 - p_1}{f(p_2) - f(p_1)} \cdot f(p_2) = 1.1667 - \frac{0.1667}{0.2414} \cdot (-0.5786) = \underline{1.5663}$$

$f(p_2) = -0.5786$

$f(p_3) = \underline{1.2763}$

(2) $\dot{y} = y - t/y \qquad w_0 = y(1) = 1.400 \qquad$ up to $t = 2 \qquad h = \frac{1}{3}$

$t_0 = 1$

$$k_{0,1} = \frac{1}{3} \cdot f(t_0, w_0) = \frac{1}{3} \cdot f(1, 1.4) = \frac{1}{3} \cdot 0.6857 \approx \underline{0.2286}$$

$$k_{0,2} = \frac{1}{3} \cdot f\left(t_0 + \frac{2}{3} \cdot \frac{1}{3}, w_0 + \frac{2}{3} 0.2286\right) = \frac{1}{3} \cdot f(1.2222, 1.5524) \approx \underline{0.2550}$$

0.7651

$$w_1 = w_0 + \frac{1}{4}(k_{0,1} + 3k_{0,2}) = 1.4 + \frac{1}{4}(0.2286 + 3 \cdot 0.2550) \approx \underline{\underline{1.6484}}$$

0.8396

$$k_{1,1} = \frac{1}{3} \cdot f(t_1, w_1) = \frac{1}{3} f(1.3333, 1.6484) = \underline{0.2799}$$

0.9873

$$k_{1,2} = \frac{1}{3} f\left(t_1 + \frac{2}{3} \cdot \frac{1}{3}, w_1 + \frac{2}{3} k_{1,1}\right) = \frac{1}{3} \cdot f(1.5555, 1.835) = \underline{0.3291}$$

$$w_2 = w_1 + \frac{1}{4}(k_{1,1} + 3k_{1,2}) = 1.6484 + \frac{1}{4}(0.2799 + 3 \cdot 0.3291) = \underline{1.9652}$$

(3) $a_0 = f(x_0) = 0.761$

$a_1 = f[x_0, x_1] = -0.5680$

$a_2 = f[x_0, x_1, x_2] = -0.8500$

$a_3 = f[x_0, x_1, x_2, x_3] = 0.5346$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{0.477 - 0.761}{1.5 - 1} = -0.5680$$

$$f[x_1, x_2] = \frac{0.620 - 0.477}{\ldots} = -0.1430$$

$f[x_1, x_2]$ $\quad 0.5 - 1.5$

$$f[x_2, x_3] = \frac{0.769 - 0.620}{2 - 0.5} = -0.3007$$

$$f[x_0, x_1, x_2] = \frac{-0.1430 - (-0.5680)}{0.5 - 1} = -0.8500$$

$$f[x_1, x_2, x_3] = \frac{-0.3007 - (-0.1430)}{2 - 1.5} = -0.3154$$

$$f[x_0, x_1, x_2, x_3] = \frac{-0.3154 - (-0.8500)}{2 - 1} = 0.5346$$

$$P(x) = 0.761 + (x-1)\left(-0.5680 + (x-1.5)\left(-0.8500 + (x-0.5)\cdot 0.5346\right)\right)$$

$$p(0.8) = 0.778$$

$$|f(x) - p(x)| \le \frac{f'''}{4!} \prod_{i=0}^{3} (x - x_i)$$

$$\le \frac{4 \cdot (0.8-0.5)(0.8-1)(0.8-1.5)(0.8-2)}{4!}$$

(4) $n = 6$ $\qquad \int_{0.2}^{1.4} \frac{1}{1+x^3}\,dx$

Trapezoid rule: $\quad T_n(f; a, b) = h\left(\frac{1}{2} f(x_0) + \sum_{i=1}^{n-1} f(x_i) + \frac{1}{2} f(x_n)\right)$

$$h = \frac{b-a}{n} = \frac{1.4 - 0.2}{6} = 0.2$$

$$T_6(f; 0.2, 1.4) = 0.2\left(\frac{1}{2} f(0.2) + f(0.4) + f(0.6) + f(0.8) + f(1.0) + f(1.2) + \frac{1}{2} f(1.4)\right)$$

$$= 0.2\left(\frac{1}{2} 0.9921 + 0.9398 + 0.8224 + 0.6614 + 0.5 + 0.3666 + \frac{1}{2} 0.2671\right)$$

$$= 0.7840$$

error estimate: $\quad \frac{b-a}{12} \left| f(a) - 2f\left(\frac{a+b}{2}\right) + f(b) \right|$

$[0.2; 0.6]$

$$E_1 = \frac{0.6 - 0.2}{12} \left| f(0.2) - 2f(0.4) + f(0.6) \right|$$

$$= \frac{0.4}{12} \left| 0.9921 - 2 \cdot 0.9398 + 0.8224 \right|$$

$$= 0.0022$$

$[0.6; 1.0]$

$$E_2 = \frac{1-0.6}{12}\left| f(0.6) + 2f(0.8) + f(1.0) \right|$$
$$= \frac{0.4}{12}\left| 0.8224 - 2 \cdot 0.6614 + 0.5 \right|$$
$$= \underline{0.000013}$$

$[1.0, 1.4]$

$$E_3 = \frac{0.4}{12}\left| f(1.0) - 2f(1.2) + f(1.4) \right|$$
$$= \frac{0.4}{12}\left| 0.5 - 2 \cdot 0.3666 + 0.2671 \right|$$
$$= \underline{0.0011}$$

total estimate error $\underline{E_T = 0.0033}$ subdivide $[0.2, 0.6]$

(5) $n = 6$ $T = 3$

$$a_0 = \frac{2}{6}\sum_{i=0}^{5} y_i \cos(0) = \frac{2}{6}(0.017 + 0.620 + 0.761 + 0.477 + 0.169 + 0.038)$$

$\underline{a_0 = 0.694}$

$$a_1 = \frac{2}{6}\sum_{i=0}^{5} y_j \cos(2\pi j/6) = \frac{2}{6}\Big(0.017 + 0.620 \cdot \overset{0.5}{\cancel{\cos(2\pi/6)}} + 0.761\, \overset{-0.5}{\cancel{\cos(4\pi/6)}} + 0.477\, \overset{-1}{\cancel{\cos(\pi)}} + 0.169\, \cos \underset{-0.5}{\cancel{(8\pi/6)}} + 0.038\, \underset{0.5}{(\cancel{10\pi/6})}\Big)$$

$\underline{a_1 = -0.1987}$

$$b_1 = \frac{2}{6}\sum_{i=0}^{5} y_j \sin(2\pi j/n) = \frac{2}{6}\Big(\overset{0}{\cancel{0.017}} + 0.620 \sin \overset{0.866}{\cancel{(2\pi/6)}} + 0.761 \overset{0.866}{\cancel{\sin(4\pi/6)}} + \overset{0}{\cancel{0.477 \sin(\pi)}} + 0.169 \sin \underset{-0.866}{\cancel{(8\pi/6)}} + 0.038 \sin \underset{-0.866}{\cancel{(10\pi/6)}}\Big)$$

$b_1 = 0.3389$

$$s_1(t) = \frac{0.694}{2} - 0.1987\cos(2\pi t/3) + 0.3389 \sin(2\pi t/3)$$

$$s_1(0.8) = \frac{0.694}{2} - 0.1987\cos(2\pi \cdot 0.8/3) + 0.3389 \sin(2\pi \cdot 0.8/3)$$

$$\underline{\underline{s_1(0.8) = 0.7048}}$$

(6) $A = \begin{bmatrix} 7 & 3 \\ 3 & 2 \end{bmatrix}$ $\mu^{(0)} = 2$ $x^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\mu I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad A - \mu I = \begin{bmatrix} 5 & 3 \\ 3 & 0 \end{bmatrix} \qquad (A-\mu I)^{-1} = \begin{bmatrix} 0 & -3 \\ -3 & 5 \end{bmatrix}$$

$$y^{(0)} = (A-\mu I)^{-1} x^{(0)} = \begin{bmatrix} 0 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

$$x^{(1)} = \frac{y^{(0)}}{\| y^{(0)} \|} = \frac{1}{\sqrt{34}} \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -0.5145 \\ 0.8575 \end{bmatrix}$$

$$y^{(1)} = \begin{bmatrix} 0 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} -0.5145 \\ 0.8575 \end{bmatrix} = \begin{bmatrix} -2.5725 \\ 5.8310 \end{bmatrix}$$

$$x^{(2)} = \frac{y^{(1)}}{\| y^{(1)} \|} = \frac{1}{\sqrt{40.6183}} \begin{bmatrix} -2.5725 \\ 5.8310 \end{bmatrix} = \begin{bmatrix} -0.4036 \\ 0.9149 \end{bmatrix}$$

$$\lambda = (A x^{(2)}) = \begin{bmatrix} 7 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -0.4036 \\ 0.9149 \end{bmatrix} = \begin{bmatrix} -0.0805 \\ 0.6190 \end{bmatrix}$$

$$\underline{\underline{\lambda = 0.6190}}$$