Bonus 7

Detvan Pavel
i6302111

b) $f(x) = e^x$ on $[-1,1]$

$$c_k = (k+1/2)\int_{-1}^{1} f(x) P_k(x)\,dx$$

$$c_0 = \frac{1}{2}\int_{-1}^{1} e^x \cdot 1\,dx \approx 1.1752$$

$$c_1 = \frac{3}{2}\int_{-1}^{1} e^x \cdot x\,dx \approx 1.1036$$

$$c_2 = \frac{5}{2}\int_{-1}^{1} e^x \cdot \left(\frac{3}{2}x^2 - \frac{1}{2}\right)dx \approx 0.3578$$

$$c_3 = \frac{7}{2}\cdot\int_{-1}^{1} e^x \cdot \left(\frac{5}{2}x^3 - \frac{3}{2}x\right)dx \approx 0.0705$$

$P_0(x) = 1$

$P_1(x) = x$

$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$

$P_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$

$$\frac{1}{2}\int_{-1}^{1} e^x \cdot 1\,dx = \frac{1}{2}(e^1 - e^{-1}) \approx \underline{1.1752} = c_0$$

$$\frac{3}{2}\int_{-1}^{1} e^x\, x\,dx = \begin{matrix} u = x & du = dx \\ dv = e^x dx & v = e^x \end{matrix} = xe^x - \int_{-1}^{1} e^x dx = \frac{3}{2}(xe^x - e^x)\Big|_{-1}^{1} =$$

$$= \frac{3}{2}\left[\cancel{(e^1 - e^1)}_{0} - (-e^{-1} - e^{-1})\right] = \frac{3}{2}(e^{-1} + e^{-1}) = \underline{1.1036} = c_1$$

$$\frac{5}{2}\int_{-1}^{1} e^x \cdot \left(\frac{3}{2}x^2 - \frac{1}{2}\right)dx = \frac{5}{2}\int_{-1}^{1} \frac{3x^2e^x}{2} - \frac{e^x}{2}\,dx = \frac{15}{4}\int_{-1}^{1} x^2e^x - \frac{5}{4}\int_{-1}^{1} e^x dx =$$

$$= 3.2958 - 2.9380 = \underline{0.3578} = c_2$$

$$\int_{-1}^{1} x^2e^x dx = \begin{matrix} u = x^2 & du = 2x\,dx \\ dv = e^x dx & v = e^x \end{matrix} = x^2e^x - 2\cdot\int xe^x dx = x^2e^x - 2\cdot\left(xe^x - \int e^x dx\right) =$$

$$= (x^2e^x - 2xe^x + 2e^x)\Big|_{-1}^{1} = e - \frac{5}{e} \approx 0.8789$$

polynomial of degree 1:

$$g(x) = 1.1752 + 1.1036 \cdot P_1(x)$$

polynomial of degree 2:

$$g(x) = 1.1752 + 1.1036 \cdot P_1(x) + 0.3578 \cdot P_2(x)$$

polynomial of degree 3:

$$g(x) = 1.1752 + 1.1036 \cdot P_1(x) + 0.3578 \cdot P_2(x) + 0.0705 \cdot P_3(x)$$

errors:

$$E_n = \left| \int_{-1}^{1} e^x dx - \sum_{k=0}^{n} c_k \right|$$

$$E_1 = \left| e - \frac{1}{e} - (1.1752 + 1.1036) \right| = \underline{\underline{0.0716}}$$

$$E_2 = \left| e - \frac{1}{e} - (1.1752 + 1.1036 + 0.3578) \right| = \underline{\underline{0.2862}}$$

$$E_3 = \left| e - \frac{1}{e} - (1.1752 + 1.1036 + 0.3578 + 0.0705) \right| = \underline{\underline{0.3567}}$$

with higher degree polynomial we get bigger error

(data may be overfitting ?)