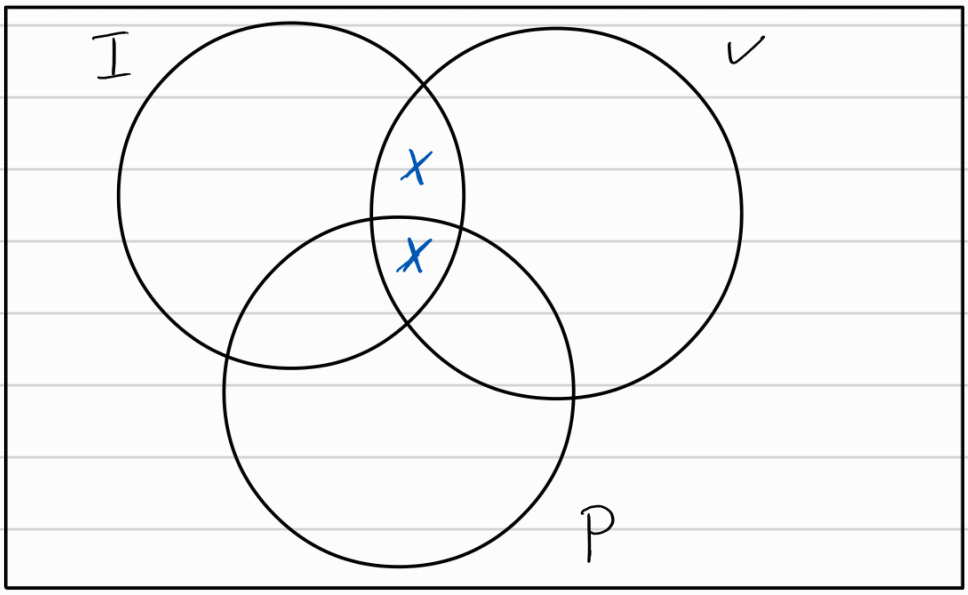
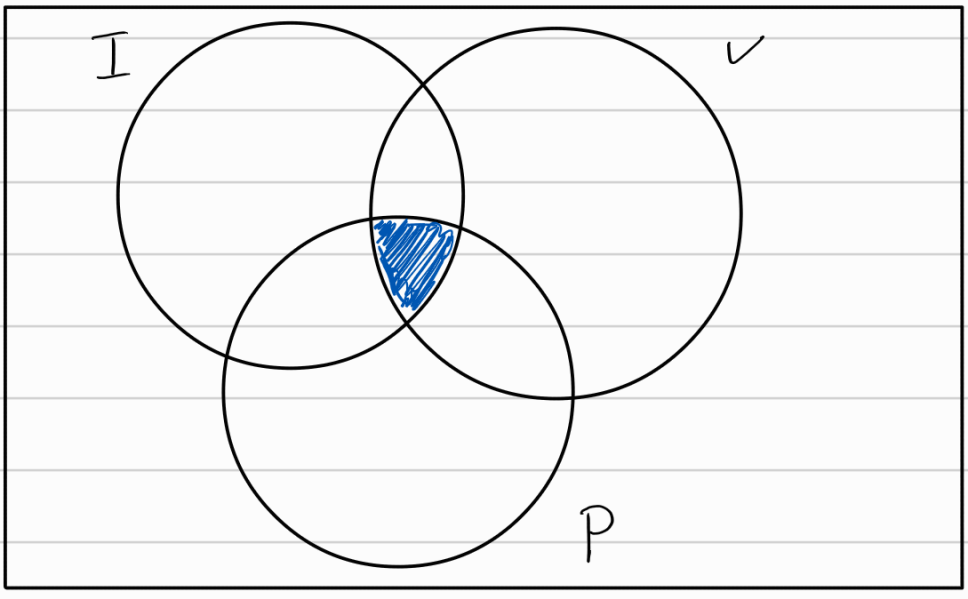
Pavel Detvan
i630211

(1) some inferences are valid

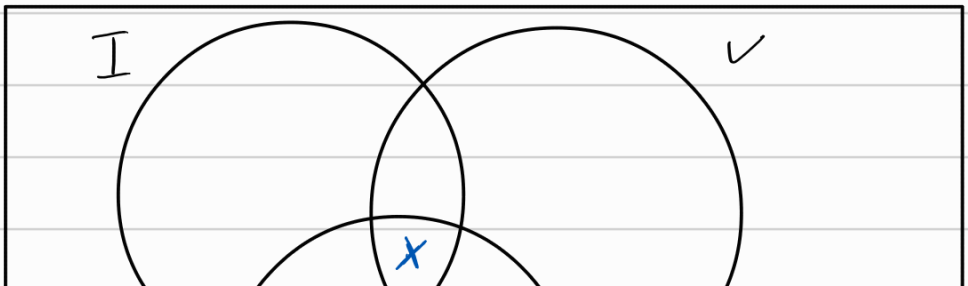


every valid inference is provable:

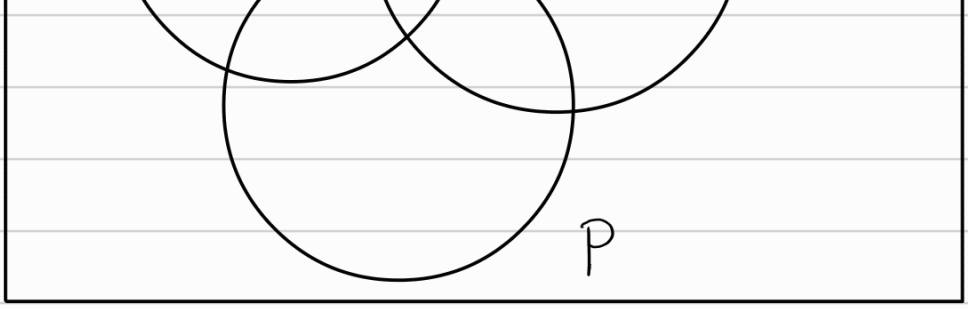


---

some provable inferences are valid:



valid ✓



$\exists x(Ix \wedge Vx), \forall x(Vx \to Px) \circ \exists x((Px \wedge Ix) \to Vx)$

$Ia \wedge Va, \forall x(Vx \to Px) \overset{+}{\circ} \exists x((Px \wedge Ix) \to Vx)$

$Ia, Va, Va \to Pa \quad \circ \quad \exists x((Px \wedge Ix) \to Vx)$

$Ia, Va, Va \to Pa \quad \circ \quad (Pa \wedge Ia) \to Va$

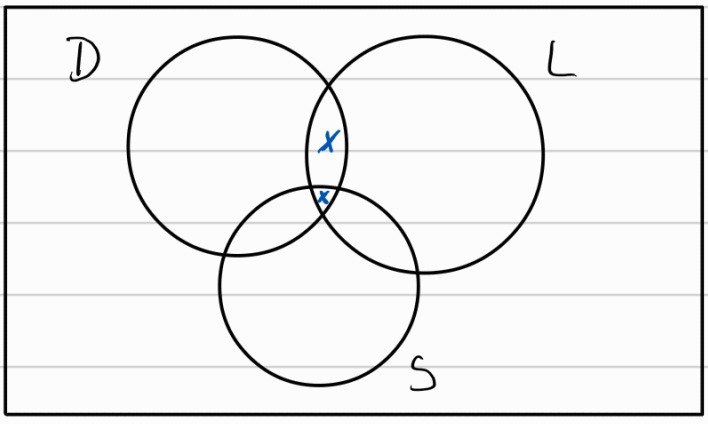
$Ia, Va \circ Va$ — X

$Ia, Va, Pa \circ (Pa \wedge Ia) \to Va$

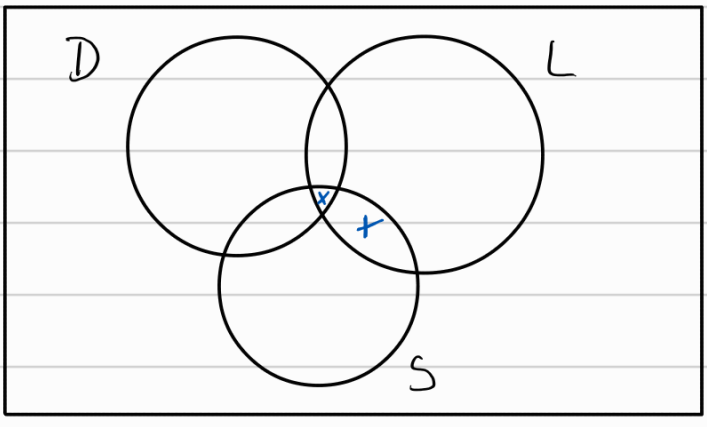
$Pa \wedge Va, Ia, Va, Pa \circ Va$ — X

- closed branches $\Rightarrow$ valid

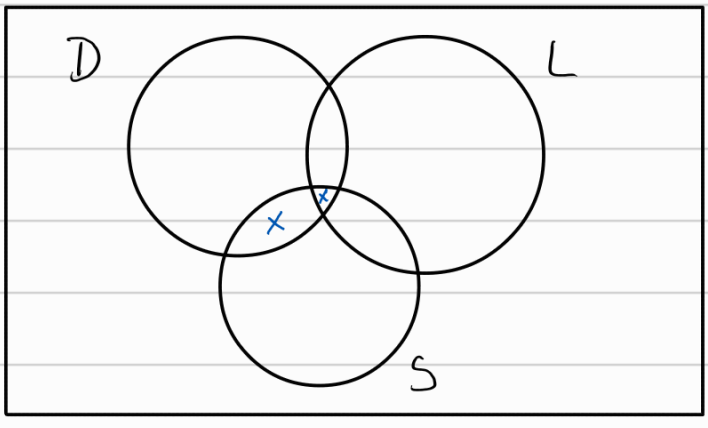
b) Some DACS students are logicians



Some logicians are smart:

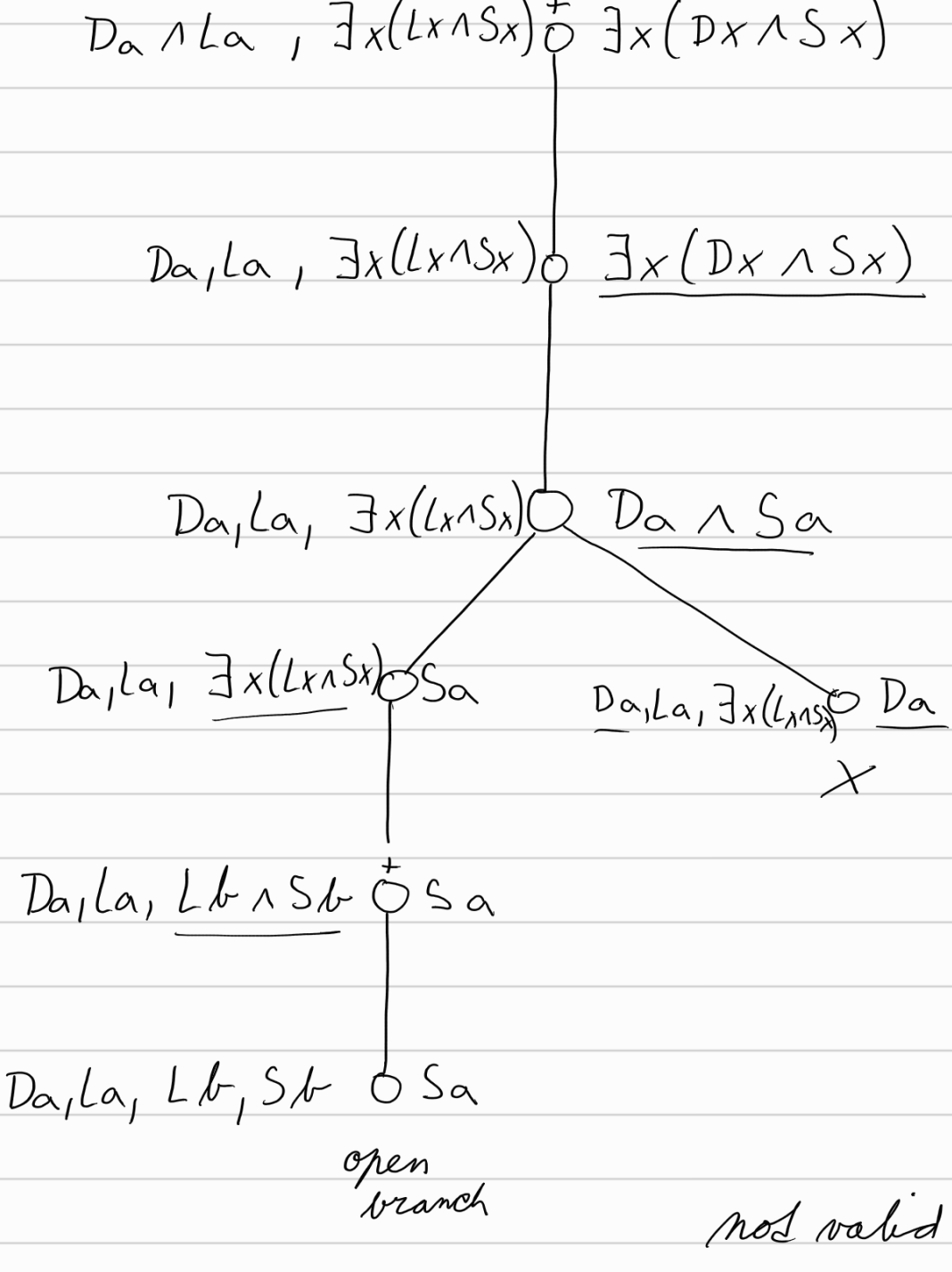


Some DACS students are smart:



not valid

$\underline{\exists x(Dx \wedge Lx)}, \exists x(Lx \wedge Sx) \not\vdash \exists x(Dx \wedge Sx)$



(2) i) $\exists x \forall y (\neg Bx \wedge \neg Rxy)$

$\forall x (\neg Rxx)$

ii) $\exists x \exists y ((Bx \wedge By) \wedge (Rxy \wedge Ryx))$

$\exists x (\neg Bx)$

iii) $\forall x (Bx)$
$\forall x (\neg Bx)$

(3) i) $\forall x(Px \to Qx) \circ \forall x(Px \vee Qx)$

$\forall x(Px \to Qx) \quad \overset{+}{\circ} \; Pa \vee Qa$

$\forall x(Px \to Qx) \circ Pa, Qa$

$Pa \to Qa \circ Pa, Qa$

$\circ Pa, Pa, Qa \qquad Qa \circ Pa, Qa$

↑ open branch $\qquad$ X

not valid

ii) $Pa, \forall x(Px \to Qx) \circ \exists x Qx$

$Pa, \forall x(Px \to Qx) \circ Qa$

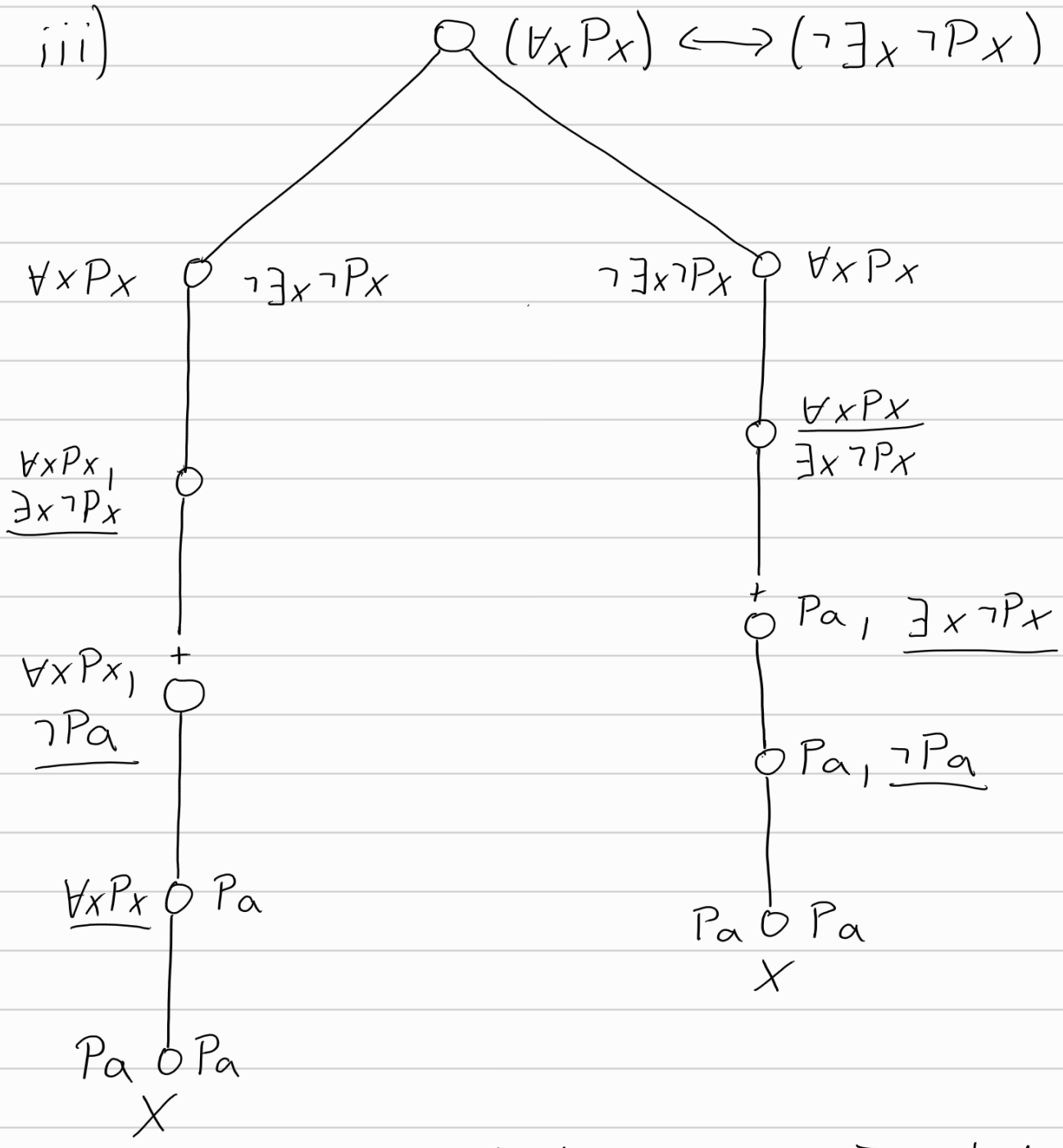
$Pa, Pa \to Qa \circ Qa$

$Pa, Qa \circ Qa \qquad Pa \circ Qa, Pa$

X $\qquad$ X

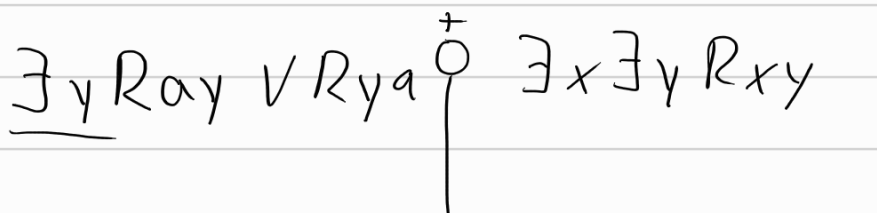
valid

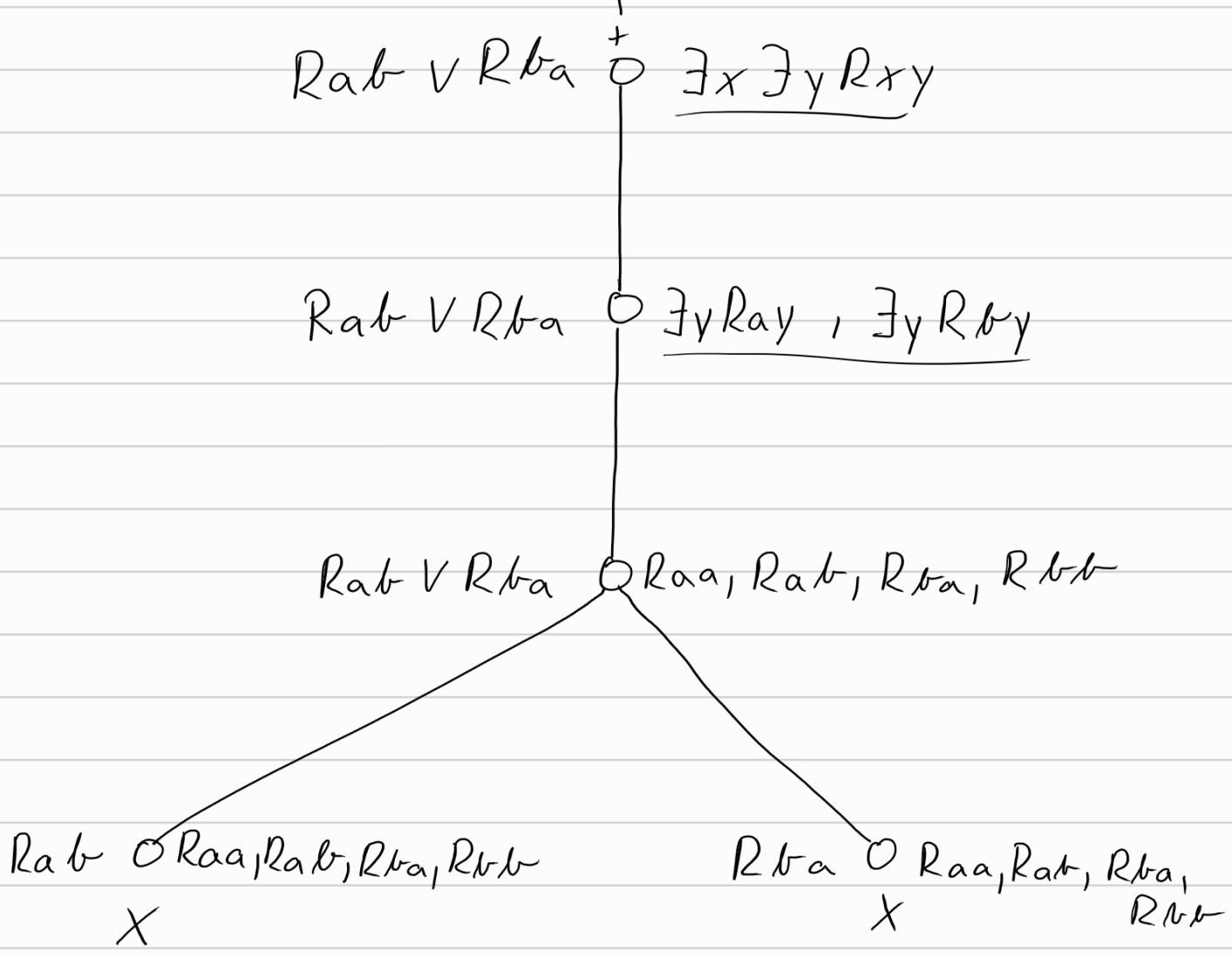
iii) $(\forall x Px) \leftrightarrow (\neg \exists x \neg Px)$



closed branches → valid

iv) $\exists x \exists y (Rxy \vee Ryx) \quad \exists x \exists y Rxy$





closed branches → valid

(4) $\forall x(Ax \to Bx) \vdash (\exists x Ax) \to (\exists x Bx)$

1	$\forall x(Ax \to Bx)$	
2	$\exists x Ax$	(ass)
3	c	(exist.)
4	Ac	($E\exists$ 2)
5	$Ac \to Bc$	($E\forall$ 1)
6	Bc	($E\to$ 4,5)
7	$\exists x Bx$	($I\exists$ 6)
8	$\exists x Bx$	($E\exists$ 3-7)
9	$(\exists x Ax) \to (\exists x Bx)$	($I\to$ 2,8)

ii) $Pa \to \forall(Qx \to Qb), Qa, \neg Qb \vDash \neg Pa$

1	$Pa \to \forall(Qx \to Qb)$	
2	Qa	
3	$\neg Qb$	
4	Pa	(ass)
5	$\forall(Qx \to Qb)$	($E\to$ 1,4)
6	$Qa \to Qb$	($E\forall$ 5)
7	Qb	($E\to$ 2,6)
8	$\bot$	($\bot$ 3,7)
9	$\neg Pa$	($I\neg$ 4-8)

iii) $\exists x Px, \forall x Px \to Qx \vDash \neg\forall x \neg Qx$

1	$\exists x Px$	
2	$\forall x Px \to Qx$	
3	$\forall x \neg Qx$	(ass)
4	Pa	a exist.
5	$Pa \to Qa$	($E\forall$ 2)
6	Qa	($E\to$ 4,5)
7	$\neg Qa$	($E\forall$ 3)
8	$\bot$	($\bot$ 6,7)
9	$\bot$	($E\exists$ 1,4-8)
10	$\neg\forall x \neg Qx$	($I\neg$ 3,9)