$\exists x Rx \quad \circ \quad \forall x Rx$

(E∃)

$+$

$Ra \quad \circ \quad \forall x Rx$

(E∀)

$+$

$Ra \quad \circ \quad Rb$

open

invalid

$\exists x \forall y (Px \to Qy) \quad \circ \quad \forall x \exists y (Py \to Qx)$

(E∃)

$+$

$\forall y (Pa \to Qy) \quad \circ \quad \forall x \exists y (Py \to Qx)$

(E∀)

$+$

$\forall y (Pa \to Qy) \quad \circ \quad \exists y (Py \to Qb)$

(E∀)

$Pa \to Qa,\ Pa \to Qb \quad \circ \quad \exists y (Py \to Qb)$

(E∃)

$Pa \to Qa,\ Pa \to Qb \quad \circ \quad Pa \to Qb,\ Pb \to Qb \qquad \times$

(E→)

$Pa \to Qa \quad \circ \quad Pa \to Qb,\ Pb \to Qb,\ Pa$

(E→)

$Pa \to Qa,\ Pa \quad \circ \quad Qb,\ Pb \to Qb,\ Pa$

$\times$

$Pa \to Qa,\ Qb \quad \circ \quad Pa \to Qb,\ Pb \to Qb$

$Pa \to Qa,\ Qb,\ Pa \quad \circ \quad Qb,\ Pb \to Qb$

$\times$

valid

$\forall x (Px \to \exists y Qy) \quad \circ \quad \exists x Qx \ \lor\ \neg \exists x Px$

(E∨)

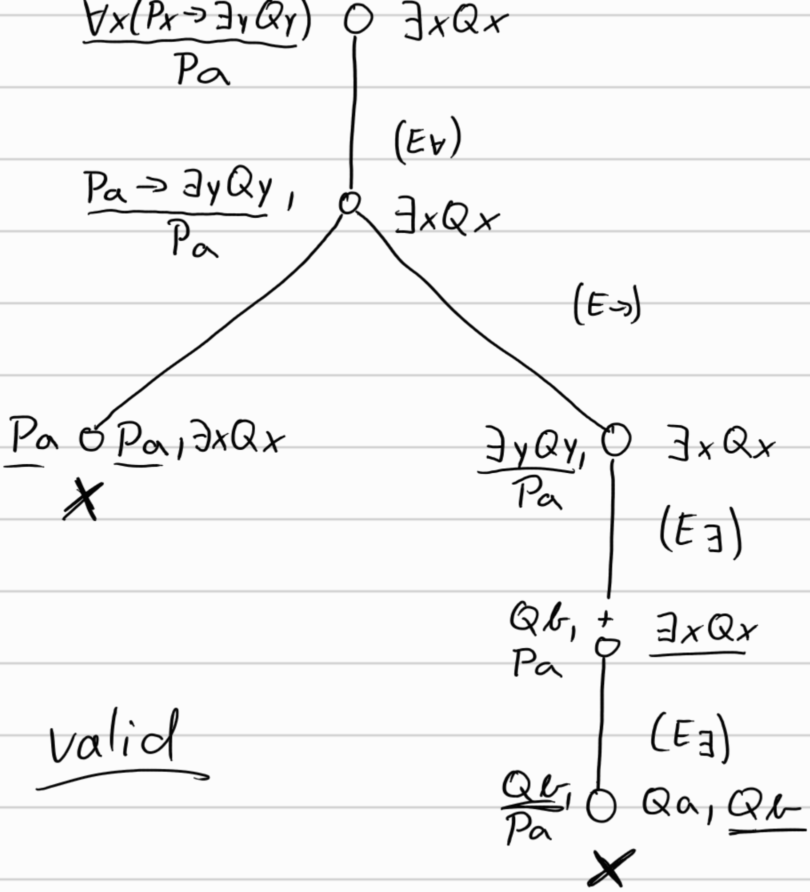
$\forall x (Px \to \exists y Qy) \quad \circ \quad \exists x Qx,\ \neg \exists x Px$

(E¬)

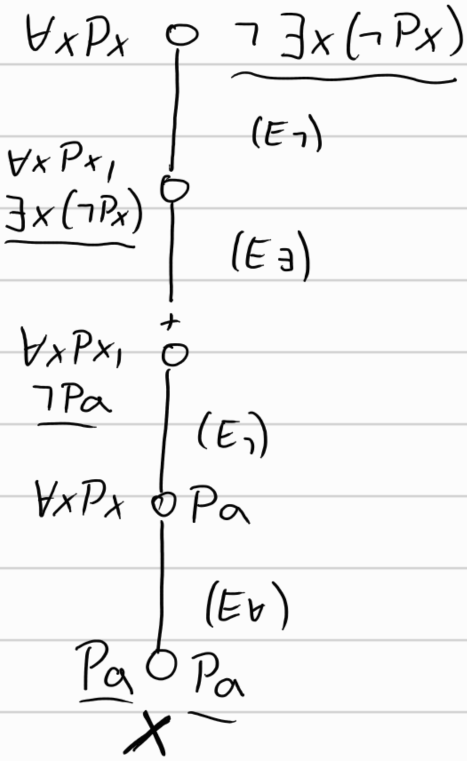
$\forall x (Px \to \exists y Qy),\ \exists x Px \quad \circ \quad \exists x Qx$

(E∃)

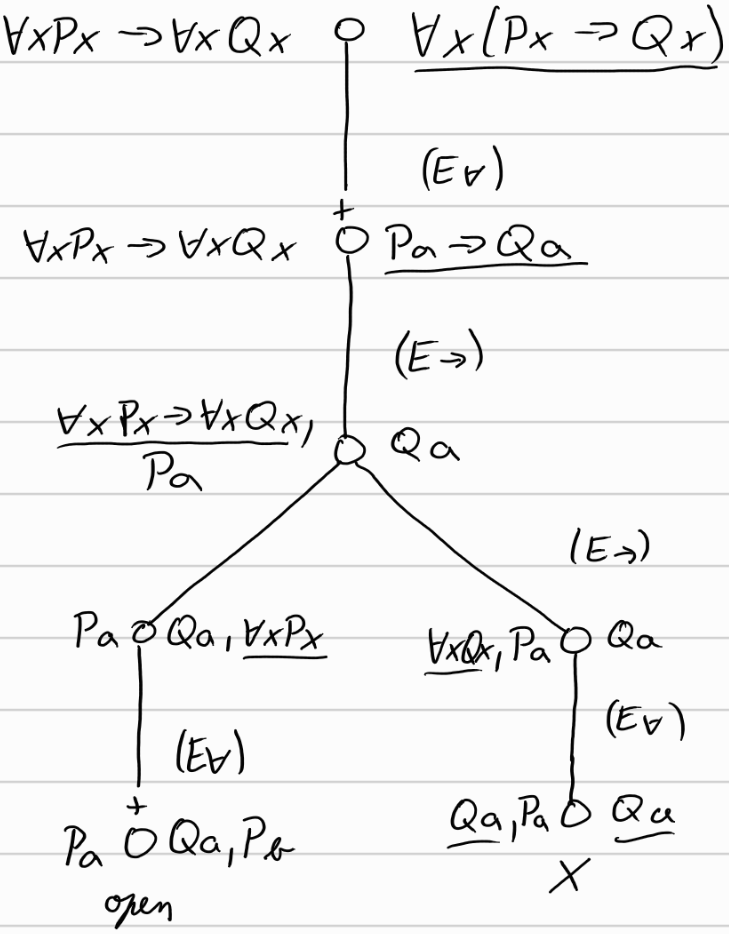
$+$



valid



valid



invalid

counterexample: $Pa, \neg Pb, \neg Qa$

eg: $U = \{a, b\}$

$P = \{a\}$

$Q = \emptyset$

$\forall x \varphi(x) \vDash \exists x \varphi(x)$

1	$\forall x \varphi(x)$	ass
2	φx	$E_{\forall}1$
3	$\exists x \varphi(x)$	$I_{\exists}2$

$\forall x (Px \Rightarrow Qx), \exists x Px \vDash \exists x Qx$

1	$\forall x (Px \Rightarrow Qx)$	
2	$\exists x Px$	
3	Pc	c – constant
4	$Pc \Rightarrow Qc$	$E_{\forall}1$
5	Qc	$E_{\Rightarrow}\ 3,4$
6	$\exists x Qx$	$I_{\exists}\ 5$
7	$\exists x Qx$	$E_{\exists}(2,3,6)$

$\forall x Px, \forall x Qx \vDash \forall x (Px \wedge Qx)$

1	$\forall x Px$	
2	$\forall x Qx$	
3	c	universal const.
4	Pc	$E_{\forall}1$
5	Qc	$E_{\forall}2$
6	$Pc \wedge Qc$	$I_{\wedge}\ 4,5$
7	$\forall x (Px \wedge Qx)$	$I_{\forall}\ 3,6$

$\exists y \forall x Rxy \vDash \forall x \exists y Rxy$

1	$\exists y \forall x Rxy$	
2	$\forall x Rxc$	c - exist. const.
3	d	generic const.
4	Rdc	$E_{\forall}2$
5	$\exists y Rdy$	$I_{\exists}4$
6	$\forall x \exists y Rxy$	$I_{\forall}\ 3,5$

1	$\forall x Px$	
2	u	generic const.
3	Pu	$E_{\forall}1$
	$\forall x Px$	$I_{\forall}\ 2,3$

6	$\forall x \exists y Rxy$	$I\forall$ 3,5
7	$\forall x \exists y Rxy$	$E\exists(1,6)$

$\forall y Py$	$I\forall$ 2,5

$$\neg \exists x Px \models \forall x \neg Px$$

1	$\neg \exists x Px$	
2	c	generic
3	Pc	(ass)
4	$\exists x Px$	$(I_\exists 3)$
5	$\bot$	$\bot(1,4)$
6	$\neg Pc$	$I_\neg(3\text{-}5)$
7	$\forall x \neg Px$	$I\forall$

$$\forall x (Px \to Qx) \models \forall x Px \to \forall x Qx$$

1	$\forall x (Px \to Qx)$	
2	$\forall x Px$	ass
3	c	universal
4	Pc	$E\forall 2$
5	$Pc \to Qc$	$E\forall 1$
6	Qc	$E\to 4,5$
7	$\forall x Qx$	$I\forall$ 3-6
8	$\forall x Px \to \forall x Qx$	$I\to$ 2,7

$$\forall x \forall y Rxy \models \forall x Rxx$$

1	$\forall x \forall y Rxy$	
2	c	universal
3	$\forall y Rcy$	$E\forall 1$
4	Rcc	$E\forall 3$

5	$\forall x Rxx$	I$\forall$ 2-4

$$\exists x (Px \land Rx), \forall x (Px \Rightarrow Qx) \vDash \exists x (Qx \land Rx)$$

1	$\exists x (Px \land Rx)$	
2	$\forall x (Px \Rightarrow Qx)$	
3	c	existential
4	$Pc \land Rc$	E$\exists$ 1
5	Pc	E$\land$ 4
6	Rc	E$\land$ 4
7	$Pc \Rightarrow Qc$	E$\forall$ 2
8	Qc	E$\Rightarrow$ 5,7
9	$Qc \land Rc$	I$\land$ 6,8
10	$\exists x (Qx \land Rx)$	I$\exists$ 9
11	$\exists x (Qx \land Rx)$	E$\exists$ 3-10