A: $m \times n$ matrix

	definition	subspace for	equal to $\mathbb{R}^n / \mathbb{R}^m / \mathbb{R}^n$ if	equal to $\{0\}$ if
$\text{Nul}(A)$	$\{x : A\underline{x} = \underline{0}\}$	$\mathbb{R}^n$	A is the zero matrix	A has a pivot in every col
$\text{Col}(A)$	$\text{Span}\{\underline{a}_1, \dots, \underline{a}_n\}$	$\mathbb{R}^m$	A has pivot in every row	A is the zero matrix
$\text{Row}(A)$	$\text{Span}\{r_1, \dots, r_m\}$	$\mathbb{R}^n$	A^T has a pivot in every row	A is the zero matrix

	dimension
$\text{Nul}(A)$	# free vars in the equation $A\underline{x} = \underline{0}$ = # nonpivot cols in A = n - rank A
$\text{Col}(A)$	# pivot cols in A = # pivot cols in A = rank A
$\text{Row}(A)$	# non-zero rows in the echelon form of A = # pivot cols in A = rank A

	finding a basis
$\text{Nul}(A)$	Find a general sol. of $A\underline{x} = \underline{0}$. Write the sol. in parametric vector form where the weights are the free vars. The corresponding vectors form a basis for Nul A
$\text{Col}(A)$	The pivot cols of A (so, of A itself and thus NOT the pivot cols of a reduced form of A
$\text{Row}(A)$	The nonzero rows of an echelon form of A

$$(\text{Row } A)^{\perp} = \text{Nul } A \quad \text{and} \quad (\text{Col } A)^{\perp} = \text{Nul } A^T$$

Equivalent statements in Linear Algebra

Let A be an $m \times n$ matrix with cols $\underline{a}_1, \underline{a}_2, \dots, a_n$

$m \geq n$: $\begin{bmatrix} A \end{bmatrix}$	$m \leq n$: $[\ A\ ]$
The following statements are equivalent:	The following statements are equivalent
① A has pivot in every col	② A has pivot in every row

1. A has pivot in every col
2. A has n pivot positions
3. There are no free var.
4. $A\underline{x} = \underline{0}$ has only a trivial sol.
5. $\{\underline{a_1}, \underline{a_2}, \dots, \underline{a_n}\}$ is linearly indep.
6. $T: \underline{x} \longmapsto A\underline{x}$ is one-to-one
7. $\text{Nul } A = \{0\}$
8. $\dim \text{Nul } A = 0$
9. $\text{rank } A = n$

a. A has pivot in every row
b. A has m pivot positions
c. The echelon form of A doesn't contain a row-all of zeros
d. $A\underline{x} = \underline{b}$ is consistent for every $\underline{b}$ in $\mathbb{R}^m$
e. $\text{Span}\{\underline{a_1}, \underline{a_2}, \dots, \underline{a_n}\} = \mathbb{R}^m$
f. $T: \underline{x} \longmapsto A\underline{x}$ is onto
g. $\text{Col } A = \mathbb{R}^m$
h. $\dim \text{Col } A = m$
i. $\text{rank } A = m$

If A is a square ($m = n$), then statements 2 and b are equivalent. Hence, the following statements are equivalent for square matrices.

* 1 - 9, a - i
* A is invertible
* There is a matrix C such that $CA = I_n$ and $AC = I_n$
* A is row equivalent to I_n
* A^T is invertible
* $\det A \neq 0$
* The cols of A form a basis for $\mathbb{R}^n$
* 0 is not an eigenvalue of A