

Euler Method

$$\frac{dy}{dt} = f(t, y) = e^{-t} - y^2 \quad y(0) = 0 \quad \text{up to } 1$$

$$t_0 = 0 \quad w_0 = y_0 = y(0) = 0$$

take $t_f = 1$ to aim to approx. $y(1)$ ($y(t)$ where $t=1$)

step size $h = 0.5$

times: $t_0 = 0 \quad t_1 = 0.5$
 $t_2 = 1.0$

$$w_0 = y(0) = 0$$

$$f(t_0, w_0) = f(0, 0) = e^{-0.0} - 0^2 = \underline{1}$$

$$w_1 = y(0.5) \approx w_0 + h f(t_0, w_0) \\ = 0 + 0.5 \cdot 1 = \underline{0.5}$$

$$f(t_1, w_1) = f(0.5, 0.5) = e^{-0.5} - 0.5^2 = \underline{0.357}$$

$$w_2 = y(1) = w_1 + h f(t_1, w_1) \\ \approx 0.5 + 0.5 \cdot 0.357 \\ \approx \underline{0.678}$$