$\boxed{1.1}$

(3) $\quad \begin{aligned} x_1 + 5x_2 &= 7 \\ x_1 - 2x_2 &= -2 \end{aligned}$

$$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 1 & -2 & -2 \end{array}\right] \underset{R_2 : R_2 - R_1}{\sim} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & -7 & -9 \end{array}\right]$$

$$\underset{R_2 : R_2 \cdot (-1/7)}{\sim} \left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 9/7 \end{array}\right] \underset{R_1 : R_1 - 5R_2}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 4/7 \\ 0 & 1 & 9/7 \end{array}\right] \quad \begin{cases} x_1 = 4/7 \\ x_2 = 9/7 \end{cases}$$

(7) $$\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \sim$$

$\rightarrow$ this means $0=1$ so no solution

(11) $$\begin{cases} \quad\quad\;\; x_2 + 4x_3 = -4 \\ x_1 + 3x_2 + 3x_3 = -2 \\ 3x_1 + 7x_2 + 5x_3 = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 4 & -4 \\ 1 & 3 & 3 & -2 \\ 3 & 7 & 5 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 3 & 7 & 5 & 6 \end{array}\right] \underset{R_3 : R_3 - 3R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & -2 & -4 & 12 \end{array}\right]$$

$$\underset{R_3 : R_3 + 2R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 4 & 4 \end{array}\right] \underset{R_3 : R_3 * 1/4}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \text{RFE}$$

$$\underset{R_1 : R_1 - 3R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -9 & 10 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_1 : R_1 + 9R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_2 : R_2 - 4R_3}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \end{array}\right] \quad \begin{cases} x_1 = 19 \end{cases}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{cases} x_2 = -8 \\ x_3 = 1 \end{cases} \quad \text{RRFE}$$

(12)

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 3 & -7 & 7 & -8 \\ -4 & 6 & 2 & 4 \end{array}\right] \underset{R_2 : R_2 - 3R_1}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ -4 & 6 & 2 & 4 \end{array}\right] \underset{R_3 : R_3 + 4R_1}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -6 & 18 & -12 \end{array}\right] \underset{R_3 : R_3 \cdot 1/6}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 2 & -5 & 4 \\ 0 & -1 & 3 & -2 \end{array}\right] \underset{R_2 : R_2 \cdot 1/2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 1 & -5/2 & 2 \\ 0 & -1 & 3 & -2 \end{array}\right] \underset{R_3 : R_3 + R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 1 & -5/2 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \text{RFE}$$

$$\underset{R_2 : R_2 + 5/2 R_3}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 4 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \underset{R_1 : R_1 + 3R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 4 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \underset{R_1 : R_1 - 4R_3}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \text{RRFE} \quad \begin{cases} x_1 = 2 \\ x_2 = 2 \\ x_3 = 0 \end{cases}$$

(19)

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right] \underset{R_4 : R_4 - 3R_1}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \underset{R_3 : R_3 + 2R_2}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \end{array}\right] \underset{R_4 : R_4 + 3R_3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \end{array}\right] \quad \text{RFE}$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -9 & 7 & -11 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 0 & 0 & 0 & -5 & 10 \end{array}\right]$$

(20)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array}\right] \underset{\substack{R_2 : R_2 \cdot 1/2 \\ R_4 : R_4 + 2R_1}}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array}\right] \underset{R_4 : R_4 - 3R_2}{\sim}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right] \underset{R_4 : R_4 + R_3}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(23)

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix} \underset{R_2 : R_2 - 3R_1}{\sim} \begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$$

$c = 6 - 3h$

if $c = 0$ $\quad h = 2$

no solution $(0 = -4)$

if $c \neq 0$ $\quad h \neq 2$

system has solution

(25)

$$\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix} \underset{R_2 : R_2 + 4R_1}{\sim} \begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix}$$

$c = h + 12$

$c x_2 = 0 \rightarrow$ solution for any c

system consistent for all h

$\boxed{1.2}$

(4)

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \underset{R_2 : R_2 - R_3}{\sim} \begin{bmatrix} 1 & 3 & 5 & 7 \\ -2 & -2 & -2 & 8 \\ 5 & 7 & 9 & 1 \end{bmatrix} \underset{R_2 \cdot 1/2}{\sim}$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 1 & 1 & 1 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -2 & -4 & -11 \end{bmatrix} \sim$$

$$\begin{bmatrix} \dots & \dots & \dots & \dots \\ 5 & 7 & 9 & 1 \end{bmatrix} \underset{R_2 : R_2 - R_1}{\sim} \begin{bmatrix} \dots & \dots & \dots & \dots \\ 5 & 7 & 9 & 1 \end{bmatrix} \quad R_3 : R_3 - 5R_1$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -2 & -4 & -11 \\ 0 & -8 & -16 & -34 \end{bmatrix} \underset{R_3 : R_3 - 4R_2}{\sim} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -2 & -4 & -11 \\ 0 & 0 & 0 & 10 \end{bmatrix} \underset{\substack{R_2 / (-2) \\ R_3 / (10)}}{\sim}$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 11/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pivot cols: 1, 2, 4

(7) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 8 & 9 & 4 \end{bmatrix} \underset{R_2 : R_2 - 4R_1}{\sim} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & -3 & -12 \end{bmatrix} \underset{R_1 : R_1 + R_2}{\sim} \begin{bmatrix} 1 & 2 & 0 & -8 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$$\begin{cases} x_1 + 2x_2 = -8 \\ x_3 = 4 \end{cases} \qquad \begin{cases} x_1 = -8 - 2x_2 \\ x_2 \text{ free} \\ x_3 = 4 \end{cases}$$

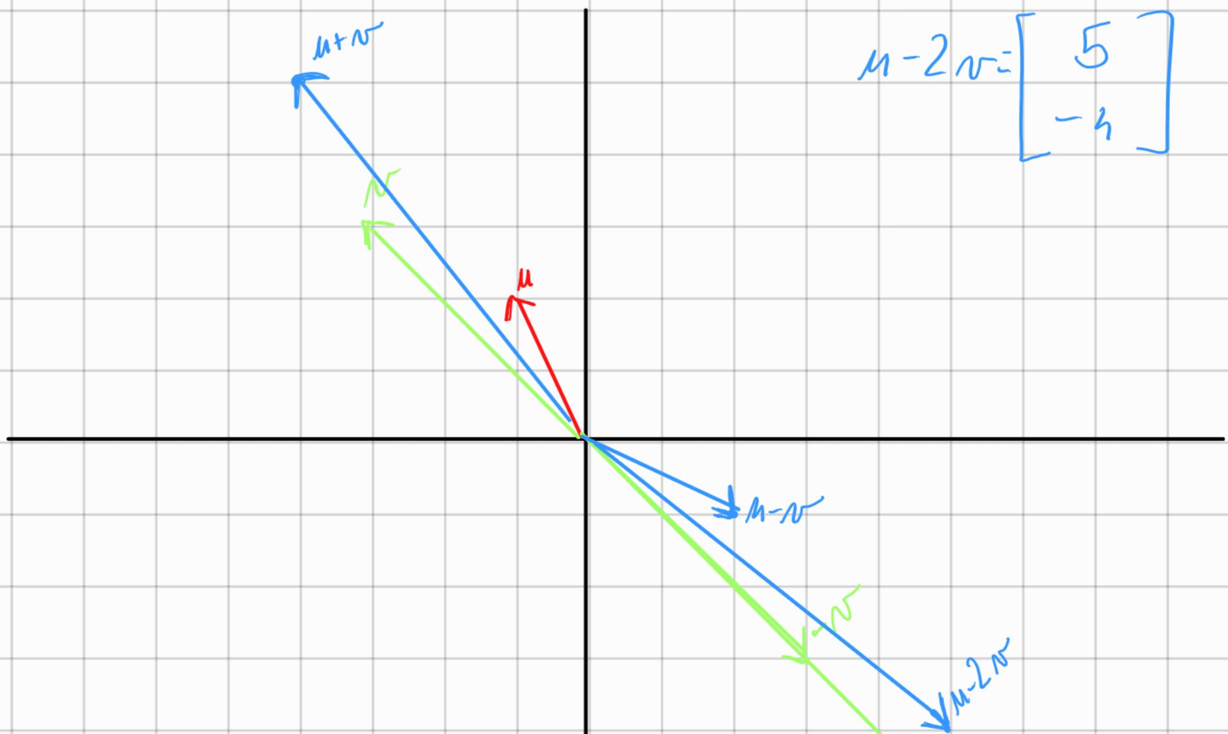
x_2 free

$\boxed{1.3}$

(3) $u = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$

$u - v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$u - 2v = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$



⑤

$$x_1\begin{bmatrix}4\\-3\\2\end{bmatrix} + x_2\begin{bmatrix}-8\\7\\0\end{bmatrix} = \begin{bmatrix}9\\-6\\-5\end{bmatrix}$$

$$\begin{bmatrix}4x_1\\-3x_1\\2x_1\end{bmatrix} + \begin{bmatrix}-8x_2\\7x_2\\0x_2\end{bmatrix} = \begin{bmatrix}9\\-6\\-5\end{bmatrix}$$

$$\begin{bmatrix}4x_1-8x_2\\-3x_1+7x_2\\2x_1\end{bmatrix} = \begin{bmatrix}9\\-6\\-5\end{bmatrix} \qquad \begin{cases}4x_1-8x_2=9\\-3x_1+7x_2=-6\\2x_1=-5\end{cases}$$

⑨

$$\begin{bmatrix}x_2+5x_3\\4x_1+6x_2-x_3\\-x_1+3x_2-8x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1\begin{bmatrix}0\\4\\-1\end{bmatrix} + x_2\begin{bmatrix}1\\6\\3\end{bmatrix} + x_3\begin{bmatrix}5\\-1\\-8\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

⑪ determine if b is a linear combination of a_1, a_2, a_3

$$a_1=\begin{bmatrix}1\\-2\\0\end{bmatrix} \quad a_2=\begin{bmatrix}0\\1\\2\end{bmatrix} \quad a_3=\begin{bmatrix}5\\-6\\8\end{bmatrix} \quad b=\begin{bmatrix}2\\-1\\6\end{bmatrix}$$

$\Rightarrow$ does the vector equation $x_1a_1+x_2a_2+x_3a_3=b$

have a solution?

$$x_1\begin{bmatrix}1\\-2\\0\end{bmatrix} + x_2\begin{bmatrix}0\\1\\2\end{bmatrix} + x_3\begin{bmatrix}5\\-6\\8\end{bmatrix} = \begin{bmatrix}2\\-1\\6\end{bmatrix}$$

$a_1 \quad a_2 \quad a_3 \quad b$

$$\left[\begin{array}{ccc|c}1&0&5&2\\-2&1&-6&-1\\0&2&8&6\end{array}\right] \underset{R_2:R_2+2R_1}{\sim} \left[\begin{array}{ccc|c}1&0&5&2\\0&1&4&3\\0&2&8&6\end{array}\right] \underset{R_3:R_3-2R_2}{\sim}$$

$$\left[\begin{array}{ccc|c}1&0&5&2\\0&1&4&3\\0&0&0&0\end{array}\right]$$

→ has a solution, so the vector eq. has a solution as well

→ b is a linear combination of a_1, a_2, a_3

(12) $$x_1\begin{bmatrix}1\\-2\\2\end{bmatrix} + x_2\begin{bmatrix}0\\5\\5\end{bmatrix} + x_3\begin{bmatrix}2\\0\\8\end{bmatrix} = \begin{bmatrix}-5\\11\\-7\end{bmatrix}$$

$a_1 \quad a_2 \quad a_3 \quad b$

$$\left[\begin{array}{ccc|c}1&0&2&-5\\-2&5&0&11\\2&5&8&-7\end{array}\right] \underset{R_2:R_2+R_3}{\sim} \left[\begin{array}{ccc|c}1&0&2&-5\\0&10&8&4\\2&5&8&-7\end{array}\right] \underset{R_3:R_3-2R_1}{\sim}$$

$$\left[\begin{array}{ccc|c}1&0&2&-5\\0&5&4&2\\0&5&4&3\end{array}\right] \underset{R_3:R_3-R_2}{\sim} \left[\begin{array}{ccc|c}1&0&2&-5\\0&5&4&2\\0&0&0&1\end{array}\right]$$

NO SOLUTION

→ b is not linear combination

(21) $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $y = \begin{bmatrix} h \\ k \end{bmatrix}$

$$[u \; v \; y] = \left[\begin{array}{cc|c} 2 & 2 & h \\ -1 & 1 & k \end{array}\right] \underset{R_2 : R_2 + \frac{1}{2}R_1}{\sim} \left[\begin{array}{cc|c} 2 & 2 & h \\ 0 & 2 & k + h/2 \end{array}\right]$$

consistent for all h and k

y is in Span$\{u, v\}$ for all h and k

(22) question ??

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 8 \\ 0 & 0 & 0 & 1 \end{array}\right] \underset{R_3 : R_3 + R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 8 \\ 1 & 2 & 3 & 8 \end{array}\right]$$

(33)

$$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \\ \begin{array}{cccc} a_1 & a_2 & a_3 & b \end{array} \end{array}$$

a) there are only 3 vectors in $\{a_1, a_2, a_3\}$ and b is not one of them

b) $$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \underset{R_3 : R_3 + 2R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{array}\right] \underset{R_3 : R_3 - 2R_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right]$$

consistent so b is in W

There are infinitely many in $W = \text{Span}\{a_1, a_2, a_3\}$

(34)

$$\begin{array}{c} \left[\begin{array}{ccc|c} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{array}\right] \\ \begin{array}{cccc} a_1 & a_2 & a_3 & b \end{array} \end{array} \underset{\substack{R_2: R_2+R_3 \\ R_1: R_1/2}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 6 & 6 & 6 \\ 1 & -2 & 1 & 3 \end{array}\right] \underset{R_3: R_3-R_1}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 6 & 6 & 6 \\ 0 & -2 & -2 & -2 \end{array}\right] \underset{R_2: R_2+3R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & -2 \end{array}\right] \underset{\text{switch } R_2 \text{ with } R_3}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

consistent

b is a linear combination of A
so b is in W

$$x_1 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$$

$x_1 = 0$, $x_2 = 0$, $x_3 = 1$

$$0a_1 + 0a_2 + 1a_3 = a_3$$

(7)

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} = A \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$$

(11)

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{array}\right] \underset{\substack{R_3 : R_3 + 2R_1 \\ R_3 : R_3/5}}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_2 : R_2 - 5R_3}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_1 : R_1 - 4R_3}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 0 & -6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \underset{R_1 : R_1 - 2R_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{cases} x_1 = 0 \\ x_2 = -3 \\ x_3 = 1 \end{cases}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$$

(17)

$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \underset{R_2 : R_2 + R_1}{\sim} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix} \underset{R_3 : R_3 + 2R_2}{\sim}$$

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 3 & -1 \end{bmatrix} \underset{\substack{\text{swap } R_3 : R_4 \\ R_3 : R_3 - 2R_1}}{\sim} \begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -6 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{R_3 : R_3 + 2R_1}{\sim}$$

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

because not every row of A contains a pivot, the equation $Ax = b$ has no solution for each b in R^4

(18)

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \underset{R_3 : R_3 - R_1}{\sim} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ -2 & -8 & 2 & -1 \end{bmatrix} \underset{R_3 : R_3 + R_2}{\sim}$$

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ -2 & -8 & 2 & -1 \end{bmatrix} \underset{\substack{R_4 : R_4 + 2R_1 \\ \text{swap } R_4 : R_3}}{\sim} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -2 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{R_3 : R_3 + 2R_2}{\sim}$$

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

because not every row of B contains a pivot position, the eq. $Bx = y$ doesn't have a solution for each y in $\mathbb{R}^4$

(21)

$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \\ \begin{matrix} v_1 & \quad v_2 & \quad v_3 \end{matrix} \end{array} \underset{R_3 : R_3 + R_1}{\sim} \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

the matrix $[v_1 \ v_2 \ v_3]$ does not have a pivot in each row, so the columns of the matrix do not span $\mathbb{R}^4$

$\{v_1, v_2, v_3\}$ doesn't span $\mathbb{R}^4$

(36)

$$u = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix} \quad v = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} \quad w = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

$3u - 5v - w = 0$

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = w$$

$x_1 u + x_2 v = w$

$3u - 5v = w$

$$\boxed{\begin{aligned} x_1 &= 3 \\ x_2 &= -5 \end{aligned}}$$

37 $x_1 q_1 + x_2 q_2 + x_3 q_3 = v$

$Qx = v$ where $Q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$ and

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$