

$$\underline{E\forall}: \frac{\forall x \varphi(x)}{\varphi(c)} \quad E\forall$$

$$Ev: \frac{\forall x (P_x \rightarrow Q_{xx})}{P_c \rightarrow Q_{cc}} \quad E\forall$$

$$\underline{I\exists}: \frac{\varphi(c)}{\exists x \varphi(x)} \quad I\exists$$

$$Ev: \frac{P_c \rightarrow Q_{cc}}{\exists x (P_x \rightarrow Q_{xx})} \quad I\exists$$

$$\underline{I\forall}: \frac{\begin{array}{|c|} \hline c \\ \vdots \\ \hline \varphi(c) \\ \hline \end{array}}{\forall x \varphi(x)} \quad I\forall \quad (\text{universal constant})$$

$$Ev: \begin{array}{|l|l|l|} \hline 1 & \forall x (P_x \rightarrow Q_x) & \\ \hline 2 & \forall x P_x & \\ \hline 3 & P_c \rightarrow Q_c & (E\forall 1) \\ \hline 4 & P_c & (E\forall 2) \\ \hline 5 & Q_c & (E\rightarrow 3,4) \\ \hline 6 & \forall x Q_x & (I\forall 3-5) \\ \hline \end{array}$$

$$\underline{E\exists}: \frac{\begin{array}{|c|} \hline \exists x \varphi_x \\ \hline \varphi(c) \\ \vdots \\ \psi \quad (\text{no } c \text{ in } \psi) \\ \hline \psi \\ \hline \end{array}}{\psi} \quad E\exists$$

$$Ev: \begin{array}{|l|l|l|} \hline 1 & \forall x (P_x \rightarrow Q_x) & \\ \hline 2 & \exists x P_x & \\ \hline 3 & P_c & (E\exists 2) \\ \hline 4 & P_c \rightarrow Q_c & (E\forall 1) \\ \hline 5 & Q_c & (E\rightarrow 3,4) \\ \hline 6 & \exists y Q_y & (I\exists 5) \\ \hline 7 & \exists y Q_y & (E\exists 3-6) \\ \hline \end{array}$$

to remember:

$$\frac{\forall x \varphi(x)}{\varphi(c)} \quad E\forall$$

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if \forall is before

$$\frac{\begin{array}{|c|} \hline c \\ \vdots \\ \hline \varphi(c) \\ \hline \end{array}}{\forall x \varphi(x)} \quad I\forall$$

indent if \forall is below

$$\frac{\varphi(c)}{\exists x \varphi(x)} \quad I\exists$$

$$\frac{\exists x \varphi(x)}{\varphi(c)} \quad E\exists$$

$\exists x \varphi(x)$	\exists
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no indent
if \exists is below

	$\varphi(c)$
	\vdots
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indent if \exists is before