$\boxed{H1}$

Absolute Error: $|p^* - p|$

Relative Error: $\dfrac{|p^* - p|}{|p|}$

a) abs error $|3.14 - \pi| = 0.00159265\ldots$

$= 1.6 \cdot 10^{-3}$ (2 sf)

relative error $\dfrac{|3.14 - \pi|}{|\pi|} = 0.0005069\ldots$

$= 5.1 \cdot 10^{-4}$ (2 sf)

$= 0.05\%$

b) abs error $|22/7 - \pi| = 0.00126\ldots$

$= 1.3 \cdot 10^{-3}$ (2 sf)

relative error $\dfrac{|22/7 - \pi|}{|\pi|} = 0.000402\ldots$

$= 4.0 \cdot 10^{-4}$ (2 sf)

$= 0.04\%$

$\boxed{H2}$ c) $2.07^3 = 8.8697\ldots \approx 8.87$ (3 sf)

abs error $|2.07^3 - 8.87| = 0.000257$

$\approx 2.57 \cdot 10^{-4}$ (3 sf)

relative error $\dfrac{|2.07^3 - 8.87|}{|2.07^3|} = 2.90 \cdot 10^{-5}$ (3 sf)

$= 2.9 \cdot 10^{-3}\ \%$

e) $p = \frac{1}{204} \approx 0.0049$

$p^* = 1.42 - 1.41 = 0.010$

abs error: $0.00510 = 5.10 \cdot 10^{-3}$

relative error: 1.04

$104\ \%$

$\boxed{HC3}$ a) $a = 1.028$ $b = -12.83$ $c = 0.07316$

$b^2 - 4ac = 160,57006$

$2a = 2.056$

$x_{12} = \frac{12.83 \pm \sqrt{160.57006}}{2.056}$

$x_1 = 12.40351 \approx \underline{12.40}$ (4sf)

$x_2 = 0.077031 \approx \underline{0.07703}$ (4sf)

x_1 abs. err. $= 0.00351 = 3.510 \cdot 10^{-3}$ (4sf)

x_1 rel. err $= 2.830 \cdot 10^{-4}$

$= 2.830 \cdot 10^{-2}\ \%$ (4sf)

b) $a = 2$ $b = -128$ $c = 1$

$b^2 - 4ac = 16376$

$2a = 4$

$$x_{12} = \frac{128 \pm \sqrt{16376}}{4}$$

$x_1 = 63.99219\ldots \approx 63.99$ (4sf)

$x_2 = 0.007813\ldots \approx 7.813 \cdot 10^{-3}$ (4sf)

x_1 rel. error $\approx 3.42229 \cdot 10^{-5}$

$3.42 \cdot 10^{-3}\ \%$

$\boxed{HC4}$ direct evaluation:

$x = 2.44$

$x^2 = 5.9536 \approx 5.95$ (3sf)

$x^3 = x^2 \cdot x = 14.5$ (3sf)

$x^4 = x^3 \cdot x = 35.4$ (3sf)

$x^5 = x^4 \cdot x = 86.4$ (3sf)

$1.01\,x^5 = 87.3$ (3sf)

$-5.26\,x^3 = -76.3$ (3sf)

$-0.0173\,x^2 = -0.103$ (3sf)

$0.839\,x = 2.05$ (3sf)

$87.3 - 76.3 - 0.103 + 2.05 - 1.91 = 11.0$ (3sf)

exact value $= 10.97481$

abs error $= 0.02519$ $\qquad 2.51 \cdot 10^{-2}$

rel. error $= 0.00229526$ $\qquad 2.30 \cdot 10^{-3}$

$= 0.23\ \%$

$\boxed{H8}$ b) $x^3 - 3x^2 - 7 = 0$ on $[1, 4]$

i) bisection method:

$f(1) = -9$
$f(4) = 15$

$\to$ opposite sign, so interval $[1,4]$ is valid

$$c = \frac{1+4}{2} = 2.5$$

$f(2.5) = -6.375$ $\to$ $\neq 0$ or not close to 0

interval $[1, 2.5]$ $\to$ not valid, $f(1)$ and $f(2.5)$ have the same sign $(-)$

interval $[2.5, 4]$ $\to$ valid

$f(2.5) = -6.375$
$f(4) = 15$

repeat:

$$c = \frac{2.5+4}{2} = 3.25$$

$f(3.25) = 0.9843$ $\neq 0$

interval $[2.5, 3.25]$ $\to$ valid

repeat:

$$c = \frac{2.5+3.25}{2} = 2.8575$$

$f(2.8575) = -3.017\ldots$

interval $[2.8575, 3.25]$

repeat: ?? rounding?

continue until?

T10

$$f(x) = x^3 - 6x^2 + 7x + 2 \qquad p_0 = 1$$
$$f'(x) = 3x^2 - 12x + 7$$

newton's method:

$$p_{n+1} = p_n - \frac{f(p_n)}{f'(p_n)}$$

$$p_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{4}{-2} = 3$$

$$p_2 = 3 - \frac{f(3)}{f'(3)} = 3 - \frac{-4}{-2} = 1$$

$$p_3 = 1 - \frac{f(1)}{f'(1)} = 3 \longrightarrow \text{repetision}$$