

$$(1) P(4.82) = 1.18x^2 - 5.82x + 1.93$$

$$x^2 = 4.82 \cdot 4.82 \approx 23.2 \quad (3sf)$$

$$1.18 \cdot 23.2 \approx 27.4 \quad (3sf)$$

$$5.82 \cdot 4.82 \approx 28.1 \quad (3sf)$$

$$P(4.82) = 27.4 - 28.1 + 1.93 = -0.700 + 1.93 = \underline{\underline{1.23}}$$

$$\text{relative error} = \frac{|1.23 - 1.291832|}{|1.291832|} = \underline{\underline{4.79\%}}$$

$$(2) P_0 = 3 \quad P_1 = 2 \quad f(x) = x^3 - x - 9 \quad 10^{-1} = 0.1$$

$$\text{recant} \quad P_{m+1} = P_m - \frac{P_m - P_{m-1}}{f(P_m) - f(P_{m-1})} f(P_m)$$

$$P_2 = P_1 - \frac{P_1 - P_0}{f(P_1) - f(P_0)} f(P_1) = 2 - \frac{2 - 3}{-3 - 15} (-3) = \underline{2.1667}$$

$$P_3 = P_2 - \frac{P_2 - P_1}{f(P_2) - f(P_1)} f(P_2) = 2.1667 - \frac{0.1667}{-0.9949 - (-3)} \cdot (-0.9949) = \underline{2.2494}$$

diff = 0.1667  
converges  
diff 0.0827 < 0.1  
stop

$$f(P_2) < 0 \quad f(P_3) > 0 \quad [2.1667, 2.2494]$$

$$(3) \frac{dy}{dt} = t/y - 1 \quad w_0 = y(0) = \frac{3}{5} \quad h = 0.25$$

$t_0 = 0 \quad t_m = 0.5 \quad 2 \text{ steps}$

$$\begin{aligned} \text{Heun 2nd order:} \quad k_{i1} &= h f(t_i, w_i) \\ k_{i2} &= h f(t_i + \frac{2}{3}h, w_i + \frac{2}{3}k_{i1}) \\ w_{i+1} &= w_i + \frac{1}{4}k_{i1} + \frac{3}{4}k_{i2} \end{aligned}$$

$$k_{01} = 0.25 \cdot f(t_0, w_0) = 0.25 f(0, \frac{3}{5}) = -0.25$$

$$k_{02} = 0.25 \cdot f(0 + \frac{2}{3} \cdot 0.25, \frac{3}{5} + \frac{2}{3}(-0.25)) = 0.25 \cdot f(0.1667, 0.4333) = -0.1538$$

$$w_1 = \frac{3}{5} + \frac{1}{4}(-0.25) + \frac{3}{4}(-0.1538) = \underline{0.4222}$$

$$k_{11} = 0.25 \cdot f(t_1, w_1) = 0.25 \cdot f(0.25, 0.4222) = -0.1020$$

$$k_{12} = 0.25 \cdot f(0.25 + \frac{2}{3} \cdot 0.25, 0.4222 + \frac{2}{3}(-0.1020)) = 0.25 \cdot f(0.4167, 0.3542) = 0.0441$$

$$w_2 = w_1 + \frac{1}{4}k_{11} + \frac{3}{4}k_{12} = 0.4222 + \frac{1}{4}(-0.1020) + \frac{3}{4}(0.0441) = \underline{\underline{0.4298}}$$

$$\text{abs. error} = 0.026$$

