1.5

(1)
$$\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ -2 & -7 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{array}\right] \underset{R_2 : R_2 + R_1}{\sim} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 4 & 2 & 7 & 0 \end{array}\right] \underset{R_3 : R_3 - 2R_1}{\sim}$$

$$\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 12 & -9 & 0 \end{array}\right] \underset{R_3 : R_3 + R_2}{\sim} \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0 \\ 0 & -12 & 9 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3 is free

nontrivial sol.

(2)
$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ -2 & 1 & -4 & 0 \\ 1 & 2 & 9 & 0 \end{array}\right] \underset{\substack{R_2 : R_2 + 2R_1 \\ R_3 : R_3 - R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 5 & 2 & 0 \end{array}\right] \underset{R_3 : R_3 + R_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & 12 & 0 \end{array}\right]$$

no free var.

only trivial solution

(3)
$$\left[\begin{array}{ccc|c} -3 & 5 & -7 & 0 \\ -6 & 7 & 1 & 0 \end{array}\right] \underset{R_2 : R_2 - 2R_1}{\sim} \left[\begin{array}{ccc|c} -3 & 5 & -7 & 0 \\ 0 & -3 & 15 & 0 \end{array}\right]$$

x_3 is free

nontrivial sols

(5)
$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \underset{R_2 : R_2 + 4R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \underset{\substack{R_3 : R_3 + R_2 \\ R_2 : R_2 / 3}}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\underset{R_1 : R_1 - 3R_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{cases} x_1 - 5x_3 = 0 \\ x_2 + 2x_3 = 0 \\ 0 = 0 \end{cases} \quad \begin{cases} x_1 = 5x_3 \\ x_2 = -2x_3 \\ x_3 \text{ is free} \end{cases}$$

parametric vector form:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

(7) $$\left[\begin{array}{cccc|c} 1 & 3 & -3 & 7 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{array}\right] \underset{R_1 : R_1 - 3R_2}{\sim} \left[\begin{array}{cccc|c} 1 & 0 & 9 & -8 & 0 \\ 0 & 1 & -4 & 5 & 0 \end{array}\right]$$

$$\begin{cases} x_1 + 9x_3 - 8x_4 = 0 \\ x_2 - 4x_3 + 5x_4 = 0 \end{cases}$$

basic var: x_1 ; x_2

free var: x_3 ; x_4

$$\begin{cases} x_1 = -9x_3 + 8x_4 \\ x_2 = 4x_3 - 5x_4 \end{cases}$$

general solution:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9x_3 + 8x_4 \\ 4x_3 - 5x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9x_3 \\ 4x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 8x_4 \\ -5x_4 \\ 0 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

(9) $$\left[\begin{array}{ccc|c} 2 & -8 & 6 & 0 \\ -1 & 4 & -3 & 0 \end{array}\right] \underset{\substack{R_1 : R_1 + 2R_2 \\ \text{swap}}}{\sim} \left[\begin{array}{ccc|c} 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 4x_2 - 3x_3$ $\qquad$ x_2 ; x_3 free

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4x_2 - 3x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

homogeneous?

$$\begin{bmatrix} 2 & -8 & 6 \\ -1 & 4 & -3 \end{bmatrix}\left(x_2\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}\right) = x_2\begin{bmatrix} 2 & -8 & 6 \\ -1 & 4 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} +$$

$$+ x_3\begin{bmatrix} 2 & -8 & 6 \\ -1 & 4 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = x_2\begin{bmatrix} 0 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(18) $\begin{cases} x_1 = 3x_4 \\ x_2 = 8 + x_4 \\ x_3 = 2 - 5x_4 \\ x_4 \text{ free} \end{cases}$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3x_4 \\ 8 + x_4 \\ 2 - 5x_4 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 3x_4 \\ x_4 \\ -5x_4 \\ x_4 \end{bmatrix} =$

$$= \underbrace{\begin{bmatrix} 0 \\ 8 \\ 2 \\ 0 \end{bmatrix}}_{p} + x_4 \underbrace{\begin{bmatrix} 3 \\ 1 \\ -5 \\ 1 \end{bmatrix}}_{q} = p + x_4 q$$

the sol. set is the line through p in the direction of q

(23) $a = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ $\quad b = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$ $\quad$ find equation of the line through a parallel to b

line: $x = a + tb$ where t represents a parameter

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -5 \\ 3 \end{bmatrix} \qquad \begin{cases} x_1 = -2 - 5t \\ x_2 = 3t \end{cases}$$

(24) $a = \begin{bmatrix} 5 \\ \dots \end{bmatrix}$ $\quad b = \begin{bmatrix} -4 \\ \dots \end{bmatrix}$ $\quad$ find equation of the line

$a = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ $\quad b = \begin{bmatrix} -4 \\ 9 \end{bmatrix}$ find equation of the line through a parallel to b

line: $\quad x = a + t\,b \quad$ where t represents a parameter

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} + t \begin{bmatrix} -4 \\ 9 \end{bmatrix} \qquad \begin{cases} x_1 = 5 - 4t \\ x_2 = -2 + 9t \end{cases}$$

(25) $p = \begin{bmatrix} 2 \\ -5 \end{bmatrix} \quad q = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

find a equation of a line M through p and q (M is parallel to vector $q - p$)

$$q - p = \begin{bmatrix} -3 - 2 \\ 1 - (-5) \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

$$x = p + t(q - p) = \begin{bmatrix} 2 \\ -5 \end{bmatrix} + t \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$

(26) $p = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \quad q = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$

find a equation of a line M through p and q (M is parallel to vector $q - p$)

$$q - p = \begin{bmatrix} 0 - (-6) \\ -4 - 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$$

$$x = p + t(q - p) = \begin{bmatrix} -6 \\ 3 \end{bmatrix} + t \begin{bmatrix} 6 \\ -7 \end{bmatrix}$$

$\boxed{1.7}$

(2) $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$ $\left[\begin{array}{ccc|c} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$

no free variables

$x_1 \underline{u} + x_2 \underline{v} + x_3 \underline{z} = 0 \; (*)$

homogeneous equation $(*)$ has trivial sol.

linearly independent

(7) $\left[\begin{array}{cccc|c} 1 & 4 & -3 & 0 & 0 \\ -2 & -7 & 5 & 1 & 0 \\ -4 & -5 & 7 & 5 & 0 \end{array}\right] \underset{R_2 : R_2 + 2R_1}{\sim} \left[\begin{array}{cccc|c} 1 & 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ -4 & -5 & 7 & 5 & 0 \end{array}\right] \underset{R_3 : R_3 + 4R_1}{\sim}$

$\left[\begin{array}{cccc|c} 1 & 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 11 & -5 & 5 & 0 \end{array}\right] \underset{R_3 : R_3 - 11R_2}{\sim} \left[\begin{array}{cccc|c} 1 & 4 & -3 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 6 & -6 & 0 \end{array}\right]$

3 rows = at most 3 pivots, but 4 vars

$\Downarrow$

at least 1 var is free

$A\underline{x} = 0$ has a nontrivial sol.

cols of A are linearly dependent

(9)

$v_1 = \begin{bmatrix} 1 \\ -3 \\ \vdots \end{bmatrix}$ $v_2 = \begin{bmatrix} -3 \\ 10 \\ \vdots \end{bmatrix}$ $v_3 = \begin{bmatrix} 2 \\ -7 \\ \vdots \end{bmatrix}$

$2 \qquad -6 \qquad h$

a) V_3 is in Span$\{V_1, V_2\}$ if and only if $x_1 \underline{V_1} + x_2 \underline{V_2} = \underline{V_3}$ has a solution

$$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 10 & -7 \\ 2 & -6 & h \end{bmatrix} \underset{\substack{R_2 : R_2 + 3R_1 \\ R_3 : R_3 - 2R_2}}{\sim} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & h-4 \end{bmatrix}$$

$0 = h - 4 \quad \Rightarrow \quad h = 4$

V_3 is in Span$\{V_1, V_2\}$ for $h = 4$

b) for $\{V_1, V_2, V_3\}$ to be lineary independent the equation $x_1 \underline{V_1} + x_2 \underline{V_2} + x_3 \underline{V_3} = 0$ must have only trivial solution

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -3 & 10 & -7 & 0 \\ 2 & -6 & h & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & h-4 & 0 \end{array}\right]$$

for $h = 4$, x_3 is free

$\Downarrow$

nontrivial sol.

$\{V_1, V_2, V_3\}$ lineary dependent when $h = 4$

(11)

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ -1 & -5 & 5 & 0 \\ 4 & 7 & h & 0 \end{array}\right] \underset{\substack{R_2 : R_2 + R_1 \\ R_3 : R_3 - 4R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & -5 & h+4 & 0 \end{array}\right] \underset{R_3 : R_3 - 5R_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & h-6 & 0 \end{array}\right]$$

$x_1 \underline{V_1} + x_2 \underline{V_2} + x_3 \underline{V_3} = 0$ has nontrivial sol. only when $h = 6$

linearly dependent if and only if $h = 6$

(13) $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 5 & -9 & h & 0 \\ -3 & 6 & -9 & 0 \end{array}\right] \underset{\substack{R_2 : R_2 - 5R_1 \\ R_3 : R_3 + 3R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & h-15 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\Rightarrow x_1 \underline{v_1} + x_2 \underline{v_2} + x_3 \underline{v_3} = 0$ has a free var.

$\Rightarrow$ nontrivial sol no matter what h is

$\Rightarrow$ linearly dependent for all h

(15) $\left[\begin{array}{cccc|c} 5 & 2 & 1 & -1 & 0 \\ 1 & 8 & 3 & 7 & 0 \end{array}\right]$

$\Rightarrow$ lin. dependent
$\Rightarrow$ free var.
$\Rightarrow$ 4 cols but only 2 rows

(29) $\begin{bmatrix} \square & * & * \\ 0 & \square & * \\ 0 & 0 & \square \end{bmatrix}$

(30) $\begin{bmatrix} \square & * \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \square \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\boxed{1.8}$

(19) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $y_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ $y_2 = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps e_1 into y_1 and e_2 into y_2

find images of: $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$x = \begin{bmatrix} 5 \\ -3 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 5e_1 - 3e_2$$

from linearity of T:

$$T(x) = T(5e_1 - 3e_2) = 5T(e_1) - 3T(e_2) = 5y_1 - 3y_2$$

$$= 5 \begin{bmatrix} 2 \\ 5 \end{bmatrix} - 3 \begin{bmatrix} -1 \\ 6 \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \end{bmatrix}$$

to find image $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = x_1 e_1 + x_2 e_2$$

$$T(x) = T(x_1 e_1 + x_2 e_2) = x_1 T(e_1) + x_2 T(e_2) = x_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$$

37 define $f : \mathbb{R} \to \mathbb{R}$
$f(x) = mx + b$

a) show that f is lin. transformation when $b = 0$

$b = 0 \quad \Rightarrow \quad f(x) = mx$

for all x, y in $\mathbb{R}$ and scalars c and d

$f(cx+dy) = m(cx+dy) = mcx + mdy = c(mx) + d(my)$
$= c \cdot f(x) + d \cdot f(y)$

f is linear

b) find a property of a linear transformation that is violated when $b \neq 0$

when $f(x) = mx + b \quad (b \neq 0)$

$f(0) = b \neq 0 \quad \leftarrow$ f not linear

or alternatively:

$f(2x) = m(2x) + b \qquad 2f(x) = 2mx + 2b$

$b \neq 0$ then $f(2x) \neq 2f(x)$

f is not linear transform.

39 let $T: R^n \to R^m$
let $\{v_1, v_2, v_3\}$ be a lin. dep. set in R^n

why set $\{T(v_1), T(v_2), T(v_3)\}$ is lin. dep.?

- suppose $\{v_1, v_2, v_3\}$ is lin. dep.
- then there exist scalars c_1, c_2, c_3 not all zero such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$
- then $T(c_1v_1 + c_2v_2 + c_3v_3) = T(0) = 0$
- since not all the weights are zero, $\{T(v_1), T(v_2), T(v_3)\}$

is lin. dep. set. $\square$

$\boxed{1.9}$

(3) $T(e_1) = -e_2 \qquad T(e_2) = e_1$

$$A = [-e_2 \;\; e_1] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

(9)

horizontal sher maps e_1 into e_1 and then the reflection in the line $x_2 = -x_1$ maps e_1 into $-e_2$

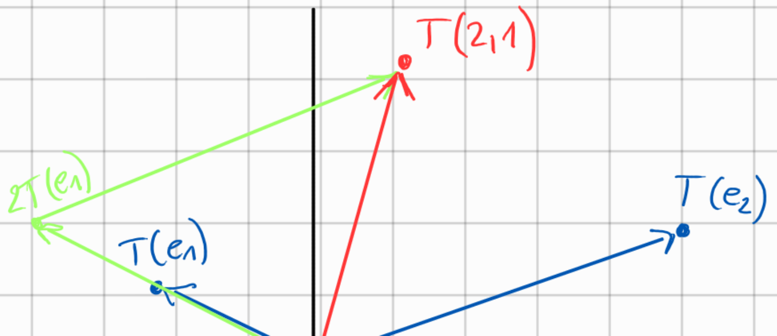
the horizontal sher maps e_2 into $e_2 - 3e_1$ and then the reflection in the line $x_2 = -x_1$ is a linear transformation. So the image of $e_2 - 3e_1$ is the same linear combination of the images of e_2 and e_1 namely:

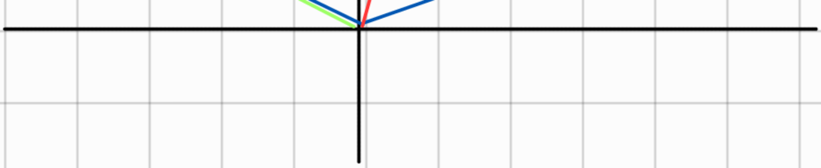
$$-e_1 - 3(-e_2) = -e_1 + 3e_2$$

$$e_1 \to e_1 \to -e_2$$
$$e_2 \to e_2 - 3e_1 \to -e_1 + 3e_2$$

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 3 \end{bmatrix}$$

(13)





sketch $T(2,1)$

since $(2,1) = 2e_1 + e_2 \to 2T(e_1) + T(e_2)$

(21) $T_1(x_1, x_2) = (x_1 + x_2, 4x_1 + 5x_2)$

find x such that $T(x) = (3, 8)$

$$T(x) = \begin{bmatrix} x_1 + x_2 \\ 4x_1 + 5x_2 \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

solve:

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & 5 & 8 \end{array}\right] \underset{R_2 : R_2 - 4R_1}{\sim} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -4 \end{array}\right] \underset{R_1 : R_1 - R_2}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -4 \end{array}\right]$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$

(35) $T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$

$$A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$$

→ the cols of A are linearly dependant (more cols than rows), so T is not one-to-one

→ A has pivot in each row, rows span $\mathbb{R}^2$

→ T maps $\mathbb{R}^3$ into $\mathbb{R}^2$

2.1

(1) $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ $\quad -2A = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix}$

$B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$ $\quad B - 2A = \begin{bmatrix} 3 & -5 & 3 \\ -7 & 2 & -7 \end{bmatrix}$
$(B + (-2A))$

$C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ $\quad AC =$ not defined (size $A \neq$ size C)

$D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$ $\quad CD = \begin{bmatrix} 1\cdot 3 + 2\cdot(-1) & 1\cdot 5 + 2\cdot 4 \\ -2\cdot 3 + 1\cdot(-1) & -2\cdot 5 + 1\cdot 4 \end{bmatrix}$

$$CD = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$$

(7) A size 5×3
AB size 5×7
B size ? $\quad \rightarrow$ B size 3×7

(8) B # rows ?
BC size 3×4

B must match # rows of AB, so B has 3 rows

(9) $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$ $\quad$ what k will make $AB = BA$

$$AB = \begin{bmatrix} 2\cdot 4 + 5\cdot 3 & 2(-5) + 5k \\ -3\cdot 4 + 1\cdot 3 & (-3)\cdot(-5) + k \end{bmatrix} = \begin{bmatrix} 23 & -10 + 5k \\ -9 & 15 + k \end{bmatrix}$$

$BA = \begin{bmatrix} 23 & 15 \\ 6 - 3k & 15 + k \end{bmatrix}$ $\quad AB = BA$ iff
$-10 + 5k = 15$ and

$6-3k=-9$

$$\Downarrow$$

$$k=5$$

(10) $AB=\begin{bmatrix}1 & -7\\ -2 & 14\end{bmatrix}$ $AC:\begin{bmatrix}1 & -7\\ -2 & 14\end{bmatrix}$ $AB=AC$ ✓

(25) $\begin{bmatrix}-1 & 2 & -1\\ 6 & -9 & 3\end{bmatrix}=AB=[Ab_1 \;\; Ab_2 \;\; Ab_3]$

$Ax=\begin{bmatrix}-1\\ 6\end{bmatrix}$ $[A \;\; Ab_1]\sim\left[\begin{array}{cc|c}1 & -2 & -1\\ 1 & 5 & 6\end{array}\right]\sim\left[\begin{array}{cc|c}1 & 0 & 7\\ 0 & 1 & 4\end{array}\right]$

$A=\begin{bmatrix}1 & -2\\ 1 & 5\end{bmatrix}$ $b_1=\begin{bmatrix}7\\ 4\end{bmatrix}$

$Ax=\begin{bmatrix}2\\ -9\end{bmatrix}$ $[A \mid Ab_2]\sim\left[\begin{array}{cc|c}1 & -2 & 2\\ 1 & 5 & -9\end{array}\right]\sim\left[\begin{array}{cc|c}1 & 0 & -8\\ 0 & 1 & 5\end{array}\right]$

$$b_2=\begin{bmatrix}-8\\ 5\end{bmatrix}$$

(27) $B=[b_1 \;\; b_2 \;\; b_3]$ 3^{rd} col of $AB=Ab_3$

$b_3=b_1+b_2$ $Ab_3=A(b_1+b_2)=Ab_1+Ab_2$

3^{rd} col is a sum of first 2 cols of AB

(28) $B=[b_1 \;\; 0 \;\; b_3]$ 2^{nd} col of AB ?

$\Rightarrow$ all zeros $\Rightarrow Ab_2=A0=0$

(29) last col of AB are all 0s, but B has no col of 0s

what we know about cols of A?

- let b_p be the last col of B
- last col of AB are 0s
- $Ab_p = 0$ is a linear dependence relation among the cols of A
- cols of A are linearly dependent