Bonus 5 3.8b

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x	0.25	0.5	0.75	1.0	0.8
y	-0.07	-0.03	0.34	1.10	—

$y = f(x)$

$$P3(x) = f(x_0) + f(x_0,x_1)\cdot(x-x_0) + f(x_0,x_1,x_2)\cdot(x-x_0)(x-x_1) + f(x_0,x_1,x_2,x_3)\cdot(x-x_0)(x-x_1)(x-x_2)$$

$$f(x_0,x_1) = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{-0.03-(-0.07)}{0.5-0.25} = \frac{4}{25} = 0.16$$

$$f(x_1,x_2) = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{0.34-(-0.03)}{0.75-0.5} = \frac{37}{25} = 1.48$$

$$f(x_2,x_3) = \frac{f(x_3)-f(x_2)}{x_3-x_2} = \frac{1.10-0.34}{1.0-0.75} = 3.04$$

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2)-f(x_0,x_1)}{x_2-x_0} = \frac{1.48-0.16}{0.75-0.25} = \frac{66}{25} = 2.64$$

$$f(x_1,x_2,x_3) = \frac{f(x_2,x_3)-f(x_1,x_2)}{x_3-x_1} = \frac{3.04-1.48}{1.0-0.5} = 3.12$$

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0} = \frac{3.12-2.64}{1.0-0.25} = 0.64$$

$$P3(x) = -0.07 + 0.16(x-0.25) + 2.64\cdot(x-0.25)(x-0.5) + 0.64\cdot(x-0.25)(x-0.5)(x-0.75)$$

at $x = 0.8$

$$P_3(0.8) = -0.07 + 0.16(0.8 - 0.25) + 2.64 \cdot (0.8 - 0.25)(0.8 - 0.5) + \\ + 0.64 \cdot (0.8 - 0.25)(0.8 - 0.5)(0.8 - 0.75) = 0.4588$$

(just to check)

$$P_3(0.8) = -0.07 + (x - 0.25)\left(0.16 + (x - 0.5)\left(2.64 + (x - 0.75) \cdot 0.64\right)\right) \\ = 0.4588 \quad \checkmark$$

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x	0.25	0.5	0.75	1.0	1.1	0.8
y	-0.07	-0.03	0.34	1.10	2.0	—

$$y = f(x)$$

$$P_4(x) = f(x_0) + f(x_0, x_1) \cdot (x - x_0) + f(x_0, x_1, x_2) \cdot (x - x_0)(x - x_1) + \\ + f(x_0, x_1, x_2, x_3) \cdot (x - x_0)(x - x_1)(x - x_2) \\ + f(x_0, x_1, x_2, x_3, x_4) \cdot (x - x_0)(x - x_1)(x - x_2)(x - x_3)$$

$$P_4(x) = f(x_0) + (x - x_0)\left(f(x_0, x_1) + (x - x_1)\left(f(x_0, x_1, x_2) + (x - x_2)\left(f(x_0, x_1, x_2, x_3) + \right.\right.\right. \\ \left.\left.\left.(x - x_3) \cdot f(x_0, x_1, x_2, x_3, x_4)\right)\right)\right)$$

$$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3} = \frac{2 - 1.1}{1.1 - 1} = 9$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2} = \frac{9 - 3.04}{1.1 - 0.75} = \frac{596}{35} \approx 17.029$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1} = \frac{\frac{596}{35} - 3.12}{1.1 - 0.5} = \frac{2434}{105} \approx 23.181$$

$$f(x_0,x_1,x_2,x_3,x_4) \approx \frac{f(x_1,x_2,x_3,x_4)-f(x_0,x_1,x_2,x_3)}{x_4-x_0} = \frac{\frac{2434}{105}-0.64}{1.1-0.25} \approx \underline{26.519}$$

$$P_4(0.8) = -0.07 + 0.16(0.8-0.25) + 2.64\cdot(0.8-0.25)(0.8-0.5) +$$
$$+ 0.64\cdot(0.8-0.25)(0.8-0.5)(0.8-0.75) +$$
$$+ 26.519\cdot(0.8-0.25)(0.8-0.5)(0.8-0.75)(0.8-1)$$
$$= \underline{0.4151}$$