(1)

a) satisfiable
b) tautology
c) satisfiable
d) satisfiable

(2) a) $a \models \underbrace{(b \wedge c) \to (a \to b)}_{s}$

| a | b | c | $b \wedge c$ | $a \to b$ | s |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 |

statement $(b \wedge c) \to (a \to b)$ is a tautology, therefore the inference is <u>valid</u>

b) $\{p \vee q, q \vee r\} \models (p \wedge r) \to \neg q$

| p | q | r | $p \vee q$ | $q \vee r$ | $p \wedge r$ | $\neg q$ | $(p \wedge r) \to \neg q$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

not valid

counterexample: $p, q, r$ are T

$\{T, T\} \vDash F$

c) $\{p \to q,\ q \to r\} \vDash p \to r$

| $p$ | $q$ | $r$ | $p \to q$ | $q \to r$ | $p \to r$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |

valid

3

- $a \Leftrightarrow b$
- $\neg b \Leftrightarrow c$
- $c \Leftrightarrow \left[\underbrace{(a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c)}_{s}\right]$

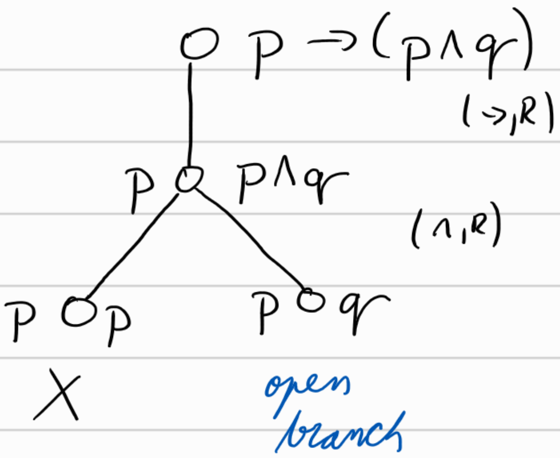
| $a$ | $b$ | $c$ | $a \Leftrightarrow b$ | $\neg b \Leftrightarrow c$ | $(a \wedge b \wedge \neg c)$ | $(a \wedge \neg b \wedge c)$ | $(\neg a \wedge b \wedge c)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

| s | c ⇔ s |
|---|---|
| 0 | 0 |
| 1 | 0 |
| 1 | 1 |
| 0 | 1 |
| 1 | 1 |
| 0 | 1 |
| 0 | 0 |
| 0 | 1 |

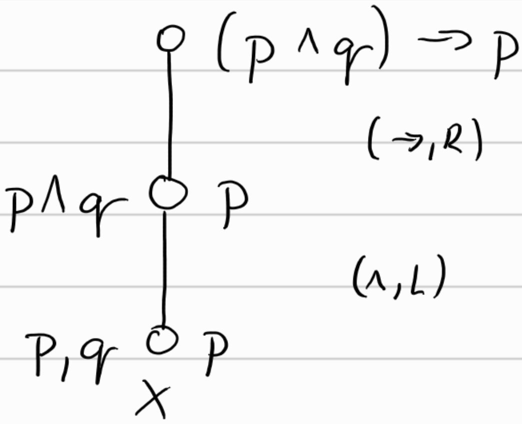
There are no possible configuration, where all three sentences are either true or false.

(4) a)



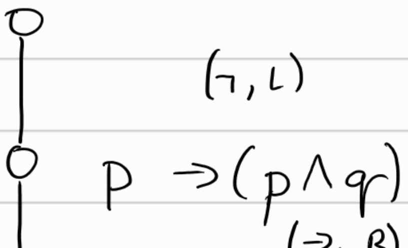

not tautology

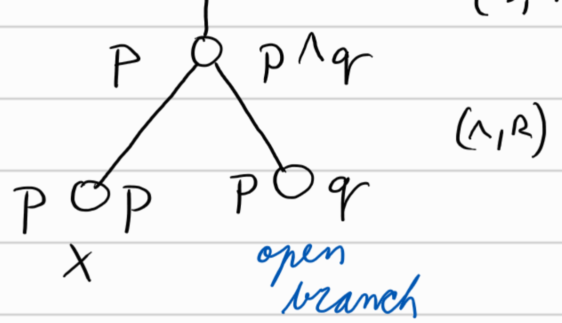
b)




tautology

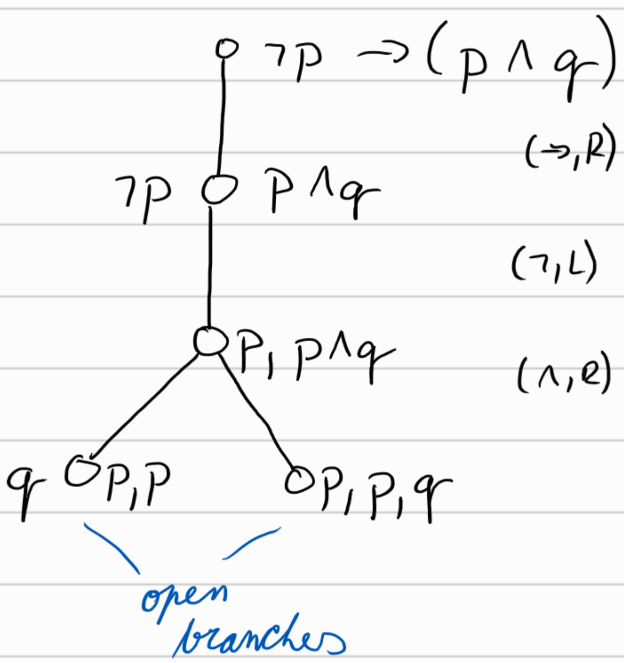
c) $\neg(p \Rightarrow (p \wedge q))$





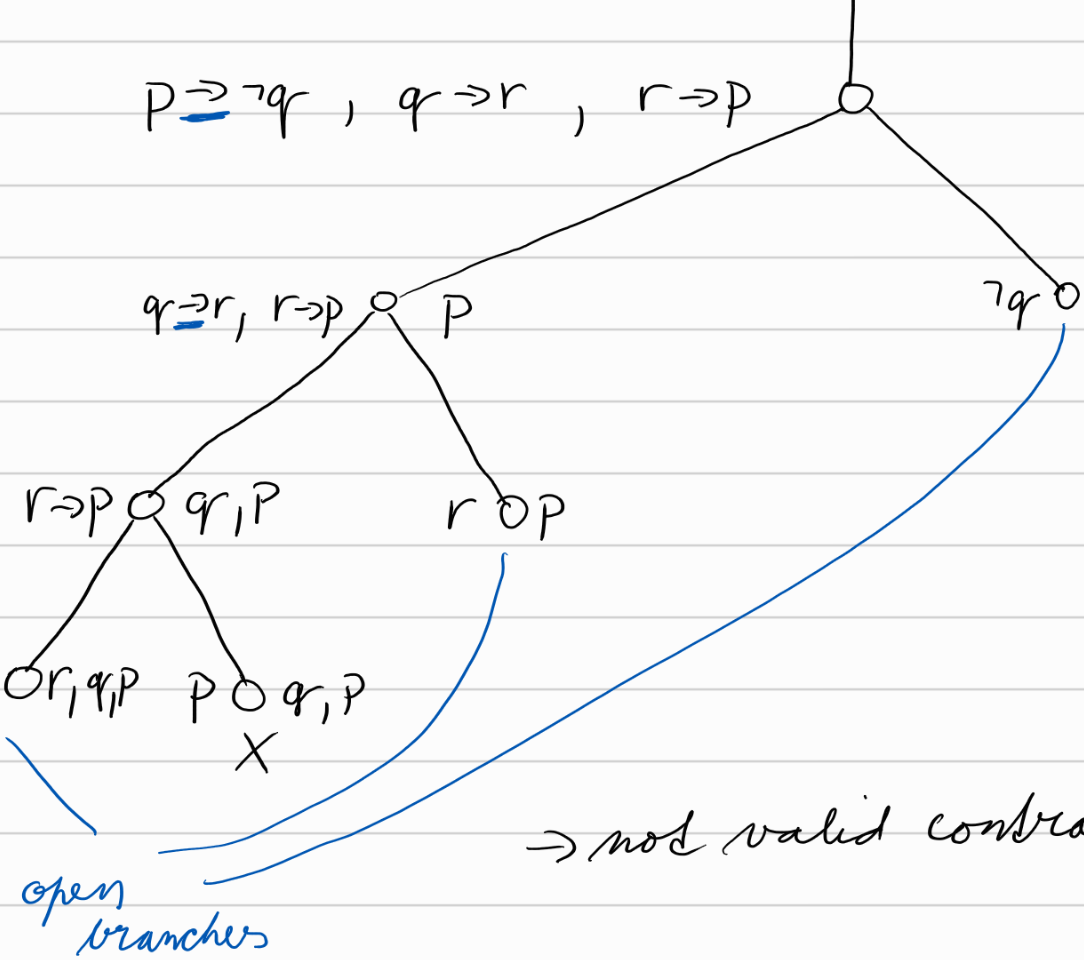

not tautology

d)



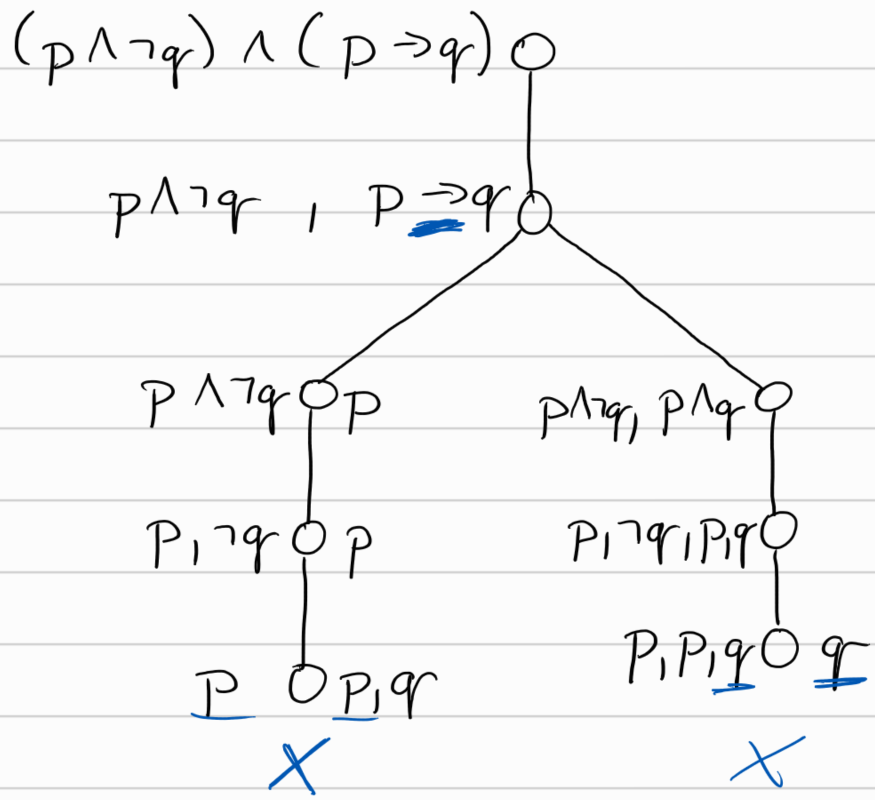

not tautology

5) a) $(p \to \neg q) \wedge (q \to r) \wedge (r \to p)$



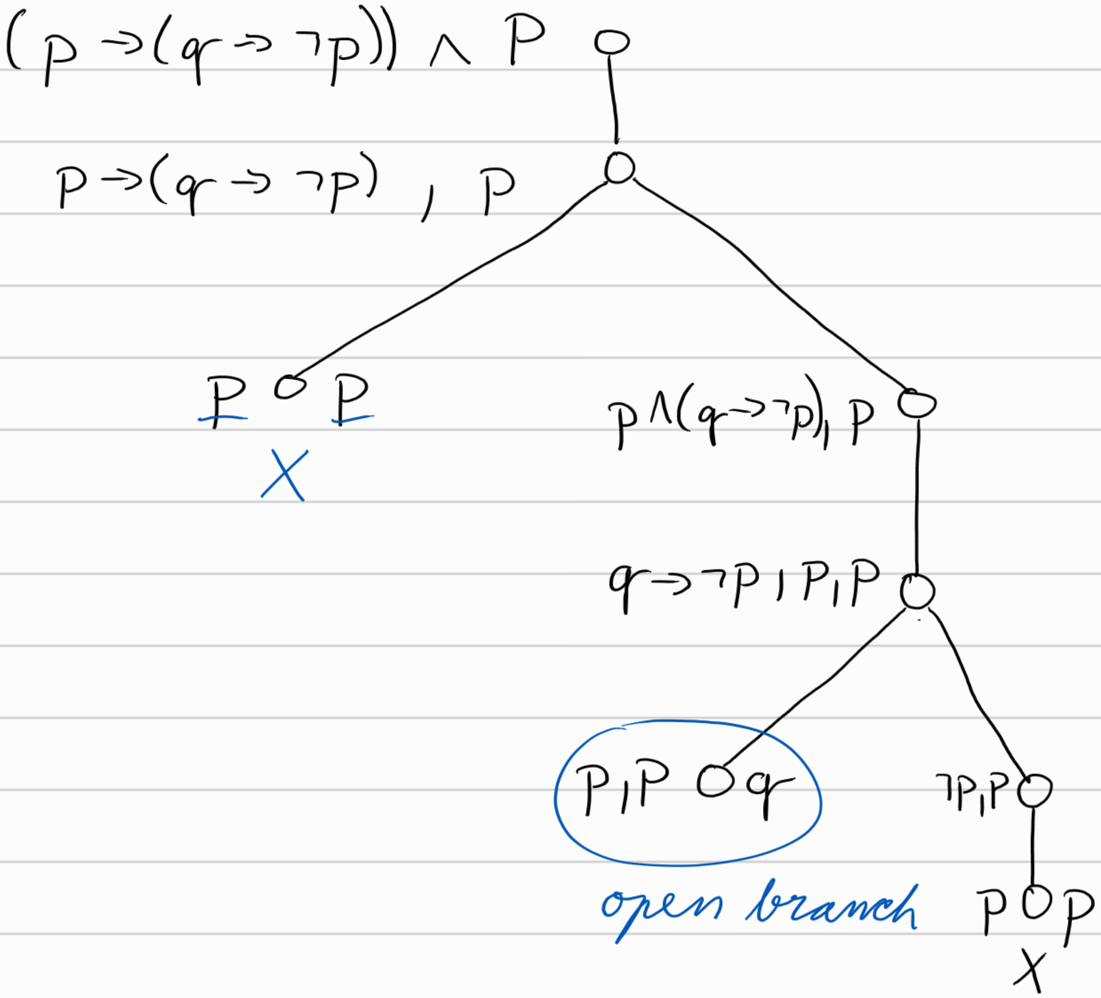

→ not valid contradiction

b)




valid contradiction

c)



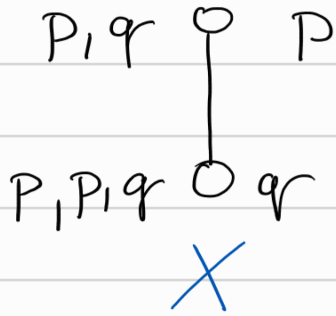

not valid contradiction

6. missing brackets

possible corrections:

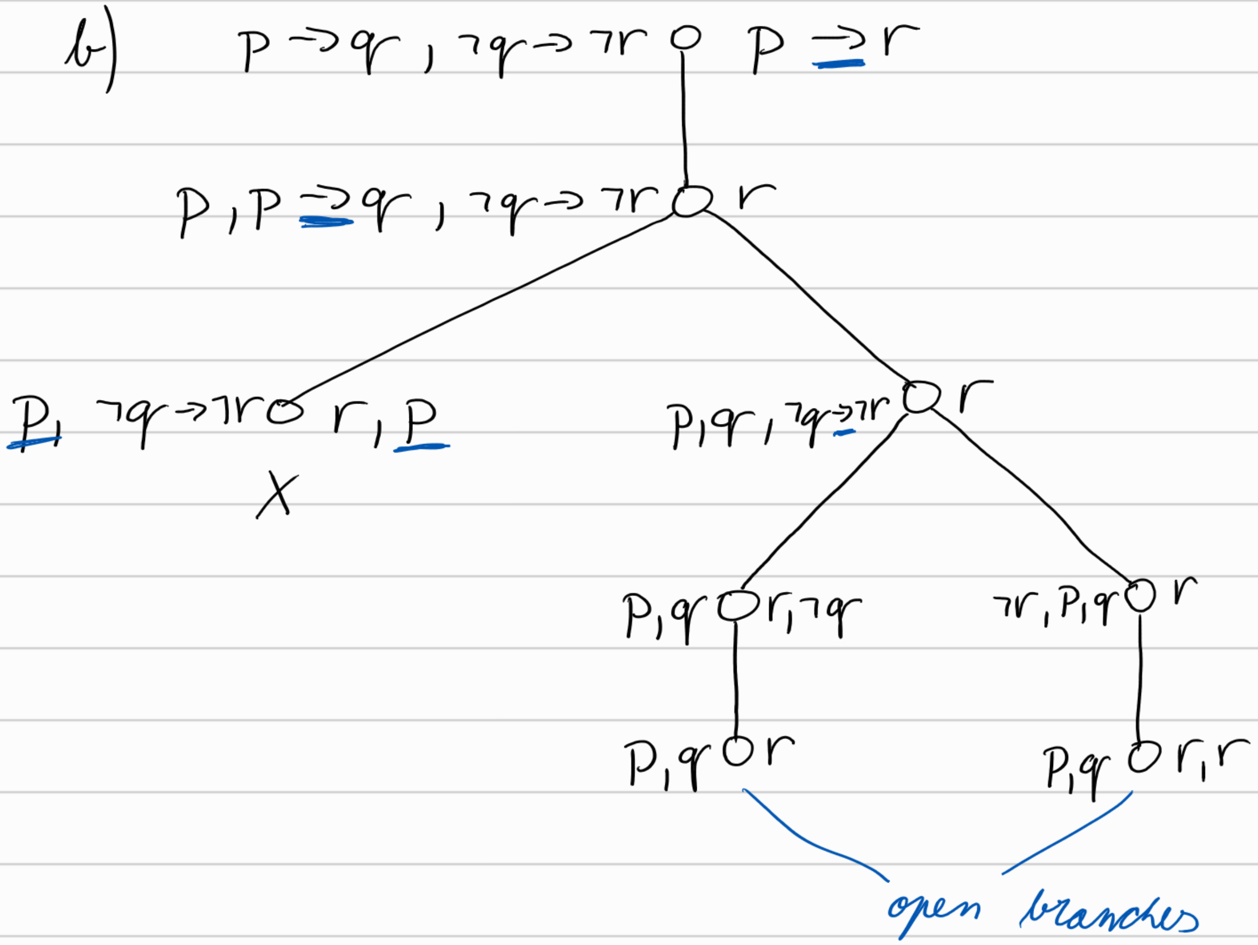
- $((p \to q) \wedge (q \to p)) \to (p \vee \neg q)$
- $(p \to q) \wedge ((q \to p) \to (p \vee \neg q))$

7. a)




valid

b)




not valid

c) $p \wedge q, \neg q$ ○ $r$

$p \wedge q \circ r, q$

$p, \cancel{q} \circ r, \cancel{q}$

$\times$

valid

8

a)

| | | |
|---|---|---|
| 1 | $\neg\neg p$ | |
| 2 | $\quad \neg p$ | ass |
| 3 | $\quad \bot$ | $E_{\neg}(1,2)$ |
| 4 | $p$ | $I_{\neg}(2,3)$ |

b)

| | | |
|---|---|---|
| 1 | $p \to \neg q$ | |
| 2 | $q$ | |
| 3 | $\quad p$ | ass |
| 4 | $\quad \neg q$ | $E_{\to}(1,3)$ |
| 5 | $\quad \bot$ | $E_{\neg}(2,4)$ |
| 6 | $\neg p$ | $I_{\neg}(3\text{-}5)$ |

c)

| | | |
|---|---|---|
| 1 | $\quad \neg(p \vee \neg p)$ | ass |
| 2 | $\quad\quad p$ | ass |
| 3 | $\quad\quad p \vee \neg p$ | $I_{\vee}(2)$ |
| 4 | $\quad\quad \bot$ | $E_{\neg}(1,3)$ |
| 5 | $\quad \neg p$ | $I_{\neg}(2\text{-}4)$ |
| 6 | $\quad p \vee \neg p$ | $I_{\vee}(5)$ |
| 7 | $\quad \bot$ | $E(1,6)$ |
| 8 | $p \vee \neg p$ | $I_{\neg}(1\text{-}7)$ |

d)

| | | |
|---|---|---|
| 1 | $p \vee \neg q$ | |
| 2 | $q$ | ass |
| 3 | $\neg p$ | ass |
| 4 | $p$ | ass |
| 5 | $\bot$ | $E_\neg(3,4)$ |
| 6 | $\neg q$ | ass |
| 7 | $\bot$ | $E_\neg(2,6)$ |
| 8 | $\bot$ | $E_\vee(1,4\text{-}7)$ |
| 9 | $p$ | |
| 10 | $q \to p$ | $I_\to(2,9)$ |

e)

| | | |
|---|---|---|
| 1 | $p \to q$ | ass |
| 2 | $\neg q$ | ass |
| 3 | $p$ | ass |
| 4 | $q$ | $E_\to(1,3)$ |
| 5 | $\bot$ | $E_\neg(2,4)$ |
| 6 | $\neg p$ | $I_\neg(2\text{-}5)$ |
| 7 | $\neg q \to \neg p$ | $I_\to(2,6)$ |

f)

| | | |
|---|---|---|
| 1 | $a \vee \neg a$ | ass |
| 2 | $\neg p \to \neg q$ | ass |
| 3 | $q$ | ass |
| 4 | $\neg p$ | ass |
| 5 | $\neg q$ | $E_\to(2,4)$ |
| 6 | $\bot$ | $E_\neg(3\text{-}5)$ |
| 7 | $p$ | $I_\neg(4\text{-}6)$ |
| 8 | $q \to p$ | $I_\to(3,7)$ |
| 9 | $(\neg p \to \neg q) \to (q \to p)$ | $I_\to(2,8)$ |