2.2

(1)

$$\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{1}{16-15} \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$$

(5)

$$\begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(7) $A\underline{x} = \underline{b}$ $\quad A = \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix}$ $\quad \underline{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$$\underline{x} = A^{-1}\underline{b} = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -18 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(41) $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \underset{\substack{R_2 : R_2 + 3R_1 \\ R_3 : R_3 - 2R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right] \underset{R_3 : R_3 + 3R_2}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \underset{\substack{R_1 : R_1 + R_3 \\ R_2 : R_2 + R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \underset{R_3 : R_3/2}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

2.3

(3) invertible?

$$\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

3×3 matrix has 3 pivot pos. and hence is invertible

(5) invertible?

$$\begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -5 \\ -4 & -9 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 3 & -5 \\ 0 & -9 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix is not invertible because it is not row equivalent to I_n

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3.1

expanding 1^{st} row:

(1)

$$\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = 3\cdot(-1)^2\cdot\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix} + 0\cdot(-1)^3\begin{bmatrix} 2 & 2 \\ 0 & -1 \end{bmatrix}$$

$$+ 4\cdot(-1)^4\cdot\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} =$$

$$= 3\cdot\begin{bmatrix} 3 & 2 \\ 5 & -1 \end{bmatrix} + 4\cdot\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} =$$

$$= 3\cdot(-3-10) + 4\cdot(10-0) = 3(-13)+4(10)$$

$$= \underline{\underline{1}}$$

expanding 2^{nd} col:

$$\begin{bmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{bmatrix} = (-1)^{1+2} \cdot 0 \begin{bmatrix} 2 & 2 \\ 0 & -1 \end{bmatrix} + (-1)^{2+2} \cdot 3 \begin{bmatrix} 3 & 4 \\ 0 & -1 \end{bmatrix} + (-1)^{3+2} \cdot 5 \cdot \begin{bmatrix} 3 & 4 \\ 2 & 2 \end{bmatrix}$$

$$= 3(-3) - 5(-2) = \underline{\underline{1}}$$

3 expansion 1^{st} row:

$$\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix} = (-1)^{1+1} \cdot 2 \cdot \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} + (-1)^{1+2} \cdot (-2) \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} + (-1)^{1+3} \cdot 3 \cdot \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= 2(-7) + 2(-5) + 3(8) = \underline{\underline{0}}$$

expansion 2^{nd} col:

$$\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix} = (-1)^{1+2} \cdot (-2) \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} + (-1)^{2+2} \cdot 1 \cdot \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} + (-1)^{3+2} \cdot 3 \cdot \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

$$= 2(-5) + 1(-5) - 3(-5) = \underline{\underline{0}}$$

5 expand 1^{st} row:

$$\begin{bmatrix} 2 & 3 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{bmatrix} = (-1)^{2} \cdot 2 \cdot \begin{bmatrix} 0 & 3 \\ 1 & 5 \end{bmatrix} + (-1)^{3} \cdot 3 \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix} + (-1)^{4} \cdot (-3) \begin{bmatrix} 4 & 0 \\ 6 & 1 \end{bmatrix} =$$

$$= 2(-3) - 3(2) - 3(4) = \underline{\underline{-24}}$$

9

$$\begin{bmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{bmatrix} = (-1)^4 \cdot 3 \cdot \begin{bmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{bmatrix} = 3 \cdot (-1)^4 \cdot 5 \cdot \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} =$$

$$= 15(1) = \underline{\underline{15}}$$

(14)

$$\begin{bmatrix} 6 & 0 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix} = (-1)^{3+5} \cdot 1 \cdot \begin{bmatrix} 6 & 0 & 2 & 4 \\ 9 & 0 & -4 & 1 \\ 2 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 \end{bmatrix} =$$

$$= 1 \cdot (-1)^{3+1} \cdot 2 \cdot \begin{bmatrix} 0 & 2 & 4 \\ 0 & -4 & 1 \\ 2 & 3 & 2 \end{bmatrix} = 2 \cdot (-1)^{3+1} \cdot 2 \cdot \begin{bmatrix} 2 & 4 \\ -4 & 1 \end{bmatrix} =$$

$$= 4(18) = \underline{\underline{72}}$$

(19) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$

$\begin{bmatrix} c & d \\ a & b \end{bmatrix} = cb - ad = -(ad - bc)$

(22) $\begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix} = d(a+kc) - c(b+kd) =$

$= ad + \cancel{kcd} - bc - \cancel{kcd} = ad - bc$ same

$\boxed{3.2}$

(5)

$$\begin{bmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ -2 & -8 & 7 \end{bmatrix} \underset{\substack{R_2 : R_2 + R_1 \\ R_3 : R_3 + 2R_1}}{\sim} \begin{bmatrix} 1 & 5 & -4 \\ 0 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \underset{R_3 : R_3 - 2R_2}{\sim} \begin{bmatrix} 1 & 5 & -4 \\ 0 & 1 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

$$= 1 \cdot 1 \cdot (-3) = \underline{\underline{-3}}$$

(9)

$$\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 0 & 5 & 3 \\ 3 & -3 & -2 & 3 \end{bmatrix} \underset{\substack{R_3 : R_3 + R_1 \\ R_4 : R_4 - 3R_1}}{\sim} \begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 7 & 3 \end{bmatrix} \underset{R_3 : R_3 + R_2}{\sim} \begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 7 & 3 \end{bmatrix}$$

$$\underset{R_4 : R_4 - R_3}{\sim} \begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 0 & -4 \end{bmatrix} = 7 \cdot (-4) = \underline{\underline{-28}}$$

(11)

$$\begin{bmatrix} 3 & 4 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 6 & 8 & -4 & -1 \end{bmatrix} \underset{R_4 : R_4 - 2R_1}{\sim} \begin{bmatrix} 3 & 4 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$$= (-1)^3 \cdot 4 \cdot \begin{bmatrix} 3 & 1 & -3 \\ -6 & -4 & 3 \\ 0 & 2 & 1 \end{bmatrix} = -4 \cdot \begin{bmatrix} 3 & 1 & -3 \\ 0 & -2 & -3 \\ 0 & 2 & 1 \end{bmatrix} =$$

$$= -4 \cdot (-1)^2 \cdot 3 \cdot \begin{bmatrix} -2 & -3 \\ 2 & 1 \end{bmatrix} = -12(4) = \underline{\underline{-48}}$$

(12) $\begin{bmatrix} -2 & 6 & 0 & 9 \\ 3 & 4 & 8 & 2 \\ 4 & 3 & 0 & 1 \\ 3 & 1 & 2 & -1 \end{bmatrix} \underset{R_2 : R_2 - 4R_4}{\sim} \begin{bmatrix} -2 & 6 & 0 & 9 \\ -9 & 0 & 0 & 6 \\ 4 & 3 & 0 & 1 \\ 3 & 1 & 2 & -1 \end{bmatrix} =$

$= (-1)^7 \cdot 2 \cdot \begin{bmatrix} -2 & 6 & 9 \\ -9 & 0 & 6 \\ 4 & 3 & 1 \end{bmatrix} \underset{R_1 : R_1 - 2R_3}{=} -2 \cdot \begin{bmatrix} -10 & 0 & 7 \\ -9 & 0 & 6 \\ 4 & 3 & 1 \end{bmatrix} =$

$= -2 \cdot (-1)^5 \cdot 3 \cdot \begin{bmatrix} -10 & 7 \\ -9 & 6 \end{bmatrix} = 6(-60+63) = 6 \cdot 3 = \underline{\underline{18}}$

(17) $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 7$

$\begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix} \underset{R_1 : R_1 - R_2}{\sim} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \underline{\underline{7}}$

(21)

$\begin{bmatrix} 1 & 3 & 6 \\ 2 & 4 & 7 \\ 0 & 5 & 8 \end{bmatrix} \underset{R_2 : R_2 - 2R_1}{\sim} \begin{bmatrix} 1 & 3 & 6 \\ 0 & -2 & -5 \\ 0 & 5 & 8 \end{bmatrix} = (-1)^2 \cdot 1 \cdot \begin{bmatrix} -2 & -5 \\ 5 & 8 \end{bmatrix} =$

$= 1(9) = \underline{\underline{9}} \quad \neq 0$ invertible

(22) $\begin{bmatrix} 4 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 0 \end{bmatrix} = (-1)^5 \cdot 1 \cdot \begin{bmatrix} 4 & 5 \end{bmatrix} =$

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & -4 & 3 \end{bmatrix} \underset{R_3:R_3-3R_2}{\sim} \begin{bmatrix} 3 & 2 & 1 \\ -8 & -10 & 0 \end{bmatrix} \quad \begin{bmatrix} -8 & -10 \end{bmatrix}$$

$$=(-1)(0)=\underline{\underline{0}}=0$$ not invertible

(24) $$\begin{bmatrix} 4 & -6 & -3 \\ 6 & 0 & -5 \\ 2 & 6 & -2 \end{bmatrix} \underset{R_3:R_3+R_1}{\sim} \begin{bmatrix} 4 & -6 & -3 \\ 6 & 0 & -5 \\ 6 & 0 & -5 \end{bmatrix} = (-1)^3 \cdot (-6) \cdot \begin{bmatrix} 6 & -5 \\ 6 & -5 \end{bmatrix} = 6 \cdot 0 = 0$$

lineary dependent

(25) $$\begin{bmatrix} 7 & -8 & 7 \\ -4 & 5 & 0 \\ -6 & 7 & -5 \end{bmatrix} = (-1)^4 \cdot 7 \cdot \begin{bmatrix} -4 & 5 \\ -6 & 7 \end{bmatrix} + (-1)^6 \cdot (-5) \cdot \begin{bmatrix} 7 & 8 \\ -4 & -5 \end{bmatrix} =$$

$$= 7(2) + (-5) \cdot (-3) = 14 + 15 = \underline{\underline{29}}$$

linary independent

$\boxed{\text{EXAM}}$

(1) $$\begin{bmatrix} 3 & -6 & 2 \\ 1 & -2 & 1 \\ -2 & 4 & -2 \end{bmatrix} \underset{R_3:R_3+2R_2}{\sim} \begin{bmatrix} 3 & -6 & 2 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \text{free var}$$

lineary dependent
not invertible

(2) $$T(e_1) = T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1/2 \end{bmatrix}$$

$$T(e_2) = T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 2 & 1/2 \\ 1/2 & 1 \end{bmatrix} \qquad P = \left(5/2 \;,\; 3/2\right)$$

(3)
$$\left[\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\substack{R_1 \leftrightarrow R_2 \\ R_3 : R_3 + 2R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 2 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right] \underset{R_2 : R_2 - 3R_1}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right] \underset{R_2 : R_2 + R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right] \underset{R_1 : R_1 + R_2}{\sim}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A\underline{x} = \underline{b} \qquad \underline{x} = A^{-1}\underline{b}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 8 \end{bmatrix}$$

(4)
$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \underset{R_1 : R_1 - R_2}{\sim} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} =$$

$$= (-1)^4 \cdot 1 \cdot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ \end{bmatrix} = (-1)^2 \cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + (-1)^4 \cdot 1 \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$$

$$= 1(1) + 1(1) = \underline{\underline{2}}$$