9.1

(1) $\left\{ \frac{2n^2}{n^2+1} \right\} = \left\{ 1, \frac{8}{5}, \frac{9}{5}, \frac{32}{17}, \dots \right\}$

bounded, positive, increasing, converges to 2

(2) $\left\{ \frac{2n}{n^2+1} \right\} = \left\{ 1, \frac{4}{5}, \frac{3}{5}, \frac{8}{17} \dots \right\}$

bounded, positive, decreasing, converges to 0

(4) $\left\{ \sin \frac{1}{n} \right\} = \left\{ \sin 1, \sin \frac{1}{2}, \sin \frac{1}{3}, \sin \frac{1}{4} \dots \right\}$

bounded, positive, decreasing, converges to 0

(5) $\left\{ \frac{n^2-1}{n} \right\} = \left\{ 0, \frac{3}{2}, \frac{8}{3}, \dots \right\}$

bounded below, positive, increasing, diverges to ∞

(8) $\left\{ \frac{(-1)^n n}{e^n} \right\} = \left\{ -\frac{1}{e}, \frac{2}{e^2}, -\frac{3}{e^3}, \dots \right\}$

bounded, alternating, converges to 0

(9) $\left\{ \frac{2^n}{n^n} \right\} = \left\{ 2, 1, \frac{8}{27}, \frac{16}{256}, \dots \right\}$

bounded, decreasing, positive, converges to 0

(10) $\left\{\frac{(n!)^2}{(2n)!}\right\} = \left\{\frac{1}{2}, \frac{1}{6}, \frac{1}{20}, \dots\right\}$

bounded, positive, decreasing, converges to 0

(13) $\{1, 1, -2, 3, 3, -4, 5, 5, -6, \dots\}$ is divergent

(14)

$$\lim \frac{5-2n}{3n-7} = \lim \frac{\frac{5}{n}-2}{3-\frac{7}{n}} = -\frac{2}{3}$$

(15)

$$\lim \frac{n^2-4}{n+5} = \lim \frac{1-\frac{4}{n^2}}{\frac{1}{n}+\frac{5}{n^2}} = \infty$$

(17)

$$\lim (-1)^n \frac{n}{n^3+1} = \lim \underbrace{(-1)^n}_{\text{depends } n \text{ even/odd}} \underbrace{\left(\frac{\frac{1}{n^2}}{1+\frac{1}{n^3}}\right)}_{0} = \underline{\underline{0}}$$

(19)

$$\lim \frac{e^n-e^{-n}}{e^n+e^{-n}} = \lim \frac{e^n-\frac{1}{e^n}}{e^n+\frac{1}{e^n}} = \lim \frac{\frac{e^{2n}-1}{e^n}}{\frac{e^{2n}+1}{e^n}} =$$

$$= \lim \frac{\cancel{e^n}(e^{2n}-1)}{\cancel{e^n}(e^{2n}+1)} = \lim \frac{1-\frac{1}{e^{2n}}}{1+\frac{1}{e^{2n}}} = \frac{1-0}{1+0} = \underline{\underline{1}}$$

(22)

$$\lim \frac{n}{\ln(n+1)} \overset{H}{=} \lim \frac{1}{\frac{1}{n+1}} =$$

$$\frac{d}{dx}\ln(x+1) = \frac{1}{x+1}$$

$$= \lim (n+1) = \underline{\underline{\infty}}$$

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$$\lim\left(\sqrt{n+1}-\sqrt{n}\right)=0$$

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$$\lim\left(\frac{n-1}{n+1}\right)^{n}$$