Bonus H4c (2.4c)

Detvan
i630211

AB2: Adams Bashfor

$$w_{i+1} = w_i + (h/2)(3f(t_i, w_i) - f(t_{i-1}, w_{i-1}))$$

for $h = 0.5$:

$$\dot{y} = (y+t)^2 - 1$$

up to $t = 1$

$w_0 = y(0) = 2/3$

$h = 0.5$

bootstrapping using RK4:

$t_0 = 0$

$$k_1 = h f(t_0, y_0) = 0.5 \cdot f(0, 2/3) = 0.5 \cdot [(2/3+0)^2 - 1] = -0.2778$$
$$k_2 = h f(t_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1) = h f(1/4, 0.5278) = -0.1975$$
$$k_3 = h f(t_0 + 1/2h, y_0 + \tfrac{1}{2}k_2) = h \cdot f(1/4, 0.15125) = -0.1655$$
$$k_4 = h f(t_0 + h, y_0 + k_3) = h \cdot f(0.5, 0.0845) = 0.0011$$

$$w_1 = y(0.5) = y(0) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \approx 0.4995$$

$t(1) = 0.5$ $\quad y = 0.4995$

predictor AB2:

$$y(1) = y + \left(\frac{h}{2}\right)(3f(t_1, y) - f(t_0, y_0))$$

$y(1) = 0.6377$

corrector AM2:

$$y(1) = y + \frac{h}{12}\left(5f(t_2, y_1) + 8(t_1, y) - f(t_0, y_0)\right)$$

$$y(1) = 0.8729$$

for $h = 0.25$

$$\dot{y} = (y+t)^2 - 1$$

up to $t = 1$

$w_0 = y(0) = 2/3$

$h = 0.25$

bootstrapping using RK4:

$t_0 = 0$

$$k_1 = h\, f(t_0, y_0) = 0.25 \cdot f(0, 2/3) = -0.1389$$
$$k_2 = h\, f(t_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1) = h\, f(1/8, 0.5972) = -0.1196$$
$$k_3 = h\, f(t_0 + 1/2h, y_0 + \tfrac{1}{2}k_2) = h \cdot f(1/8, 0.6069) = -0.1161$$
$$k_4 = h\, f(t_0 + h, y_0 + k_3) = h \cdot f(0.25, 0.5506) = -0.0898$$

$$w_1 = y(0.5) = y(0) + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \approx 0.5500$$

$t_1 = 0.25$ $\qquad w1 = y(0.25) = 0.5500$

predictor AB2:

$$w_2 = y(0.5) = w_1 + \left(\frac{h}{2}\right)\left(3f(t_1, w_1) - f(t_0, w_0)\right)$$

$$w_2 = y(0.5) = 0.4844$$

corrector AM2: $t_2 = 0.5$

$$w_2 = y(0.5) = w_1 + \frac{h}{12}\left(5f(t_2, w_1) + 8(t_1, w_1) - f(t_0, w_0)\right)$$

$$w_2 = y(0.5) = 0.4984$$

predictor AB2:

$$w_3 = y(0.75) = w_2 + \left(\frac{h}{2}\right)\left(3f(t_2, w_2) - f(t_1, w_1)\right)$$

$$w_3 = y(0.75) = 0.5521$$

corrector AM2: $t_3 = 1$

$$w_3 = y(0.75) = w_2 + \frac{h}{12}\left(5f(t_3, w_3) + 8(t_2, w_2) - f(t_1, w_1)\right)$$

$$w_3 = y(0.75) = 0.5750$$

predictor AB2: $t_3 = 1$

$$w_4 = y(1) = w_3 + \left(\frac{h}{2}\right)\left(3f(t_3, w_3) - f(t_2, w_2)\right)$$

$$w_4 = y(1) = 0.8589$$

corrector AM2: $t_4 = 1.25$

$$w_4 = y(1) = w_2 + \frac{h}{12}\left(5f(t_4, w_3) + 8(t_3, w_3) - f(t_2, w_2)\right)$$

$$w_4 = y(1) = 0.9568$$