Bonus 6 4.12c

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x	$f(x)$	$f'(x)$ est.
2.5	3.939	-2.352
2.75	3.351	-2.662
3.0	2.608	-3.286
3.25	1.708	-2.784
3.5	1.216	-1.968

estimate:

$$\int_{x_0}^{x_m} \sqrt{1+f'(x)^2}\, dx$$

$x_0 = 2.5$
$x_m = 3.5$

finite-difference estimates:

est. 1st point using forward finite difference:

$$f'(2.5) = \frac{f(2.75)-f(2.5)}{0.25} = \frac{3.351-3.939}{0.25} \approx -2.352$$

others estimate with centered difference:

$$f'(2.75) = \frac{f(3.0)-f(2.5)}{2\cdot 0.25} = \frac{2.608-3.939}{0.5} = -2.662$$

$$f'(3.0) = \frac{f(3.25)-f(2.75)}{2\cdot 0.25} = \frac{1.708-3.351}{0.5} = -3.286$$

$$f'(3.25) = \frac{f(3.5)-f(3.0)}{2\cdot 0.25} = \frac{1.216-2.608}{0.5} = -2.784$$

last approximate with backward difference:

$$f'(3.5) = \frac{f(3.5)-f(3.25)}{0.25} = \frac{1.216-1.708}{0.25} = -1.968$$

$$\int_{2.5}^{3.5} \sqrt{1+f'(x)^2}\, dx \approx T(f,[2.5,3.5]) = h\left(\tfrac{1}{2}f(2.5)+f(2.75)+f(3.0)+ + f(3.25)+\tfrac{1}{2}f(3.5)\right)$$

$$= 0.25\left(\tfrac{1}{2}\cdot 2.556 + 2.844 + 3.435 + 2.960 + \tfrac{1}{2}\cdot 2.207\right)$$

$$= 2.905$$

$$f(x) = \sqrt{1 + f'(x)^2}$$

$$f(x) = \sqrt{1 + f'(x)^2}$$

$$f(2.5) = \sqrt{1 + f'(2.5)^2} = \sqrt{1 + (-2.352)^2} \approx 2.556$$

$$f(2.75) = \sqrt{1 + f'(2.75)^2} = \sqrt{1 + (-2.662)^2} \approx 2.844$$

$$f(3.0) = \sqrt{1 + f'(3.0)^2} = \sqrt{1 + (-3.286)^2} \approx 3.435$$

$$f(3.25) = \sqrt{1 + f'(3.25)^2} = \sqrt{1 + (-2.785)^2} \approx 2.960$$

$$f(3.5) = \sqrt{1 + f'(3.5)^2} = \sqrt{1 + (-1.968)^2} \approx 2.207$$