# Midpoint Method

$\dot{y} = e^{-t} - y^2$ $\quad y(0) = 0$ $\quad$ up to $t = 1$, $h = 0.5$

times: $t_0 = 0$ $\quad t_1 = 0.5$ $\quad t_2 = 1$

$$w_1 = w_0 + h f\left(t_0 + \tfrac{1}{2}h,\ w_0 + \tfrac{1}{2}h f(t_0, w_0)\right)$$

$$f(t_0, w_0) = f(0,0) = 1$$

$$f\left(t_0 + \tfrac{1}{2}h,\ w_0 + \tfrac{1}{2}h \underbrace{f(t_0, w_0)}_{=1}\right) = f\left(t_0 + \tfrac{1}{2}h,\ w_0 + \tfrac{1}{2}h\right) = f(0.25, 0.25)$$
$$= e^{-0.25} - 0.25^2$$
$$= \underline{0.7163}$$

$$w_1 = w_0 + h f\left(t_0 + \tfrac{1}{2}h,\ w_0 + \tfrac{1}{2}h f(t_0, w_0)\right)$$
$$= 0 + 0.5 \cdot 0.7163 = \underline{0.3582}$$

$$w_2 = w_1 + h f\left(t_1 + \tfrac{1}{2}h,\ w_1 + \tfrac{1}{2}h f(t_1, w_1)\right)$$

$$f(t_1, w_1) = f(0.5, 0.3582) = 0.4783$$
$$f\left(t_1 + \tfrac{1}{2}h,\ w_1 + \tfrac{1}{2}h \underbrace{f(t_1, w_1)}_{=0.4783}\right) = f(0.75, 0.4777)$$
$$= \underline{0.2442}$$

$$w_2 = w_1 + h f\left(t_1 + \tfrac{1}{2}h,\ w_1 + \tfrac{1}{2}h f(t_1, w_1)\right)$$
$$= 0.3582 + 0.5 \cdot 0.2442 = \underline{0.4802}$$