# Offline Diversity Maximization Under Imitation Constraints



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## **M**OTIVATION

## **Diverse**

- Robust solutions
- Multiple options

[DIAYN, DADS, DOMINO]

online setting

## Offline

- Use large datasets
- Safe learning

[AWAC, BC, CRR, IQL, CQL] single expert, not diverse

### **Imitation**

- No reward engineering
- Human demonstrations

[GAIL, SMODICE] single expert, not diverse

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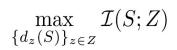
## **Imitation**

- No reward engineering
- Human demonstrations

[GAIL, SMODICE] single expert, not diverse

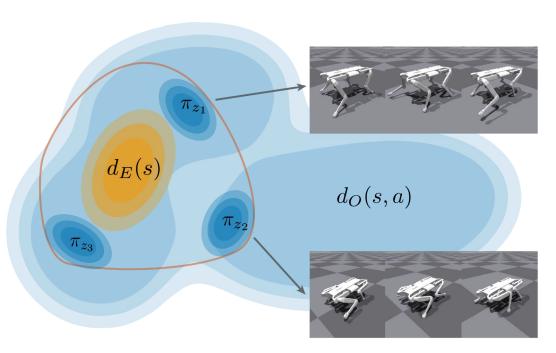
Propose: principled algorithm for Diverse Offline Imitation (DOI) learning

## PROBLEM FORMULATION



subject to

$$D_{\mathrm{KL}}(d_z(S)||d_E(S)) \le \varepsilon \quad \forall z$$



## Input:

state-action behavior dataset

$$\mathcal{D}_O \sim d_O(s, a)$$

state-only expert dataset

$$\mathcal{D}_E \sim d_E(s)$$

## RELAXED PROBLEM FORMULATION

$$\max_{\{d_z(S)\}_{z\in Z}} \mathcal{I}(S;Z)$$

Mutual Information: Variational Lower Bound

$$\mathcal{I}(S; Z) \geq \sum_{z} \mathbb{E}_{d_{z}(s)} \left[ \frac{\log(|Z|q(z|s))}{|Z|} \right]$$

q(z|s) train a skill-discriminator

## RELAXED PROBLEM FORMULATION

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q(z|s) train a skill-discriminator

subject to  $D_{\mathrm{KL}}(d_z(S)||d_E(S)) \le \varepsilon \quad \forall z$ 

#### **SMODICE** expert (offline)

$$d_{\widetilde{E}}(S, A) \approx \arg \min_{d(s,a)} D_{\mathrm{KL}}(d(S)||d_{E}(S))$$

subject to  $\mathrm{D_{KL}}(d_z(S,A)||d_{\widetilde{E}}(S,A)) \leq \varepsilon \quad \forall z$ 

# **A**LGORITHMIC **A**PPROACH

# (LAGRANGE)

$$\max_{\substack{d_z(s,a)\\q(z|s)}} \min_{\lambda \ge 0} \sum_{z} \mathbb{E}_{d_z(s)} \left[ \frac{\log(|Z|q(z|s))}{|Z|} \right] + \left[ \sum_{z} \lambda_z \left[ \epsilon - \mathrm{D_{KL}} \left( d_z(S,A) || d_{\widetilde{E}}(S,A) \right) \right] \right]$$

**Diversity** 

**Imitation** 

## **ALGORITHMIC APPROACH**

# (FENCHEL)

$$\max_{\substack{d_z(s,a)\\q(z|s)}} \min_{\lambda \geq 0} \sum_{z} \mathbb{E}_{d_z(s)} \left[ \frac{\log\left(|Z|q(z|s)\right)}{|Z|} \right] + \sum_{z} \lambda_z \left[ \epsilon - \mathrm{D_{KL}}\left(d_z(S,A) || d_{\widetilde{E}}(S,A)\right) \right]$$

$$\max_{\substack{d_z(s,a)\\q(z|s)\\q(z|s)}} \min_{\lambda > 0} \sum_{z} \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}}\left(d_z(S,A) || d_O(S,A)\right) \right\}$$

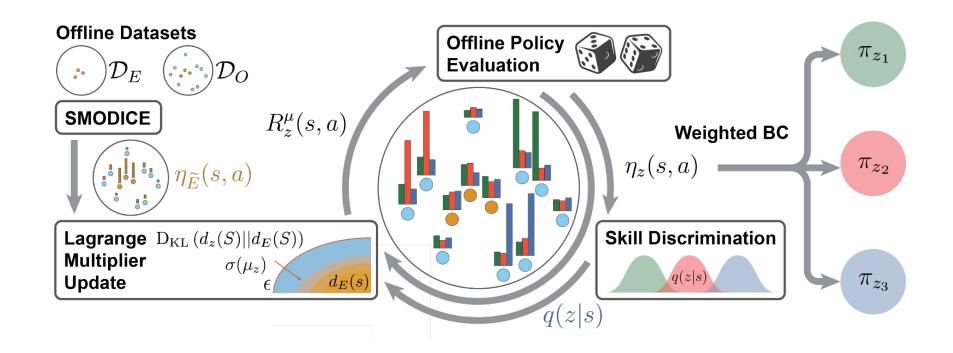
$$\eta_z(s,a) = \frac{d_z(s,a)}{d_O(s,a)}$$

Regularized RL Problem

## **ALGORITHMIC APPROACH**

$$\max_{\substack{d_z(s,a)\\q(z|s)}} \min_{\lambda \geq 0} \sum_z \mathbb{E}_{d_z(s)} \left[ \frac{\log\left(|Z|q(z|s)\right)}{|Z|} \right] + \sum_z \lambda_z \left[ \epsilon - \mathrm{D_{KL}} \left( d_z(S,A) || d_{\widetilde{E}}(S,A) \right) \right] \\ \max_{\substack{d_z(s,a)\\q(z|s)}} \min_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \max_{\substack{d_z(s,a)\\q(z|s)}} \min_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\lambda \geq 0} \sum_z \lambda_z \left\{ \epsilon + \mathbb{E}_{d_z(s,a)} \left[ R_z^{\lambda}(s,a) \right] - \mathrm{D_{KL}} \left( d_z(S,A) || d_O(S,A) \right) \right\} \\ \prod_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\substack{d_z(s,a)\\q(z|s)}} \sum_{\substack{d_$$

## **ALTERNATING OPTIMIZATION SCHEME**



# **E**XPERIMENTS

#### I. Locomotion Task (SIM & REAL)



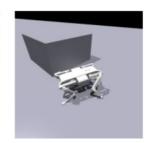


Solo12

#### II. Obstacle Navigation Task (SIM)



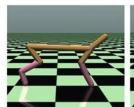




III. D4RL Envs (SIM)





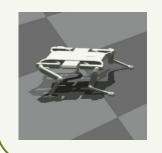


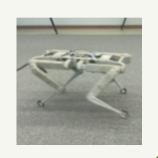


Ant

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## I. Locomotion Task (SIM & REAL)



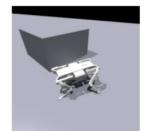


Solo12

#### II. Obstacle Navigation Task (SIM)



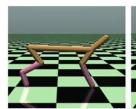




III. D4RL Envs (SIM)









Half-Cheetah

Ant

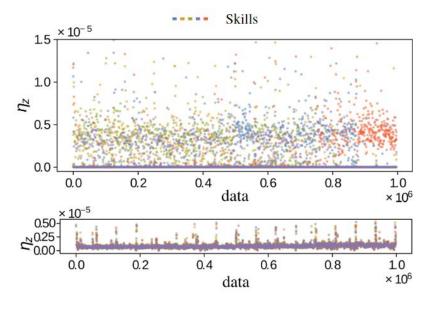
# I. LOCOMOTION TASK (SIM)

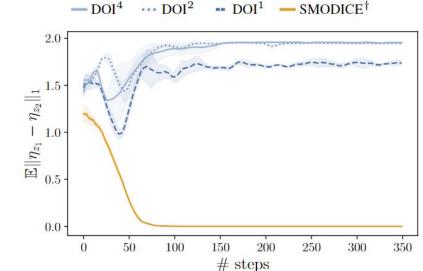
(Expected Importance Ratios)

**Offline** Evaluation

1) DOI skills well-separate data

2) Constraint level  $\varepsilon$  controls ratio distance

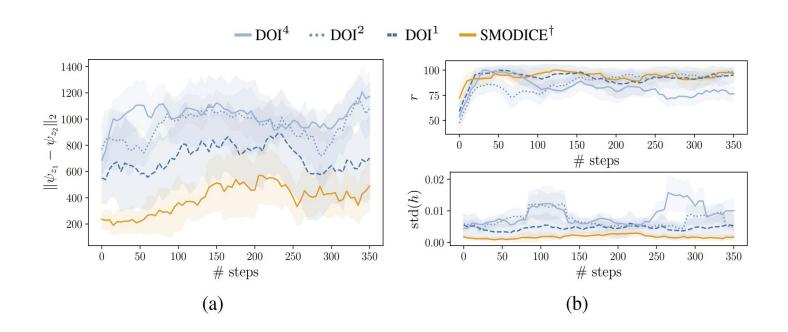




# I. LOCOMOTION TASK (SIM)

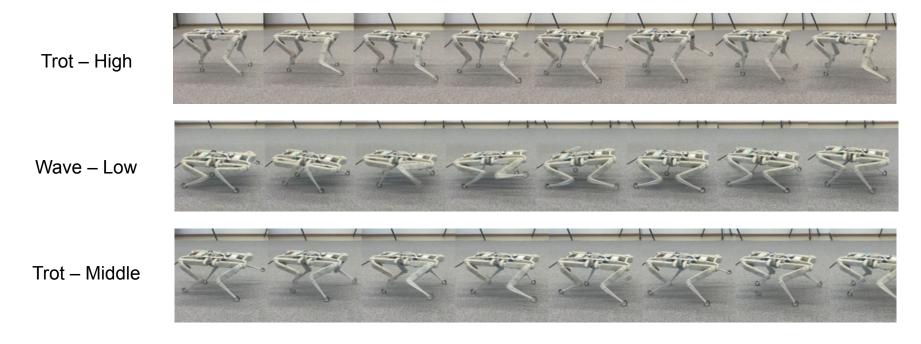
Online (Monte Carlo)
Evaluation

**3)** Relaxed constraints yield increased diversity, albeit at the expense of performance loss.



# I. LOCOMOTION TASK (REAL)

4) DOI skills trained in SIM (with domain randomization) are successfully deployed in the Real System



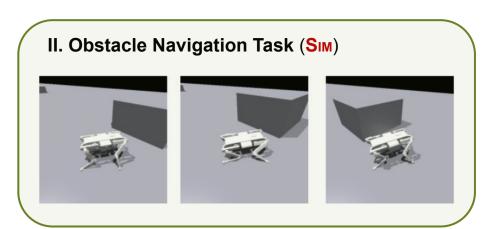
# **EXPERIMENTS**

#### I. Locomotion Task (SIM & REAL)



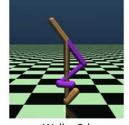


Solo12



III. D4RL Envs (SIM)









Walker2d

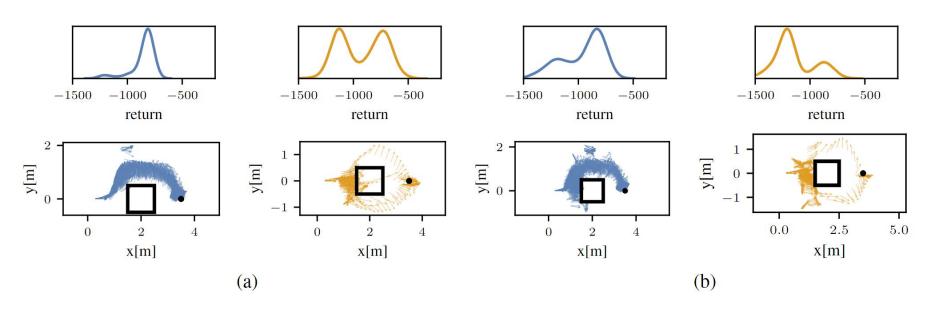
2d Half-Cheetah

Ant

# II. OBSTACLE NAVIGATION TASK (SIM)

Online (Monte Carlo)
Evaluation

**5)** SMODICE expert struggles with out-of-distribution (higher) box heights, while a robust DOI skill successfully navigates by detouring (to the left side).



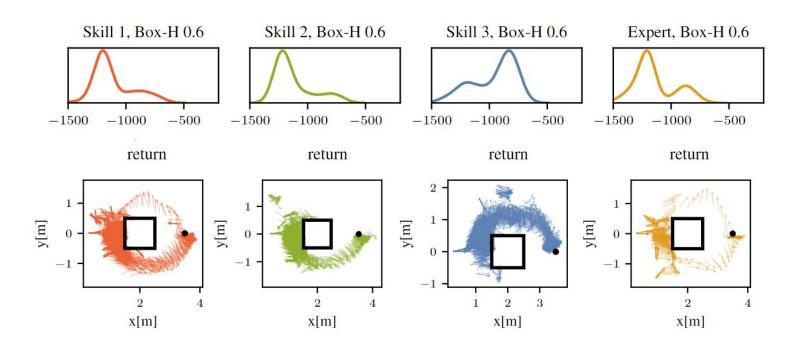
Box Height 0.3 m

Box Height 0.6 m

# II. OBSTACLE NAVIGATION TASK (SIM)

Limitation:

**6)** Not all learned DOI skills are robust. Selection is required.



## CONCLUSION

#### Principled algorithm (DOI)

Offline Diversity maximization under Imitation constraints



**Project Website** 

#### **Experiments**

Show **DOI**'s effectiveness on:

- Solo12 tasks (Locomotion & Obstacle Navigation)
- Standard D4RL environments

#### Limitation

Agent's performance is sensitive to relaxing the imitation constraints