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Максимальное значение

Доминанта работы №2

Задание 1

a) $f(x) = x^2 - 30x + 1$ $f'(x) = 2x - 30$

$f'(x) = 0 : 2x - 30 = 0$

$x_0 = 15$

$f(15) = -224$

$(15, -224)$ - критический
точка

$f''(x) = 2 > 0 \Rightarrow (15, -224)$ - точка минимума

b) $f(x) = -x^3 + 5x - 7x^2 + 17$

$f'(x) = -3x^2 + 5 - 14x$

$f'(x) = 0 : -3x^2 + 5 - 14x = 0$

$3x^2 + 14x - 5 = 0$

$\Delta = 14^2 - 4 \cdot 3 \cdot (-5) = 256 \quad \sqrt{\Delta} = \pm 16$

$x_1 = \frac{-14 + 16}{2 \cdot 3} = \frac{2}{6} = \frac{1}{3} \quad x_2 = \frac{-14 - 16}{2 \cdot 3} = -\frac{50}{6} = -5$

$f(x_1) = -\frac{1}{27} + \frac{5}{3} - \frac{7}{9} + 17 = -\frac{1 + 45 - 21}{27} + 17 = 17\frac{23}{27}$

$$f(x_2) = 125 + 25 - 175 + 17 = -58$$

$$\left(\frac{1}{3}, 17 \frac{23}{27}\right), (-5, -58)$$

Критические точки

$$f''(x) = -6x - 14$$

$$f''\left(\frac{1}{3}\right) = -2 - 14 = -16 < 0 \Rightarrow$$

$$\Rightarrow \left(\frac{1}{3}, 17 \frac{23}{27}\right) - \text{точка максимума}$$

$$f''(-5) = (-6) \cdot (-5) - 14 = 30 - 14 = 16 > 0 \Rightarrow$$

$$\Rightarrow (-5, -8) - \text{точка минимума}$$

c) $h(x) = x^2 - \frac{1}{x} \quad h'(x) = 2x + \frac{1}{x^2}$

$$2x + \frac{1}{x^2} = 0 \Leftrightarrow \frac{1}{x^2} = -2x \Leftrightarrow -2x^3 = 1 \Leftrightarrow x = -\sqrt[3]{\frac{1}{2}}$$

$$h\left(-\sqrt[3]{\frac{1}{2}}\right) = \frac{1}{\sqrt[3]{4}} - \sqrt[3]{2} = \frac{1 - \sqrt[3]{8}}{\sqrt[3]{4}} \quad \left(-\frac{1}{\sqrt[3]{2}}, \frac{1 - \sqrt[3]{8}}{\sqrt[3]{4}}\right) -$$

Критическая точка

$$f''(x) = 2 - \frac{2}{x^3} \quad f''\left(-\frac{1}{\sqrt[3]{2}}\right) = 2 + \frac{2}{\frac{1}{2}} = 6 > 0 \Rightarrow$$

$$\Rightarrow \left(-\frac{1}{\sqrt[3]{2}}, \frac{1-\sqrt[3]{8}}{\sqrt[3]{4}}\right) - \text{максимум}$$

d) $y(x) = \frac{1}{x^5} + x^{12} + 8$

$$y'(x) = -\frac{5}{x^6} + 14x^{11} \quad -\frac{5}{x^6} = -14x^{11}$$

$$x^{11} = \frac{5}{14} \Leftrightarrow x = \pm \sqrt[11]{\frac{5}{14}}$$

$$\left(\sqrt[11]{\frac{5}{14}}, \sqrt[11]{\frac{14}{5}} + \sqrt[11]{\frac{5}{14}} + 8\right) - \text{критическая}\\ \text{максимум}$$

$$y''(x) = \frac{30}{x^7} + 272x^{10}$$

(формально это не максимум)

$$y''\left(\sqrt[11]{\frac{5}{14}}\right) > 0 \Rightarrow \text{максимум}$$

$$\left(-\sqrt[11]{\frac{5}{14}}, -\sqrt[11]{\frac{15}{5}} - \sqrt[11]{\frac{5}{14}} + 8\right) - \text{критическая}\\ \text{максимум}$$

$$y'' \left(-\sqrt[22]{\frac{5}{17}} \right) = -162,4 < 0 \Rightarrow \text{максимум}$$

Задание 3

a) $f''(x) = 2 \quad f'''(x) = 0 \Rightarrow \emptyset$

b) $g''(x) = -6x - 14 : -6x - 14 = 0$
 $x = -\frac{14}{6} = -2\frac{1}{3}$

$$g\left(-2\frac{1}{3}\right) \approx 20$$

$$g'''(x) = -6 \neq 0 \Rightarrow \left(-2\frac{1}{3}, 20\right) - \text{максимум}$$

c) $h''(x) = 2 - \frac{2}{x^3} : 2 - \frac{2}{x^3} = 0$

$$2x^3 = 2$$

$$x = 1$$

$$h(1) = 0$$

$$h'''(x) = -\frac{6}{x^4}$$

$$h'''(1) = -6 < 0 \Rightarrow (1, 0) - \text{минимум}$$

d) $y''(x) = \frac{30}{x^4} + 272x^{16}$

$$y''(x) = 0 : \frac{30}{x^7} + 272x^{16} = 0$$

$$272x^{16} = -\frac{30}{x^7}$$

$$272x^{23} = -30$$

$$x^{23} \approx -0,11$$

$$x \approx \sqrt[23]{-0,11} \approx 0,91$$

$$y(0,91) = 9,81$$

$$y'''(x) = -\frac{210}{x^8} + 4352x^{15}$$