

Домашняя контрольная работа по матанализу 2

$$1. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{k}}{n\sqrt{n} + k\sqrt{k}}$$

$$\sum_{k=1}^n \frac{\sqrt{k}}{n\sqrt{n} + k\sqrt{k}} = \frac{1}{n} \sum_{k=1}^n \frac{n\sqrt{k}}{n\sqrt{n} + k\sqrt{k}} = \frac{\sqrt{k}}{\sqrt{n} + \frac{k\sqrt{k}}{n}} = \frac{1}{\frac{\sqrt{n}}{\sqrt{k}} + \frac{k}{n}} = \frac{1}{n} \sum_{k=1}^n \frac{1}{\left(\frac{k}{n}\right)^{-\frac{1}{2}} + \frac{k}{n}} = \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right)$$
$$f(x) = \frac{1}{x^{-\frac{1}{2}} + x} = \frac{\sqrt{x}}{1 + x\sqrt{x}}$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx = \int_0^1 \frac{\sqrt{x}}{1 + x\sqrt{x}} dx$$

$$\int \frac{\sqrt{x}}{1 + x\sqrt{x}} dx = \left[t = \sqrt{x} \right] = \int \frac{2t^2}{1 + t^3} dt = 2 \left(\int \frac{1}{3(t+1)} dt + \int \frac{2t-1}{3(1-t+t^2)} dt \right) =$$

$$= 2 \left(\frac{1}{3} \ln|t+1| + \int \frac{2t-1}{3(1-t+t^2)} dt \right) = \left[\frac{u=1-t+t^2}{du=(2t-1)dt} \right] = 2 \left(\frac{1}{3} \ln|t+1| + \frac{1}{3} \ln|u| \right) =$$

$$= \frac{2}{3} (\ln|t+1| + \ln|1-t+t^2|) =$$

$$= \frac{2}{3} (\ln|\sqrt{x}+1| + \ln|1-\sqrt{x}+x|)$$

$$\int_0^1 f(x) dx = \frac{2}{3} (\ln|\sqrt{x}+1| + \ln|1-\sqrt{x}+x|) \Big|_0^1 = \frac{2}{3} (\ln 2 + \ln 1 - \ln 1 - \ln 1) = \frac{2 \ln 2}{3}$$

Ответ: $\frac{2 \ln 2}{3}$

$$2. y = \sqrt{4-x^2}, y=0, 0 \leq x \leq \frac{\pi}{2}$$

$$\int \sqrt{4-x^2} dx = \left[\begin{array}{l} x = 2 \sin t \\ dx = 2 \cos t dt \\ t = \arcsin\left(\frac{x}{2}\right) \end{array} \right] = 2 \int \cos^2 t dt = 4 \int \frac{1 + \cos 2t}{2} dt = \sin 2t + 2t$$

$$\int_0^{\frac{\pi}{2}} \sqrt{4-x^2} dx = (\sin 2t + 2t) =$$

$$= \left(\sin \left(2 \arcsin \left(\frac{x}{2} \right) \right) + 2 \arcsin \left(\frac{x}{2} \right) \right) = 2 \sin \left(\arcsin \left(\frac{x}{2} \right) \right) \cos \left(\arcsin \left(\frac{x}{2} \right) \right) + 2 \arcsin \left(\frac{x}{2} \right) \Big|_0^{\frac{\pi}{2}} =$$

$$= \left(2 \frac{x}{2} \sqrt{1 - \frac{x^2}{4}} + 2 \arcsin \left(\frac{x}{2} \right) \right) \Big|_0^{\frac{\pi}{2}} = \left(\frac{x}{2} \sqrt{4-x^2} + 2 \arcsin \left(\frac{x}{2} \right) \right) \Big|_0^{\frac{\pi}{2}} =$$

$$= 2 \arcsin \left(\frac{\pi}{4} \right) + \frac{\pi}{4} \sqrt{4 - \frac{\pi^2}{4}}$$

Ответ: $2 \arcsin \left(\frac{\pi}{4} \right) + \frac{\pi}{4} \sqrt{4 - \frac{\pi^2}{4}}$