Домашняя контрольная работа по матанализу 2

1. $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\dfrac{\sqrt{k}}{n\sqrt{n}+k\sqrt{k}}$

$$\sum_{k=1}^{n}\frac{\sqrt{k}}{n\sqrt{n}+k\sqrt{k}}=\frac{1}{n}\sum_{k=1}^{n}\frac{n\sqrt{k}}{n\sqrt{n}+k\sqrt{k}}=\frac{\sqrt{k}}{\sqrt{n}+\frac{k\sqrt{k}}{n}}=\frac{1}{\frac{\sqrt{n}}{\sqrt{k}}+\frac{k}{n}}=\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\left(\frac{k}{n}\right)^{-\frac{1}{2}}+\frac{k}{n}}=\frac{1}{n}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)$$

$$f(x)=\frac{1}{x^{-\frac{1}{2}}+x}=\frac{\sqrt{x}}{1+x\sqrt{x}}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)=\int_0^1 f(x)dx=\int_0^1\frac{\sqrt{x}}{1+x\sqrt{x}}dx$$

$$\int\frac{\sqrt{x}}{1+x\sqrt{x}}dx=\begin{bmatrix}t=\sqrt{x}\\ dt=\frac{dx}{2t}\end{bmatrix}=\int\frac{2t^2}{1+t^3}dt=2\left(\int\frac{1}{3(t+1)}dt+\int\frac{2t-1}{3(1-t+t^2)}dt\right)=$$

$$=2\left(\frac{1}{3}\ln|t+1|+\int\frac{2t-1}{3(1-t+t^2)}dt\right)=\begin{bmatrix}u=1-t+t^2\\ du=(2t-1)dt\end{bmatrix}=2\left(\frac{1}{3}\ln|t+1|+\frac{1}{3}\ln|u|\right)=$$

$$=\frac{2}{3}\left(\ln|t+1|+\ln|1-t+t^2|\right)=$$

$$=\frac{2}{3}\left(\ln|\sqrt{x}+1|+\ln|1-\sqrt{x}+x|\right)$$

$$\int_0^1 f(x)dx=\frac{2}{3}\left(\ln|\sqrt{x}+1|+\ln|1-\sqrt{x}+x|\right)\Big|_0^1=\frac{2}{3}\left(\ln 2+\ln 1-\ln 1-\ln 1\right)=\frac{2\ln 2}{3}$$

Ответ: $\frac{2\ln 2}{3}$

2. $y=\sqrt{4-x^2}, y=0, 0\le x\le\frac{\pi}{2}$

$$\int\sqrt{4-x^2}dx=\begin{bmatrix}x=2\sin t\\ dx=2\cos t dt\\ t=\arcsin(\frac{x}{2})\end{bmatrix}=2\int\cos^2 t dt=4\int\frac{1+\cos 2t}{2}dt=\sin 2t+2t$$

$$\int_0^{\frac{\pi}{2}}\sqrt{4-x^2}dx=(\sin 2t+2t)=$$

$$=\left(\sin\left(2\arcsin\left(\frac{x}{2}\right)\right)+2\arcsin\left(\frac{x}{2}\right)=2\sin\left(\arcsin\left(\frac{x}{2}\right)\right)\cos\left(\arcsin\left(\frac{x}{2}\right)\right)+2\arcsin\left(\frac{x}{2}\right)\right)\Big|_0^{\frac{\pi}{2}}=$$

$$=\left(2\frac{x}{2}\sqrt{1-\frac{x^2}{4}}+2\arcsin\left(\frac{x}{2}\right)\right)\Big|_0^{\frac{\pi}{2}}=\left(\frac{x}{2}\sqrt{4-x^2}+2\arcsin\left(\frac{x}{2}\right)\right)\Big|_0^{\frac{\pi}{2}}=$$

$$=2\arcsin\left(\frac{\pi}{4}\right)+\frac{\pi}{4}\sqrt{4-\frac{\pi^2}{4}}$$

Ответ: $2\arcsin\left(\frac{\pi}{4}\right)+\frac{\pi}{4}\sqrt{4-\frac{\pi^2}{4}}$