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Контролна работа II

Павел Сарпов, фн: 62393, група: 5

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1 задача

$X \sim N(163; 5,6^2)$, избираме 20 случайно

$S_n \approx N(20.163; 20^2.5,6^2)$

1) $$\mathbb{P}\left(\frac{S_n}{20} > 165\right) \approx \mathbb{P}\left(N(0,1) > \frac{165-163}{5,6}\right) =$$

$$\approx 1 - \Phi(0,36) = 1 - 0,6406 = 0,3594$$

2) $X_1, X_2, \dots, X_{20} \sim X$

$$\mathbb{P}(X > 180) = 1 - \mathbb{P}(X \le 180) \approx 1 - \Phi\left(\frac{163-180}{5,6}\right) \approx$$

$$\approx 1 - 0,0012 = 0,9988$$

$$\Rightarrow \mathbb{P}(\text{има над } 180) = \sum_{k=1}^{20} \binom{20}{k} (0,9988)^k (0,0012)^{20-k}$$

2 задача

X, Y $\quad f_{X,Y}(x,y) = \begin{cases} ce^{-y} & \text{за } 0 < x < y < \infty \\ 0 & \text{иначе} \end{cases}$

1) $$1 = \int_0^y \int_x^\infty ce^{-y}\,dy\,dx = c\int_0^y \left[-e^{-y}\right]_x^\infty dx =$$

$$= c\int_0^y e^{-x}\,dx = c(-e^{-y}+1)$$

$$\Rightarrow c = \frac{1}{1-e^{-y}} = \frac{e^y}{e^y-1}$$

~~$Cov(X,Y) = EXY - EXEY$~~

$$EXY = \int_0^y \int_x^\infty xy \cdot ce^{-y}\,dy\,dx = c\int_0^y x\left[-e^{-y}(y+1)\right]_x^\infty dx =$$

$$= c\int_0^y xe^{-x}(x+1)\,dx =$$

$$= c\left[-e^{-x}(x^2+2x+2) - e^{-x}(x+1)\right]_0^y =$$

$$= c\left[3 - e^{-y}(y^2+3y+3)\right]$$

$$f_X(x) = \int_x^\infty ce^{-y}\,dy = ce^{-x}$$

$$f_Y(y) = \int_0^y ce^{-y}\,dx = cye^{-y}$$

$$\Rightarrow \mathbb{E}X = \int_0^y x c e^{-x} dx = c\left[-e^{-x}(x+1)\right]_0^y =$$

$$= c\left[1 - e^{-y}(y+1)\right]$$

$$\mathbb{E}Y = \int_x^\infty y c y e^{-y} dy = c\left[-e^{-y}(y^2+2y+2)\right]_x^\infty$$

$$= c e^{-x}(x^2+2x+2)$$

$$\Rightarrow \mathrm{Cov}(X, Y) = c\left[3 - e^{-y}(y^2+3y+3)\right] -$$

$$- c^2 e^{-x}\left[1 - e^{-y}(y+1)\right](x^2+2x+2)$$

$$\text{2)}$$

$$\mathbb{D}X = \mathbb{E}X^2 - (\mathbb{E}X)^2 = \int_0^y x^2 c e^{-x} dx - c^2\left[1 - e^{-y}(y+1)\right] =$$

$$= c\left[-e^{-x}(x^2+2x+2)\right]_0^y - c^2\left[1 - e^{-y}(y+1)\right]$$

$$= c\left[2 - e^{-y}(y^2+2y+2)\right] - c^2\left[1 - e^{-y}(y+1)\right]$$

$$\mathbb{D}Y = \mathbb{E}Y^2 - (\mathbb{E}Y)^2 = \int_x^\infty y^2 c y e^{-y} dy - c^2 \cdot e^{-2x}(x^2+2x+2) =$$

$$= c\left[-e^{-y}(y^3+3y^2+6y+6)\right]\Big|_x^\infty - c^2 e^{-2x}(x^2+2x+2)$$

$$= c e^{-x}(x^3+3x^2+6x+6) - c^2 e^{-2x}(x^2+2x+2)$$

$$\mathrm{Cor}(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathbb{D}X}\sqrt{\mathbb{D}Y}} =$$

$$= \frac{c\left[3-e^{-y}(y^2+3y+3)\right] - c^2e^{-x}\left[1-e^{-y}(y+1)\right](x^2+2x+1)}{\sqrt{\left[c\left(2-e^{-y}(y^2+2y+2)\right)-c^2\left[1-e^{-y}(y+1)\right]\right]\left[ce^{-x}(x^3+3x^2+6x+6) - c^2e^{-2x}(x^2+2x+2)\right]}}$$

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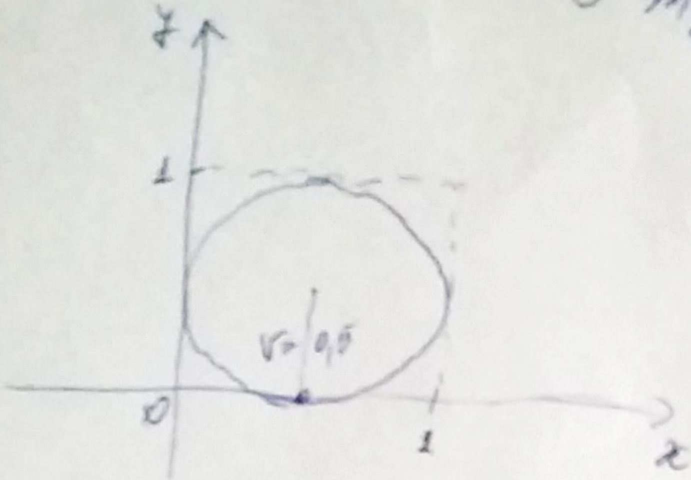
2) $$\mathbb{E}(X \mid Y=1) = \int_0^y x \frac{f_{X,Y}(x,1)}{f_Y(1)}\,dx =$$

$$= \int_0^y x \frac{ce^{-1}}{ce^{-1}}\,dx = \frac{y^2}{2}$$

3 задача



$0 \le x, y \le 1$

$P(r \le 0{,}5) = \frac{\pi 0{,}5^2}{1} =$

$= \pi 0{,}5^2$

4 задача

$X, Y \; ; \; X \perp\!\!\!\perp Y \; ; \; X \sim U(0,1), \; Y \sim Exp(1)$

1) $P(X \le a) = P(Y \ge a) = 1 - P(Y \le a)$

$$\frac{a-0}{1-0} = 1 - (1 - e^{-1a})$$

$a = e^{-a} \Rightarrow$ не съществува такова a

2) $f_{X/Y} = ?$ $\quad \begin{cases} Z_1 = \frac{X}{Y} \\ Z_2 = X \end{cases} \Leftrightarrow \begin{cases} X = Z_2 \\ Y = \frac{Z_2}{Z_1} \end{cases}$

$$f_{X,Y}(x,y) \overset{X \perp\!\!\!\perp Y}{=} f_X(x) f_Y(y) = 1 \cdot e^{y} = e^{y} = f_{Z_2, \frac{Z_2}{Z_1}}$$