Метод сеток решения смешанной задачи для уравнения гиперболического типа

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Явный метод

$$\frac{\partial^2 U(x,t)}{\partial t^2} = \frac{\partial^2 U(x,t)}{\partial x^2},\ 0 \le x \le 1,\ 0 \le t \le 1.$$

$$U(0,t) = \frac{1}{1+\alpha},\ U(1,t) = \frac{1}{t+\alpha+1}$$

$$U(x,0) = \frac{1}{x+\alpha} = \alpha(x),\ \frac{\partial U(x,0)}{\partial t} = -\frac{1}{(x+\alpha)^2} = \beta(x)$$

In[1]:=
```
h = 0.1
τ = 0.05
α = 0.5 + 0.1 * 8
maxM = 1/h
maxN = 1/τ
```

Out[1]= 0.1

Out[2]= 0.05

Out[3]= 1.3

Out[4]= 10.

Out[5]= 20.

In[51]:=
```
U = ConstantArray[0, {11, 21}];
```

In[52]:=
$$\gamma_0[t\_] := \frac{1}{1+\alpha}$$
$$\gamma_1[t\_] := \frac{1}{t+1+\alpha}$$
$$A[x\_] := \frac{1}{\alpha+x}$$
$$B[x\_] := \frac{-1}{(x+\alpha)^2}$$
$$s = \frac{\tau^2}{h^2};$$
$$\phi[x\_, t\_] := \left(-\alpha^2 t + 1\right) e^{-\alpha x}$$

In[58]:=
```
X = (# h & /@ Range[0, maxM])
T = (# τ & /@ Range[0, maxN])
```

Out[58]=
```
{0., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.}
```

Out[59]=
```
{0., 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4,
 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95, 1.}
```

In[60]:=
```
U[[1]] = (γ0[#] & /@ T)
U[[maxM + 1]] = (γ1[#] & /@ T)
```

Out[60]=
```
{0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783,
 0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783,
 0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783, 0.434783}
```

Out[61]=
```
{0.434783, 0.425532, 0.416667, 0.408163, 0.4, 0.392157, 0.384615,
 0.377358, 0.37037, 0.363636, 0.357143, 0.350877, 0.344828, 0.338983,
 0.333333, 0.327869, 0.322581, 0.31746, 0.3125, 0.307692, 0.30303}
```

In[62]:=
$$U[\![;; , 1]\!] = A[\#] \&/@ X$$

$$U[\![;; , maxN + 1]\!] = \left(A[\#] + \tau * B[\#] + \frac{\tau^2}{2} (A''[\#])\right) \&/@ X$$

Out[62]=
```
{0.769231, 0.714286, 0.666667, 0.625, 0.588235,
 0.555556, 0.526316, 0.5, 0.47619, 0.454545, 0.434783}
```

Out[63]=
```
{0.740783, 0.689687, 0.645185, 0.606079, 0.571443,
 0.540552, 0.51283, 0.487813, 0.465123, 0.44445, 0.425536}
```

In[64]:=
```
U // MatrixForm
```

Out[64]//MatrixForm=

0.769231	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783
0.714286	0	0	0	0	0	0	0	0
0.666667	0	0	0	0	0	0	0	0
0.625	0	0	0	0	0	0	0	0
0.588235	0	0	0	0	0	0	0	0
0.555556	0	0	0	0	0	0	0	0
0.526316	0	0	0	0	0	0	0	0
0.5	0	0	0	0	0	0	0	0
0.47619	0	0	0	0	0	0	0	0
0.454545	0	0	0	0	0	0	0	0
0.434783	0.425532	0.416667	0.408163	0.4	0.392157	0.384615	0.377358	0.37037

In[65]:=
```
For[i = 2, i ≤ maxM, i++,
 For[j = 1, j ≤ maxN, j++, U[[i]][[j + 1]] = s U[[i + 1, j]] + 2 (1 - s) U[[i, j]] + s U[[i - 1, j]]]]
```

In[66]:=
```
U // MatrixForm
```

Out[66]//MatrixForm=

0.769231	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783	0.434783
0.714286	1.4304	2.2543	3.49015	5.34391	8.12457	12.2955	18.552	27.9367
0.666667	1.33482	2.35983	4.10332	7.02752	11.8773	19.847	32.8444	53.9047
0.625	1.25123	2.21054	3.90577	6.88449	12.0836	21.0947	36.6039	63.1169
0.588235	1.17749	2.07904	3.6712	6.48325	11.446	20.1899	35.5585	62.4888
0.555556	1.11197	1.96233	3.46326	6.11268	10.7898	19.0463	33.6169	59.3149
0.526316	1.05336	1.85804	3.27764	5.78227	10.2016	17.9998	31.7613	56.0462
0.5	1.00063	1.76428	3.11093	5.4858	9.67427	17.0618	30.0927	53.0793
0.47619	0.952922	1.67954	2.96038	5.2183	9.1989	16.2169	28.5908	50.4094
0.454545	0.909561	1.70896	3.08749	5.47336	9.61462	16.8197	29.3799	51.3119
0.434783	0.425532	0.416667	0.408163	0.4	0.392157	0.384615	0.377358	0.37037

In[67]:=
```
{U // Transpose}[[1]][[2 ;;]]
```

Out[67]=
```
{{0.434783, 1.4304, 1.33482, 1.25123, 1.17749, 1.11197, 1.05336,
   1.00063, 0.952922, 0.909561, 0.425532}, {0.434783, 2.2543, 2.35983,
   2.21054, 2.07904, 1.96233, 1.85804, 1.76428, 1.67954, 1.70896, 0.416667},
  {0.434783, 3.49015, 4.10332, 3.90577, 3.6712, 3.46326, 3.27764,
   3.11093, 2.96038, 3.08749, 0.408163}, {0.434783, 5.34391, 7.02752,
   6.88449, 6.48325, 6.11268, 5.78227, 5.4858, 5.2183, 5.47336, 0.4},
  {0.434783, 8.12457, 11.8773, 12.0836, 11.446, 10.7898, 10.2016, 9.67427,
   9.1989, 9.61462, 0.392157}, {0.434783, 12.2955, 19.847, 21.0947,
   20.1899, 19.0463, 17.9998, 17.0618, 16.2169, 16.8197, 0.384615},
  {0.434783, 18.552, 32.8444, 36.6039, 35.5585, 33.6169, 31.7613, 30.0927,
   28.5908, 29.3799, 0.377358}, {0.434783, 27.9367, 53.9047, 63.1169,
   62.4888, 59.3149, 56.0462, 53.0793, 50.4094, 51.3119, 0.37037},
  {0.434783, 42.0138, 87.8412, 108.152, 109.512, 104.595, 98.898, 93.6305,
   88.884, 89.6629, 0.363636}, {0.434783, 63.1294, 142.265, 184.188,
   191.306, 184.27, 174.496, 165.17, 156.734, 156.806, 0.357143},
  {0.434783, 94.8027, 229.18, 311.848, 333.007, 324.232, 307.811, 291.379,
   276.393, 274.482, 0.350877}, {0.434783, 142.313, 367.471, 525.067,
   577.472, 569.599, 542.774, 514.022, 487.434, 480.909, 0.344828},
  {0.434783, 213.578, 586.785, 879.468, 997.474, 998.766, 956.561, 906.726,
   859.657, 843.308, 0.338983}, {0.434783, 320.476, 933.571, 1465.9,
   1716.08, 1747.52, 1684.53, 1599.23, 1516.17, 1479.96, 0.333333},
  {0.434783, 480.822, 1480.48, 2432.24, 2940.59, 3050.3, 2963.68, 2819.98,
   2674.06, 2599.07, 0.327869}, {0.434783, 721.342, 2340.92, 4018.48,
   5018.95, 5310.59, 5208.09, 4970.89, 4716.08, 4567.2, 0.322581},
  {0.434783, 1082.12, 3691.71, 6612.95, 8533.04, 9220.63, 9139.79, 8758.35,
   8316.84, 8029.9, 0.31746}, {0.434783, 1623.29, 5808.1, 10 842.3,
   14 452.8, 15 964.2, 16 014.8, 15 422.5, 14 664.9, 14 124.1, 0.3125},
  {0.434783, 2435.04, 9117.98, 17 715.5, 24 389.8, 27 559.5, 28 013.3, 27 137.4,
   25 852.9, 24 852.5, 0.307692}, {0.740783, 3652.68, 14 285.7, 28 852.8,
   41 013.6, 47 436.7, 48 909.8, 47 709.5, 45 563.7, 43 742., 0.425536}}
```

In[68]:=
```
TableForm[
 Join[{{"u^n_0", "u^n_1", "u^n_2", "u^n_3", "u^n_4", "u^n_5", "u^n_6", "u^n_7", "u^n_8", "u^n_9", "u^n_10"}},
  (U // Transpose)[[2 ;;]]]]
```

Out[68]//TableForm=

u^n_0	u^n_1	u^n_2	u^n_3	u^n_4	u^n_5	u^n_6	u^n_7
0.434783	1.4304	1.33482	1.25123	1.17749	1.11197	1.05336	1.00063
0.434783	2.2543	2.35983	2.21054	2.07904	1.96233	1.85804	1.76428
0.434783	3.49015	4.10332	3.90577	3.6712	3.46326	3.27764	3.11093
0.434783	5.34391	7.02752	6.88449	6.48325	6.11268	5.78227	5.4858
0.434783	8.12457	11.8773	12.0836	11.446	10.7898	10.2016	9.67427
0.434783	12.2955	19.847	21.0947	20.1899	19.0463	17.9998	17.0618
0.434783	18.552	32.8444	36.6039	35.5585	33.6169	31.7613	30.0927
0.434783	27.9367	53.9047	63.1169	62.4888	59.3149	56.0462	53.0793
0.434783	42.0138	87.8412	108.152	109.512	104.595	98.898	93.6305
0.434783	63.1294	142.265	184.188	191.306	184.27	174.496	165.17
0.434783	94.8027	229.18	311.848	333.007	324.232	307.811	291.379
0.434783	142.313	367.471	525.067	577.472	569.599	542.774	514.022
0.434783	213.578	586.785	879.468	997.474	998.766	956.561	906.726
0.434783	320.476	933.571	1465.9	1716.08	1747.52	1684.53	1599.23
0.434783	480.822	1480.48	2432.24	2940.59	3050.3	2963.68	2819.98
0.434783	721.342	2340.92	4018.48	5018.95	5310.59	5208.09	4970.89
0.434783	1082.12	3691.71	6612.95	8533.04	9220.63	9139.79	8758.35
0.434783	1623.29	5808.1	10 842.3	14 452.8	15 964.2	16 014.8	15 422.5
0.434783	2435.04	9117.98	17 715.5	24 389.8	27 559.5	28 013.3	27 137.4
0.740783	3652.68	14 285.7	28 852.8	41 013.6	47 436.7	48 909.8	47 709.5