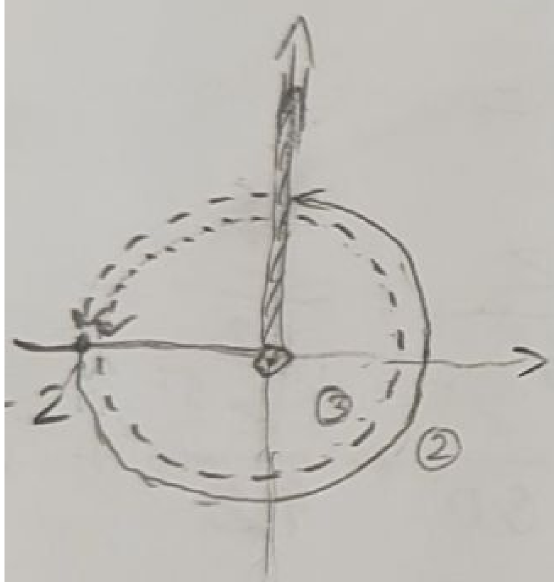
5.4

$$f(z) = \left|\frac{\psi(z)}{\psi(z_0)}\right|^n \cdot e^{i \cdot n \cdot \Delta \arg \psi} \cdot f_0 \qquad f_0 = 2^{\frac{1}{3}} \cdot e^{\frac{1}{3}\pi i}$$



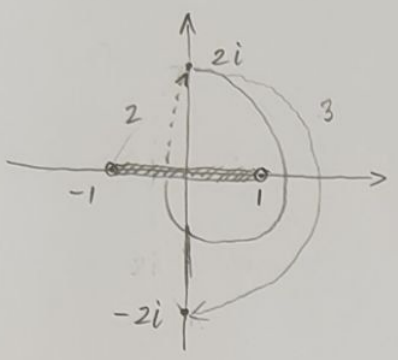
$$n = \frac{1}{3}$$

$$\Delta \arg \psi_2 = 2\pi$$

$$f(z_2) = f_0 \cdot e^{\frac{2}{3}\pi i} = 2^{\frac{1}{3}} \cdot e^{\pi i} = -2^{\frac{1}{3}}$$

$$\Delta \arg \psi_3 = 4\pi$$

$$f(z_3) = f_0 \cdot e^{\frac{4}{3}\pi i} = 2^{\frac{1}{3}} \cdot e^{\frac{5}{3}\pi i}$$



$$g(z) = \sqrt{1-z^2}$$

$$g(z_1) = \sqrt{5}$$

$$\Delta \arg \psi_3 = -2\pi$$

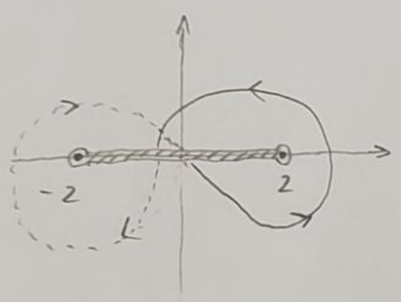
$$g(z_3)\ \sqrt{5}\cdot e^{-\pi i} = -\sqrt{5}$$

$$\Delta \arg \psi_2 = -2\pi$$

$$g(z_2) = -\sqrt{5}$$

5.6

$$g(z) = \sqrt{z^2-4}$$



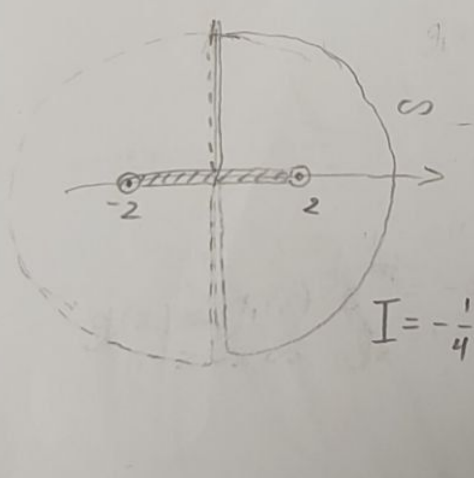
$$I = \int_\gamma \frac{dz}{g(z) - 3z} = \int_\gamma \frac{dz}{\sqrt{z^2-4} - 3z} =$$

$$= \int_\gamma \frac{\sqrt{z^2-4} + 3z}{z^2 - 4 - 9z^2} dz = -\frac{1}{4}\underbrace{\int_\gamma \frac{\sqrt{z^2-4} + 3z}{4z+1} dz}_{= I^*}$$

$$f(z) = \frac{\sqrt{z^2-4} + 3z}{4z+1} = \frac{z\sqrt{1+\left(-\frac{4}{z^2}\right)} + 3z}{1-(-4z)} =$$

$$= \left(z\cdot\left(1 - \frac{2}{z^2} - \frac{2}{z^4} + \ldots\right) + 3z\right)\cdot\left(1 - 4z + 16z^2 + \ldots\right) =$$

$$= \left(4z - \frac{2}{z} - \frac{2}{z^3} + \ldots\right)\left(1 - 4z + 16z^2 + \ldots\right)$$



$$\operatorname*{res}_{z=0} f(z) = -2 \qquad \operatorname*{res}_{z=\infty} f(z) = 2$$

$$I = -\frac{1}{4} I^* = -\frac{1}{4}\left(-2\pi i \sum_j \operatorname*{res}_j f(z)\right) = \frac{\pi i}{2}\left(\frac{1}{2}\operatorname*{res}_{z=\infty} f(z)\right) = \frac{1}{2}\pi i$$

5.1

$$I_0(z) = \int_0^\infty t^z e^{-t} dt \qquad z > -1$$

$$I(z) = \frac{t^{z+1}}{z+1} e^{-t}\Big|_0^\infty + \frac{1}{z+1}\int_0^\infty t^{z+1} e^{-t} dt = \frac{1}{z+1}\int_0^\infty t^{z+1} e^{-t} dt =$$

$$= \frac{1}{z+1}\left(\frac{t^{z+2}}{z+2} e^{-t}\Big|_0^\infty + \frac{1}{z+2}\int_0^\infty t^{z+2} e^{-t} dt\right) = \ldots =$$

$$= \frac{1}{(z+1)(z+2)(z+3)} \cdot \int_0^\infty t^{z+3} \cdot e^{-t} dt \qquad (z > -4)$$

$$I^*(z) = \frac{1}{(z+1)(z+2)} \int_0^\infty t^{z+3} \cdot e^{-t} dt$$

$$I(z) = \frac{1}{z+3} \cdot I^*(z) \qquad \operatorname*{res}_{z=-3} I(z) = I^*(-3) = \frac{1}{2}\int_0^\infty e^{-t} dt = \frac{1}{2}$$

5.2

$$f(z) = \sum_{n=0}^\infty \frac{z^n}{(n+1)(n+2)} = \sum_{n=0}^\infty \left(\frac{1}{n+1} - \frac{1}{n+2}\right) z^n =$$

$$= \underbrace{\sum_{n=0}^\infty \frac{z^n}{n+1}}_{S_1} - \underbrace{\sum_{n=0}^\infty \frac{z^n}{n+2}}_{S_2}$$

$$S_1 = 1 + \frac{z}{2} + \frac{z^2}{3} + \frac{z^3}{4} + \ldots \qquad S_2 = \frac{1}{2} + \frac{z}{3} + \frac{z^2}{4} + \frac{z^3}{5} + \ldots$$

$$S_2 = \frac{1}{z} \cdot (S_1 - 1)$$

$$f(z) = S_1 - S_2 = S_1 - \frac{1}{z}(S_1 - 1) = \frac{1}{z} + S_1\left(1 - \frac{1}{z}\right) =$$

5.10

$$f(z)=\int_1^z\left(\frac{1}{\omega}+\frac{\alpha}{\omega^3}\right)\cdot\cos\omega\ d\omega=\underbrace{\int_1^z\frac{1}{\omega}\cos\omega\ d\omega}_{=I_1}+\underbrace{\int_1^z\frac{\alpha}{\omega^3}\cdot\cos\omega\ d\omega}_{=I_2}$$

$$I_1=\ln\omega\cdot\cos\omega\Big|_1^z+\int_1^z\ln\omega\cdot\sin\omega\ d\omega=\ln z\cdot\cos z+\int_1^z\ln\omega\cdot\sin\omega\ d\omega$$

$$I_2=-\frac{1}{2}\frac{\alpha}{\omega^2}\cdot\cos\omega\Big|_1^z-\frac{\alpha}{2}\int_1^z\frac{\sin\omega}{\omega^2}d\omega=-\frac{1}{2}\frac{\alpha}{z^2}\cdot\cos z+\frac{\alpha}{2}\cos(1)+$$

$$+\frac{\alpha}{2\omega}\cdot\sin\omega\Big|_1^z-\frac{\alpha}{2}\int_1^z\frac{1}{\omega}\cos\omega\ d\omega=-\frac{1}{2}\frac{\alpha}{z^2}\cos z+\frac{\alpha}{2}\cos(1)+\frac{\alpha}{2z}\sin z-$$

$$-\frac{\alpha}{2}\cdot\sin(1)-\frac{\alpha}{2}\ln\omega\cdot\cos\omega\Big|_1^z-\frac{\alpha}{2}\int_1^z\ln\omega\cdot\sin\omega\ d\omega$$

$$I_1+I_2=\ln z\cdot\cos z-\frac{\alpha}{2}\ln z\cdot\cos z-\frac{1}{2}\frac{\alpha}{z^2}\cos z+\frac{\alpha}{2}\cos(1)+\frac{\alpha}{2z}\sin z-$$

$$-\frac{\alpha}{2}\cdot\sin(1)$$

$\ln z\cdot\cos z\cdot\left(1-\frac{\alpha}{2}\right)$ – должно быть однозначным

$$\Rightarrow \underline{\alpha=2}$$

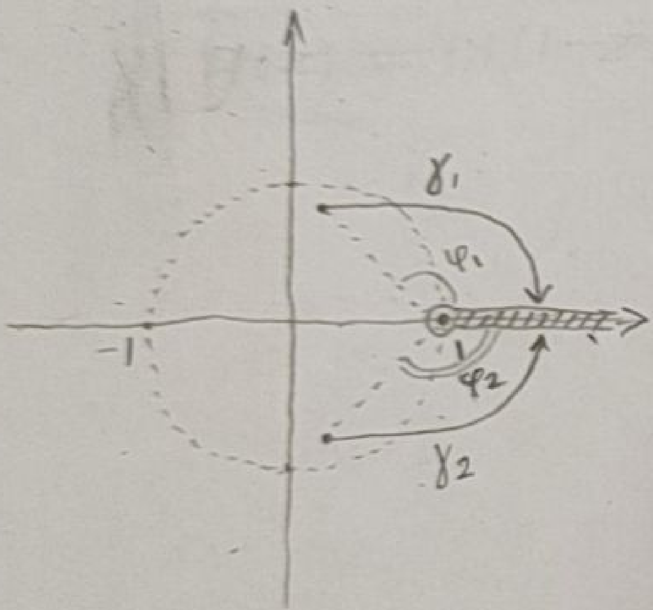
5.2

$$f(z)=\sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)}z^n=\sum_{n=0}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)z^n=$$

$$=\sum_{n=0}^{\infty}\frac{z^n}{n+1}-\sum_{n=0}^{\infty}\frac{z^n}{n+2}=\frac{1}{z}\sum_{n=0}^{\infty}\frac{z^{n+1}}{n+1}-\frac{1}{z^2}\sum_{n=0}^{\infty}\frac{z^{n+2}}{n+2}=$$

$$=\frac{1}{z}\sum_{n=1}^{\infty}\frac{z^n}{n}-\frac{1}{z^2}\sum_{n=2}^{\infty}\frac{z^n}{n}=\frac{1}{z}\left(-\ln(1-z)\right)-\frac{1}{z^2}\left(-\ln(1-z)-z\right)=$$

$$=-\frac{1}{z}\cdot\ln(1-z)+\frac{1}{z^2}\ln(1-z)+\frac{1}{z}=\frac{1}{z^2}\left[\ln(1-z)-z\cdot\ln(1-z)+z\right]$$



$$g(z)=\ln(1-z)$$

$$\gamma_1:\quad g(2+i0)=\ln\left|\frac{1+i0}{x_0}\right|-i\pi+\ln(x_0)=-ix_0$$

$$f_{\gamma_1}(2+i0)=\frac{1}{4}\left[-i\pi+2i\pi+2\right]=\frac{1}{2}+\frac{i\pi}{4}$$

$$\gamma_2:\quad g(2-i0)=\ln\left|\frac{1-i0}{x_0}\right|+i0+\ln(x_0)=i\pi$$

$$f_{\gamma_2}(2-i0)=\frac{1}{4}\left[i\pi-2i\pi+2\right]=\frac{1}{2}-\frac{i\pi}{4}$$