N1 S_{ij} – тензор $\quad S_{ij} = S_{ji}$

$$S'_{ij} = \sum_{kl} \alpha_{ik} \alpha_{jl} S_{kl}$$

$$S'_{ji} = \sum_{kl} \alpha_{jk} \alpha_{il} S_{kl} = \sum_{kl} \alpha_{il} \alpha_{jk} S_{lk} =$$

$$= \sum_{kl} \alpha_{ik} \alpha_{jl} S_{kl} = S'_{ij}$$

N2

$$\Pi' = \alpha \Pi \alpha^T$$

$$\Pi'_{ij} = \sum_{kl} \alpha_{ik} \alpha_{jl} \Pi_{kl}$$

$$(\Pi \alpha^T)_{ij} = \sum_{k} \Pi_{ik} (\alpha^T)_{kj}$$

$$[\alpha (\Pi \alpha^T)]_{ij} = \sum_{l} \alpha_{il} \sum_{k} \Pi_{lk} (\alpha^T)_{kj} =$$

$$= \sum_{l} \alpha_{il} \sum_{k} \Pi_{lk} \alpha_{jk} = \sum_{lk} \alpha_{il} \alpha_{jk} \Pi_{lk} =$$

$$= \sum_{kl} \alpha_{ik} \alpha_{jl} \Pi_{kl} = \Pi'_{ij}$$

N3 $\quad \varepsilon_{ij} = -\varepsilon_{ji} \quad \varepsilon_{ii} = 0 \quad \varepsilon_{12} = 1$

$$\alpha = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\varepsilon' = \alpha \varepsilon \alpha^T \qquad \varepsilon'_{ij} = \sum_{kl} \alpha_{ik} \alpha_{jl} \varepsilon_{kl}$$

$$\varepsilon'_{11} = \cos\theta \cdot \sin\theta - \sin\theta \cdot \cos\theta = 0 = \varepsilon_{11}$$

$$\varepsilon'_{12} = \cos^2\theta + \sin^2\theta = 1 = \varepsilon_{12}$$

$$\varepsilon'_{21} = -\sin^2\theta - \cos^2\theta = -1 = \varepsilon_{21}$$

$$\varepsilon'_{22} = -\sin\theta \cdot \cos\theta + \cos\theta \cdot \sin\theta = 0 = \varepsilon_{22}$$

$$\varepsilon' = \varepsilon$$

N4 A_{ij} B_{ij}

$$A = i_1 p_1 + i_2 p_2 + i_3 p_3 \qquad B = i_1' p_1' + i_2' p_2' + i_3' p_3'$$

1) $C = AB = (i_1 p_1 + i_2 p_2 + i_3 p_3)(i_1' p_1' + i_2' p_2' + i_3' p_3') =$

$= i_1 i_1' p_1 p_1' + i_1 i_2' p_1 p_2' + \ldots$

$C = i_i i_j' p_i p_j' = i_i i_j' (i_1 p_{i1} + i_2 p_{i2} + i_3 p_{i3})(i_1' p_{j1}' + i_2' p_{j2}' + i_3' p_{j3}')$

$= i_i i_j' \cdot \sum_{kl} i_k i_l' p_{ik} p_{jl}'$

$C = i_i i_j' i_k i_l' p_{ik} p_{jl}' \qquad C_{ijkl} = p_{ik} p_{jl}'$

2) $C_{ijkl} = A_{ij} B_{kl} = p_{ij} p_{kl}$

$D = \sum_j i_i i_j i_j i_l p_{ij} p_{jl} =$

$= i_i i_l \sum_j p_{ij} p_{jl} \qquad D_{il} = p_{ij} p_{jl}$

3) $D = \sum_i D_{ii}$

$D' = \sum_i D_{ii}' = \sum_i \sum_{kl} \alpha_{ik} \alpha_{il} D_{kl} =$

$= \sum_i \sum_{kl} \alpha_{ki}^T \alpha_{il} D_{kl} = \sum_{kl} \delta_{kl} D_{kl} = \sum_i D_{ii}$

N5 $\varphi(r) = \varphi(x_1, x_2, x_3) \qquad D_{ij} = \frac{\partial^2 \varphi}{\partial x_i \partial x_j}$

$D = \left(i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + i_3 \frac{\partial}{\partial x_3}\right)\left(i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + i_3 \frac{\partial}{\partial x_3}\right) \varphi =$

$= \left(\sum_{kl} i_k i_l \frac{\partial^2}{\partial x_k \partial x_l}\right) \varphi$

N6

$$A = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} \qquad \omega = (\omega_1, \omega_2, \omega_3) - \text{вектор?}$$

$$\omega_k = \frac{1}{2}\sum_{ij} A_{ij}\,\varepsilon_{ijk}$$

$$\omega_k' = \frac{1}{2}\sum_{ij} A_{ij}'\,\varepsilon_{ijk}' = \frac{1}{2}\sum_{ij}\sum_{pqrst} \alpha_{ip}\alpha_{jq}\alpha_{ir}\alpha_{js}\alpha_{kt} A_{pq}\,\varepsilon_{rst} =$$

$$= \frac{1}{2}\sum_{pqrst}\left(\sum_i \alpha_{ip}\alpha_{ir}\right)\cdot\left(\sum_j \alpha_{jq}\alpha_{js}\right)\alpha_{kt} A_{pq}\,\varepsilon_{rst} =$$

$$= \frac{1}{2}\sum_{pqrst} \delta_{pr}\delta_{qs}\,\alpha_{kt} A_{pq}\,\varepsilon_{rst} = \sum_t \alpha_{kt}\cdot\frac{1}{2}\left(\sum_{pq} A_{pq}\,\varepsilon_{pqt}\right) =$$

$$= \sum_t \alpha_{kt}\,\omega_t$$

N7

1) $$\sum_j A_{ij}\,\delta_{jk} = A_{ik} \qquad \sum_i A_{ij}\,\delta_{ik} = A_{kj}$$

$$\sum_{ij} A_{ij}\,\delta_{ij} = A_{ii}$$

2) $$\sum_k \delta_{ik}\delta_{kj} = \delta_{ij} \qquad \sum_{ik}\delta_{ik}\delta_{ik} = \sum_{ik}\delta_{ik} = n$$

$$\sum_{ik}\delta_{ik}\delta_{ki} = \sum_{ik}\delta_{ik} = n$$

N8

$$\varepsilon_{ij}\,\varepsilon_{lm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} = \begin{cases} 1 & ijml \in \{1212,\ 2121\} \\ -1 & ijml \in \{1221,\ 2112\} \\ 0 & \end{cases} = \varepsilon_{ij}\,\varepsilon_{lm}$$

N9

1) $\varepsilon_{ijk}\varepsilon_{lmn} = \delta_{il}\delta_{jm}\delta_{kn} - \delta_{il}\delta_{jn}\delta_{km} + \delta_{im}\delta_{jn}\delta_{kl} -$
$- \delta_{im}\delta_{jl}\delta_{kn} + \delta_{in}\delta_{jl}\delta_{km} - \delta_{in}\delta_{jm}\delta_{kl}$

2) $\sum\limits_{k} \varepsilon_{ijk}\varepsilon_{lmk} = 3\cdot\delta_{il}\delta_{jm} - \delta_{il}\delta_{jm} + \delta_{im}\delta_{jl} -$
$- 3\cdot\delta_{im}\delta_{jl} + \delta_{im}\delta_{jl} - \delta_{il}\delta_{jm} =$
$= \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

3) $\sum\limits_{jk} \varepsilon_{ijk}\varepsilon_{ljk} = 3\,\delta_{il} - \delta_{il} = 2\,\delta_{il}$

N10

A_{ij} - антисимметричный тензор $A_{ij} = -A_{ji}$
S_{ij} - симметричный тензор $S_{ij} = S_{ji}$

$$\sum_{ij} A_{ij} S_{ij} = \sum_{ij} A_{ij} S_{ji} = -\sum_{ij} A_{ji} S_{ji} =$$
$$= -\sum_{ij} A_{ij} S_{ij} \Rightarrow \sum_{ij} A_{ij} S_{ij} = 0$$

N11 Π_{ij} - тензор $\vec{a}$ - вектор

1) $b_i = \sum\limits_{j} \Pi_{ij}\, a_j$

$$b'_i = \sum_{j} \Pi'_{ij}\, a'_j = \sum_{jlmn} \alpha_{il}\alpha_{jm}\Pi_{lm}\,\alpha_{jn} a_n =$$
$$= \sum_{jlmn} \alpha_{il}\alpha_{jm}\alpha_{jn}\Pi_{lm}\, a_n = \sum_{lmn} \alpha_{il}\delta_{mn}\Pi_{lm}\, a_n =$$
$$= \sum_{lm} \alpha_{il}\Pi_{lm}\, a_m = \sum_{l} \alpha_{il} \sum_{m} \Pi_{lm}\, a_m =$$
$$= \sum_{l} \alpha_{il}\, b_l$$

2) $c_j = \sum\limits_{i} \Pi_{ij}\, a_i$

$$c'_j = \sum_{i} \Pi'_{ij}\, a'_i = \sum_{ilmn} \alpha_{il}\alpha_{jm}\alpha_{in}\Pi_{lm}\, a_n =$$
$$= \sum_{lmn} \alpha_{jm}\delta_{ln}\Pi_{lm}\, a_n = \sum_{lm} \alpha_{jm}\Pi_{lm}\, a_l =$$
$$= \sum_{m} \alpha_{jm} \sum_{l} \Pi_{lm}\, a_l = \sum_{m} \alpha_{jm}\, c_m$$