N1 $$\frac{E_z^2}{E_2^2} - 2\frac{E_z E_y}{E_1 E_2} \cdot \cos\alpha + \frac{E_y^2}{E_1^2} = \sin^2\alpha$$

1) $$a = \frac{\sin\alpha}{\sqrt{\left(\frac{\cos\theta}{E_1}\right)^2 + 2\frac{\cos\theta \cdot \sin\theta}{E_1 E_2}\cos\alpha + \left(\frac{\sin\theta}{E_2}\right)^2}}$$

$$b = \frac{\sin\alpha}{\sqrt{\left(\frac{\sin\theta}{E_1}\right)^2 - 2\frac{\sin\theta \cdot \cos\theta}{E_1 \cdot E_2}\cos\alpha + \left(\frac{\cos\theta}{E_2}\right)^2}}$$

2) $E_y = E_y' \cdot \cos\theta + E_z' \cdot \sin\theta$

$E_z = -E_y' \cdot \sin\theta + E_z' \cdot \cos\theta$

$$E_z^2 - 2\frac{E_2}{E_1} E_z E_y \cdot \cos\alpha + \frac{E_2^2}{E_1^2} E_y^2 = E_2^2 \cdot \sin^2\alpha$$

$$E_y'^2 \cdot \sin^2\theta - 2E_y' E_z' \cdot \sin\theta \cdot \cos\theta + E_z'^2 \cdot \cos^2\theta -$$

$$-2\frac{E_z}{E_1}\cos\alpha \cdot \left[-E_y'^2 \cdot \sin\theta\cos\theta + E_y' E_z' \cdot \cos^2\theta - E_y' E_z' \cdot \sin^2\theta + E_z'^2 \cdot \sin\theta \cdot \cos\theta\right]$$

$$+\frac{E_2^2}{E_1^2} \cdot \left[E_y'^2 \cdot \cos^2\theta + 2E_y' E_z' \cdot \sin\theta \cdot \cos\theta + E_z'^2 \cdot \sin^2\theta\right] = E_2^2 \cdot \sin^2\alpha$$

$$\frac{E_y'^2}{E_2^2} \cdot \sin^2\theta - \frac{E_y' E_z'}{E_2^2} \cdot \sin 2\theta + \frac{E_z'^2}{E_2^2} \cdot \cos^2\theta - 2\frac{\cos\alpha}{E_1 E_2}\left[-\frac{1}{2}E_y'^2 \cdot \sin 2\theta + \right.$$

$$\left. + \frac{1}{2}E_z'^2 \cdot \sin 2\theta + E_y' E_z' \cdot \cos 2\theta\right] + \frac{E_y'^2}{E_1^2} \cdot \cos^2\theta +$$

$$+ \frac{E_y' E_z'}{E_2^2} \cdot \sin 2\theta + \frac{E_z'^2}{E_2^2} \cdot \sin^2\theta = \sin^2\alpha \qquad \ldots$$

$$tg\, 2\theta = \frac{2E_1 E_2 \cdot \cos\alpha}{E_2^2 - E_1^2}$$

3) Круговая поляризация наступает при: $\alpha = \frac{\pi}{2} + \pi k \quad k \in \mathbb{Z} \quad E_2 = E_1$

N2

$$f = t^2 \qquad t \in [-\pi; \pi]$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cdot \cos nt + b_n \cdot \sin nt)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} t^2 \, dt = \frac{1}{3\pi} t^3 \Big|_{-\pi}^{\pi} = \frac{1}{3\pi} \cdot 2\pi^3 = \frac{2}{3}\pi^2$$

$$a_n = \frac{2}{\pi} \int_{0}^{\pi} t^2 \cdot \cos nt \, dt = \left| \begin{matrix} u = t^2 & du = 2t\,dt \\ dv = \cos nt\,dt & v = \frac{1}{n}\sin nt \end{matrix} \right| =$$

$$= \frac{2t^2}{\pi n} \sin nt \Big|_{0}^{\pi} - \frac{4}{\pi n} \int_{0}^{\pi} t \cdot \sin nt \, dt = \left| \begin{matrix} u = t & du = dt \\ dv = \sin nt\,dt & v = \frac{-1}{n}\cos nt \end{matrix} \right| =$$

$$= \frac{4t}{\pi n^2} \cdot \cos nt \Big|_{0}^{\pi} - \frac{2}{\pi n^2} \int_{0}^{\pi} \cos nt \, dt = \frac{(-1)^n \cdot 4}{n^2}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} t^2 \cdot \sin nt \, dt = \ldots = 0$$

$$f(t) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 4}{n^2} \cdot \cos nt = t^2$$

$$f(\pi) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 4}{n^2} \cdot (-1)^n = \pi^2$$

$$4 \cdot \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2}{3}\pi^2 \qquad \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

N 3 Неподвижная система отсчёта:

$$v_0 = u \qquad v_- = v \qquad v_+ = 0$$

Движущаяся система отсчета (с зарядом):

$$v_0' = 0 \qquad v_-' = \frac{v-u}{1-\frac{vu}{c^2}} \qquad v_+' = -u$$

Движущаяся система отсчёта (с электронами):

$$v_-'' = 0$$

$$p_+' = \frac{p_+}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\begin{cases} p_- = \dfrac{p_-''}{\sqrt{1-\frac{v^2}{c^2}}} \\ p_-' = \dfrac{p_-''}{\sqrt{1-\frac{1}{c^2}\left(\frac{v-u}{1-\frac{uv}{c^2}}\right)^2}} \end{cases} \Rightarrow p_-' = \frac{\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{1}{c^2}\left(\frac{v-u}{1-\frac{uv}{c^2}}\right)^2}} p_-$$

$$p' = p_+' + p_-' = \frac{p_+}{\sqrt{1-\frac{u^2}{c^2}}} - \frac{\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{1}{c^2}\left(\frac{v-u}{1-\frac{uv}{c^2}}\right)^2}} p_+ =$$

$$= \frac{uv}{\sqrt{1-\frac{u^2}{c^2}}\, c^2} p_+$$

$$E' = \frac{2p'S}{R} = \frac{2uv\, p_+ \cdot S}{\sqrt{1-\frac{u^2}{c^2}} \cdot c^2 \cdot R} = \frac{1}{c^2} \frac{2uI}{\sqrt{1-\frac{u^2}{c^2}}} \frac{1}{R}$$

$$F' = qE' = \frac{2uq \cdot I}{c^2\sqrt{1-\frac{u^2}{c^2}} \cdot R}$$

N 4

1) $(\vec{a}\nabla)\cdot\vec{r}$ $\qquad \vec{r} = (x \ y \ z)$

$$\left(a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}\right)(x \ y \ z) =$$

$$= \begin{pmatrix} a_x \frac{\partial x}{\partial x} + a_y \frac{\partial x}{\partial y} + a_z \frac{\partial x}{\partial z} \\ a_x \frac{\partial y}{\partial x} + a_y \frac{\partial y}{\partial y} + a_z \frac{\partial y}{\partial z} \\ a_x \frac{\partial z}{\partial x} + a_y \frac{\partial z}{\partial y} + a_z \frac{\partial z}{\partial z} \end{pmatrix} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = \vec{a}$$

2) $\nabla \times (a(r) \times b)$

Докажем $\vec{a} \times (\vec{b} \times \vec{c})$

Выберем правый ортонормированный базис $\vec{e}_1, \vec{e}_2, \vec{e}_3$ так:

$\vec{a} = \alpha_1 \vec{e}_1 + \alpha_2 \vec{e}_2 + \alpha_3 \vec{e}_3$

$\vec{b} = \beta_1 \vec{e}_1 + \beta_2 \vec{e}_2$

$\vec{c} = \gamma_1 \vec{e}_1$

$\times$	e_1	e_2	e_3
e_1	0	$+e_3$	$-e_2$
e_2	$-e_3$	0	$+e_1$
e_3	$+e_2$	$-e_1$	0

$\vec{e}_i \cdot \vec{e}_j = \delta_{ij}$

$$\vec{b} \times \vec{c} = (\beta_1 \vec{e}_1 + \beta_2 \vec{e}_2) \times \gamma_1 \vec{e}_1 =$$

$$= \beta_1 \gamma_1 (\vec{e}_1 \times \vec{e}_1) + \beta_2 \gamma_1 (\vec{e}_2 \times \vec{e}_1) = -\beta_2 \gamma_1 \vec{e}_3$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\alpha_1 \vec{e}_1 + \alpha_2 \vec{e}_2 + \alpha_3 \vec{e}_3) \times (-\beta_2 \gamma_1 \vec{e}_3) =$$

$$= +\alpha_1 \beta_2 \gamma_1 \cdot \vec{e}_2 - \alpha_2 \beta_2 \gamma_1 \cdot \vec{e}_1$$

$$(\vec{a}\cdot\vec{c})\cdot\vec{b} - (\vec{a}\cdot\vec{b})\cdot\vec{c} = \alpha_1\gamma_1\cdot\vec{b} - (\alpha_1\beta_1 + \alpha_2\beta_2)\,\vec{c} =$$

$$= \alpha_1\beta_1\gamma_1\cdot\vec{e}_1 + \alpha_1\beta_2\gamma_1\vec{e}_2 - \alpha_1\beta_1\gamma_1\cdot\vec{e}_1 - \alpha_2\beta_2\gamma_1\cdot\vec{e}_1 =$$

$$= \alpha_1\beta_2\gamma_1\cdot\vec{e}_2 - \alpha_2\beta_2\gamma_1\cdot\vec{e}_1$$

$$\Rightarrow \quad \vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\cdot\vec{b} - (\vec{a}\cdot\vec{b})\cdot\vec{c} =$$

$$= (\vec{c}\cdot\vec{a})\cdot\vec{b} - (\vec{a}\cdot\vec{b})\cdot\vec{c}$$

$$\nabla\times(a(r)\times b) = (b\nabla)\cdot a(r) - b(\nabla\cdot a(r))$$

N5

1) $$\nabla \times f(r)\cdot r = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(r)\cdot x & f(r)\cdot y & f(r)\cdot z \end{vmatrix} =$$

$$= \left(\frac{\partial}{\partial y}(f(r)\cdot z) - \frac{\partial}{\partial z}(f(r)\cdot y)\right) i + \left(\frac{\partial}{\partial z}(f(r)\cdot x) - \frac{\partial}{\partial x}(f(r)\cdot z)\right) j +$$

$$+ \left(\frac{\partial}{\partial x}(f(r)\cdot y) - \frac{\partial}{\partial y}(f(r)\cdot x)\right) k =$$

$$= \left(z\cdot\frac{\partial f(r)}{\partial y} - y\cdot\frac{\partial f(r)}{\partial z}\right) i + \left(x\cdot\frac{\partial f(r)}{\partial z} - z\cdot\frac{\partial f(r)}{\partial x}\right) j + \left(y\frac{\partial f(r)}{\partial x} - x\cdot\frac{\partial f(r)}{\partial y}\right) k =$$

$$= \begin{pmatrix} \frac{\partial f(r)}{\partial z} - \frac{\partial f(r)}{\partial y} \\ \frac{\partial f(r)}{\partial x} - \frac{\partial f(r)}{\partial z} \\ \frac{\partial f(r)}{\partial y} - \frac{\partial f(z)}{\partial x} \end{pmatrix} \cdot \vec{r}$$

2) $$\nabla \times (a \times r) = (a\nabla)\cdot r - r\cdot(\nabla\cdot a) = a$$

N 6

$$\frac{1}{c^2(x)}\frac{\partial^2 f}{\partial t^2}-\frac{\partial^2 f}{\partial x^2}=0$$

1) $\frac{\partial f}{\partial t}=i\omega\cdot f \qquad \frac{\partial}{\partial t}(i\cdot\omega\cdot f)=-\omega^2 f$

$$\frac{\omega^2}{c^2(x)}\cdot f+\frac{\partial^2 f}{\partial x^2}=0 \qquad \frac{\omega}{c(x)}=k(x)$$

$$\frac{\partial^2 f}{\partial x^2}+k^2(x)\cdot f=0$$

2) $c(x+\lambda)-c(x)=\frac{dc(x)}{dx}\lambda=\frac{dc(x)}{dx}\frac{2\pi\cdot c(x)}{\omega}$

$$\frac{c(x+\lambda)-c(x)}{c(x)}=\frac{2\pi}{\omega}\frac{dc(x)}{dt}<<1 \quad (\text{по условию})$$

$$\frac{2\pi}{\omega}\frac{d}{dx}\left(\frac{\omega}{k(x)}\right)=\frac{2\pi}{\omega}\left(-\frac{\omega}{k^2}\right)\cdot\frac{dk(x)}{dx}=-2\pi\frac{dk(x)}{dx}\frac{1}{k^2}$$

$$\frac{dk(x)}{dx}\frac{1}{k^2}<<1$$

3) $f=e^{S(x)}$:

$$\frac{\partial^2 e^{S(x)}}{\partial x^2}+k^2(x)\cdot e^{S(x)}=0$$

$$\frac{d^2S}{dx^2}+\left(\frac{dS}{dx}\right)^2+k^2(x)=0$$

4) $\frac{d^2S}{dx^2}<<k^2(x)$:

$$\left(\frac{dS}{dx}\right)^2+k^2(x)=0$$

$$\frac{dS}{dx}=\pm i\,k(x) \qquad S=\pm i\int k(x)\,dx$$

$$\frac{d^2S}{dx^2}=\pm i\,\frac{dk(x)}{dx}<<k^2(x)$$

№7

$$\nabla \cdot \vec{E} = 4\pi \rho$$

$$\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot (\nabla \times \vec{H}) = 0 = \nabla \cdot \left(\frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\right) = \frac{4\pi}{c}\nabla \vec{j} + \frac{1}{c}\nabla \cdot \frac{\partial \vec{E}}{\partial t} =$$

$$= \frac{4\pi}{c}\nabla \vec{j} + \frac{1}{c}\frac{\partial}{\partial t}(\nabla \cdot \vec{E}) = \frac{4\pi}{c}\nabla \vec{j} + \frac{1}{c}\frac{\partial}{\partial t}\cdot(4\pi \rho)$$

$$\frac{\partial \rho}{\partial t} + \nabla \vec{j} = 0$$

№8

$$\vec{H} = \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times \vec{A}) =$$

$$= -\frac{1}{c}\nabla \times \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times \left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = 0$$

$$\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\nabla \varphi \qquad \vec{E} = -\nabla \varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\Rightarrow \quad A' = A + \nabla \psi \qquad \varphi' = \varphi - \frac{\partial \psi}{\partial t}$$

N9

$$B_z(z) = B_0 - \alpha z$$

1) $\nabla \cdot \vec{B} = 0 = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \qquad (B_y = 0)$

$$\frac{\partial B_x}{\partial x} - \alpha = 0$$

$$B_x = \alpha x + C(z)$$

$x=0 \quad z=0: \quad B_x = B_z \qquad C(z) = B_0$

$$B_x = B_0 + \alpha x$$

2) $\frac{dx}{B_0 + \alpha x} = \frac{dz}{B_0 - \alpha z} \qquad \frac{dz}{dx} = \frac{B_0 - \alpha z}{B + \alpha x}$