① $\cdot\ \mathrm{Re}[e^{\lambda(it-t^2)}] = \mathrm{Re}[e^{-\lambda t^2}\cdot e^{i\lambda t}] =$

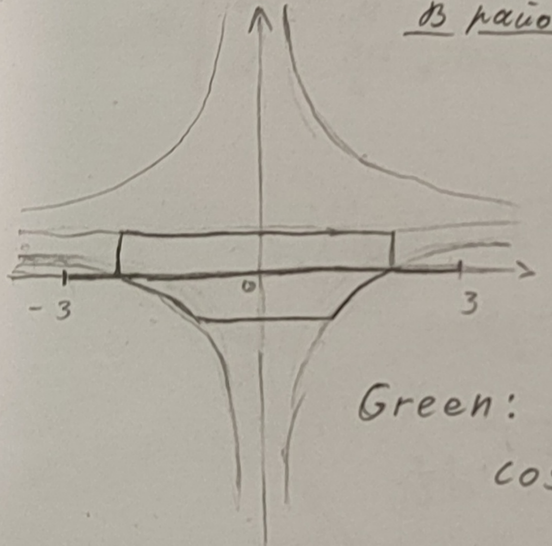
$t = x+iy \qquad t^2 = x^2-y^2+2ixy \qquad \lambda\in\mathbb{R}$

$= \mathrm{Re}[e^{-\lambda(x^2-y^2)-2i\lambda xy}\cdot e^{-y\lambda+i\lambda x}] =$

$= \mathrm{Re}[e^{\lambda(y^2-x^2)}\cdot(\cos(2\lambda xy)-i\cdot\sin(2\lambda xy))\cdot e^{-y\lambda}\cdot(\cos(\lambda x)+i\cdot\sin(\lambda x))] =$

$= e^{\lambda(y^2-y-x^2)}\cdot(\cos(2\lambda xy)\cdot\cos(\lambda x)+\sin(2\lambda xy)\cdot\sin(\lambda x)) =$

$= e^{\lambda(y^2-y-x^2)}\cdot\cos[2\lambda xy-\lambda x] = e^{\lambda(y^2-y-x^2)}\cdot\cos[\lambda x(2y-1)]$



В районе $x=0$:

Red: $y=0 \quad \lambda=32$

$\cos[32x]=0$

$32x = \frac{\pi}{2}+\pi k \qquad k\in\mathbb{Z}$

$x = \frac{\pi}{64}+\frac{\pi}{32}k$

Green: $y=\frac{1}{2} \quad \lambda=32$

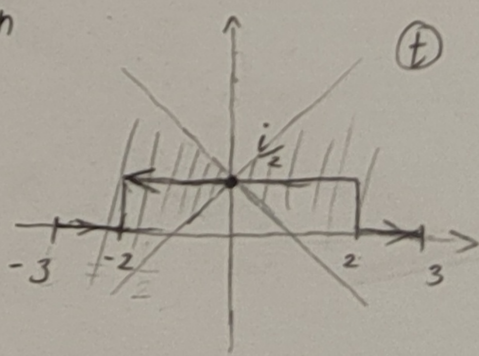
$\cos[32x\cdot(2\cdot\frac{1}{2}-1)] = \cos[0] = 1$

Black: $y=-\frac{1}{2} \quad \lambda=32$

$\cos[32x(2\cdot\frac{-1}{2}-1)] = \cos[64x]$

$x = \frac{\pi}{128}+\frac{\pi}{64}k \quad k\in\mathbb{Z}$

$\Rightarrow$ a – Red, b – Black, c – Green



$\cdot\ I(\lambda) = \int\limits_{\text{Green}} \frac{e^{\lambda(it-t^2)}}{1+t^2}\,dt$

$\frac{d}{dt}(\lambda[it-t^2]) = \lambda[i-2t] = 0$

$t = \frac{i}{2}$

$\frac{d^2}{dt^2}[it-t^2] = -2$

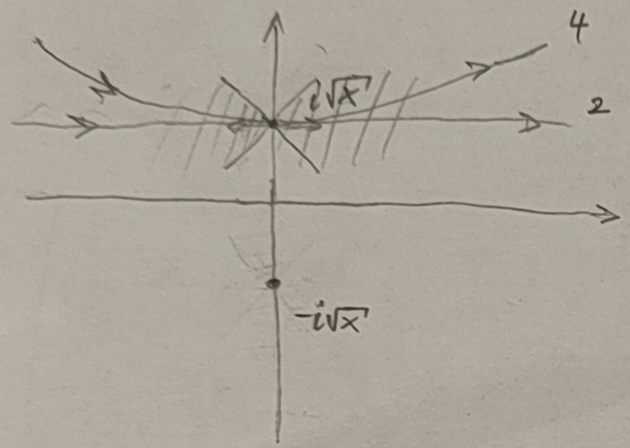
По методу Лапласа $(\lambda\to+\infty)$:

$I(\lambda)\approx\sqrt{\frac{\pi}{\lambda}}\,e^{-\frac{\lambda}{4}}\cdot\left[\frac{4}{3}+O(\lambda^{-1})\right]$

②

$$Ai(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(xt+\frac{t^3}{3})}dt$$

$$\frac{d^2}{dt^2}\left[i(xt+\frac{t^3}{3})\right]\Big|_{t=\pm i\sqrt{x}} = 2it\Big|_{t=\pm i\sqrt{x}} = \mp 2\sqrt{x}$$



По контуру 2:

$$i(xt+\frac{t^3}{3}) \approx (-x^{\frac{3}{2}}+\frac{1}{3}x^{\frac{3}{2}}) + \frac{(t-i\sqrt{x})^2}{2}\cdot(-2\sqrt{x}) =$$

$$= -\frac{2}{3}x^{\frac{3}{2}} - \sqrt{x}(t-i\sqrt{x})^2$$

$$\Rightarrow \underset{x\to+\infty}{Ai(x)} \approx \frac{1}{2\pi}e^{-\frac{2}{3}x}\cdot\int_{-\infty}^{+\infty} e^{-\sqrt{x}(t-i\sqrt{x})^2}dt$$

(3)

$$Ai(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\left(xt+\frac{t^3}{3}\right)}dt \qquad x \to -\infty$$

$$t_0 = \pm\sqrt{|x|}$$

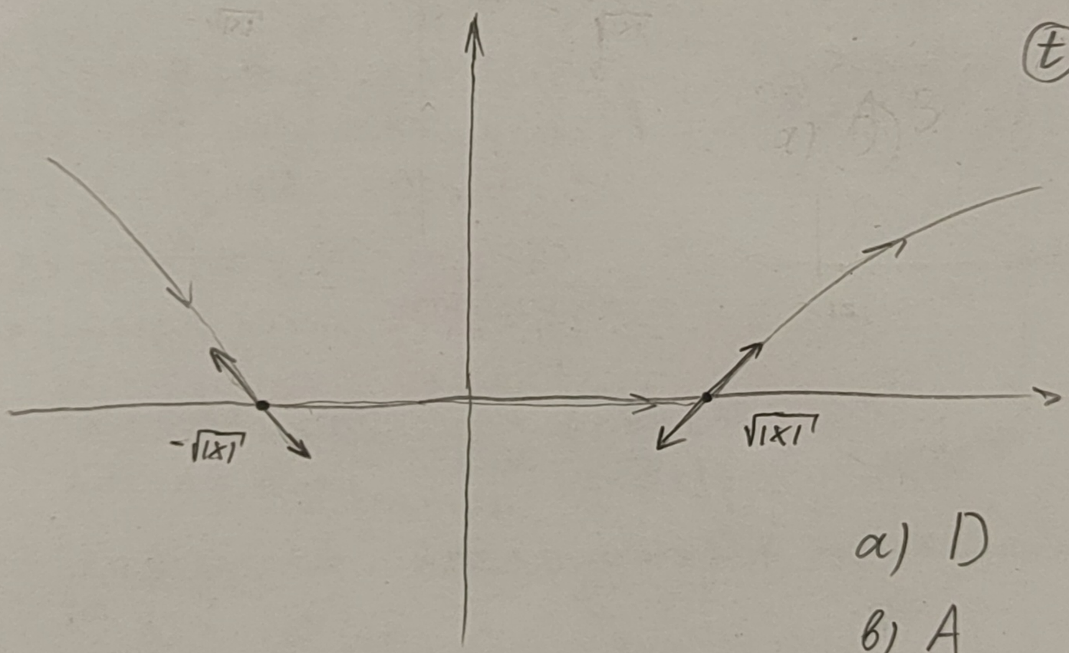
Условие наискорейшего спуска

$$\arg\left[f''(x,t)\big|_{t=t_0}\right] + 2\alpha = \pi + 2\pi k \quad k\in\mathbb{Z}$$

$$f''(x,t) = 2i\cdot t$$

$$\left[\begin{array}{l}\arg\left[2i(\sqrt{|x|})\right]+2\alpha = \pi+2\pi k\\ \arg\left[2i(-\sqrt{|x|})\right]+2\alpha = \pi+2\pi k\end{array}\right. \Rightarrow \left[\begin{array}{l}\frac{\pi}{2}+2\alpha = \pi+2\pi k\\ -\frac{\pi}{2}+2\alpha = \pi+2\pi k\end{array}\right. \Rightarrow$$

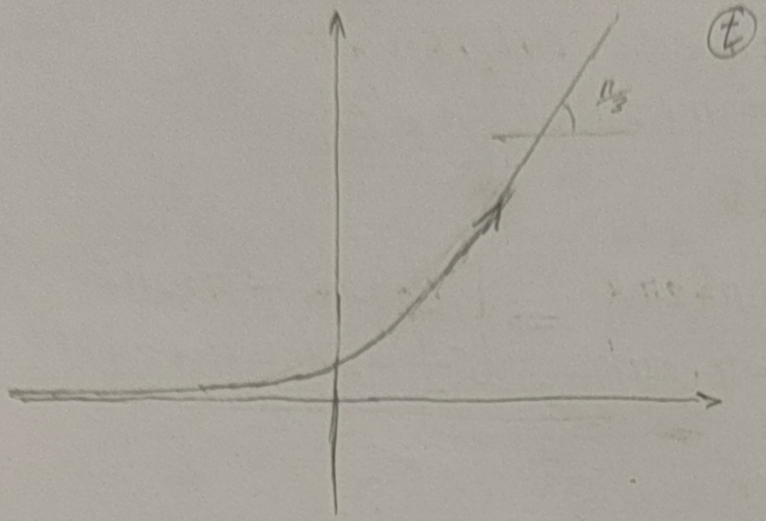
$$\Rightarrow \left[\begin{array}{l}\alpha = \frac{\pi}{4}+\pi k\\ \alpha = \frac{3\pi}{4}+\pi k\end{array}\right.$$



а) D

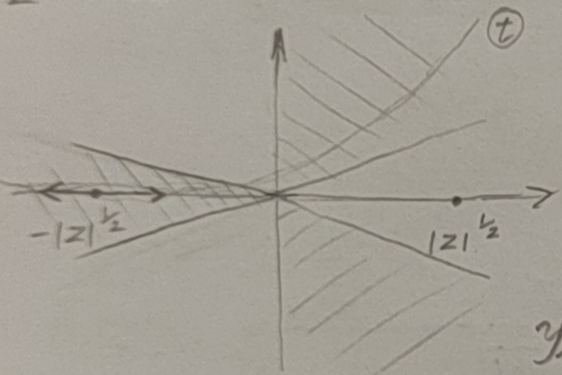
в) A

(4) $I(z) = \int\limits_{-\infty}^{e^{\pi i/3}\cdot\infty} e^{\left(\frac{t^3}{3} - zt\right)}\, dt$ $\qquad |z| \gg 1$



$\left(\frac{t^3}{3} - zt\right)' = t^2 - z = 0 \qquad t = \pm\sqrt{z}$

$z \to +\infty$



При $|t| \to \infty$ интеграл сходится при $Re\left(\frac{t^3}{3}\right) \to -\infty$

$t = \rho \cdot e^{i\varphi}$ $\quad Re\left(\frac{t^3}{3}\right) = \frac{\rho^3}{3}\cos(3\varphi)$

$\cos(3\varphi) < 0 \Rightarrow \frac{\pi}{6} + \frac{2\pi}{3}k < \varphi < \frac{\pi}{2} + \frac{2\pi}{3}k$

$\Rightarrow |z|^{1/2} \cdot e^{i\pi}$ – точка перевала

Условие наискорейшего спуска:

$\arg\left[\left(\frac{t^3}{3} - zt\right)''\Big|_{t=|z|^{1/2}e^{i\pi}}\right] + 2\alpha = \pi + 2\pi k \qquad k \in \mathbb{Z}$

$\pi + 2\alpha = \pi + 2\pi k \Rightarrow \alpha = \pi k$

(4.2)

$$f(z,t)=\frac{t^3}{3}-zt$$

$t_0=-|z|^{\frac12}$ $\quad t=|z|^{\frac12}\cdot s$ $\quad s_0=-1$

$f(z,s)=\frac{|z|^{\frac32}}{3}(s^3-3s)$ $\quad f''(z,s)=2|z|^{\frac32}\cdot s$

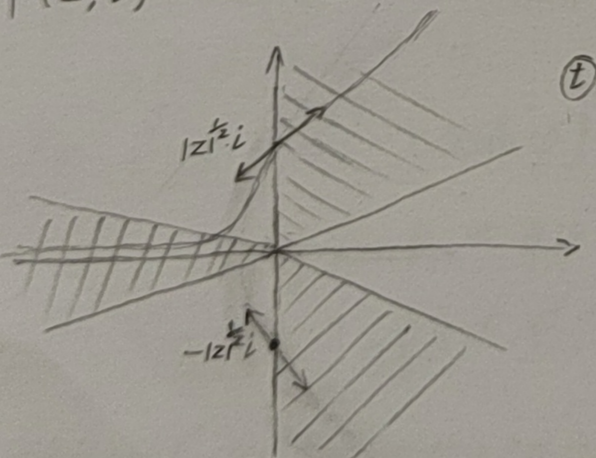
$$I(s)=\int\limits_{-\infty}^{e^{\frac{\pi}{3}i}\cdot\infty} e^{\frac{|z|^{3/2}}{3}(s^3-3s)}|z|^{\frac12}ds$$

$$f(z,s)\approx f(z,s_0)+\frac{f''(z,s)|_{s=s_0}}{2}(s-s_0)^2=$$
$$=\frac23|z|^{\frac32}-|z|^{\frac32}(s+1)^2$$

$$I(s)\approx\int\limits_{-1-\varepsilon}^{-1+\varepsilon}|z|^{\frac12}\cdot e^{\frac23|z|^{\frac32}-|z|^{3/2}(s+1)^2}ds=\begin{vmatrix} x=s+1\\ ds=dx\end{vmatrix}=$$
$$=|z|^{\frac12}\cdot e^{\frac23|z|^{\frac32}}\int\limits_{-\varepsilon}^{\varepsilon}e^{-|z|^{\frac32}x^2}dx=|z|^{\frac12}e^{\frac23|z|^{\frac32}}\cdot\frac{\sqrt{2\pi}}{\sqrt{2|z|^{\frac32}}}=$$
$$=|z|^{-\frac14}\sqrt{\pi}\,e^{\frac23|z|^{\frac32}}$$

$z\to-\infty$ $\quad z=|z|e^{i\pi}$

$f'(z,t)=t^2-z=t^2-|z|e^{i\pi}=0$ $\quad t^2=|z|e^{i\pi}$ $\quad t_0=\pm|z|^{\frac12}e^{i\frac{\pi}{2}}$



Условие наискорейшего спуска:

$$\arg\left[\left(\frac{t^3}{3}-zt\right)''\Big|_{t=\pm|z|^{\frac12}e^{i\frac{\pi}{2}}}\right]+2\alpha=\pi+2\pi k$$

$$\arg\left[2\cdot(\pm|z|^{\frac12})e^{i\frac{\pi}{2}}\right]+2\alpha=\pi+2\pi k$$

$$\left[\begin{array}{l}\frac{\pi}{2}+2\alpha=\pi+2\pi k\\ -\frac{\pi}{2}+2\alpha=\pi+2\pi k\end{array}\right.\Rightarrow\left[\begin{array}{l}\alpha=\frac{\pi}{4}+\pi k\\ \alpha=\frac{3\pi}{4}+\pi k\end{array}\right.$$

$\alpha=\frac{\pi}{4}$ $\quad t_0=|z|^{\frac12}e^{i\frac{\pi}{2}}=|z|^{\frac12}\cdot i$

$t=|z|^{\frac12}i\cdot s$ $\quad s_0=1$ $\quad f(z,s)=-\frac{i|z|^{\frac32}}{3}(s^3+3s)$ $\quad f''(z,s)=-2i|z|^{\frac32}\cdot s$

$$I(s)=\int\limits_{-i\infty}^{e^{-\frac{i\pi}{6}}\infty}e^{-\frac{i|z|^{\frac32}}{3}(s^3+s)}|z|^{\frac12}i\cdot ds$$

$$f(z,s) \approx f(z,s_0) + \frac{f''(z,s_0)}{2}(s-s_0)^2 =$$

$$= -\frac{4}{3} i|z|^{\frac{3}{2}} - i|z|^{\frac{3}{2}}(s-1)^2$$

$$I(s) \approx \int_{1-\varepsilon}^{1+\varepsilon} i|z|^{\frac{1}{2}} \cdot e^{-\frac{4}{3}i|z|^{\frac{3}{2}} - i|z|^{\frac{3}{2}}(s-1)^2} ds = \left| \begin{matrix} x = s-1 \\ ds = dx \end{matrix} \right| =$$

$$= i|z|^{\frac{1}{2}} e^{-\frac{4}{3}i|z|^{\frac{3}{2}}} \int_{-\varepsilon}^{\varepsilon} e^{-i|z|^{\frac{3}{2}}x^2} dx = i|z|^{\frac{1}{2}} e^{-\frac{4}{3}i|z|^{\frac{3}{2}}} \frac{\sqrt{2\pi}}{\sqrt{2i|z|^{\frac{3}{2}}}} =$$

$$= e^{\frac{\pi}{4}i} |z|^{-\frac{1}{4}} \sqrt{\pi} \cdot e^{-\frac{4}{3}i|z|^{\frac{3}{2}}}$$

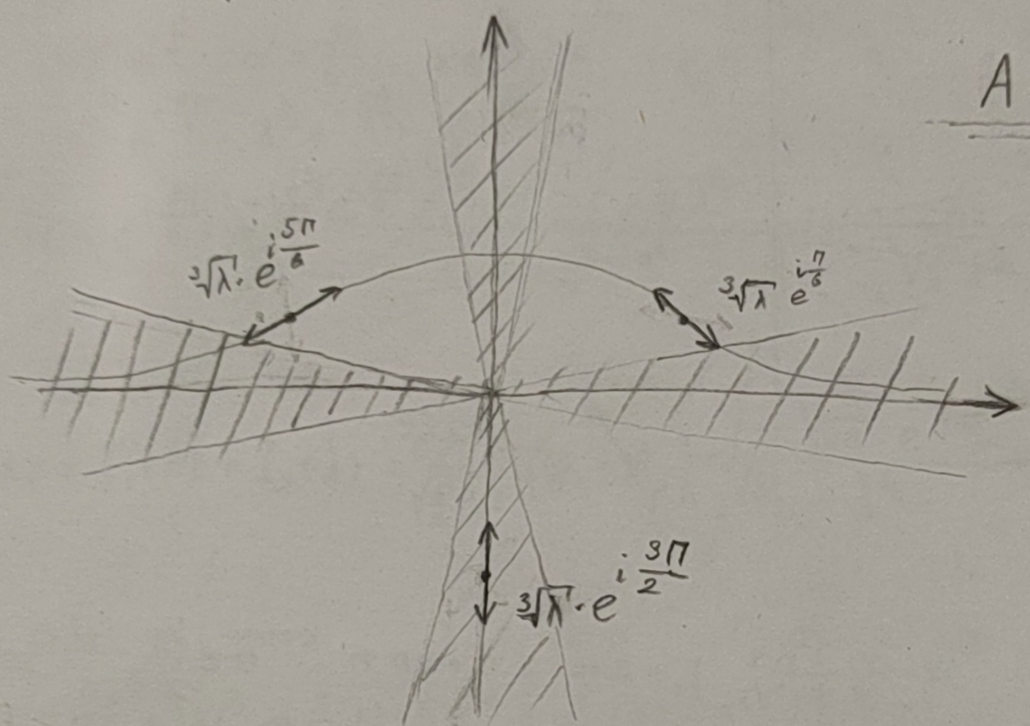
⑤
$$I(\lambda) = \int_{-\infty}^{+\infty} e^{-\frac{x^4}{4} + i\lambda x}\, dx$$

$$f(x,\lambda) = -\frac{x^4}{4} + i\lambda x \qquad \frac{\partial f}{\partial x} = -x^3 + i\lambda = 0$$

- $\lambda \to +\infty$

$$x^3 = \lambda \cdot e^{i\left(\frac{\pi}{2} + 2\pi k\right)} \qquad k \in \mathbb{Z}$$

$$x_p = \sqrt[3]{\lambda} \cdot e^{i\left(\frac{\pi}{6} + \frac{2}{3}\pi k\right)}$$



A + 4

$x \to +\infty$: $f(x,\lambda) \longrightarrow -\frac{x^4}{4}$

для сходимости $\quad Re\left(\frac{x^4}{4}\right) \longrightarrow +\infty$

$$x = \rho \cdot e^{i\varphi}$$

$$Re\left(\frac{x^4}{4}\right) = \frac{\rho^4}{4}\cdot\cos(4\varphi) \quad \Rightarrow \quad \cos(4\varphi) > 0$$

$$-\frac{\pi}{2} + 2\pi k < 4\varphi < \frac{\pi}{2} + 2\pi k, \quad k \in \mathbb{Z}$$

$$-\frac{\pi}{8} + \frac{\pi}{2}k < \varphi < \frac{\pi}{8} + \frac{\pi}{2}k$$

$e^{i\pi}$

направление наискорейшего спуска

$$\frac{\partial^2 f}{\partial x^2} = -3x^2 \qquad \arg\left(\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_p}\right) + 2\alpha = \pi + 2\pi n \qquad n \in \mathbb{Z}$$

$x_p = \sqrt[3]{\lambda}\cdot e^{i\frac{\pi}{6}} \qquad \pi + \frac{\pi}{3} + 2\alpha = \pi + 2\pi n \qquad \alpha = -\frac{\pi}{6} + \pi n$

$x_p = \sqrt[3]{\lambda}\cdot e^{i\frac{5\pi}{6}} \qquad \pi + \frac{5\pi}{3} + 2\alpha = \pi + 2\pi n \qquad \alpha = -\frac{5\pi}{6} + \pi n$

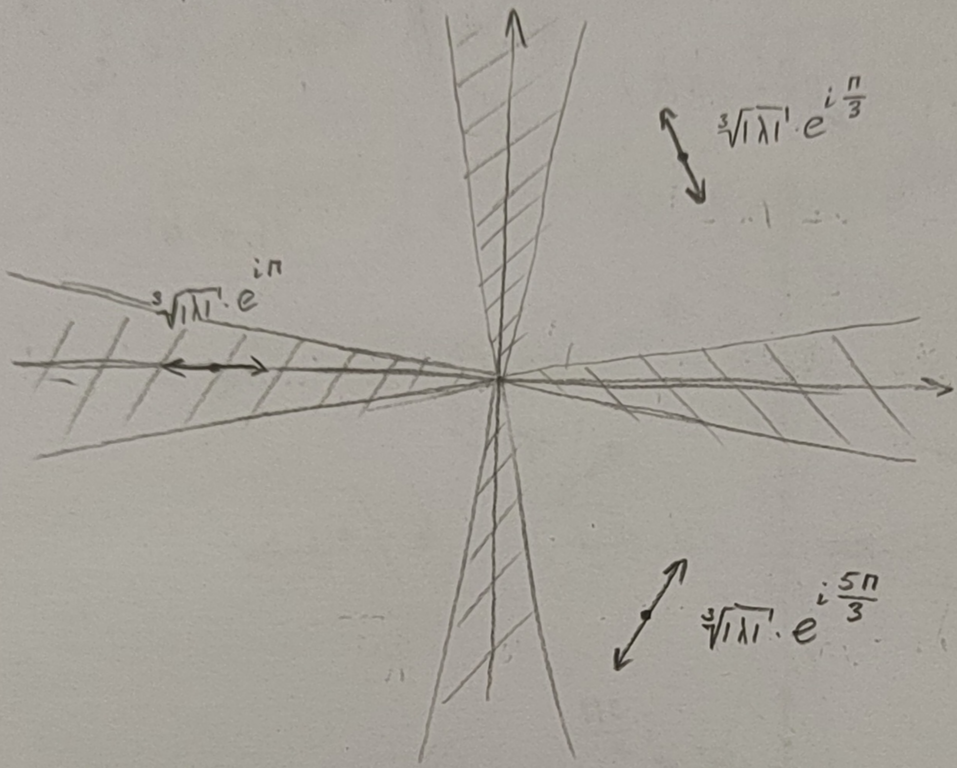
$x_p = \sqrt[3]{\lambda}\cdot e^{i\frac{3\pi}{2}} \qquad \pi + 3\pi + 2\alpha = \pi + 2\pi n \qquad \alpha = -\frac{3\pi}{2} + \pi n$

• $\lambda = i|\lambda|$, $|\lambda| \to +\infty$ $\qquad f(x,|\lambda|) = -\frac{x^4}{4} - |\lambda|\cdot x$

$\frac{\partial f}{\partial x} = -x^3 - |\lambda| = 0 \qquad x^3 = -|\lambda| = |\lambda|\cdot e^{i(\pi+2\pi k)} \qquad k \in \mathbb{Z}$

$$x_p = \sqrt[3]{|\lambda|}\cdot e^{i\left(\frac{\pi}{3}+\frac{2}{3}\pi k\right)}$$

B+7



$\frac{\partial^2 f}{\partial x^2} = -3x^2 \qquad \arg\left(\frac{\partial^2 f}{\partial x^2}\Big|_{x_p}\right) + 2\alpha = \pi + 2\pi n \qquad n \in \mathbb{Z}$

$x_p = \sqrt[3]{|\lambda|}\, e^{i\frac{\pi}{3}} \qquad \pi + \frac{2\pi}{3} + 2\alpha = \pi + 2\pi n \qquad \alpha = -\frac{\pi}{3} + \pi n$

$x_p = \sqrt[3]{|\lambda|}\cdot e^{i\pi} \qquad \pi + 2\pi + 2\alpha = \pi + 2\pi n \qquad \alpha = -\pi + \pi n$

$x_p = \sqrt[3]{|\lambda|}\cdot e^{i\frac{5\pi}{3}} \qquad \pi + \frac{10\pi}{3} + 2\alpha = \pi + 2\pi n \qquad \alpha = -\frac{5\pi}{3} + \pi n$

5.2

- $\lambda \to +\infty \quad x_{p_1} = \sqrt[3]{\lambda} \cdot e^{i\frac{5\pi}{6}} \quad \alpha = \frac{\pi}{6} \quad x = \sqrt[3]{\lambda} \cdot e^{i\frac{5\pi}{6}} + v \cdot e^{i\frac{\pi}{6}}$

$$f \approx f(\sqrt[3]{\lambda} \cdot e^{i\frac{5\pi}{6}}, \lambda) + f''(\sqrt[3]{\lambda} \cdot e^{i\frac{5\pi}{6}}, \lambda) \cdot \frac{v^2 \cdot e^{i\frac{\pi}{3}}}{2} = -\frac{\lambda^{\frac{4}{3}}}{4} \cdot e^{i\frac{10\pi}{3}} + \lambda^{\frac{4}{3}} e^{i\frac{4\pi}{3}} - \frac{3}{2}\lambda^{\frac{2}{3}} \cdot v^2$$

$$I_1(\lambda) = \underbrace{e^{i\frac{\pi}{6}} \cdot e^{-\frac{\lambda^{4/3}}{4} e^{i\frac{10\pi}{3}} + \lambda^{4/3} e^{i\frac{4\pi}{3}}}}_{A_1} \cdot \int_{-1}^{1} e^{-\frac{3}{2}\lambda^{\frac{2}{3}} v^2} dv = \frac{A_1}{\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}}} \cdot \int_{-1}^{1} e^{-\frac{3}{2}\lambda^{\frac{2}{3}} v^2} d\left(\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}} \cdot v\right) =$$

$$= \frac{A_1}{\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}}} \cdot \sqrt{\pi}$$

$x_{p_2} = \sqrt[3]{\lambda} \cdot e^{i\frac{\pi}{6}} \quad \alpha = \frac{5\pi}{6} \quad x = \sqrt[3]{\lambda} e^{i\frac{\pi}{6}} + v \cdot e^{i\frac{5\pi}{6}}$

$$f \approx -\frac{\lambda^{\frac{4}{3}}}{4} e^{i\frac{2\pi}{3}} + \lambda^{\frac{4}{3}} e^{i\frac{2\pi}{3}} - \frac{3}{2}\lambda^{\frac{2}{3}} v^2$$

$$I_2(\lambda) = \underbrace{e^{i\frac{5\pi}{6}} \cdot e^{-\frac{\lambda^{4/3}}{4} e^{i\frac{2\pi}{3}} + \lambda^{4/3} e^{i\frac{2\pi}{3}}}}_{A_2} \cdot \int_{-1}^{1} e^{-\frac{3}{2}\lambda^{\frac{2}{3}} v^2} dv = \frac{A_2}{\sqrt{\frac{3}{2}}\lambda^{\frac{2}{3}}} \int_{-1}^{1} e^{-\frac{3}{2}\lambda^{\frac{2}{3}} v^2} d\left(\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}} v\right) =$$

$$= \frac{A_2}{\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}}} \sqrt{\pi}$$

$$I(\lambda) = I_1(\lambda) + I_2(\lambda) = \frac{\sqrt{\pi}}{\sqrt{\frac{3}{2}}\lambda^{\frac{1}{3}}} (A_1 + A_2) = \frac{2}{\sqrt{3}} \frac{\sqrt{2\pi} \cdot e^{-\frac{3}{8}\lambda^{\frac{4}{3}}}}{\lambda^{\frac{1}{3}}} \cdot \cos\left(\frac{3}{8}\sqrt{3}\lambda^{\frac{4}{3}} - \frac{\pi}{6}\right)$$

- $\lambda = i|\lambda| \quad |\lambda| \to +\infty \quad x_p = \sqrt[3]{|\lambda|} \cdot e^{i\pi} \quad \alpha = 0 \quad x = \sqrt[3]{|\lambda|} \cdot e^{i\pi} + v$

$$f \approx -\frac{|\lambda|^{\frac{4}{3}}}{4} + |\lambda|^{\frac{4}{3}} - \frac{3}{2}|\lambda|^{\frac{2}{3}} \cdot v^2 = \frac{3}{4}|\lambda|^{\frac{4}{3}} - \frac{3}{2}|\lambda|^{\frac{2}{3}} v^2$$

$$I(\lambda) = e^{\frac{3}{4}|\lambda|^{\frac{4}{3}}} \cdot \int_{-1}^{1} e^{-\frac{3}{2}|\lambda|^{\frac{2}{3}} v^2} dv = \frac{e^{\frac{3}{4}|\lambda|^{\frac{4}{3}}}}{\sqrt{\frac{3}{2}}|\lambda|^{\frac{1}{3}}} \int_{-1}^{1} e^{-\frac{3}{2}|\lambda|^{\frac{2}{3}} v^2} d\left(\sqrt{\frac{3}{2}}|\lambda|^{\frac{1}{3}} v\right) =$$

$$= \frac{e^{\frac{3}{4}|\lambda|^{\frac{4}{3}}}}{\sqrt{\frac{3}{2}}|\lambda|^{\frac{1}{3}}} \sqrt{\pi} = \frac{1}{\sqrt{3}} \frac{\sqrt{2\pi} \cdot e^{\frac{3}{4}|\lambda|^{\frac{4}{3}}}}{|\lambda|^{\frac{1}{3}}}.$$

⑥

$$I(\lambda) = \int_{-\infty}^{+\infty} e^{-\lambda(x^2-3ix)} \cdot F(x)\, dx$$

- $f = -\lambda(x^2 - 3ix) \qquad \frac{\partial f}{\partial x} = -\lambda(2x - 3i) = 0 \qquad x_p = \frac{3}{2}i$

- $F(x) = \int_0^{+\infty} \frac{(1+ix)\, y^{ix}}{(1+y)^{2+2ix}} e^{-y} dy$

$$F(x_p) = \int_0^{+\infty} \frac{(1-\frac{3}{2})\, y^{-\frac{3}{2}}}{(1+y)^{2-3}} e^{-y} dy = \int_0^{+\infty} (-\tfrac{1}{2}) \frac{1+y}{y^{\frac{3}{2}}} e^{-y} dy =$$

$$= -\frac{1}{2}\int_0^{+\infty} [y^{-\frac{3}{2}} + y^{-\frac{1}{2}}] e^{-y} dy = -\frac{1}{2}\int_0^{+\infty} y^{-\frac{3}{2}} e^{-y} dy - \frac{1}{2}\int_0^{+\infty} y^{-\frac{1}{2}} e^{-y} dy$$

$$\boxed{\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt}$$

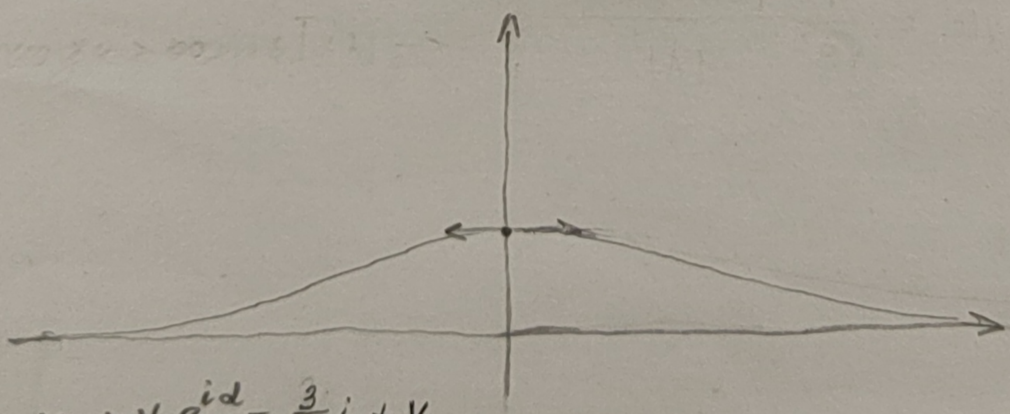
$$-\frac{1}{2}\int_0^{+\infty} y^{a-1} e^{-y} dy - \frac{1}{2}\int_0^{+\infty} y^{b-1} e^{-y} dy = -\frac{1}{2}\left[\Gamma(a) + \Gamma(b)\right]$$

$x_p = \frac{3}{2}i: \quad a = -\frac{1}{2}, \quad b = \frac{1}{2}$

$$F_{con}(x_p) = -\frac{1}{2}\left[\Gamma(-\tfrac{1}{2}) + \Gamma(\tfrac{1}{2})\right] = -\frac{1}{2}\left[-2\sqrt{\pi} + \sqrt{\pi}\right] = \frac{\sqrt{\pi}}{2}$$

- $\lambda \to +\infty \qquad \arg\left(\frac{\partial^2 f}{\partial x^2}\Big|_{x_p}\right) + 2\alpha = \pi + 2\pi k \quad k \in \mathbb{Z}$

$$\pi + 2\alpha = \pi + 2\pi k \qquad \alpha = \pi k \qquad k=0 \Rightarrow \alpha = 0$$



$$x = x_p + v \cdot e^{i\alpha} = \frac{3}{2}i + v$$

$$I(\lambda) = \int_{-1}^{1} e^{-\lambda(-\frac{9}{4} + 3iv + v^2 + \frac{9}{2} - 3iv)} \cdot \frac{\sqrt{\pi}}{2} dv = \frac{\sqrt{\pi}}{2} \cdot e^{-\lambda \cdot \frac{9}{4}} \int_{-1}^{1} e^{-\lambda v^2} dv =$$

$$= \frac{\sqrt{\pi}}{2} \cdot e^{-\frac{9}{4}\lambda} \frac{1}{\sqrt{\lambda}} \int_{-1}^{1} e^{-\lambda v^2} d(\sqrt{\lambda} \cdot v) = \frac{\pi}{2} e^{-\frac{9}{4}\lambda} \frac{1}{\sqrt{\lambda}}$$

$$\bullet\ F(x) = \int_0^{+\infty} \frac{(1+ix)\, y^{ix}}{(1+y)^{2+2ix}} e^{-y} dy =$$

$$= \int_0^{+\infty} (1+ix)\left[\frac{1}{(1+y)^{2+2ix}} - 1\right] y^{ix} e^{-y} dy + \int_0^{+\infty} (1+ix) y^{ix} e^{-y} dy$$

$$(1+ix) \int_0^{+\infty} y^{(1+ix)-1} e^{-y} dy = (1+ix)\, \Gamma(1+ix) = \Gamma(2+ix)$$

$$F_{con}(x) = \int_0^{+\infty} (1+ix)\left[\frac{1}{(1+y)^{2+2ix}} - 1\right] y^{ix} e^{-y} dy + \Gamma(2+ix)$$

$\Downarrow$

сходится при $\operatorname{Im} x < 2$

Полюсы $\Gamma(2+ix)$: $x = i(2+n)$ $n \in \mathbb{N}$

(7) $I(\lambda) = \int\limits_{-\infty}^{+\infty} \cos(\lambda \cdot \cos x)\, \frac{\sin x}{x}\, dx \qquad \lambda \to +\infty$

- $I(\lambda) = Re\left[\int\limits_{-\infty}^{+\infty} e^{i\lambda \cdot \cos x} \cdot \frac{\sin x}{x}\, dx \right]$

$f = i\lambda \cdot \cos x \qquad \frac{\partial f}{\partial x} = -i\lambda \cdot \sin x = 0 \qquad \sin x = 0$

$x_{p_k} = \pi k, \quad k \in \mathbb{Z}$

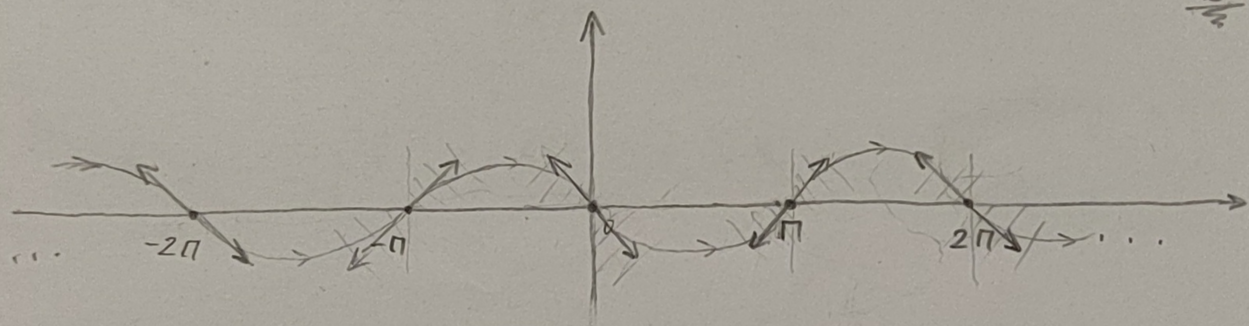
$\arg\left(\frac{\partial^2 f}{\partial x^2}\Big|_{x_p} \right) + 2\alpha = \pi + 2\pi n \qquad n \in \mathbb{Z}$

$\arg\left(-i\lambda \cdot \cos(\pi k)\right) + 2\alpha = \pi + 2\pi n$

$\arg\left(e^{i\frac{3\pi}{2}} \cdot \lambda \cdot e^{i\pi k}\right) + 2\alpha = \pi + 2\pi n$

$\frac{3\pi}{2} + \pi k + 2\alpha = \pi + 2\pi n \qquad \alpha_k = -\frac{\pi}{4} - \frac{\pi}{2}k + \pi n$

B



$x_{p_k} = \pi k \implies \frac{\sin x_{p_k}}{x_{p_k}} = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$

$k=0$:

$x = v \cdot e^{-i\frac{\pi}{4}} \qquad f \approx i\lambda - i\frac{\lambda}{2} v^2 e^{-i\frac{\pi}{2}} = i\lambda - \frac{\lambda}{2} v^2$

$\implies I(\lambda) \approx Re\left[e^{-i\frac{\pi}{4}} \cdot e^{i\lambda} \int\limits_{-1}^{1} e^{-\frac{\lambda}{2} v^2} dv \right] = Re\left[\sqrt{\frac{2}{\lambda}}\, e^{-i\left(\frac{\pi}{4} - \lambda\right)} \int\limits_{-1}^{1} e^{-\frac{\lambda}{2} v^2} d\left(\sqrt{\frac{\lambda}{2}}\, v\right) \right] =$

$= Re\left[\sqrt{\frac{2\pi}{\lambda}}\, e^{i\left(\lambda - \frac{\pi}{4}\right)} \right] = \sqrt{\frac{2\pi}{\lambda}} \cdot \cos\left(\lambda - \frac{\pi}{4}\right) = I_{lead}(\lambda)$

$$\bullet \int_{-\infty}^{+\infty} \frac{\sin x}{x} e^{i\lambda\cdot\cos x} dx$$

$$x_{pk} = \pi k \qquad x_k = \pi k + v\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}$$

$$f = i\cdot\lambda\cdot\cos x$$

$$f_k \approx f(x_{pk}) + \frac{f''(x_{pk})}{2}(x_k - x_{pk})^2 + \sum_{n=3}^{+\infty} \frac{f^{(n)}(x_{pk})}{n!}(x_k - x_{pk})^n =$$

$$= i\lambda\cdot(-1)^k - \frac{i\lambda}{2}(-1)^k\cdot v^2\cdot e^{i\frac{\pi}{2}(-1)^{k+1}} + \sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!} v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}} =$$

$$= i\lambda\cdot(-1)^k - \frac{\lambda}{2}v^2 + i\lambda\cdot\sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!} v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}}$$

$$F = \frac{\sin x}{x}$$

$$F_k \approx \sum_{n=0}^{+\infty}\frac{F^{(n)}(x_{pk})}{n!}(x_k - x_{pk})^n$$

$$e^{i\lambda(-1)^k}\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}\cdot \int_{-1}^{1}\left[\sum_{n=0}^{+\infty}\frac{F^{(n)}(x_{pk})}{n!} v^n\cdot e^{i\frac{n\pi}{4}(-1)^{k+1}}\right]\cdot e^{-\frac{\lambda}{2}v^2 + i\lambda\cdot\sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!}v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}}} \cdot dv$$

$$= e^{i\lambda(-1)^k}\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}\cdot \int_{-1}^{1}\left[\sum_{n=0}^{+\infty}\frac{F^{(n)}(x_{pk})}{n!} v^n\cdot e^{i\frac{n\cdot\pi}{4}(-1)^{k+1}}\right]\cdot e^{-\lambda\left(\frac{v^2}{2} - i\sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!}v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}}\right)} \cdot dv$$

$$u = \left(\frac{v^2}{2} - i\sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!}v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}}\right)^{1/2} =$$

$$= \frac{v}{\sqrt{2}}\left(1 - \frac{2i}{v^2}\sum_{n=2}^{+\infty}\frac{(-1)^n}{(2n)!}v^{2n}\cdot e^{i\frac{\pi}{2}n(-1)^{k+1}}\right)^{1/2} =$$

$$= \frac{v}{\sqrt{2}}\left(1 + 2i\sum_{n=1}^{+\infty}\frac{(-1)^n}{(2n+2)!}v^{2n}\cdot e^{i\frac{\pi}{2}(n+1)\cdot(-1)^{k+1}}\right)^{1/2} \approx \frac{v}{\sqrt{2}}\left(1 + i\sum_{n=1}^{+\infty}\frac{(-1)^n}{(2n+2)!}v^{2n}\cdot e^{i\frac{\pi}{2}(n+1)(-1)^{k+1}} + \ldots\right)$$

$$= \frac{v}{\sqrt{2}}\left(1 + \frac{i}{24}v^2 + \ldots\right) = \frac{v}{\sqrt{2}} + \frac{i}{24\sqrt{2}}v^3 + \ldots$$

$$v = \sqrt{2}u - \frac{i}{6\sqrt{2}}u^3 + \ldots$$

$$e^{i\lambda(-1)^k}\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}\cdot\int_{-1}^{1}\left[\sum_{n=0}^{+\infty}\frac{F^{(n)}(x_{pk})}{n!}e^{i\frac{n\pi}{4}(-1)^{k+1}}\cdot\left(\sqrt{2}u-\frac{i}{6\sqrt{2}}u^3+\dots\right)^n\right]\cdot e^{-\lambda u^2}\cdot\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2+\dots\right)du\approx$$

$$\approx e^{i\lambda(-1)^k}\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}\cdot\int_{-1}^{1}\left[F(x_{pk})+F'(x_{pk})\cdot e^{i\frac{\pi}{4}(-1)^{k+1}}\cdot\left(\sqrt{2}u-\frac{i}{6\sqrt{2}}u^3\right)+\frac{(-1)^{k+1}i}{2}\cdot F''(x_{pk})\cdot\left(\sqrt{2}u-\frac{i}{6\sqrt{2}}u^3\right)^2\right]\cdot e^{-\lambda u^2}\cdot\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2\right)du$$

$$F(x_{pk})=\frac{\sin(x_{pk})}{x_{pk}}=\begin{cases}1, & k=0\\ 0, & k\neq 0\end{cases}$$

$$F'(x_{pk})=\frac{\cos(x_{pk})}{x_{pk}}-\frac{\sin(x_{pk})}{(x_{pk})^2}=\begin{cases}0, & k=0\\ \frac{(-1)^k}{\pi k}, & k\neq 0\end{cases}$$

$$F''(x_{pk})=-\frac{\sin(x_{pk})}{x_{pk}}-2\cdot\frac{\cos(x_{pk})}{(x_{pk})^2}+2\cdot\frac{\sin(x_{pk})}{(x_{pk})^3}=\begin{cases}-\frac{1}{3}, & k=0\\ -2\cdot\frac{(-1)^k}{(\pi k)^2}, & k\neq 0\end{cases}$$

$$\int_{-\infty}^{+\infty}\frac{\sin x}{x}\cdot e^{i\lambda\cdot\cos x}dx\approx e^{i\lambda}\cdot e^{-i\frac{\pi}{4}}\cdot\int_{-1}^{1}e^{-\lambda u^2}\cdot\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2\right)du+$$

$$+\frac{i}{6}e^{i\lambda}\cdot e^{-i\frac{\pi}{4}}\cdot\int_{-1}^{1}\left(\sqrt{2}u-\frac{i}{6\sqrt{2}}u^3\right)^2\cdot e^{-\lambda u^2}\cdot\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2\right)du+$$

$$+2i\cdot\sum_{k=1}^{+\infty}\int_{-1}^{1}\frac{1}{(\pi k)^2}\left(\sqrt{2}u-\frac{i}{6\sqrt{2}}u^3\right)^2\cdot e^{-\lambda u^2}\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2\right)du$$

$$\int_{-1}^{1}e^{-\lambda u^2}\left(\sqrt{2}-\frac{i}{2\sqrt{2}}u^2\right)du=\sqrt{2}\int_{-1}^{1}e^{-\lambda u^2}du-\frac{i}{2\sqrt{2}}\int_{-1}^{1}u^2\cdot e^{-\lambda u^2}du=$$

$$=\sqrt{\frac{2}{\lambda}}\sqrt{\pi}-\frac{i}{2\sqrt{2}}\int_{0}^{1}u\cdot e^{-\lambda u^2}du^2=\sqrt{\frac{2\pi}{\lambda}}-\frac{i}{2\sqrt{2}}\int_{0}^{1}t^{\frac{1}{2}}e^{-\lambda t}dt=$$

$$=\sqrt{\frac{2\pi}{\lambda}}-\frac{i}{2\sqrt{2}}\frac{1}{\lambda}\frac{1}{\sqrt{\lambda}}\int_{0}^{1}(\lambda t)^{\frac{1}{2}}\cdot e^{-(\lambda t)}\cdot d(\lambda t)=\sqrt{\frac{2\pi}{\lambda}}-\frac{i}{2\sqrt{2}}\frac{1}{\lambda^{\frac{3}{2}}}\Gamma\left(\frac{3}{2}\right)=$$

$$=\sqrt{\frac{2\pi}{\lambda}}-\frac{e^{i\frac{\pi}{2}}}{4\sqrt{2}}\frac{\sqrt{\pi}}{\lambda^{\frac{3}{2}}}\Rightarrow I_{sub}=\frac{1}{8}\frac{\sqrt{2\pi}}{\lambda^{\frac{3}{2}}}\sin\left(\lambda-\frac{\pi}{4}\right)$$