

Notes on the Rabi's Cat

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November 10, 2022

1 Model

Parity violating Rabi model

$$\hat{H} = \omega \hat{b}^\dagger \hat{b} + \omega R (\hat{J}_z + j) + 2\sqrt{R}\lambda \left[(\hat{b}^\dagger + \hat{b}) \hat{J}_z - i\delta (\hat{b}^\dagger - \hat{b}) \hat{J}_y \right] + \mu (\hat{b}^\dagger + \hat{b}) (\hat{J}_z + j), \quad (1)$$

- $\hat{\mathbf{J}} = (\hat{J}_x, \hat{J}_y, \hat{J}_z)$ is the quasispin (qubit) operator with size $j = 1/2$.
- \hat{b}, \hat{b}^\dagger are the field (oscillator) annihilation and creation operators.
- ω is the field frequency.
- $R \gg 1$ is the detuning between the quasispin and the oscillator, and plays the role of the size parameter of the system.
- λ is the quasispin-field interaction strength.
- δ interpolates between the ($\delta = 1$) Jaynes-Cummings regime, ($\delta = 0$) Rabi regime and ($\delta = -1$) anti-Jaynes-Cummings regime.
- μ governs the strength of parity-violation.

2 Critical structure

We focus on the excited-state quantum phase transition (ESQPT) at $E = 0$. Its character is given by the index k of the corresponding nondegenerate stationary point in the infinite- R limit of the Hamiltonian (1), and it changes with increasing interaction strength λ at two critical points

$$\lambda_c = \frac{\omega}{2}, \quad (2)$$

$$\lambda_0 = \frac{\lambda_c}{|\delta|}. \quad (3)$$

- $\lambda < \lambda_c$ (normal phase, N): $k = 0$, stable dynamics,
- $\lambda_c < \lambda < \lambda_0$ (superradiant phase I, S1): $k = 1$, unstable dynamics,
- $\lambda > \lambda_0$ (superradiant phase II, S2): $k = 2$, stable dynamics.

3 Numerical study

In the calculations, the following values of the parameters are taken:

$$\omega = 1, \quad \delta = \frac{1}{2}, \quad \lambda = \frac{3}{4}, \quad (4)$$

so the system is situated in the middle of phase S1. We evolve the initially factorized state $|\psi(0)\rangle = |-\frac{1}{2}\rangle \otimes |0\rangle$ (both quasispin and field are in their lowest state) and calculate expectation values $\langle\psi(t)|\bullet|\psi(t)\rangle$ of the following operators:

- J_x (first component of the quasispin operator),
- $\hat{q} = \frac{1}{2\sqrt{jR}} (\hat{b}^\dagger + \hat{b})$ (coordinate).

The expectation value of both operators vanishes at all times if the parity is conserved $\mu = 0$.

4 Results

The numerical results for the expectation values of operators \hat{q}, \hat{J}_x sensitive to the parity violation are displayed in Figures 1—4. Note that similar behaviour is observed for operators \hat{q}, \hat{J}_y . The product μR is kept constant, which makes the curves roughly coincide with one another. The value of this product is chosen so that the first significant minimum of $q(t)$ is the deepest.

Scaling $t' = st$ in Figures 1 and 2 makes the first pronounced minimum of q at $t_{\min} \approx 13R^{0.1}$ sit approximately at the same time t' .

The first small dip, observed in Figure 1 (b) at $t' \approx 7$ is caused by different depths of the left and right well. This difference is getting smaller with decreasing μ , and in the limit $R \rightarrow \infty, \mu R = \text{const}$ vanishes. On the other hand, the well-pronounced minimum at $t' \approx 20$ is caused by the phase difference between the orbits in the left and right well, and its magnitude doesn't change with μ , provided the product μR is kept constant. The same is observed for the first derivative $q'(t)$ in Figure 3.

Figure 4 shows that the first important extreme of $q(t)$ and $J_x(t)$ occurs for a fixed R approximately at the same time t . There exists the smallest μ_0 for which the phenomenon is present (the centre of the leftmost lowest blue region in the second panel), and then there is a sequence of alternating minima-maxima both in q and J_x at $\mu = k\mu_0, k = 1, 2, 3, 4, \dots$. This is related to the difference of phases between wavepackets in both wells.

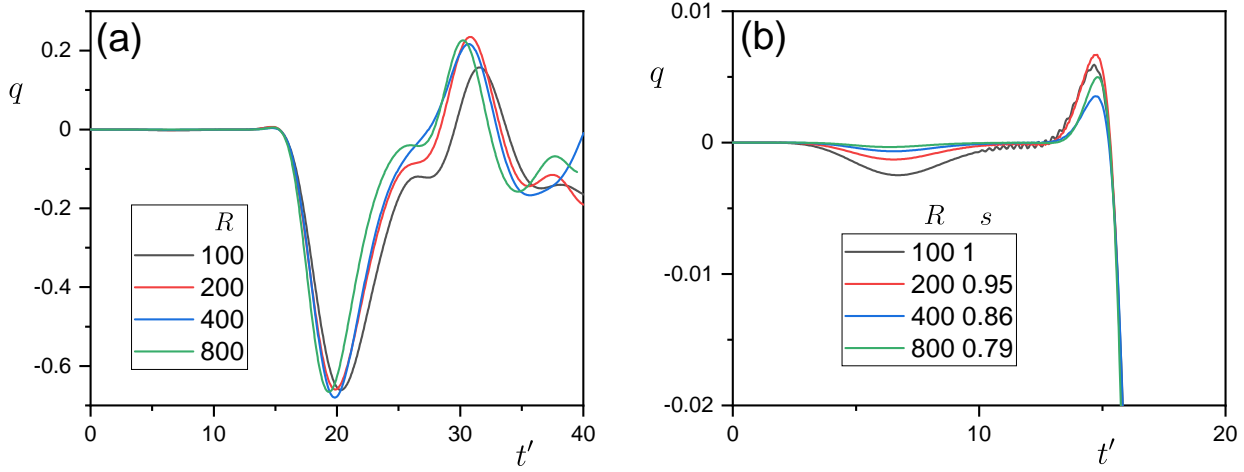


Figure 1: Expectation value $q(t) = \langle \psi(t) | \hat{q} | \psi(t) \rangle$ for different system sizes R and for a perturbative value of μ reciprocal to the system size $\mu = 0.13/R$. The magnitude of the parity violation is observed to be independent of the system size. (a) Full image. (b) Detail to the pre-parity-violation. Note that in both panels the time is rescaled via $t' = st$ and s is indicated in the legend of panel (b).

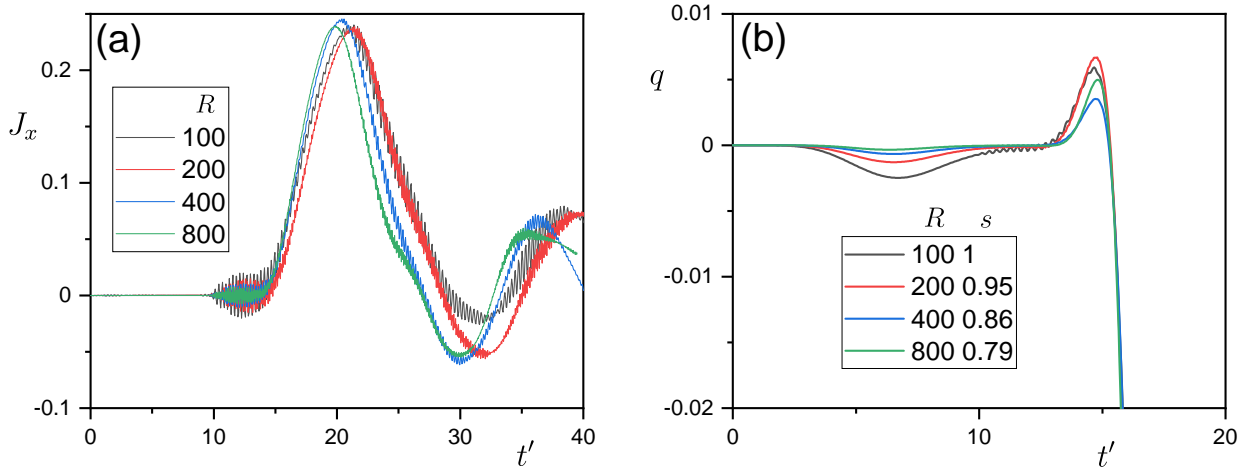


Figure 2: The same as in Figure 1, but here for expectation value $J_x(t) = \langle \psi(t) | \hat{J}_x | \psi(t) \rangle$. Note the fast oscillations caused by the detuning given by R .

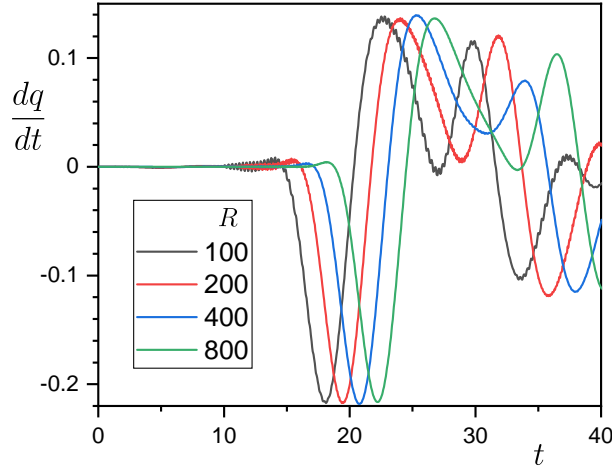


Figure 3: The first derivative of $q(t)$.

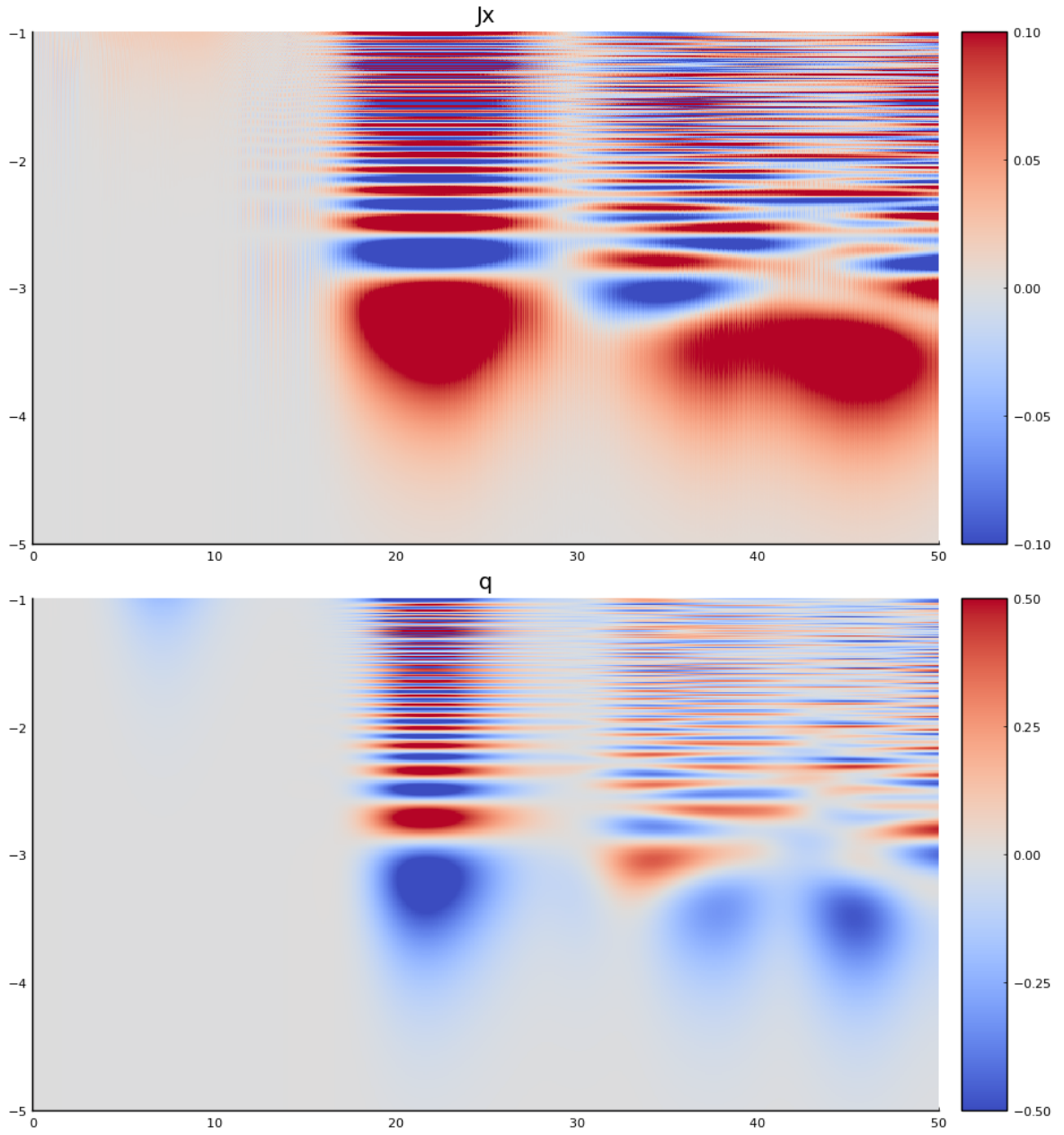


Figure 4: Full image of J_x and q as a function of t (vertical axis) and $\log_{10} \mu$ (horizontal axis) for $R = 200$.