Notes on the Rabi's Cat

Das Pavel

October 11, 2023

1 Model

Parity violating Rabi model

$$\hat{\mathsf{H}} = \omega \hat{\mathsf{b}}^\dagger \hat{\mathsf{b}} + \omega R \left(\hat{\mathsf{J}}_z + j \right) + 2\sqrt{R}\lambda \left[\left(\hat{\mathsf{b}}^\dagger + \hat{\mathsf{b}} \right) \hat{\mathsf{J}}_z - \mathrm{i}\delta \left(\hat{\mathsf{b}}^\dagger - \hat{\mathsf{b}}^\dagger \right) \hat{\mathsf{J}}_y \right] + \mu \left(\hat{\mathsf{b}}^\dagger + \hat{\mathsf{b}} \right) \left(\hat{\mathsf{J}}_z + j \right), \tag{1}$$

- $\hat{\mathbf{J}} = \left(\hat{\mathsf{J}}_x, \hat{\mathsf{J}}_y, \hat{\mathsf{J}}_z \right)$ is the quasispin (qubit) operator with size $j = 1/2$.
- $\hat{\mathsf{b}}, \hat{\mathsf{b}}^\dagger$ are the field (oscillator) annihilation and creation operators.
- ω is the field frequency.
- $R \gg 1$ is the detuning between the quasispin and the oscillator and plays the role of the size parameter of the system.
- λ is the quasispin-field interaction strength.
- δ interpolates between the ($\delta = 1$) Jaynes-Cummings regime, ($\delta = 0$) Rabi regime and ($\delta = -1$) anti-Jaynes-Cummings regime.
- μ governs the strength of parity-violation.

2 Critical structure

We focus on the excited-state quantum phase transition (ESQPT) at $E = 0$. Its character is given by the index k of the corresponding nondegenerate stationary point in the infinite-R limit of the Hamiltonian (1), and it changes with increasing interaction strength λ at two critical points

$$\lambda_c = \frac{\omega}{2}, \tag{2}$$

$$\lambda_0 = \frac{\lambda_c}{|\delta|}. \tag{3}$$

- $\lambda < \lambda_c$ (normal phase, N): $k = 0$, stable dynamics,
- $\lambda_c < \lambda < \lambda_0$ (superradiant phase I, S1): $k = 1$, unstable dynamics,
- $\lambda > \lambda_0$ (superradiant phase II, S2): $k = 2$, stable dynamics.

3 Numerical study

In the calculations, the following values of the parameters are taken:

$$\omega = 1, \qquad \delta = \frac{1}{2}, \qquad \lambda = \frac{3}{4}, \tag{4}$$

so the system is situated in the middle of phase S1. We evolve the initially factorized state $|\psi(0)\rangle = \left|-\frac{1}{2}\right\rangle \otimes |0\rangle$ (both quasispin and field are in their lowest state) and calulate expectation values $\langle\psi(t)|\bullet|\psi(t)\rangle$ of the following operators:

- J_x (first component of the quasispin operator),
- $\hat{\mathsf{q}} = \frac{1}{2\sqrt{jR}}\left(\hat{\mathsf{b}}^\dagger + \hat{\mathsf{b}}\right)$ (coordinate related to the field).

The expectation value of both operators vanishes at all times if the parity is conserved, *i.e.*, $\mu = 0$.

4 Results

The numerical results for the expectation values of operators $\hat{\mathsf{q}}, \hat{\mathsf{J}}_x$ sensitive to the parity violation are displayed in Figures 1—4. Note that similar behaviour is observed for operators $\hat{\mathsf{p}}, \hat{\mathsf{J}}_y$. The product μR is kept constant, which makes the curves roughly coincide with one another. The value of this product is chosen so that the first significant minimum of $q(t)$ is the deepest.

Scalling $t' = st$ in Figures 1 and 2 makes the first pronounced minimum of q at $t_{\min} \approx 13R^{0.1}$ sit approximately at the same time t'.

The first small dip, observed in Figure 1 (b) at $t' \approx 7$ is caused by different depths between the left and right Hamiltonian wells. This difference gets smaller with decreasing μ, and in the limit $R \to \infty$, $\mu R = \text{const}$ vanishes. On the other hand, the well-pronounced minimum at $t' \approx 20$ is caused by the phase difference between the orbits in the left and right well, and its magnitude doesn't change with μ, provided the product μR is kept constant. The same is observed for the first derivative $q'(t)$ in Figure 3.

Figure 4 shows that the first important extreme of $q(t)$ and $J_x(t)$ occurs, for a given R, approximately at the same time $t_{\min}$. There exists the smallest μ_0 for which the phenomenon is present (the centre of the leftmost lowest blue region in the second panel), and then there is a sequence of alternating minima-maxima both in q and J_x at $\mu = k\mu_0$, $k = 1, 2, 3, 4, \ldots$ .. This is related to the difference of phases between the wavepackets in both wells.

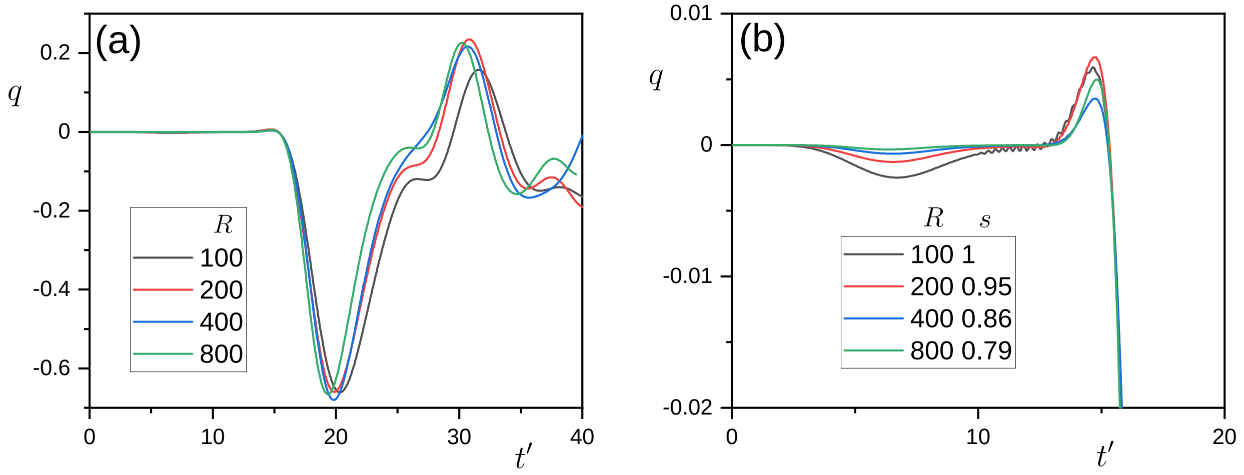

Figure 1: Expectation value $q(t) = \langle\psi(t)|\hat{\mathrm{q}}|\psi(t)\rangle$ for different system sizes R and for a perturbative value of μ reciprocal to the system size, $\mu = 0.13/R$. The magnitude of the parity violation is observed to be independent of the system size. (a) Full image. (b) Detail of the pre-parity-violation. Note that in both panels the time is rescalled via $t' = st$ and s is indicated in the legend of panel (b).

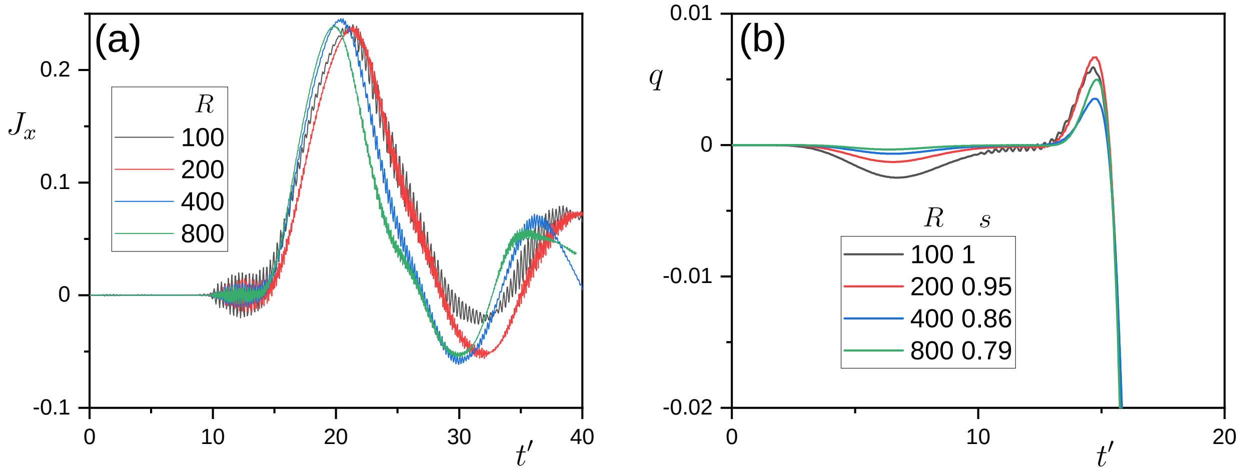

Figure 2: The same as in Figure 1, but here for expectation value $J_x(t) = \left\langle\psi(t)\middle|\hat{\mathrm{J}}_x\middle|\psi(t)\right\rangle$. Note the fast oscillations caused by the detuning given by parameter R.

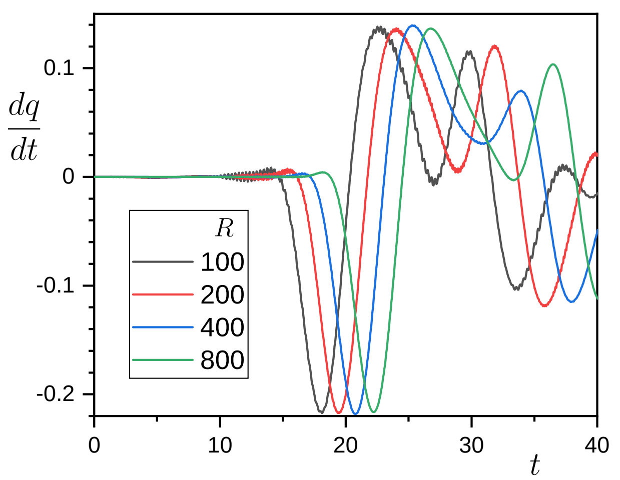

Figure 3: The first derivative of $q(t)$.

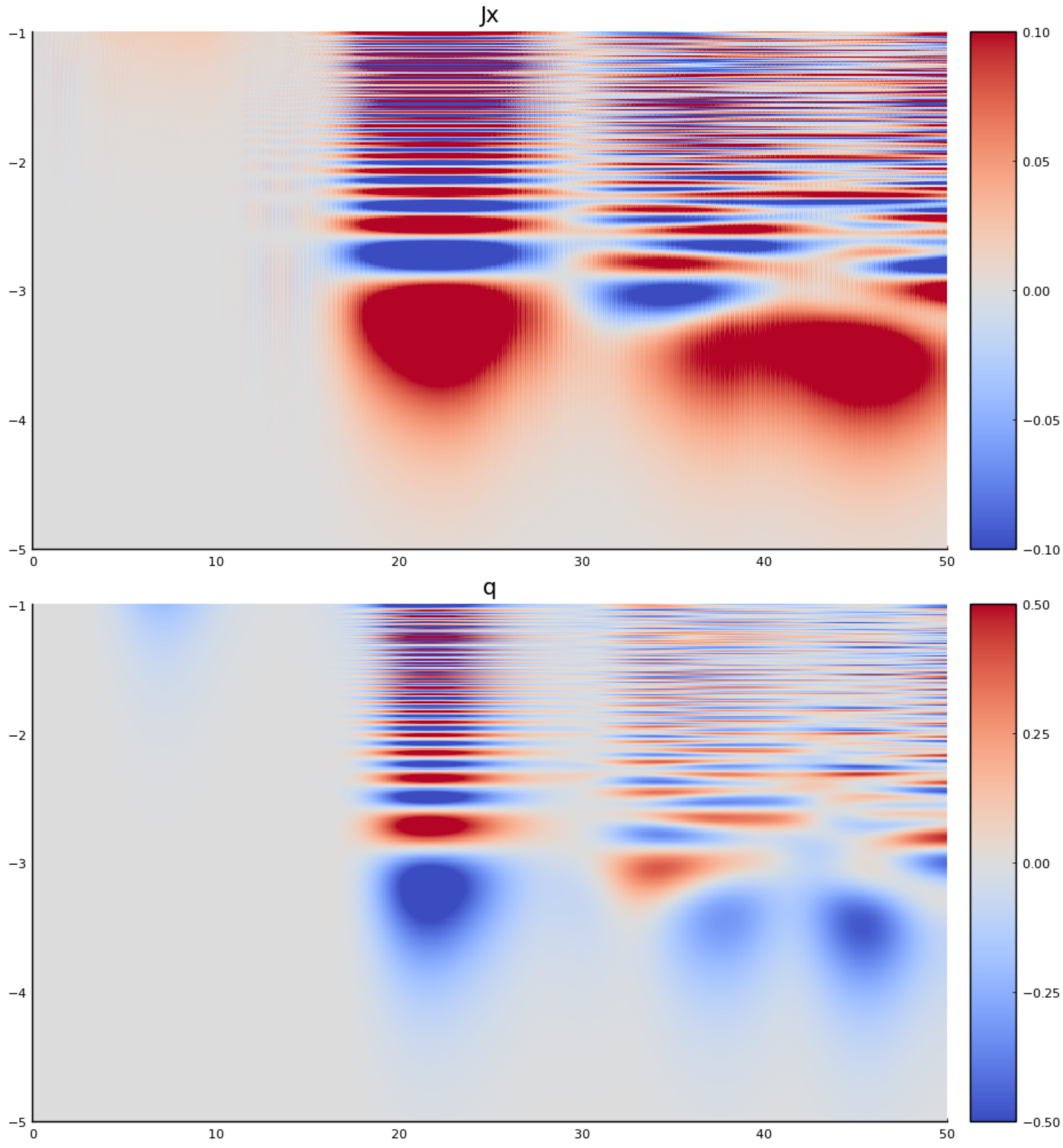

Figure 4: Full image of J_x and q as a function of t (horizontal axis) and $\log_{10}\mu$ (vertical axis) for $R = 200$.