Assignment 2 (Due on the week September 21- 26)

1. Prove that $|x+y+z| \leqslant |x|+|y|+|z|$ for all numbers x, y, and z.

2. Show that a *convergent* sequence in $\mathbb{R}^n$ can have only one accumulating point, and therefore only one limit.

3. Show that the positive orthant

$$\mathbb{R}^n_+ = \{(x_1, x_2, \ldots, x_n) \,|\, x_i > 0, i = 1, 2, \ldots, n\}$$

   is an open subset of $\mathbb{R}^n$ by finding a formula for ε in terms of the x_i's.

4. Prove that every convergent sequence in $\mathbb{R}^n$ is bounded.

5. Given two sets S_1 and S_2 in $\mathbb{R}^n$ define their sum by

$$S_1 + S_2 = \{x \in \mathbb{R}^n \colon x = x_1 + x_2,\, x_1 \in S_1, x_2 \in S_2\}.$$

   Prove that if S_1 and S_2 are compact, then $S_1 + S_2$ is also compact.