MOR. Retake — 15.09.2015

1. A problem with the mixed constraints is given: $3x_1x_2 - x_2^3 \to \max$
   subject to $2x_1 + 5x_2 \geq 20$, $x_1 - 2x_2 = 5$, $x_1, x_2 \geq 0$.
   (a) Check NDCQ conditions.
   (b) Form the Langrangian function.
   (c) Solve the maximization problem by the use of Langrange method or otherwise.
   (d) Justify that the maximum point was found.
2. Consider the linear programming problem with parameter β and nonnegative x_i:
   $$\begin{aligned} 3x_1 + x_2 - x_3 + 4x_4 &\to \max \\ x_1 + x_2 - x_3 + x_4 &\leq 1 \\ \beta x_1 + x_2 + x_3 + 2x_4 &\leq 2 \end{aligned}$$
   (a) Find the optimal values of the primal variables for $\beta = 6$.
   (b) Find the function $\phi(\beta)$, where $\phi(\beta)$ is the maximum value of the objective function for fixed value of β.
   (c) Sketch the graph of $\phi(\beta)$.
3. Find the general solution of the differential equation $y'' + 6y' + 9y = xe^{-2x} + \sin(x)$
4. Consider the system of difference equations
   $$\begin{cases} x_{t+1} = 2x_t - 4y_t \\ y_{t+1} = x_t - 3y_t + 3 \end{cases}$$
   (a) Solve the system
   (b) Find the equilibrium solution and check whether it's stable
5. Find all pure and mixed Nash equilibria in the following bimatrix game:

	d	e	f
a	4;5	1;4	1;1
b	2;8	5;0	0;4
c	0;3	2;2	5;7

6. There is an auction of a painting with two players. The value of the painting for the first player is a random variable v_1, for the second player — v_2. The random variables v_1 and v_2 are independent and identically distributed from 0 to 1 million dollars with density function $f(t) = 2t$. Each player makes the bid b_i knowing only his own value of the painting. The player who makes the highest bid gets the painting and pays his bid.

   Find a Nash equilibrium where each player uses linear strategy of the form $b_i = k \cdot v_i$.