Variant 1. 2016-10-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = x^3+2xz-3z^3-y^2$. Using the total differential find the approximate value of $f(1.01, 0.99, 1.02)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + z^3 = 3 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1, 1, 1)$
   (b) Find $y'(z)$ and $x'(z)$

3. Consider the function $f(u) = g(x, y)$ where $x = u^2$ and $y = \cos u$. The function g has continuous second derivatives everywhere. Find $f'(u)$ and $f''(u)$.

4. The curve on the plane is defined by the equation $x^4 + y^2 + y^4 = 3$.
   (a) Find a vector that is orthogonal to the curve at the point $(1, 1)$
   (b) Find a vector that is parallel to the curve at the point $(1, 1)$

5. The function g is monotonic. The function f is the inverse of the function g. Find $f'(1)$ if it is known that $g(10) = 1$, $g'(10) = 5$, $g(1) = -2$, $g'(1) = 4$.

6. Let x be a vector, $x \in \mathbb{R}^n$, and A be $n \times n$ matrix of constants. Consider the function $f(x) = x^T Ax$.
   (a) Clearly state the Young's theorem
   (b) Express the Hesse matrix of f using A and A^T.

SECTION B

7. Consider the function $f(x, y) = \sqrt[3]{x^3 + y^3}$.
   (a) (7 points) Find $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$. Is the function $f(x, y)$ continuously differentiable everywhere?
   (b) (3 points) Find equation of the tangent plane to the graph of $z = f(x, y)$ at the origin.
   (c) (10 points) Let $\Delta f = f(x, y) - f(0, 0)$. Compare Δf with the df (total differential) at the origin. Base your comparison on the existence of the limit $\lim_{x\to0,y\to0} \frac{\Delta f - df}{\sqrt{x^2+y^2}}$ as $x \to 0$ and $y \to 0$.

8. Cournot duopoly produces good Y, where $Y = y_1 + y_2$. Here y_1 is the output of the first firm and y_2 is the output of the second firm. The inverse demand on good is given by the formula $p(Y) = 1/Y$, where is $p(Y)$ the price per unit. The total costs of the firms are $TC_1(y_1) = 2y_1$ and $TC_2(y_2) = y_2$, respectively. Let the profit of the first firm be $\pi_1 = p(y_1 + y_2)y_1 - 2y_1$ and the profit of the second firm be $\pi_2 = p(y_1 + y_2)y_2 - y_2$.
   (a) (8 points) Write down the system of the first-order conditions $\begin{cases} \frac{\partial \pi_1}{\partial y_1} = 0 \\ \frac{\partial \pi_2}{\partial y_2} = 0 \end{cases}$ and solve it.
   (b) (12 points) Government decides to impose a per unit tax t on both firms. It will increase costs for them by ty_1 and ty_2 respectively. Rewrite the system of first-order conditions accounting for the tax. Find $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ by referring to the appropriate IFT. Check that IFT conditions are verifiable here.

Variant 2. 2016-10-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = x^3 + 2xz - 3z^3 - 2y^2$. Using the total differential find the approximate value of $f(1.01, 0.99, 1.02)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 2z^3 = 4 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1, 1, 1)$
   (b) Find $y'(z)$ and $x'(z)$

3. Consider the function $f(u) = g(x, y)$ where $x = 2u^2$ and $y = \cos u$. The function g has continuous second derivatives everywhere. Find $f'(u)$ and $f''(u)$.

4. The curve on the plane is defined by the equation $x^4 + y^2 + 2y^4 = 4$.
   (a) Find a vector that is orthogonal to the curve at the point $(1, 1)$
   (b) Find a vector that is parallel to the curve at the point $(1, 1)$

5. The function g is monotonic. The function f is the inverse of the function g. Find $f'(2)$ if it is known that $g(10) = 2$, $g'(10) = 5$, $g(2) = -2$, $g'(2) = 4$.

6. Let x be a vector, $x \in \mathbb{R}^n$, and A be $n \times n$ matrix of constants. Consider the function $f(x) = x^T Ax$.
   (a) Clearly state the Young's theorem
   (b) Express the Hesse matrix of f using A and A^T.

SECTION B

7. Consider the function $f(x, y) = \sqrt[3]{x^3 + y^3}$.
   (a) (7 points) Find $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$. Is the function $f(x, y)$ continuously differentiable everywhere?
   (b) (3 points) Find equation of the tangent plane to the graph of $z = f(x, y)$ at the origin.
   (c) (10 points) Let $\Delta f = f(x, y) - f(0, 0)$. Compare Δf with the df (total differential) at the origin. Base your comparison on the existence of the limit $\lim_{x\to 0, y\to 0} \frac{\Delta f - df}{\sqrt{x^2+y^2}}$ as $x \to 0$ and $y \to 0$.

8. Cournot duopoly produces good Y, where $Y = y_1 + y_2$. Here y_1 is the output of the first firm and y_2 is the output of the second firm. The inverse demand on good is given by the formula $p(Y) = 1/Y$, where is $p(Y)$ the price per unit. The total costs of the firms are $TC_1(y_1) = 2y_1$ and $TC_2(y_2) = y_2$, respectively. Let the profit of the first firm be $\pi_1 = p(y_1 + y_2)y_1 - 2y_1$ and the profit of the second firm be $\pi_2 = p(y_1 + y_2)y_2 - y_2$.
   (a) (8 points) Write down the system of the first-order conditions $\begin{cases} \frac{\partial \pi_1}{\partial y_1} = 0 \\ \frac{\partial \pi_2}{\partial y_2} = 0 \end{cases}$ and solve it.
   (b) (12 points) Government decides to impose a per unit tax t on both firms. It will increase costs for them by ty_1 and ty_2 respectively. Rewrite the system of first-order conditions accounting for the tax. Find $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ by referring to the appropriate IFT. Check that IFT conditions are verifiable here.

Variant 3. 2016-10-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = x^3 + 2xz - 3z^3 - 3y^2$. Using the total differential find the approximate value of $f(1.01, 0.99, 1.02)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 3z^3 = 5 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1, 1, 1)$

   (b) Find $y'(z)$ and $x'(z)$

3. Consider the function $f(u) = g(x, y)$ where $x = 3u^2$ and $y = \cos u$. The function g has continuous second derivatives everywhere. Find $f'(u)$ and $f''(u)$.

4. The curve on the plane is defined by the equation $x^4 + y^2 + 3y^4 = 5$.

   (a) Find a vector that is orthogonal to the curve at the point $(1, 1)$

   (b) Find a vector that is parallel to the curve at the point $(1, 1)$

5. The function g is monotonic. The function f is the inverse of the function g. Find $f'(3)$ if it is known that $g(10) = 3$, $g'(10) = 5$, $g(3) = -2$, $g'(3) = 4$.

6. Let x be a vector, $x \in \mathbb{R}^n$, and A be $n \times n$ matrix of constants. Consider the function $f(x) = x^T Ax$.

   (a) Clearly state the Young's theorem

   (b) Express the Hesse matrix of f using A and A^T.

SECTION B

7. Consider the function $f(x, y) = \sqrt[3]{x^3 + y^3}$.

   (a) (7 points) Find $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$. Is the function $f(x, y)$ continuously differentiable everywhere?

   (b) (3 points) Find equation of the tangent plane to the graph of $z = f(x, y)$ at the origin.

   (c) (10 points) Let $\Delta f = f(x, y) - f(0, 0)$. Compare Δf with the df (total differential) at the origin. Base your comparison on the existence of the limit $\lim_{x\to0,y\to0} \frac{\Delta f - df}{\sqrt{x^2+y^2}}$ as $x \to 0$ and $y \to 0$.

8. Cournot duopoly produces good Y, where $Y = y_1 + y_2$. Here y_1 is the output of the first firm and y_2 is the output of the second firm. The inverse demand on good is given by the formula $p(Y) = 1/Y$, where is $p(Y)$ the price per unit. The total costs of the firms are $TC_1(y_1) = 2y_1$ and $TC_2(y_2) = y_2$, respectively. Let the profit of the first firm be $\pi_1 = p(y_1 + y_2)y_1 - 2y_1$ and the profit of the second firm be $\pi_2 = p(y_1 + y_2)y_2 - y_2$.

   (a) (8 points) Write down the system of the first-order conditions $\begin{cases} \frac{\partial \pi_1}{\partial y_1} = 0 \\ \frac{\partial \pi_2}{\partial y_2} = 0 \end{cases}$ and solve it.

   (b) (12 points) Government decides to impose a per unit tax t on both firms. It will increase costs for them by ty_1 and ty_2 respectively. Rewrite the system of first-order conditions accounting for the tax. Find $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ by referring to the appropriate IFT. Check that IFT conditions are verifiable here.

Variant 4. 2016-10-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x,y,z) = x^3 + 2xz - 3z^3 - 4y^2$. Using the total differential find the approximate value of $f(1.01, 0.99, 1.02)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 4z^3 = 6 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1,1,1)$
   (b) Find $y'(z)$ and $x'(z)$

3. Consider the function $f(u) = g(x,y)$ where $x = 4u^2$ and $y = \cos u$. The function g has continuous second derivatives everywhere. Find $f'(u)$ and $f''(u)$.

4. The curve on the plane is defined by the equation $x^4 + y^2 + 4y^4 = 6$.
   (a) Find a vector that is orthogonal to the curve at the point $(1,1)$
   (b) Find a vector that is parallel to the curve at the point $(1,1)$

5. The function g is monotonic. The function f is the inverse of the function g. Find $f'(4)$ if it is known that $g(10) = 4$, $g'(10) = 5$, $g(4) = -2$, $g'(4) = 4$.

6. Let x be a vector, $x \in \mathbb{R}^n$, and A be $n \times n$ matrix of constants. Consider the function $f(x) = x^T Ax$.
   (a) Clearly state the Young's theorem
   (b) Express the Hesse matrix of f using A and A^T.

SECTION B

7. Consider the function $f(x,y) = \sqrt[3]{x^3 + y^3}$.
   (a) (7 points) Find $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$. Is the function $f(x,y)$ continuously differentiable everywhere?
   (b) (3 points) Find equation of the tangent plane to the graph of $z = f(x,y)$ at the origin.
   (c) (10 points) Let $\Delta f = f(x,y) - f(0,0)$. Compare Δf with the df (total differential) at the origin. Base your comparison on the existence of the limit $\lim_{x\to 0, y\to 0} \frac{\Delta f - df}{\sqrt{x^2+y^2}}$ as $x \to 0$ and $y \to 0$.

8. Cournot duopoly produces good Y, where $Y = y_1 + y_2$. Here y_1 is the output of the first firm and y_2 is the output of the second firm. The inverse demand on good is given by the formula $p(Y) = 1/Y$, where is $p(Y)$ the price per unit. The total costs of the firms are $TC_1(y_1) = 2y_1$ and $TC_2(y_2) = y_2$, respectively. Let the profit of the first firm be $\pi_1 = p(y_1 + y_2)y_1 - 2y_1$ and the profit of the second firm be $\pi_2 = p(y_1 + y_2)y_2 - y_2$.
   (a) (8 points) Write down the system of the first-order conditions $\begin{cases} \frac{\partial \pi_1}{\partial y_1} = 0 \\ \frac{\partial \pi_2}{\partial y_2} = 0 \end{cases}$ and solve it.
   (b) (12 points) Government decides to impose a per unit tax t on both firms. It will increase costs for them by ty_1 and ty_2 respectively. Rewrite the system of first-order conditions accounting for the tax. Find $\frac{dy_1}{dt}$ and $\frac{dy_2}{dt}$ by referring to the appropriate IFT. Check that IFT conditions are verifiable here.