

**Variant 1.** 2017-09-21. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

### SECTION A

1. Find indefinite integrals

(a)  $\int e^{2x} \cos^2 x \, dx;$

(b)  $\int \frac{x-3}{x^2-6x+9} \, dx.$

2. Solve the differential equation

$$y''' - y'' + 6y' - 6 = 42 \sin(x\sqrt{6}).$$

3. Solve the difference equation

$$y_{t+2} - 8y_{t+1} + 16y_t = 5t.$$

4. The function  $f(x, y)$  is non-constant and homogeneous. It is also known that  $h(x, y) = f'_x(x, y) + 3x^2y$  is homogeneous of degree 3. Find the value of  $\frac{xf'_x(x, y) + yf'_y(x, y)}{f(x, y)}.$

5. Solve the following linear programming problem:

$$\begin{cases} 2x_1 + 2x_2 + 3x_3 \rightarrow \min \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ 3x_1 + 5x_2 + x_3 \geq 8 \\ 5x_1 + 3x_2 + x_3 \geq 9 \end{cases}.$$

6. Maximize the function

$$11 + 10x_1 - x_1^2 - 3x_2 + 8x_3 - x_3^2$$

subject to constraints  $2x_1 - x_2 + 4x_3 \leq 10$  and  $x_2 \leq 100.$

### SECTION B

7. Solve the Euler equation  $x^2y'' - xy' - 3y = 0$  on the interval  $(0; \infty)$  by using the substitution  $x = e^t$  or otherwise. The answer should be written as a function of  $x$ .
8. Three players play the following game. Simultaneously each of them chooses one possible bet: either 1\$ or 2\$. A player is declared winner if his bet is unique and wins one rouble independently of the amount of the bet. For example, if players have chosen 1, 2 and 1 then their corresponding payoffs are 0, 1 and 0.

(a) Find all Nash equilibria in pure strategies.

(b) Find symmetric Nash equilibrium in mixed strategies.