Variant 1. 2017-10-27. Please, don't forget to write the variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x,y) = x^3 + 2x - 3xy^3 - y^2$. Using the total differential find the approximate value of $f(0.98, 1.99)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 2z^3 = 4 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1,1,1)$;

   (b) Find $y'(z)$ if possible.

3. If possible find the limit
$$\lim_{x\to 0, y\to 0} \frac{\exp(x^2+y^2) - 1}{x^2 + y^2 + 3|x| + |y|};$$

4. The surface in $\mathbb{R}^3$ is defined by the equation $x^3 + 2y^3 + 3z^3 + zxy = 7$.

   (a) Find a unit vector that is orthogonal to the tangent plane at the point $(x = 1, y = 1, z = 1)$.

   (b) Find the equation of the tangent plane.

5. Consider the function $f(x,y) = x^2 + y^3 + xy$, the vector $v = (1,2)$ and the point $A = (-1,-1)$.

   (a) Find the gradient of f at the point A.

   (b) Find the directional derivative of f at the point A in the direction given by v.

6. Let x_n be a sequence in $\mathbb{R}^2$ given by
$$x_n = \begin{pmatrix} \cos(2\pi n/3) \\ \sin(2\pi(n^2-1)/3n) \end{pmatrix}$$
   (a) Find the accumulation points of this sequence.

   (b) Find the limit of this sequence if it exists.

SECTION B

7. The production function is given by $q(K,L) = (K^\rho + L^\rho)^{1/\rho}$, where $K > 0$, $L > 0$, $\rho \le 1$ and $\rho \ne 0$.

   (a) MRTS (marginal rate of technical substitution) is defined as MRTS $= -\frac{dK}{dL}\,|_{q(K,L)=const}$. Using implicit function theorem find MRTS and express your answer as a function of K/L alone.

   (b) Let $t = K/L$. Find the derivative $\sigma = \frac{d\ln t}{d\ln \mathrm{MRTS}}$.

   (c) Suggest at least one production function $q(K,L)$ such that $\sigma = 1$.

8. The closed first quadrant is denoted by $\bar{\mathbb{R}}^2_+$. Consider the function $F(x,y) = xy - (x^p/p + y^q/q)$ defined on $\bar{\mathbb{R}}^2_+$, where $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.

   (a) Find the set $S \subset \bar{\mathbb{R}}^2_+$ such that $\partial F/\partial x = 0$ and $\partial F/\partial y = 0$ at the same time. Sketch the set S.

   (b) What are the possible values of the function $F(x,y)$ for $(x,y) \in S$?

   (c) Is it true that the sign of $F(x,y)$ is the same for all $(x,y) \notin S$?

Variant 2. 2017-10-27. Please, don't forget to write the variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x,y) = 2x^3 + 2x - 3xy^3 - y^2$. Using the total differential find the approximate value of $f(0.98, 1.99)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 3z^3 = 5 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1, 1, 1)$;
   (b) Find $y'(z)$ if possible.

3. If possible find the limit
$$\lim_{x\to 0, y\to 0} \frac{\exp(x^2 + y^2) - 1}{x^2 + y^2 + 4|x| + |y|};$$

4. The surface in $\mathbb{R}^3$ is defined by the equation $2x^3 + 2y^3 + 3z^3 + zxy = 8$.
   (a) Find a unit vector that is orthogonal to the tangent plane at the point $(x = 1, y = 1, z = 1)$.
   (b) Find the equation of the tangent plane.

5. Consider the function $f(x,y) = x^2 + y^3 + xy$, the vector $v = (2, 1)$ and the point $A = (-1, -1)$.
   (a) Find the gradient of f at the point A.
   (b) Find the directional derivative of f at the point A in the direction given by v.

6. Let x_n be a sequence in $\mathbb{R}^2$ given by
$$x_n = \begin{pmatrix} 2\cos(2\pi n/3) \\ \sin(2\pi(n^2 - 1)/3n) \end{pmatrix}$$
   (a) Find the accumulation points of this sequence.
   (b) Find the limit of this sequence if it exists.

SECTION B

7. The production function is given by $q(K, L) = (K^\rho + L^\rho)^{1/\rho}$, where $K > 0$, $L > 0$, $\rho \leq 1$ and $\rho \neq 0$.
   (a) MRTS (marginal rate of technical substitution) is defined as MRTS $= -\frac{dK}{dL}|_{q(K,L)=const}$. Using implicit function theorem find MRTS and express your answer as a function of K/L alone.
   (b) Let $t = K/L$. Find the derivative $\sigma = \frac{d\ln t}{d\ln \text{MRTS}}$.
   (c) Suggest at least one production function $q(K, L)$ such that $\sigma = 1$.

8. The closed first quadrant is denoted by $\bar{\mathbb{R}}^2_+$. Consider the function $F(x, y) = xy - (x^p/p + y^q/q)$ defined on $\bar{\mathbb{R}}^2_+$, where $p > 1$, $q > 1$ and $(p-1)(q-1) = 1$.
   (a) Find the set $S \subset \bar{\mathbb{R}}^2_+$ such that $\partial F/\partial x = 0$ and $\partial F/\partial y = 0$ at the same time. Sketch the set S.
   (b) What are the possible values of the function $F(x, y)$ for $(x, y) \in S$?
   (c) Is it true that the sign of $F(x, y)$ is the same for all $(x, y) \notin S$?

Variant 3. 2017-10-27. Please, don't forget to write the variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x,y) = 3x^3+2x-3xy^3-y^2$. Using the total differential find the approximate value of $f(0.98, 1.99)$.

2. Consider the system
$$\begin{cases} x^3+y^3+4z^3=6 \\ x+x^3+2y+3y^2+xyz+z^3=9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1,1,1)$;
   (b) Find $y'(z)$ if possible.

3. If possible find the limit
$$\lim_{x\to 0, y\to 0} \frac{\exp(x^2+y^2)-1}{x^2+y^2+5|x|+|y|};$$

4. The surface in $\mathbb{R}^3$ is defined by the equation $3x^3+2y^3+3z^3+zxy=9$.
   (a) Find a unit vector that is orthogonal to the tangent plane at the point $(x=1, y=1, z=1)$.
   (b) Find the equation of the tangent plane.

5. Consider the function $f(x,y)=3x^2+y^3+xy$, the vector $v=(1,2)$ and the point $A=(-1,-1)$.
   (a) Find the gradient of f at the point A.
   (b) Find the directional derivative of f at the point A in the direction given by v.

6. Let x_n be a sequence in $\mathbb{R}^2$ given by
$$x_n = \begin{pmatrix} 3\cos(2\pi n/3) \\ \sin(2\pi(n^2-1)/3n) \end{pmatrix}$$
   (a) Find the accumulation points of this sequence.
   (b) Find the limit of this sequence if it exists.

SECTION B

7. The production function is given by $q(K,L) = (K^\rho + L^\rho)^{1/\rho}$, where $K>0$, $L>0$, $\rho \le 1$ and $\rho \neq 0$.
   (a) MRTS (marginal rate of technical substitution) is defined as MRTS $= -\frac{dK}{dL}\,|_{q(K,L)=const}$. Using implicit function theorem find MRTS and express your answer as a function of K/L alone.
   (b) Let $t=K/L$. Find the derivative $\sigma = \frac{d\ln t}{d\ln \mathrm{MRTS}}$.
   (c) Suggest at least one production function $q(K,L)$ such that $\sigma = 1$.

8. The closed first quadrant is denoted by $\bar{\mathbb{R}}^2_+$. Consider the function $F(x,y) = xy - (x^p/p + y^q/q)$ defined on $\bar{\mathbb{R}}^2_+$, where $p>1$, $q>1$ and $(p-1)(q-1)=1$.
   (a) Find the set $S \subset \bar{\mathbb{R}}^2_+$ such that $\partial F/\partial x = 0$ and $\partial F/\partial y = 0$ at the same time. Sketch the set S.
   (b) What are the possible values of the function $F(x,y)$ for $(x,y) \in S$?
   (c) Is it true that the sign of $F(x,y)$ is the same for all $(x,y) \notin S$?

Variant 4. 2017-10-27. Please, don't forget to write the variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y) = 4x^3 + 2x - 3xy^3 - y^2$. Using the total differential find the approximate value of $f(0.98, 1.99)$.

2. Consider the system
$$\begin{cases} x^3 + y^3 + 5z^3 = 7 \\ x + x^3 + 2y + 3y^2 + xyz + z^3 = 9 \end{cases}$$
   (a) Check whether the functions $y(z)$ and $x(z)$ are defined at a point $(1, 1, 1)$;

   (b) Find $y'(z)$ if possible.

3. If possible find the limit
$$\lim_{x\to 0, y\to 0} \frac{\exp(x^2 + y^2) - 1}{x^2 + y^2 + 6|x| + |y|};$$

4. The surface in $\mathbb{R}^3$ is defined by the equation $4x^3 + 2y^3 + 3z^3 + zxy = 10$.

   (a) Find a unit vector that is orthogonal to the tangent plane at the point $(x = 1, y = 1, z = 1)$.

   (b) Find the equation of the tangent plane.

5. Consider the function $f(x, y) = 4x^2 + y^3 + xy$, the vector $v = (1, 2)$ and the point $A = (-1, -1)$.

   (a) Find the gradient of f at the point A.

   (b) Find the directional derivative of f at the point A in the direction given by v.

6. Let x_n be a sequence in $\mathbb{R}^2$ given by
$$x_n = \begin{pmatrix} 4\cos(2\pi n/3) \\ \sin(2\pi(n^2 - 1)/3n) \end{pmatrix}$$
   (a) Find the accumulation points of this sequence.

   (b) Find the limit of this sequence if it exists.

SECTION B

7. The production function is given by $q(K, L) = (K^\rho + L^\rho)^{1/\rho}$, where $K > 0$, $L > 0$, $\rho \leq 1$ and $\rho \neq 0$.

   (a) MRTS (marginal rate of technical substitution) is defined as MRTS $= -\frac{dK}{dL}\,|_{q(K,L)=const}$. Using implicit function theorem find MRTS and express your answer as a function of K/L alone.

   (b) Let $t = K/L$. Find the derivative $\sigma = \frac{d\ln t}{d\ln \text{MRTS}}$.

   (c) Suggest at least one production function $q(K, L)$ such that $\sigma = 1$.

8. The closed first quadrant is denoted by $\bar{\mathbb{R}}^2_+$. Consider the function $F(x, y) = xy - (x^p/p + y^q/q)$ defined on $\bar{\mathbb{R}}^2_+$, where $p > 1$, $q > 1$ and $(p - 1)(q - 1) = 1$.

   (a) Find the set $S \subset \bar{\mathbb{R}}^2_+$ such that $\partial F/\partial x = 0$ and $\partial F/\partial y = 0$ at the same time. Sketch the set S.

   (b) What are the possible values of the function $F(x, y)$ for $(x, y) \in S$?

   (c) Is it true that the sign of $F(x, y)$ is the same for all $(x, y) \notin S$?