Name, group no:

. . . .

1. [10 points] Find the local maxima and minima of the function $f(x, y) = x^4 + 2y^4 - xy$.

Name, group no:

. . . .

2. [10 points] Find all critical points of the function $z = z(x, y)$ implicitly defined by the equation

$$x^2 + y^2 + z^2 - xz - yz + x + y + 4z + 1 = 0 \tag{1}$$

Name, group no:

. . . .

3. [10 points] Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 5x + 4y + 8z$ subject to $x^2 + y^2 + z^2 = 1$.

Name, group no:

. . . .

4. [10 points] For the function $f(x, y) = 2xy + 3$ find the level curve and the equation for its tangent at the point $(1, 2)$.

Name, group no:

. . . .

5. [10 points] Use the chain rule to find $f'(x)$ and $f''(x)$ for $f(x) = u(a, b, x)$ where $a = \sin(x)$ and $b = x^3$.

Name, group no:

. . . .

6. [10 points] Consider the function $f(x, y) = x^2 + y^3 - xy + 3y$ at the point $(2; 1)$. Find all the directions in which the growth rate of the function constitutes 80% of the maximal possible growth rate at that point.

Name, group no:

. . . .

7. [20 points] Consider an expenditure minimization problem for the agent whose utility function is $u(x_1,\ x_2) = \sqrt{x_1 x_2}$. Let $\bar{u}$ be a prescribed level of utility. Then find solution to the problem

$$\begin{cases} p_1 x_1 + p_2 x_2 \to \min \\ \sqrt{x_1 x_2} = \bar{u} \end{cases},$$

where $x_1,\ x_2 \geq 0$. Let $e = p_1 \tilde{x}_1 + p_2 \tilde{x}_2$ be the expenditure function, $\tilde{x}_1$ and $\tilde{x}_2$ being the solutions of the minimization problem. By using the appropriate envelope theorem find $\frac{\partial e}{\partial p_1}$ and $\frac{\partial e}{\partial p_2}$.

Name, group no:

. . . .

8. [20 points] For what values of p, q is the function $f(x, y) = x^p + y^q$ convex or concave. Consider only $x > 0$, $y > 0$.