Assignment 6 (Due on the week October 26– 31)

1. Consider the equation $x^3 + 3y^2 + 4xz^2 - 3z^2y = 1$. Does this equation define z as a function of x and y. If so, compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at this point.

   (a) In a neighborhood of $x = 1$, $y = 1$?

   (b) In a neighborhood of $x = 1$, $y = 0$?

   (c) In a neighborhood of $x = 0.5$, $y = 0$?

2. Consider $3x^2yz + xyz^2 = 30$ as defining x as an implicit function of y and z around the point $x = 1$, $y = 3$, $z = 2$. If y increases to 3.2 and z remains at 2, use the Implicit Function Theorem to estimate the corresponding x.

3. (a) Find all points (x_0, y_0), where $y' = 0$, if $y(x)$ is an implicit function given by an equation $\ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x}$.

   (b) Calculate $y''(x)$ at these points.

4. Prove that $y''(x) \equiv 0$ if $y(x)$ is an implicit function given by an equation $y = 2x\operatorname{arctg} \frac{y}{x}$. Where does $y(x)$ exist?

5. Find all partial derivatives of the first and second order of the composite function $w = f(x, y)$, where $x = u^2 + v^2$, $y = uv$.