Math for Economists, Fall-exam retake, 25.01.2016

1. Find the total differential for the function $f(x, y) = x^2y^2 + xy^2 + 2x + 4y$. Using the total differential find approximately $f(1.001, 1.999)$

2. The system of equations defines $x(z)$ and $y(z)$:

$$\begin{cases} x^2 + zxy + y^2 + 6z + y^3 = 10 \\ y^3x^2 + 3x + 2y + z = 7 \end{cases}$$

   Find $x'(z)$ at the point $x = 1$ and $y = 1$.

3. Find the local maxima and minima of the function $f(x, y) = x^4 + 2y^4 - xy$. Determine whether the extrema you have found are global or local.

4. Calculate all partial derivatives of the first and second order of u with respect to x and y if $u = f(\xi, \eta)$ and $\xi = x + xy$, $\eta = x/y$.

5. Use Lagrange multipliers to find the height and radius of a cylinder with the least possible surface area among those with a volume of 6π m^3. Make sure you check the second order condition for minimisation.

6. Consider the function $f(x) = h(x) - ax$, where the function h is twice differentiable and $h''(x) < 0$ for all x. The global maximum of f is denoted by $x^*(a)$.
   (a) Find dx^*/da
   (b) It is known that for $a = 1$ the optimal point is $x^* = 3$ and the value of maximum is 2016. What is the approximate value of maximum for $a = 1.01$?

7. Two simple independent problems :)
   (a) **(10 points)** Find all values of the parameter λ such that the function $f = 2x^2 + 3y^2 + z^2 + 4xy - 2xz - 2\lambda yz$ is convex.
   (b) **(10 points)** Write down the equation of the tangent plane to a surface $z = x^3 - y^3$ at the point $(-1; 1; -2)$.

8. **(20 points)** Consider a problem of finding the extremal values of the function $f(x, y) = e^x + e^y + cx + cy$ under the constraint $x + y = c$, where c is a positive parameter.
   (a) Find out what kind of a problem you need to set: a problem of maximization or minimization?
   (b) Let $f(x*(c), y*(c))$ be the *value function* of the problem. If c slightly increases and becomes $c + \Delta c$, estimate the change in $f(x*(c), y*(c))$. Your answer should contain c and Δc only.