Variant 1. 26 May 2015. Please, don't forget to write you variant number. Total duration of the exam is 120 min. Good luck! :)

SECTION A. You need to solve BOTH problems.

1. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -x_1 - 2x_2 + 3x_3$ subject to constraints: $2(x_1 + 1)^2 + x_2^2 + 3(x_3 - 1)^2 \leq 5$ and $x_1, x_2, x_3 \geq 0$. Find the point(s) of maximum and maximum value of f. Justify your answer by reference to Weierstrass theorem if it is relevant.

2. For all real values of parameter β that lies within the range $-1 < \beta < 0$ maximize linear function $2x_1 - x_2 + 8x_3 - 19$ subject to constraints $x_1 \geq x_2 + x_3 + \beta$, $2x_1 + x_2 + 4x_3 \leq \beta + 1$ and $x_1, x_2, x_3 \geq 0$. You are not asked to find the maximizer.

SECTION B. You need to solve TWO problems of your choice.

3. Find the general solution of equation $y'' - 4y' + 8y = \sin 2x + e^{-2x}$.

4. Solve the initial value problem for the system of difference equations

$$\begin{cases} x_{t+1} = -x_t + 2y_t + 7 \\ y_{t+1} = -2x_t + 2y_t \end{cases}$$

   where $x_0 = y_0 = 0$.

5. By variation of parameters solve $y'' + y = \frac{1}{\sin^2 x}$.

SECTION C. You need to solve BOTH problems

6. Andrey and Boris play the following game. Each player throws a fair coin and observes the result of his own toss. Then simultaneously Andrey guesses the result of Boris' toss and Boris guesses the result of Andrey's toss. They receive one dollar each if at least one guess was correct and receive nothing otherwise.

   (a) Find all pure Nash equilibria of this game

   (b) Are the Nash equilibria Pareto-optimal?

   (c) What is the probability of at least one correct guess in the Nash equilibria?

7. Anna and Bella play the simplified version of Battleship game. Anna places a two-decker destroyer ship on the 1×4 grid. Then Bella has one shot. Bella does not know where the Anna's ship is located. If Bella hits the Anna's ship then Bella wins the game, otherwise Anna wins.

   (a) Find at least one Nash equilibria of this game

   (b) What is the probability that Anna wins in the Nash equilibria?

Variant 2. 26 May 2015. Please, don't forget to write you variant number. Total duration of the exam is 120 min. Good luck! :)

SECTION A. You need to solve BOTH problems.

1. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -2x_1 - x_2 + 4x_3$ subject to constraints: $2(x_1 + 2)^2 + x_2^2 + 4(x_3 - 1)^2 \leq 12$ and $x_1, x_2, x_3 \geq 0$. Find the point(s) of maximum and maximum value of f. Justify your answer by reference to Weierstrass theorem if it is relevant.

2. For all real values of parameter α that lies within the range $0 < \alpha < 1$ maximize linear function $x_1 - 5x_2 + 4x_3 - 9$ subject to constraints $x_1 + 1 \geq x_2 + x_3 + \alpha$, $2x_1 + x_2 + 4x_3 \leq \alpha$ and $x_1, x_2, x_3 \geq 0$. You are not asked to find the maximizer.

SECTION B. You need to solve TWO problems of your choice.

3. Find the general solution of equation $y'' + 4y' + 8y = \cos 2x + e^{2x}$.

4. Solve the initial value problem for the system of difference equations

$$\begin{cases} x_{t+1} = -x_t + 2y_t + 7 \\ y_{t+1} = -2x_t + 2y_t \end{cases}$$

   where $x_0 = y_0 = 0$.

5. By variation of parameters solve $y'' + y = \frac{1}{\sin^2 x}$.

SECTION C. You need to solve BOTH problems

6. Andrey and Boris play the following game. Each player throws a fair coin and observes the result of his own toss. Then simultaneously Andrey guesses the result of Boris' toss and Boris guesses the result of Andrey's toss. They receive one dollar each if at least one guess was correct and receive nothing otherwise.

   (a) Find all pure Nash equilibria of this game

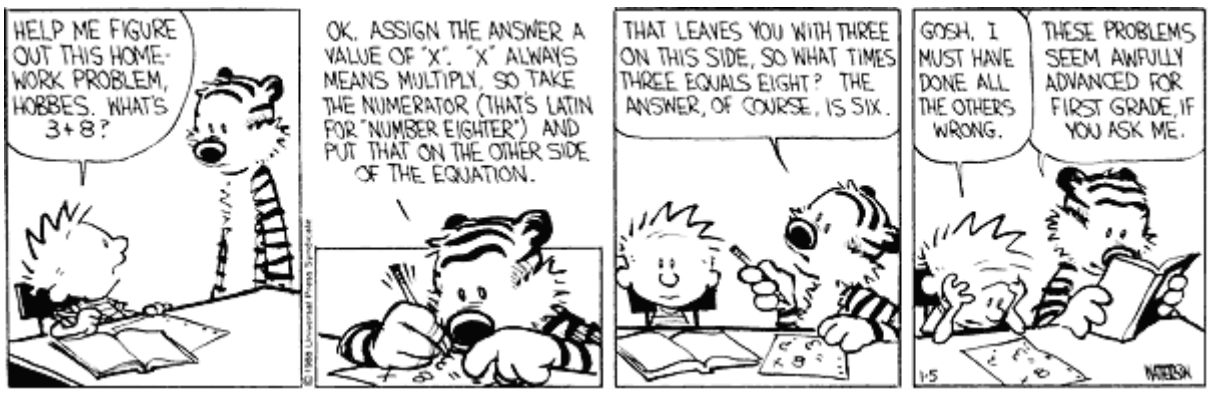
   (b) Are the Nash equilibria Pareto-optimal?

   (c) What is the probability of at least one correct guess in the Nash equilibria?

7. Anna and Bella play the simplified version of Battleship game. Anna places a two-decker destroyer ship on the 1×4 grid. Then Bella has one shot. Bella does not know where the Anna's ship is located. If Bella hits the Anna's ship then Bella wins the game, otherwise Anna wins.

   (a) Find at least one Nash equilibria of this game

   (b) What is the probability that Anna wins in the Nash equilibria?



Variant 3. 26 May 2015. Please, don't forget to write you variant number. Total duration of the exam is 120 min. Good luck! :)

SECTION A. You need to solve BOTH problems.

1. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -x_1 - 3x_2 + 2x_3$ subject to constraints: $(x_1 + 1)^2 + 2x_2^2 + 3(x_3 - 1)^2 \leq 4$ and $x_1, x_2, x_3 \geq 0$. Find the point(s) of maximum and maximum value of f. Justify your answer by reference to Weierstrass theorem if it is relevant.

2. For all real values of parameter α that lies within the range $-1 < \alpha < 0$ maximize linear function $x_1 - 2x_2 + 4x_3 - 7$ subject to constraints $x_1 \geq x_2 + x_3 + \alpha$, $2x_1 + x_2 + 4x_3 \leq \alpha + 1$ and $x_1, x_2, x_3 \geq 0$. You are not asked to find the maximizer.

SECTION B. You need to solve TWO problems of your choice.

3. Find the general solution of equation $y'' - 2y' + 2y = \sin x + e^{-x}$.

4. Solve the initial value problem for the system of difference equations

$$\begin{cases} x_{t+1} = 2x_t + y_t + 10 \\ y_{t+1} = -10x_t + 2y_t \end{cases}$$

where $x_0 = y_0 = 0$.

5. By variation of parameters solve $y'' + y = -\operatorname{ctg}^2 x$.

SECTION C. You need to solve BOTH problems

6. Andrey and Boris play the following game. Each player throws a fair coin and observes the result of his own toss. Then simultaneously Andrey guesses the result of Boris' toss and Boris guesses the result of Andrey's toss. They receive one dollar each if at least one guess was correct and receive nothing otherwise.

   (a) Find all pure Nash equilibria of this game

   (b) Are the Nash equilibria Pareto-optimal?

   (c) What is the probability of at least one correct guess in the Nash equilibria?

7. Anna and Bella play the simplified version of Battleship game. Anna places a two-decker destroyer ship on the 1×4 grid. Then Bella has one shot. Bella does not know where the Anna's ship is located. If Bella hits the Anna's ship then Bella wins the game, otherwise Anna wins.

   (a) Find at least one Nash equilibria of this game

   (b) What is the probability that Anna wins in the Nash equilibria?

Variant 4. 26 May 2015. Please, don't forget to write you variant number. Total duration of the exam is 120 min. Good luck! :)

SECTION A. You need to solve BOTH problems.

1. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -5x_1 - 3x_2 + x_3$ subject to constraints: $(x_1 + 1)^2 + 3x_2^2 + (x_3 - 1)^2 \leq 2$ and $x_1, x_2, x_3 \geq 0$. Find the point(s) of maximum and maximum value of f. Justify your answer by reference to Weierstrass theorem if it is relevant.

2. For all real values of parameter β that lies within the range $0 < \beta < 1$ maximize linear function $2x_1 - 7x_2 + 8x_3 - 17$ subject to constraints $x_1 + 1 \geq x_2 + x_3 + \beta$, $2x_1 + x_2 + 4x_3 \leq \beta$ and $x_1, x_2, x_3 \geq 0$. You are not asked to find the maximizer.

SECTION B. You need to solve TWO problems of your choice.

3. Find the general solution of equation $y'' + 2y' + 2y = \cos x + e^x$.

4. Solve the initial value problem for the system of difference equations

$$\begin{cases} x_{t+1} = x_t + 3y_t + 7 \\ y_{t+1} = -2x_t + y_t \end{cases}$$

where $x_0 = y_0 = 0$.

5. By variation of parameters solve $y'' - 4y = (15 - 16x^2)\sqrt{x}$.

SECTION C. You need to solve BOTH problems

6. Andrey and Boris play the following game. Each player throws a fair coin and observes the result of his own toss. Then simultaneously Andrey guesses the result of Boris' toss and Boris guesses the result of Andrey's toss. They receive one dollar each if at least one guess was correct and receive nothing otherwise.

   (a) Find all pure Nash equilibria of this game

   (b) Are the Nash equilibria Pareto-optimal?

   (c) What is the probability of at least one correct guess in the Nash equilibria?

7. Anna and Bella play the simplified version of Battleship game. Anna places a two-decker destroyer ship on the 1×4 grid. Then Bella has one shot. Bella does not know where the Anna's ship is located. If Bella hits the Anna's ship then Bella wins the game, otherwise Anna wins.

   (a) Find at least one Nash equilibria of this game

   (b) What is the probability that Anna wins in the Nash equilibria?