SECTION A

1. Find the gradient of $f(x,y) = \frac{x^2y}{\sqrt{x^2+y^2}}$ at the point $M(2,1)$. Compute the derivative of f at M in direction of the vector $\{-1,1\}$.

2. The system of equations defines $x(z)$ and $y(z)$:

$$\begin{cases} x^2 + zxy + y^2 + 5z + y^3 = 9 \\ y^3x^2 + 3x + 2y + z = 7 \end{cases}$$

   Find $x'(z)$ and $y'(z)$ at the point $x = 1$ and $y = 1$. State the implicit function theorem.

3. For the function $f(x,y) = x^3y^5 + x^2 - y^3 + xy$ find first order Taylor approximation at the point $(1,1)$ and second order Taylor approximation at the same point.

4. Find all stationary points of $f(x,y) = -2y^3 + 24y - x^2e^y$. Classify them as local minimum, maximum or saddle point.

5. Find the constrained extrema of the function $f(x,y) = 2x^2 + x + y + y^2$ subject to $2x^2 + y^2 = 5$.

6. For each value of a determine whether the function $f(x,y,z) = x^2 + xz + ayz + z^2$ is concave, convex, strictly concave, strictly convex.

SECTION B

7. Robinson Crusoe produces nuts (good x) and corn (good y) using two factors of production: labor L and land T in accordance with the production functions, namely $x = \sqrt{L_xT_x}$ and $y = \sqrt{L_yT_y}$, where the L_x, L_y, T_x, T_y are the quantities of labor and land employed by Crusoe in the production processes. The overall supply of labor and supply of the cultivated land equal 1. In order to find the **Production Possibilities Frontier** in the Crusoes economy the following problem should be solved:

$$(A)\ \begin{cases} \sqrt{L_xT_x} \to \max \\ \sqrt{L_yT_y} = y = const \\ L_x + L_y = 1,\ T_x + T_y = 1 \end{cases}$$

   In this problem all variables take nonnegative values and $0 \le y \le 1$.

   (a) (15 points) Solve this maximization problem by the Lagrange multiplier method. You may consider only the case when L_x, L_y, T_x, $T_y > 0$ and $0 < y < 1$.

   (b) (5 points) Let L_x^*, L_y^*, T_x^*, T_y^* be the solution of problem (A) (optimal combination of factors). Then the produced quantities of nuts and corn equal $x = \sqrt{L_x^*T_x^*}$ and $y = \sqrt{L_y^*T_y^*}$. Find the relationship between x and y in the form of some equation $G(x,y) = 0$.

8. (continuation) Crusoe does not care about his leisure time and consumes nuts and corn. His utility function is $u(x,y) = \frac{1}{3}\ln x + \frac{2}{3}\ln y$.

   (a) (10 points) By solving utility maximization problem: $\begin{cases} u(x,u) \to \max \\ G(x,y) = 0 \end{cases}$, find his optimal consumption bundle $(\tilde{x}, \tilde{y})$.

   (b) (10 points) Crusoe has plowed more land in the amount of dT (a small number). By using Envelope theorem find $\partial\tilde{x}/\partial T$. Hint. Firstly apply the theorem to problem (A).

SECTION A

1. Find the gradient of $f(x, y) = \frac{x^2 y}{\sqrt{x^2+y^2}}$ at the point $M(1, 2)$. Compute the derivative of f at M in direction of the vector $\{-1, 1\}$.

2. The system of equations defines $x(z)$ and $y(z)$:

$$\begin{cases} x^2 + zxy + y^2 + 4z + y^3 = 8 \\ y^3 x^2 + 3x + 2y + z = 7 \end{cases}$$

   Find $x'(z)$ and $y'(z)$ at the point $x = 1$ and $y = 1$. State the implicit function theorem.

3. For the function $f(x, y) = x^3 y^5 + x^2 - y^3 + 2xy$ find first order Taylor approximation at the point $(1, 1)$ and second order Taylor approximation at the same point.

4. Find all stationary points of $f(x, y) = -y^3 + 12y - x^2 e^y$. Classify them as local minimum, maximum or saddle point.

5. Find the constrained extrema of the function $f(x, y) = 2x^2 + 2x + y + y^2$ subject to $2x^2 + y^2 = 5$.

6. For each value of a determine whether the function $f(x, y, z) = x^2 + xz + ayz + z^2$ is concave, convex, strictly concave, strictly convex.

SECTION B

7. Robinson Crusoe produces nuts (good x) and corn (good y) using two factors of production: labor L and land T in accordance with the production functions, namely $x = \sqrt{L_x T_x}$ and $y = \sqrt{L_y T_y}$, where the L_x, L_y, T_x, T_y are the quantities of labor and land employed by Crusoe in the production processes. The overall supply of labor and supply of the cultivated land equal 1. In order to find the **Production Possibilities Frontier** in the Crusoes economy the following problem should be solved:

$$(A) \begin{cases} \sqrt{L_x T_x} \to \max \\ \sqrt{L_y T_y} = y = const \\ L_x + L_y = 1,\ T_x + T_y = 1 \end{cases}$$

   In this problem all variables take nonnegative values and $0 \le y \le 1$.

   (a) (15 points) Solve this maximization problem by the Lagrange multiplier method. You may consider only the case when L_x, L_y, T_x, $T_y > 0$ and $0 < y < 1$.

   (b) (5 points) Let L_x^*, L_y^*, T_x^*, T_y^* be the solution of problem (A) (optimal combination of factors). Then the produced quantities of nuts and corn equal $x = \sqrt{L_x^* T_x^*}$ and $y = \sqrt{L_y^* T_y^*}$. Find the relationship between x and y in the form of some equation $G(x, y) = 0$.

8. (continuation) Crusoe does not care about his leisure time and consumes nuts and corn. His utility function is $u(x, y) = \frac{1}{3} \ln x + \frac{2}{3} \ln y$.

   (a) (10 points) By solving utility maximization problem: $\begin{cases} u(x, u) \to \max \\ G(x, y) = 0 \end{cases}$, find his optimal consumption bundle $(\tilde{x}, \tilde{y})$.

   (b) (10 points) Crusoe has plowed more land in the amount of dT (a small number). By using Envelope theorem find $\partial \tilde{x} / \partial T$. Hint. Firstly apply the theorem to problem (A).

SECTION A

1. Find the gradient of $f(x,y) = \frac{x^2 y}{\sqrt{x^2+y^2}}$ at the point $M(3,1)$. Compute the derivative of f at M in direction of the vector $\{-1,1\}$.

2. The system of equations defines $x(z)$ and $y(z)$:

$$\begin{cases} x^2 + zxy + y^2 + 3z + y^3 = 7 \\ y^3x^2 + 3x + 2y + z = 7 \end{cases}$$

   Find $x'(z)$ and $y'(z)$ at the point $x = 1$ and $y = 1$. State the implicit function theorem.

3. For the function $f(x,y) = x^3y^5 + x^2 - y^3 + 3xy$ find first order Taylor approximation at the point $(1,1)$ and second order Taylor approximation at the same point.

4. Find all stationary points of $f(x,y) = 2y^3 - 24y + x^2e^y$. Classify them as local minimum, maximum or saddle point.

5. Find the constrained extrema of the function $f(x,y) = 2x^2 + 3x + y + y^2$ subject to $2x^2 + y^2 = 5$.

6. For each value of a determine whether the function $f(x,y,z) = 2014 - x^2 - xz - ayz - z^2$ is concave, convex, strictly concave, strictly convex.

SECTION B

7. Robinson Crusoe produces nuts (good x) and corn (good y) using two factors of production: labor L and land T in accordance with the production functions, namely $x = \sqrt{L_xT_x}$ and $y = \sqrt{L_yT_y}$, where the L_x, L_y, T_x, T_y are the quantities of labor and land employed by Crusoe in the production processes. The overall supply of labor and supply of the cultivated land equal 1. In order to find the **Production Possibilities Frontier** in the Crusoes economy the following problem should be solved:

$$(A)\ \begin{cases} \sqrt{L_xT_x} \to \max \\ \sqrt{L_yT_y} = y = const \\ L_x + L_y = 1,\ T_x + T_y = 1 \end{cases}$$

   In this problem all variables take nonnegative values and $0 \le y \le 1$.

   (a) (15 points) Solve this maximization problem by the Lagrange multiplier method. You may consider only the case when L_x, L_y, T_x, $T_y > 0$ and $0 < y < 1$.

   (b) (5 points) Let L_x^*, L_y^*, T_x^*, T_y^* be the solution of problem (A) (optimal combination of factors). Then the produced quantities of nuts and corn equal $x = \sqrt{L_x^*T_x^*}$ and $y = \sqrt{L_y^*T_y^*}$. Find the relationship between x and y in the form of some equation $G(x,y) = 0$.

8. (continuation) Crusoe does not care about his leisure time and consumes nuts and corn. His utility function is $u(x,y) = \frac{1}{3}\ln x + \frac{2}{3}\ln y$.

   (a) (10 points) By solving utility maximization problem: $\begin{cases} u(x,u) \to \max \\ G(x,y) = 0 \end{cases}$, find his optimal consumption bundle $(\tilde{x},\tilde{y})$.

   (b) (10 points) Crusoe has plowed more land in the amount of dT (a small number). By using Envelope theorem find $\partial\tilde{x}/\partial T$. Hint. Firstly apply the theorem to problem (A).

SECTION A

1. Find the gradient of $f(x,y) = \frac{x^2y}{\sqrt{x^2+y^2}}$ at the point $M(2,2)$. Compute the derivative of f at M in direction of the vector $\{-1,1\}$.

2. The system of equations defines $x(z)$ and $y(z)$:

$$\begin{cases} x^2 + zxy + y^2 + 2z + y^3 = 6 \\ y^3x^2 + 3x + 2y + z = 7 \end{cases}$$

   Find $x'(z)$ and $y'(z)$ at the point $x = 1$ and $y = 1$. State the implicit function theorem.

3. For the function $f(x,y) = x^3y^5 + x^2 - y^3 + 4xy$ find first order Taylor approximation at the point $(1,1)$ and second order Taylor approximation at the same point.

4. Find all stationary points of $f(x,y) = y^3 - 12y + x^2e^y$. Classify them as local minimum, maximum or saddle point.

5. Find the constrained extrema of the function $f(x,y) = 2x^2 + 4x + y + y^2$ subject to $2x^2 + y^2 = 5$.

6. For each value of a determine whether the function $f(x,y,z) = 2014 - x^2 - xz - ayz - z^2$ is concave, convex, strictly concave, strictly convex.

SECTION B

7. Robinson Crusoe produces nuts (good x) and corn (good y) using two factors of production: labor L and land T in accordance with the production functions, namely $x = \sqrt{L_xT_x}$ and $y = \sqrt{L_yT_y}$, where the L_x, L_y, T_x, T_y are the quantities of labor and land employed by Crusoe in the production processes. The overall supply of labor and supply of the cultivated land equal 1. In order to find the **Production Possibilities Frontier** in the Crusoes economy the following problem should be solved:

$$(A)\ \begin{cases} \sqrt{L_xT_x} \to \max \\ \sqrt{L_yT_y} = y = const \\ L_x + L_y = 1,\ T_x + T_y = 1 \end{cases}$$

   In this problem all variables take nonnegative values and $0 \le y \le 1$.

   (a) (15 points) Solve this maximization problem by the Lagrange multiplier method. You may consider only the case when L_x, L_y, T_x, $T_y > 0$ and $0 < y < 1$.

   (b) (5 points) Let L_x^*, L_y^*, T_x^*, T_y^* be the solution of problem (A) (optimal combination of factors). Then the produced quantities of nuts and corn equal $x = \sqrt{L_x^*T_x^*}$ and $y = \sqrt{L_y^*T_y^*}$. Find the relationship between x and y in the form of some equation $G(x,y) = 0$.

8. (continuation) Crusoe does not care about his leisure time and consumes nuts and corn. His utility function is $u(x,y) = \frac{1}{3}\ln x + \frac{2}{3}\ln y$.

   (a) (10 points) By solving utility maximization problem: $\begin{cases} u(x,u) \to \max \\ G(x,y) = 0 \end{cases}$, find his optimal consumption bundle $(\tilde{x},\tilde{y})$.

   (b) (10 points) Crusoe has plowed more land in the amount of dT (a small number). By using Envelope theorem find $\partial\tilde{x}/\partial T$. Hint. Firstly apply the theorem to problem (A).