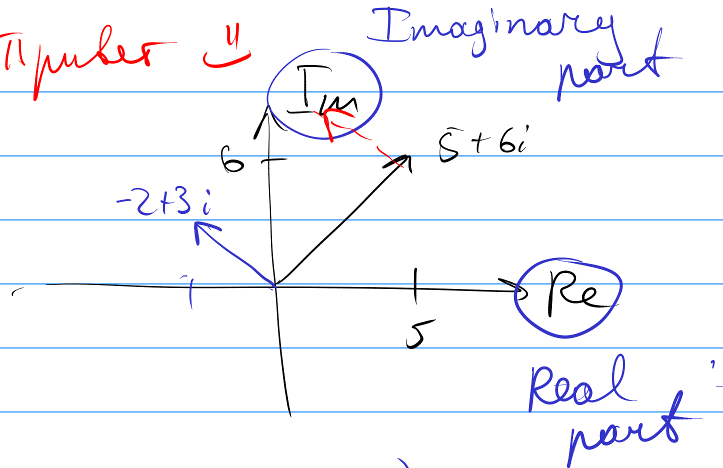
Сем 5

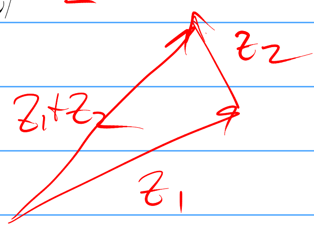
!! Привет !!



$z_1 = 5+6i$ ~

$z_2 = -2+3i$

$(5+6i)+(-2+3i) = (3+9i)$



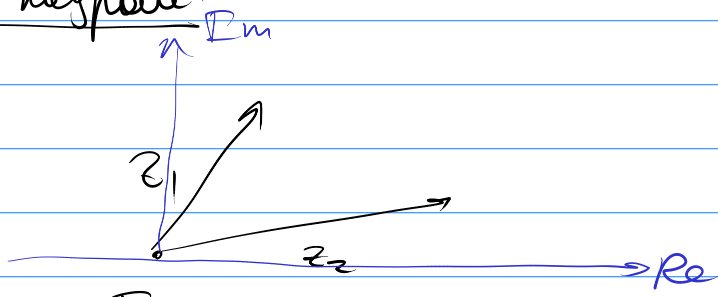
умножение

техники: $i^2 = -1$ и расскрытие скобок.

$$(5+6i)\cdot(-2+3i) = -10 - 12i + 15i + 18\underbrace{i^2}_{=-1} =$$

$$= -10 - 18 + 3i = -28 + 3i$$

геометрия за корени



(length / abs. value)

дължина (модуль) $|z|$

ъгъл с оста Re $\operatorname{Arg}(z)$

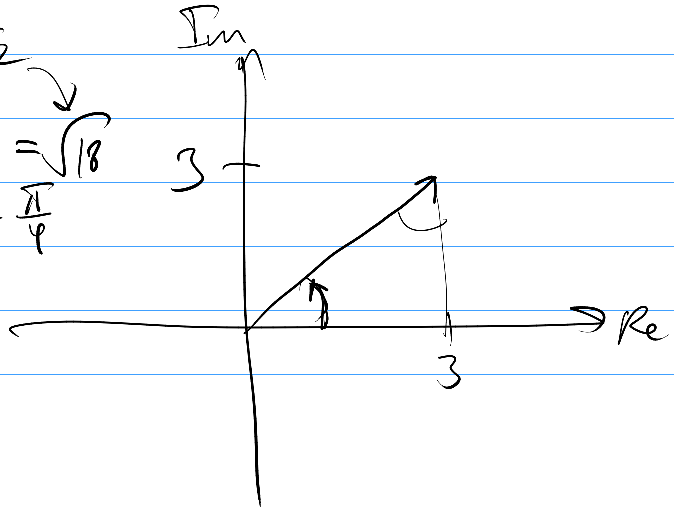
аргумент

(argument)

Упр.

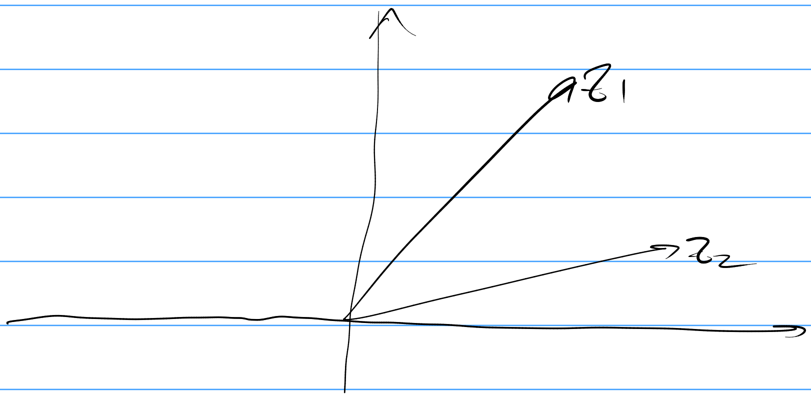
$$|3+3i| = \sqrt{3^2+3^2} = \sqrt{18} \quad (3\sqrt{2})$$

$$\operatorname{Arg}(3+3i) = 45^\circ = \frac{\pi}{4}$$



$\begin{pmatrix}3\\3\end{pmatrix} \sim 3+3i$

Геометрия умножения комплексных чисел



Опр

$w = z_1 \cdot z_2$

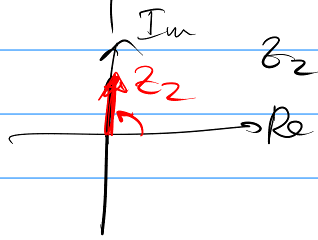
$|w| = |z_1| \cdot |z_2|$

$Arg(w) = Arg(z_1) + Arg(z_2)$

длины сомножителей → перемножить (с точностью до 2π)
аргументы (углы) сомн-лей → сложить

$z_1 = 2+2i$
$z_2 = 3i$

$|z_1| = \sqrt{2^2+2^2} = \sqrt{8}$
$Arg\, z_1 = \frac{\pi}{4} = 45^\circ$

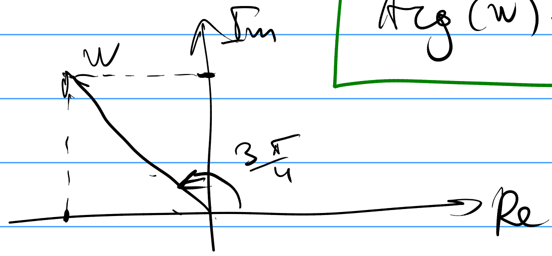


z_2

$|z_2| = 3$
$Arg(z_2) = 90^\circ = \frac{\pi}{2}$

$(2+2i)\cdot(3i) = ?$

$|w| = \sqrt{8}\cdot 3$
$Arg(w) = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$



$Re(w) = \sqrt{8}\cdot 3 \cdot \cos\left(\frac{3\pi}{4}\right) = \sqrt{8}\cdot 3\cdot\left(-\frac{\sqrt{2}}{2}\right) = -\frac{4\cdot 3}{2} = -6$

$Im(w) = \sqrt{8}\cdot 3 \cdot \sin\left(\frac{3\pi}{4}\right) = \sqrt{8}\cdot 3 \cdot \frac{\sqrt{2}}{2} = +6$

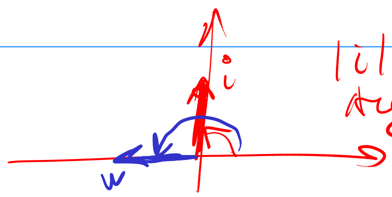
$(2+2i)\cdot(3i) = -6+6i$

технич. правило

$6i + 6i^2 = 6i - 6 = -6+6i$

$i^2 \stackrel{?}{=} -1$

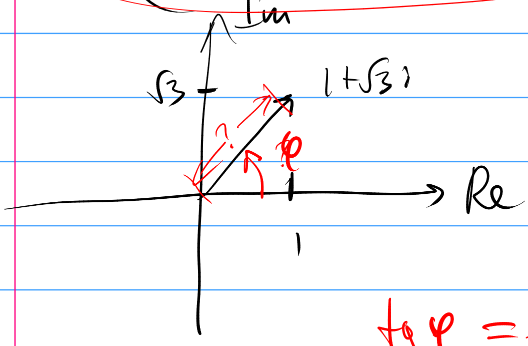
$\boxed{w = -1 + 0\cdot i}$



$|i| = 1$
$Arg(i) = \frac{\pi}{2}$

$i \cdot i = w$
$|w| = 1\cdot 1 = 1.$
$Arg(w) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$(1+\sqrt{3}i)^{2022}$? $= w$ → $|w| = 2^{2022}$, $\operatorname{Arg} w = 0$ $\quad w = 2^{2022} + 0\cdot i$



$|1+\sqrt{3}i| = \sqrt{1^2+\sqrt{3}^2} = 2$

$\operatorname{Arg}(1+\sqrt{3}i) = \frac{\pi}{3}$

$\operatorname{tg}\varphi = \sqrt{3}/1 = \sqrt{3} \qquad \varphi = 60^\circ = \frac{\pi}{3}$

$|w| = \underbrace{2\cdot 2\cdot 2\cdot 2\cdot \ldots\ldots \cdot 2}_{2022 \text{ раза}} = 2^{2022}$

$\operatorname{Arg} w \in [0;2\pi)$

$\operatorname{Arg} w = \underbrace{\frac{\pi}{3}+\frac{\pi}{3}+\ldots\ldots+\frac{\pi}{3}}_{2022 \text{ раза}} = \frac{2022\pi}{3} = \pmod{2\pi}$

$= 674\pi = 0\pi \pmod{2\pi}$

$\boxed{3.5\pi = 1.5\pi \pmod{2\pi}}$

Упр $\quad z^7 = 3 \qquad z \in \mathbb{C}$?

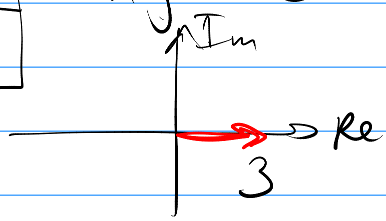
$|z|$?

$\operatorname{Arg}(z)$?

$z\cdot z\cdot z\cdot z\ldots z = 3$

$|3| = 3$

$\operatorname{Arg} 3 = 0^\circ$

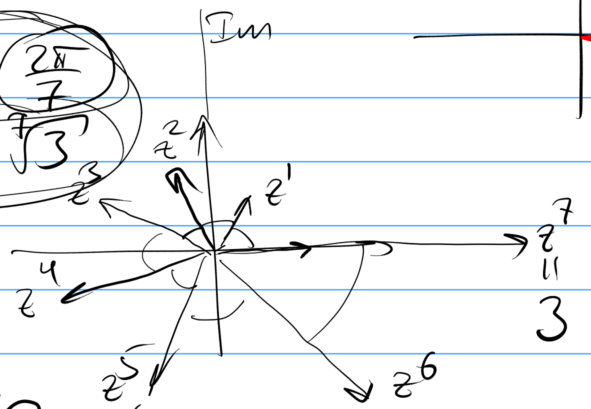


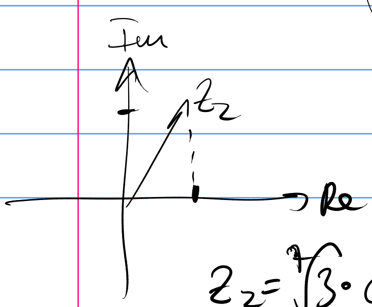
$\boxed{\text{решение 1}}$ $\quad \operatorname{Arg} z = 0.$

$|z| = \sqrt[7]{3} \qquad \boxed{z_1 = \sqrt[7]{3} + 0\cdot i}$

$\boxed{\text{решение 2}}$ $\quad \operatorname{Arg} z = \frac{2\pi}{7}$

$|z| = \sqrt[7]{3}$





$z_2 = \sqrt[7]{3}\cdot\cos\left(\frac{2\pi}{7}\right) + i\sqrt[7]{3}\cdot\sin\left(\frac{2\pi}{7}\right)$

решение 3

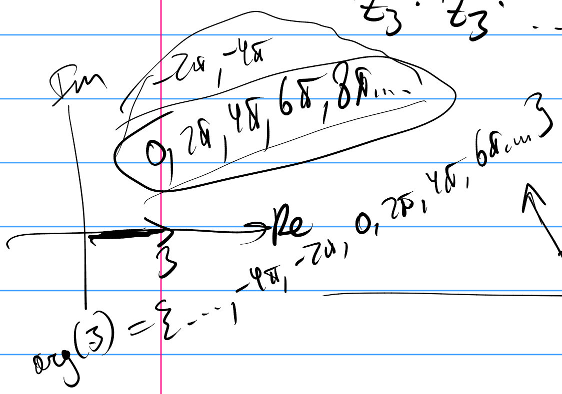
$Arg(3) = 0^\circ \in [0; 2\pi)$

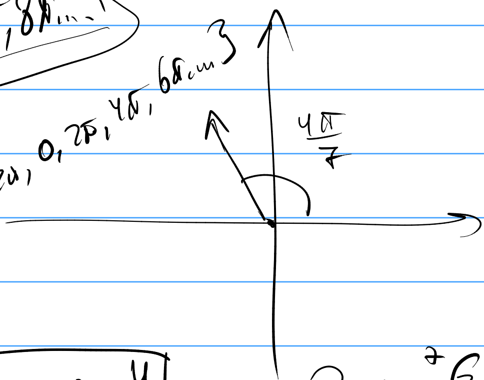
$arg(3) = \{\ldots -2\pi, 0, 2\pi, 4\pi, 6\pi, 8\pi \ldots\}$

$Arg\, z = \frac{4\pi}{7}$

$|z| = \sqrt[7]{3}$

$z_3 \cdot z_3 \cdot \ldots \cdot z_3 = \begin{cases} \text{длина } \sqrt[7]{3}\cdot\sqrt[7]{3}\cdot\ldots\cdot\sqrt[7]{3} = 3 \\ \text{угол } \frac{4\pi}{7}+\frac{4\pi}{7}+\ldots+\frac{4\pi}{7} = 4\pi = 0 \mod 2\pi \end{cases}$

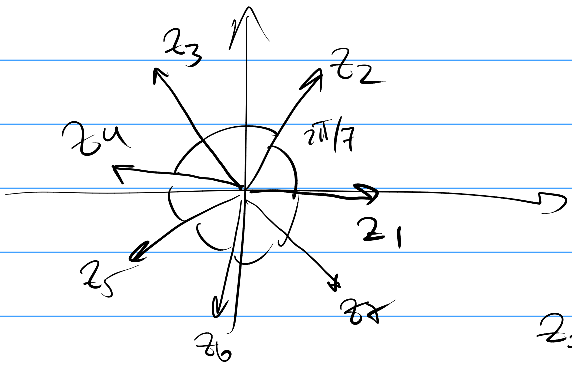




$z_3 = \sqrt[7]{3}\cdot\cos\left(\frac{4\pi}{7}\right) + i\cdot\sqrt[7]{3}\cdot\sin\left(\frac{4\pi}{7}\right)$

решение 4

$z_4 = \sqrt[7]{3}\cdot\cos\left(\frac{6\pi}{7}\right) + i\cdot\sqrt[7]{3}\cdot\sin\left(\frac{6\pi}{7}\right)$



$z_1 = \sqrt[7]{3}$

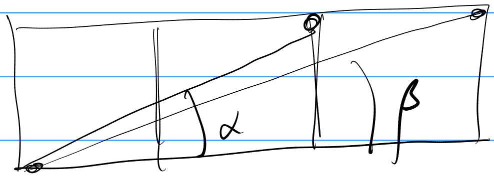
$z_7 = \sqrt[7]{3}\cdot\cos\left(\frac{12\pi}{7}\right) + i\cdot\sqrt[7]{3}\cdot\sin\left(\frac{12\pi}{7}\right)$

$z_8 = \sqrt[7]{3}\cdot\cos\left(\frac{14\pi}{7}\right) + i\sqrt[7]{3}\cdot\sin\left(\frac{14\pi}{7}\right) = z_1$

все корни $k = 1, \ldots, 7$

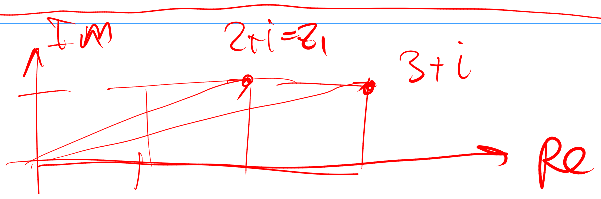
$z_k = \sqrt[7]{3}\cdot\left(\cos\left(\frac{k\cdot 2\pi}{7}\right) + i\sin\left(\frac{k\cdot 2\pi}{7}\right)\right)$

Упр



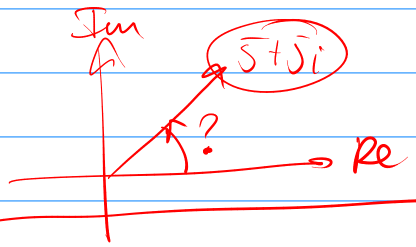
← квадратики

$\alpha + \beta\ ? = \frac{\pi}{4}$



$$\boxed{\frac{\pi}{4}} = \operatorname{Arg}(5+5i) = \operatorname{Arg}(z_1) + \operatorname{Arg}(z_2)$$

$$\overbrace{(2+i)}^{z_1} \cdot \overbrace{(3+i)}^{z_2} = 6 + 2i + 3i + \underbrace{i^2}_{-1} = 5+5i$$



$$\operatorname{Arg}(5+5i) = 45^\circ$$

$$\frac{\pi}{4} = \alpha + \beta$$