0.1 MFE, retake

Part A.

1. Give an example of
   (a) a function $f(x, y)$ that has a non-zero gradient in all the points except the point $(0, 5)$ and zero gradient in the point $(0, 5)$
   (b) a function $g(x, y)$ that has no gradient on the line $x = 3y$ and a non-zero gradient when $x \neq 3y$
2. The population of a certain country grows exponentially, $N_t = N_{1990} \cdot \exp(r(t - 1990))$. The population was 70 million in 1990 and 80 million in 2000, what will be the population in 2014?
3. Find and classify the extrema of the function $f(x, y) = x^2 - y^2$ subject to $x^2 + y^2 = 1$.
4. Given the system
$$\begin{cases} xe^{u+v} + 2uv = 1 \\ ye^{u-v} - \frac{u}{1+v} = 2x \end{cases}$$
find du and dv at $x_0 = 1$, $y_0 = 2$, $u_0 = 0$, $v_0 = 0$.
5. Consider the objective function $f(x, y) = 4kx^3 + k^2xy + 3ky^4 - 13x - 13y$. The point $(x, y) = (1, 1)$ is the maximum of the function. Find the value of k
6. In the macroeconomic linear IS-LM model for the closed economy $Y = \bar{C} + m(Y - T) + G + \bar{I} - ar$ and $\bar{L} + bY - cr = M_s$, where M_s is money supply, r — interest rate, G — government expenditures, T — lump sum tax and the constant parameters $\bar{C} > 0$, $0 < m < 1$, $\bar{I} > 0$, $a > 0$, $\bar{L} > 0$, $b > 0$, $c > 0$. Find the formulas for dr/dT, dY/dT. Assume that government expenditures and money supply are fixed exogenous variables.

Part B.

7. The production function of a firm is given by $y = \sqrt{x_1} + \sqrt{x_2}$, where x_1 and x_2 are the factors of production. Given the factor prices $w_1 = 5w$, $w_2 = w > 0$ find the total costs function of the firm.
8. A consumer splits her time $\bar{L}$ hours a week between labor and leisure. Her utility function is represented by $u(c, l) = c^{\alpha}l^{1-\alpha}$, where c is the amount of consumption and l is leisure in hours and $0 < \alpha < 1$. The weekly budget constraint is written as $pc + wl = w\bar{L}$, where p is the price of consumption, w is the hourly wage rate.
   (a) Find the consumer's optimal bundle (c^*, l^*). Justify your answer by checking second-order conditions or otherwise.
   (b) Let $\alpha = 1/4$, $\bar{L} = 168$, $p = 10$, $w = 5$. Using Envelope Theorem estimate the change in the maximum value of her utility if the wage rate has decreased by 0.5.