Assignment 13 (Due on the week December 14 – 19)

1. Let $f(x, y) = -x^2 - y^2$, and we seek to maximize that function subject to constraint $(x-1)^3 = y^2$. Solve that problem with and without the additional Lagrange multiplier λ_0.

2. Find the critical points in the problem of constrained optimization and classify them using the second-order conditions: $f(x, y, z) = xyz \to extr$, subject to $x^2 + y^2 + z^2 = 1$, $x + y + z = 0$.

3. The weekly production of a factory depends on the amounts of capital and labor it employs by the formula $q(k, l) = \sqrt{kl}$. The cost of capital is \$4 per unit and the cost of labor is \$1. Find the minimum weekly cost of producing $q = 200$. How the cost of production changes if the factory has to produce $q = 202$?

4. A firm's inventory $I(t)$ is depleted at a constant rate per unit time, i.e. $I(t) = x - \delta t$, where x is an amount of good reordered by the firm whenever the level of inventory is zero. The order is fulfilled immediately. The annual requirement for the commodity is A and the firm orders the commodity n times a year where $A = nx$. The firm incurs two types of inventory costs: a holding cost and an ordering cost. Since the average stock of inventory is $x/2$ the holding cost equals $C_h x/2$, the cost of placing one order is C_0 and with n orders a year this cost equals $C_0 n$.

   (a) Minimize the cost of inventory $C = C_h x/2 + C_0 n$ by choice of x and n subject to the constraint $A = nx$ by the Lagrange multiplier method.

   (b) Using the envelope theorem interpret the Lagrange multiplier.

5. Use the Lagrange multipliers to find the dimensions of a rectangular box with the least possible surface area among those with a volume of 27 m^3. Check the second-order conditions. Evaluate the change in the minimal surface area if the volume drops by 0.5 m^3. Compare your estimate with the direct computation.