**Variant 1**. Mock. 25 March 2015. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

**SECTION A**

1. Find all the complex roots of the equation $(z+i)^3 = 1+i$.
2. Solve the differential equation $-2x^2y' = x^2 + y^2$ with initial condition $y(e) = e$.
3. Consider the equation $y^3 + xy + 3x^2 + 2x^3 = 7$.
   (a) Does this equation define the implicit function $y(x)$ at a point $(x = 1, y = 1)$?
   (b) If the function $y(x)$ is defined find its second order Taylor expansion
4. The function $f(x, y)$ for positive $x$ and $y$ is defined as
$$f(x,y) = x^{42}y^a + x^{b+1}\sqrt{y+x} + \frac{1}{y^a x^b}$$
   (a) Find the values of $a$ and $b$ such that $f$ is homogeneous
   (b) For the values of $a$ and $b$ you have found find the degree of homegeneity of $x\frac{\partial^2 f}{\partial y^2} + y\frac{\partial^2 f}{\partial x^2}$
5. Consider two vectors, $\vec{x} = (1, 0, -1)$ and $\vec{y} = (1, 1, -2)$. Find a vector $\vec{z}$ with maximal length (called *first principal component*) such that $\vec{z}$ is a linear combination of $\vec{x}$ and $\vec{y}$, i.e. $\vec{z} = a\vec{x} + b\vec{y}$ with weights satisfying the condition $a^2 + b^2 = 1$.
6. The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$.
   (a) Find explicit formula for $F_n$
   (b) Find the "golden ratio", $\phi = \lim_{n\to\infty} F_{n+1}/F_n$
   (c) Is it true that $F_n$ is the closest integer to $\phi^n/\sqrt{5}$?

**SECTION B**

7. It is known that functions 1, $x$ and $x^2$ are particular solutions of the second-order linear differential equation $a(x)y'' + b(x)y' + y = 1$, where $a(x)$ and $b(x)$ are continuous functions.
   (a) Find the general solution of this equation
   (b) Find $a(x)$ and $b(x)$
8. Let $f(x)$ be a concave function defined on $[0; \infty)$ and $f(0) = 0$. Is it true that for $k \geq 1$ the following inequality holds: $kf(x) \geq f(kx)$?

**Variant 2**. Mock. 25 March 2015. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

**SECTION A**

1. Find all the complex roots of the equation $(z-i)^3 = 1+i$.

2. Solve the differential equation $-2x^2y' = x^2 + y^2$ with initial condition $y(e) = e$.

3. Consider the equation $y^3 + xy + 3x^2 + 3x^3 = 8$.
   (a) Does this equation define the implicit function $y(x)$ at a point $(x = 1, y = 1)$?
   (b) If the function $y(x)$ is defined find its second order Taylor expansion

4. The function $f(x, y)$ for positive $x$ and $y$ is defined as
$$f(x,y) = x^{42}y^a + x^{b+2}\sqrt{y+x} + \frac{1}{y^a x^b}$$
   (a) Find the values of $a$ and $b$ such that $f$ is homogeneous
   (b) For the values of $a$ and $b$ you have found find the degree of homegeneity of $x\frac{\partial^2 f}{\partial y^2} + y\frac{\partial^2 f}{\partial x^2}$

5. Consider two vectors, $\vec{x} = (-1, 0, 1)$ and $\vec{y} = (1, 1, -2)$. Find a vector $\vec{z}$ with maximal length (called *first principal component*) such that $\vec{z}$ is a linear combination of $\vec{x}$ and $\vec{y}$, i.e. $\vec{z} = a\vec{x} + b\vec{y}$ with weights satisfying the condition $a^2 + b^2 = 1$.

6. The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$.
   (a) Find explicit formula for $F_n$
   (b) Find the "golden ratio", $\phi = \lim_{n\to\infty} F_{n+1}/F_n$
   (c) Is it true that $F_n$ is the closest integer to $\phi^n/\sqrt{5}$?

**SECTION B**

7. It is known that functions 1, $x$ and $x^2$ are particular solutions of the second-order linear differential equation $a(x)y'' + b(x)y' + y = 1$, where $a(x)$ and $b(x)$ are continuous functions.
   (a) Find the general solution of this equation
   (b) Find $a(x)$ and $b(x)$

8. Let $f(x)$ be a concave function defined on $[0; \infty)$ and $f(0) = 0$. Is it true that for $k \geq 1$ the following inequality holds: $kf(x) \geq f(kx)$?

**Variant 3**. Mock. 25 March 2015. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

**SECTION A**

1. Find all the complex roots of the equation $(z+i)^3 = 1 - i$.
2. Solve the differential equation $-2x^2y' = x^2 + y^2$ with initial condition $y(e) = e$.
3. Consider the equation $y^3 + xy + 3x^2 + 4x^3 = 9$.
   (a) Does this equation define the implicit function $y(x)$ at a point $(x = 1, y = 1)$?
   (b) If the function $y(x)$ is defined find its second order Taylor expansion
4. The function $f(x, y)$ for positive $x$ and $y$ is defined as
$$f(x,y) = x^{42}y^a + x^{b+3}\sqrt{y+x} + \frac{1}{y^a x^b}$$
   (a) Find the values of $a$ and $b$ such that $f$ is homogeneous
   (b) For the values of $a$ and $b$ you have found find the degree of homegeneity of $x\frac{\partial^2 f}{\partial y^2} + y\frac{\partial^2 f}{\partial x^2}$
5. Consider two vectors, $\vec{x} = (1, 0, -1)$ and $\vec{y} = (-1, -1, 2)$. Find a vector $\vec{z}$ with maximal length (called *first principal component*) such that $\vec{z}$ is a linear combination of $\vec{x}$ and $\vec{y}$, i.e. $\vec{z} = a\vec{x} + b\vec{y}$ with weights satisfying the condition $a^2 + b^2 = 1$.
6. The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$.
   (a) Find explicit formula for $F_n$
   (b) Find the "golden ratio", $\phi = \lim_{n\to\infty} F_{n+1}/F_n$
   (c) Is it true that $F_n$ is the closest integer to $\phi^n/\sqrt{5}$?

**SECTION B**

7. It is known that functions 1, $x$ and $x^2$ are particular solutions of the second-order linear differential equation $a(x)y'' + b(x)y' + y = 1$, where $a(x)$ and $b(x)$ are continuous functions.
   (a) Find the general solution of this equation
   (b) Find $a(x)$ and $b(x)$
8. Let $f(x)$ be a concave function defined on $[0; \infty)$ and $f(0) = 0$. Is it true that for $k \geq 1$ the following inequality holds: $kf(x) \geq f(kx)$?

**Variant 4**. Mock. 25 March 2015. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

**SECTION A**

1. Find all the complex roots of the equation $(z-i)^3 = 1-i$.

2. Solve the differential equation $-2x^2y' = x^2 + y^2$ with initial condition $y(e) = e$.

3. Consider the equation $y^3 + xy + 3x^2 + 5x^3 = 10$.

   (a) Does this equation define the implicit function $y(x)$ at a point $(x = 1, y = 1)$?

   (b) If the function $y(x)$ is defined find its second order Taylor expansion

4. The function $f(x, y)$ for positive $x$ and $y$ is defined as

$$f(x,y) = x^{42}y^a + x^{b+4}\sqrt{y+x} + \frac{1}{y^a x^b}$$

   (a) Find the values of $a$ and $b$ such that $f$ is homogeneous

   (b) For the values of $a$ and $b$ you have found find the degree of homegeneity of $x\frac{\partial^2 f}{\partial y^2} + y\frac{\partial^2 f}{\partial x^2}$

5. Consider two vectors, $\vec{x} = (-1, 0, 1)$ and $\vec{y} = (-1, -1, 2)$. Find a vector $\vec{z}$ with maximal length (called *first principal component*) such that $\vec{z}$ is a linear combination of $\vec{x}$ and $\vec{y}$, i.e. $\vec{z} = a\vec{x} + b\vec{y}$ with weights satisfying the condition $a^2 + b^2 = 1$.

6. The Fibonacci sequence is defined as $F_n = F_{n-1} + F_{n-2}$ with initial conditions $F_0 = 0$ and $F_1 = 1$.

   (a) Find explicit formula for $F_n$

   (b) Find the "golden ratio", $\phi = \lim_{n\to\infty} F_{n+1}/F_n$

   (c) Is it true that $F_n$ is the closest integer to $\phi^n/\sqrt{5}$?

**SECTION B**

7. It is known that functions 1, $x$ and $x^2$ are particular solutions of the second-order linear differential equation $a(x)y'' + b(x)y' + y = 1$, where $a(x)$ and $b(x)$ are continuous functions.

   (a) Find the general solution of this equation

   (b) Find $a(x)$ and $b(x)$

8. Let $f(x)$ be a concave function defined on $[0; \infty)$ and $f(0) = 0$. Is it true that for $k \geq 1$ the following inequality holds: $kf(x) \geq f(kx)$?