

1. Give an example of
 - (a) a function $f(x, y)$ that has a non-zero gradient in all the points except the point $(0, 5)$ and zero gradient in the point $(0, 5)$
 - (b) a function $g(x, y)$ that has no gradient on the line $x = 3y$ and a non-zero gradient when $x \neq 3y$
2. The population of a certain country grows exponentially, $N_t = N_{1990} \cdot \exp(r(t - 1990))$. The population was 70 million in 1990 and 80 million in 2000, what will be the population in 2014?
3. Find and classify the extrema of the function $f(x, y) = x^2 - y^2$ subject to $x^2 + y^2 = 1$.
4. Given the system
$$\begin{cases} xe^{u+v} + 2uv = 1 \\ ye^{u-v} - \frac{u}{1+v} = 2x \end{cases}$$
find du and dv at $x_0 = 1, y_0 = 2, u_0 = 0, v_0 = 0$.
5. Consider the objective function $f(x, y) = 4kx^3 + k^2xy + 3ky^4 - 13x - 13y$. The point $(x, y) = (1, 1)$ is the maximum of the function. Find the value of k
6. In the macroeconomic linear IS-LM model for the closed economy $Y = \bar{C} + m(Y - T) + G + \bar{I} - ar$ and $\bar{L} + bY - cr = M_s$, where M_s is money supply, r — interest rate, G — government expenditures, T — lump sum tax and the constant parameters $\bar{C} > 0$, $0 < m < 1$, $\bar{I} > 0$, $a > 0$, $\bar{L} > 0$, $b > 0$, $c > 0$. Find the formulas for dr/dT , dY/dT . Assume that government expenditures and money supply are fixed exogenous variables.