Variant 1. 2016-12-27. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \sin(e^{2x} - e^{3y})$ at a point $x = 0$, $y = 0$.

2. Find the limit
$$\lim_{x\to 0, y\to 0} \frac{x^2y^2}{x^2 + 3y^8}$$

3. The function f is defined by $f(x, y) = x^3 + 5xy^2$. Consider the graph G of the function f
   (a) Find a vector that is orthogonal to the surface of G at the $x = 1$, $y = 1$.
   (b) Find a vector that is parallel to the surface of G at the $x = 1$, $y = 1$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+9z$ subject to $x^2 + y^2 + z^2 = 1$.

5. Consider the sets $B_n = (-1/n, (n + 1)/n)$, and the set $A = \cap_{n=1}^{\infty} B_n$.
   (a) Is the set A bounded? Open? Closed? Compact? Convex?
   (b) Sketch the set $A \times A$.

6. Find the local maxima of the function $f(x, y) = (12 - x)x \sin y + x^2 \sin y \cos y$. Check whether these local maxima are the global ones.

SECTION B

7. Let $u(c_t)$ be utility function of consumption c_t at time t which is discrete, $t \in \mathbb{N}$, ($\mathbb{N}$ — set of natural numbers). Function u is continuously differentiable and strictly concave for $c > 0$, $u(0) = 0$, $u'(c) > 0$, $\lim_{c\to 0+} u'(c) = +\infty$.
   (a) (5 points) Consider maximization problem: $\sum_{t=1}^{T} u(c_t) \to \max$ subject to $\sum_{t=1}^{T} c_t = s$, $c_t \geq 0$, where the parameter s is positive. Let $T = 2$. Show that if (c_1^*, c_2^*) is the optimal bundle then $c_1^* = c_2^*$.
   (b) (7 points) Generalize this result for any natural T. You may refer to the Lagrange method.
   (c) (8 points) Let $(c_1^*, c_2^*, c_3^*, \ldots, c_T^*)$ be the optimal bundle. Find the limit of $\sum_{t=1}^{T} u(c_t^*)$ as $T \to \infty$ or show that it does not exist.

8. In the method of least squares the straight line $a + bx$ is fit to the data $\{(x_i, y_i),\ i \in 1, 2, \ldots, n\}$, by minimizing the sum $S = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$ with respect to a and b.
   (a) (15 points) Using first-order conditions find optimal a and b. Under what conditions does the solution for a and b exist?

       Hint: you may find Cauchy-Schwartz inequality useful here.
   (b) (5 points) Show that the sufficient conditions are met.

Variant 2. 2016-12-27. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \sin(e^{3x} - e^{3y})$ at a point $x = 0$, $y = 0$.

2. Find the limit
$$\lim_{x\to 0, y\to 0} \frac{x^2y^2}{x^2 + 5y^8}$$

3. The function f is defined by $f(x, y) = x^3 + 2xy^2$. Consider the graph G of the function f
   (a) Find a vector that is orthogonal to the surface of G at the $x = 1$, $y = 1$.
   (b) Find a vector that is parallel to the surface of G at the $x = 1$, $y = 1$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+3z$ subject to $x^2 + y^2 + z^2 = 1$.

5. Consider the sets $B_n = (-2/n, (n + 2)/n)$, and the set $A = \cap_{n=1}^{\infty} B_n$.
   (a) Is the set A bounded? Open? Closed? Compact? Convex?
   (b) Sketch the set $A \times A$.

6. Find the local maxima of the function $f(x, y) = (12 - x)x \sin y + x^2 \sin y \cos y$. Check whether these local maxima are the global ones.

SECTION B

7. Let $u(c_t)$ be utility function of consumption c_t at time t which is discrete, $t \in \mathbb{N}$, ($\mathbb{N}$ — set of natural numbers). Function u is continuously differentiable and strictly concave for $c > 0$, $u(0) = 0$, $u'(c) > 0$, $\lim_{c\to 0+} u'(c) = +\infty$.
   (a) (5 points) Consider maximization problem: $\sum_{t=1}^{T} u(c_t) \to \max$ subject to $\sum_{t=1}^{T} c_t = s$, $c_t \geq 0$, where the parameter s is positive. Let $T = 2$. Show that if (c_1^*, c_2^*) is the optimal bundle then $c_1^* = c_2^*$.
   (b) (7 points) Generalize this result for any natural T. You may refer to the Lagrange method.
   (c) (8 points) Let $(c_1^*, c_2^*, c_3^*, \ldots, c_T^*)$ be the optimal bundle. Find the limit of $\sum_{t=1}^{T} u(c_t^*)$ as $T \to \infty$ or show that it does not exist.

8. In the method of least squares the straight line $a + bx$ is fit to the data $\{(x_i, y_i),\ i \in 1, 2, \ldots, n\}$, by minimizing the sum $S = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$ with respect to a and b.
   (a) (15 points) Using first-order conditions find optimal a and b. Under what conditions does the solution for a and b exist?

       Hint: you may find Cauchy-Schwartz inequality useful here.
   (b) (5 points) Show that the sufficient conditions are met.

Variant 3. 2016-12-27. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \sin(e^{4x} - e^{3y})$ at a point $x = 0$, $y = 0$.

2. Find the limit
$$\lim_{x\to 0, y\to 0} \frac{x^2y^2}{x^2 + 8y^8}$$

3. The function f is defined by $f(x, y) = x^3 + 3xy^2$. Consider the graph G of the function f
   (a) Find a vector that is orthogonal to the surface of G at the $x = 1$, $y = 1$.
   (b) Find a vector that is parallel to the surface of G at the $x = 1$, $y = 1$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+4z$ subject to $x^2 + y^2 + z^2 = 1$.

5. Consider the sets $B_n = (-3/n, (n + 3)/n)$, and the set $A = \cap_{n=1}^{\infty} B_n$.
   (a) Is the set A bounded? Open? Closed? Compact? Convex?
   (b) Sketch the set $A \times A$.

6. Find the local maxima of the function $f(x, y) = (12 - x)x \sin y + x^2 \sin y \cos y$. Check whether these local maxima are the global ones.

SECTION B

7. Let $u(c_t)$ be utility function of consumption c_t at time t which is discrete, $t \in \mathbb{N}$, ($\mathbb{N}$ — set of natural numbers). Function u is continuously differentiable and strictly concave for $c > 0$, $u(0) = 0$, $u'(c) > 0$, $\lim_{c\to 0+} u'(c) = +\infty$.
   (a) (5 points) Consider maximization problem: $\sum_{t=1}^{T} u(c_t) \to \max$ subject to $\sum_{t=1}^{T} c_t = s$, $c_t \geq 0$, where the parameter s is positive. Let $T = 2$. Show that if (c_1^*, c_2^*) is the optimal bundle then $c_1^* = c_2^*$.
   (b) (7 points) Generalize this result for any natural T. You may refer to the Lagrange method.
   (c) (8 points) Let $(c_1^*, c_2^*, c_3^*, \ldots, c_T^*)$ be the optimal bundle. Find the limit of $\sum_{t=1}^{T} u(c_t^*)$ as $T \to \infty$ or show that it does not exist.

8. In the method of least squares the straight line $a + bx$ is fit to the data $\{(x_i, y_i),\ i \in 1, 2, \ldots, n\}$, by minimizing the sum $S = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$ with respect to a and b.
   (a) (15 points) Using first-order conditions find optimal a and b. Under what conditions does the solution for a and b exist?
   Hint: you may find Cauchy-Schwartz inequality useful here.
   (b) (5 points) Show that the sufficient conditions are met.

Variant 4. 2016-12-27. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \sin(e^{5x} - e^{3y})$ at a point $x = 0$, $y = 0$.

2. Find the limit
$$\lim_{x \to 0, y \to 0} \frac{x^2 y^2}{x^2 + 6y^8}$$

3. The function f is defined by $f(x, y) = x^3 + 4xy^2$. Consider the graph G of the function f
   (a) Find a vector that is orthogonal to the surface of G at the $x = 1$, $y = 1$.
   (b) Find a vector that is parallel to the surface of G at the $x = 1$, $y = 1$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+5z$ subject to $x^2 + y^2 + z^2 = 1$.

5. Consider the sets $B_n = (-4/n, (n + 4)/n)$, and the set $A = \cap_{n=1}^{\infty} B_n$.
   (a) Is the set A bounded? Open? Closed? Compact? Convex?
   (b) Sketch the set $A \times A$.

6. Find the local maxima of the function $f(x, y) = (12 - x)x \sin y + x^2 \sin y \cos y$. Check whether these local maxima are the global ones.

SECTION B

7. Let $u(c_t)$ be utility function of consumption c_t at time t which is discrete, $t \in \mathbb{N}$, ($\mathbb{N}$ — set of natural numbers). Function u is continuously differentiable and strictly concave for $c > 0$, $u(0) = 0$, $u'(c) > 0$, $\lim_{c \to 0+} u'(c) = +\infty$.
   (a) (5 points) Consider maximization problem: $\sum_{t=1}^{T} u(c_t) \to \max$ subject to $\sum_{t=1}^{T} c_t = s$, $c_t \geq 0$, where the parameter s is positive. Let $T = 2$. Show that if (c_1^*, c_2^*) is the optimal bundle then $c_1^* = c_2^*$.
   (b) (7 points) Generalize this result for any natural T. You may refer to the Lagrange method.
   (c) (8 points) Let $(c_1^*, c_2^*, c_3^*, \ldots, c_T^*)$ be the optimal bundle. Find the limit of $\sum_{t=1}^{T} u(c_t^*)$ as $T \to \infty$ or show that it does not exist.

8. In the method of least squares the straight line $a + bx$ is fit to the data $\{(x_i, y_i),\ i \in 1, 2, \ldots, n\}$, by minimizing the sum $S = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$ with respect to a and b.
   (a) (15 points) Using first-order conditions find optimal a and b. Under what conditions does the solution for a and b exist?
   Hint: you may find Cauchy-Schwartz inequality useful here.
   (b) (5 points) Show that the sufficient conditions are met.