# MOR. Retake — 07.09.2015

1. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -x_1 - 2x_2 + 3x_3$ subject to constraints: $2(x_1 + 1)^2 + x_2^2 + 3(x_3 - 1)^2 \leq 5$ and $x_1, x_2, x_3 \geq 0$. Find the point(s) of maximum and maximum value of $f$. Justify your answer by reference to Weierstrass theorem if it is relevant or otherwise. Carefully state any theorem you use.

2. For all real values of parameter $\beta$ that lies within the range $-1 < \beta < 0$ maximize linear function $2x_1 - x_2 + 8x_3 - 19$ subject to constraints $x_1 \geq x_2 + x_3 + \beta$, $2x_1 + x_2 + 4x_3 \leq \beta + 1$ and $x_1, x_2, x_3 \geq 0$. You are not asked to find the maximizer.

3. Find the general solution of the differential equation $y'' + 6y' + 9y = xe^{-2x} + \cos(x)$

4. Consider the system of difference equations

$$\begin{cases} x_{t+1} = 2x_t - 4y_t \\ y_{t+1} = x_t - 3y_t + 3 \end{cases}$$

   (a) Solve the system

   (b) Find the equilibrium solution and check whether it's stable

5. Find all pure and mixed Nash equilibria in the following bimatrix game:

| | d | e | f |
|---|---|---|---|
| a | 4;5 | 1;4 | 1;1 |
| b | 2;8 | 5;0 | 0;4 |
| c | 0;3 | 2;2 | 5;7 |

6. There is an auction of a painting with two players. The value of the painting for the first player is a random variable $v_1$, for the second player — $v_2$. The random variables $v_1$ and $v_2$ are independent and identically distributed from 0 to 1 million dollars with density function $f(t) = 2t$. Each player makes the bid $b_i$ knowing only his own value of the painting. The player who makes the highest bid gets the painting and pays his bid.

   Find a Nash equilibrium where each player uses linear strategy of the form $b_i = k \cdot v_i$.