Variant 1. 2017-01-20. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \cos(e^{2x} - 1) - \cos(e^{3y} - 1)$ at a point $x = 0$, $y = 0$.

2. Find the limit or prove that it does not exist

$$\lim_{x \to 0, y \to 0} \frac{x^2 y^2}{x^4 + 3y^4}$$

3. Consider the sphere given by $x^2 + y^2 + z^2 = 1$. Find the equation of the tangent plane to the sphere at the point $x = 1/\sqrt{3}$, $y = 1/\sqrt{3}$, $z = -1/\sqrt{3}$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+9z$ subject to $x^2 + y^2 + 4z^2 = 1$.

5. Consider the set A on the plane (x, y) given by the inequality

$$\frac{(x^2 + y^2 - 3)(x^2 + y^2 - 10)}{x^2 + y^2 - 10} \geq 0$$

   (a) Is the set A closed? open? bounded? convex? compact?

   (b) If possible represent the set A in the form $A = B_1 \times B_2$ where each set $B_i \subset \mathbb{R}$.

6. Find and classify the critical poinst of the function $f(x, y) = \exp(-x^2 - 6y^2 + 2xy + 2y)$. Check whether these local extrema are the global ones.

SECTION B

7. Short-run total costs of a firm are given by

$$STC(q, K) = q^2 + 3qK + 4K^2 - K + \frac{1}{16},$$

   where q is the output and K is the amount of capital fixed in the short-run. In the long-run the firm can always adjust the capital in order to minimize costs. Use the appropriate envelope theorem to find $MC = (TC)'$ long-run marginal costs.

8. Solve the constrained minimization problem in two variables: $x^2 + y^2 \to \min$ subject to constraint $(x - 1)^3 = y^2$. Check firstly whether the method of Lagrange multipliers is valid to apply.

Variant 2. 2017-01-20. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x, y) = \cos(e^{2x} - 1) - \cos(e^{3y} - 1)$ at a point $x = 0$, $y = 0$.

2. Find the limit or prove that it does not exist

$$\lim_{x \to 0, y \to 0} \frac{x^2 y^2}{x^4 + 5y^4}$$

3. Consider the sphere given by $x^2 + y^2 + z^2 = 1$. Find the equation of the tangent plane to the sphere at the point $x = 1/\sqrt{3}$, $y = -1/\sqrt{3}$, $z = -1/\sqrt{3}$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 2x+3y+3z$ subject to $x^2 + y^2 + 4z^2 = 1$.

5. Consider the set A on the plane (x, y) given by the inequality

$$\frac{(x^2 + y^2 - 5)(x^2 + y^2 - 10)}{x^2 + y^2 - 10} \geq 0$$

   (a) Is the set A closed? open? bounded? convex? compact?

   (b) If possible represent the set A in the form $A = B_1 \times B_2$ where each set $B_i \subset \mathbb{R}$.

6. Find and classify the critical poinst of the function $f(x, y) = \exp(-x^2 - 4y^2 + 2xy + 2y)$. Check whether these local extrema are the global ones.

SECTION B

7. Short-run total costs of a firm are given by

$$STC(q, K) = q^2 + 3qK + 4K^2 - K + \frac{1}{16},$$

   where q is the output and K is the amount of capital fixed in the short-run. In the long-run the firm can always adjust the capital in order to minimize costs. Use the appropriate envelope theorem to find $MC = (TC)'$ long-run marginal costs.

8. Solve the constrained minimization problem in two variables: $x^2 + y^2 \to \min$ subject to constraint $(x - 1)^3 = y^2$. Check firstly whether the method of Lagrange multipliers is valid to apply.