This is a practice exam. Be honest with yourself. Close all the books and notes. Set a timer for 120 minutes. Try to solve proposed problems. We will post the solution in a couple of days.

Good luck! We believe in you!

Section A

1. (10 points) Let $z \in \mathbb{C}$. Sketch on the complex plane the set given by the equation
$$z\bar{z} + (1+i)z + (1-i)\bar{z} = 1$$

2. (10 points) The function $f(x, y, z)$ satisfies the equation
$$\frac{2f'_x}{yz} + \frac{2f'_y}{xz} + \frac{2f'_z}{xy} = \frac{f}{xyz}$$
What is the degree of homogeneity of the function $g(x, y, z) = f'_x + f'_y + f'_z$?

3. (10 points) Find the minimal value of $x_1 + 2x_2 + 3x_3 + x_4$ for non-negative values of all x_i given that $x_1 + 2x_2 + 2x_3 + x_4 \geq 3$ and $3x_1 + 2x_2 + x_3 - x_4 \geq 4$.

4. (10 points) Consider the static bi-matrix game

	d	e	f
a	1, 1	2, -1	4, 3
b	2, 1	1, 6	-1, 2
c	5, 6	2, 5	0, 2

   (a) Iteratively eliminate all strictly dominated strategies.

   (b) Find all Nash Equilibria in pure and mixed strategies.

5. (10 points) Find the global maximum of $5x + 2y + 3z$ given that $x^2 + y^2 + z^2 = 6$.

6. (10 points) Let's condsider the modified sequence of Fibonacci numbers. The recurrent equation stays the same, $F_n = F_{n-1} + F_{n-2}$ but we start with $F_0 = -1$ and $F_1 = 4$. Find the explicit formula of F_n for $n \geq 1$.

Section B

7. (20 points) Find the global minimum of the function
$$2x^2 + 4y^2 + xy + 8z^2 + 2yz,$$
subject to $x + y + z \geq 1$, $x + y + 1.5z \geq 1.2$.

8. (20 points)

   Solve the following differential equation
$$y'''' + 7y'' - 18y' + 10y = x + e^x$$