Assignment 23 (20 April)

1. Solve the following differential-equation system and analyse the time path:

$$\begin{cases} x'(t) - x(t) - 12y(t) = -60 \\ y'(t) + x(t) + 6y(t) = 36 \end{cases}$$

   with $x(0) = 13$, $y(0) = 4$.

2. Solve the following difference-equation system and analyse the time path

$$\begin{cases} x_{t+1} + x_t + 2y_t = 24 \\ y_{t+1} + 2x_t - 2y_t = 9 \end{cases}$$

   with $x_0 = 10$, $y_0 = 9$.

3. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 4 & -1 & -1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

   and find the eigenvectors corresponding to each eigenvalue. Hence find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

   (a) Use your result to find the sequences x_t, y_t, z_t, such that $x_0 = 1$, $y_0 = 3$, $z_0 = 2$ and for $t > 0$,

$$\begin{cases} x_{t+1} = 4x_t - y_t - z_t \\ y_{t+1} = x_t + 2y_t - z_t \\ z_{t+1} = x_t - y_t + 2z_t \end{cases}$$

   (b) Use your result to find the functions $x(t)$, $y(t)$, $z(t)$ such that $x(0) = 1$, $y(0) = 3$, $z(0) = 2$ and

$$\begin{cases} x' = 4x - y - z \\ y' = x + 2y - z \\ z' = x - y + 2z \end{cases}$$

4. Given the pay-off bi-matrix

	e	f	g	h
a	1,1	5,3	3,2	3,4
b	1,0	1,2	2,5	2,6
c	3,3	3,5	3,4	4,4
d	2,2	0,3	2,2	2,2

   Find all Nash equilibria in pure and mixed strategies. Check whether they are Pareto-optimal.

5. Given the pay-off matrix of a zero-sum game

	c	d	e
a	-1	-3	0
b	-4	2	-5

   Find all Nash equilibria in pure and mixed strategies. Check whether they are Pareto-optimal.

Assignment 24 (27 April)

1. A policymaker desires to double in 10 periods of time the value of GDP Y_t produced in period t. Evolution of GDP over time is given by equation $2Y_{t+2} - 3Y_{t+1} + Y_t = 2^t + t$. For which values of Y_0 and Y_1 the GDP will eventually double?

2. Solve the system of ODE
$$\begin{cases} \dot{x} = 3x - 2y \\ \dot{y} = 2x - y \end{cases}$$

3. Let production function $F(K, L)$ be twice continuous differentiable and homogeneous of the first degree. Show that its Hessian matrix has a zero determinant.

4. Two candidates, A and B, compete in an election. Of the 100 citizens, k support candidate A and $m = 100-k$ support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs $2 - c$, $1 - c$, and $-c$ in these three cases, where $0 < c < 1$.

   (a) For $k = 50$, find the set of Nash equilibria in pure strategies. (Is the action profile in which everyone votes a Nash equilibrium? Is there any Nash equilibrium in which the candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?)

   (b) What is the set of Nash equilibria in pure strategies for $k < 50$?

5. General A is defending territory accessible by two mountain passes against an attack by general B. General A has three divisions at her disposal, and general B has two divisions. Each general allocates her divisions between the two passes. General A wins the battle at a pass if and only if she assigns at least as many divisions to the pass as does general B; she successfully defends her territory if and only if she wins the battle at both passes. Find all the mixed strategy equilibria.

Additional Computer Home Assignment (27 April)

You may use any open source software, R is recommended but not mandatory. Please provide not only the answers but also the code.

1. Find all the eigenvalues and eigenvectors of the following matrix

$$\begin{pmatrix} 5 & 2 & -1 \\ 2 & 3 & 6 \\ -1 & 6 & -2 \end{pmatrix}$$

2. Draw the solution of the second order equation

$$y'' + (x+1) \cdot y' + y = \arctan x$$

with initial conditions $y(0) = 0$, $y'(0) = 1$.

3. Find numerically the global minimum of the function $f(x, y) = x^4 + y^8 + 2xy - 4x + xy^2$.

4. Find at least one Nash Equilibrium of the following zero-sum game:

	e	f	g
a	2	-3	0
b	0	3	-5
c	-4	0	2

5. Masha and Sasha play the following game. Masha writes two numbers on two small sheets of paper. The number on the first sheet is uniformly distributed on $[0; 1]$, the number on the second sheet is just the first number squared. Sasha selects one of the two sheets at random with equal probabilities. He looks at the number and may either keep it or take the other number. Sasha maximises his expected payoff. Find his optimal strategy and the maximal expected payoff.