1. (10 points) Consider the set $A = \cup_{n=1}^{\infty} \left\{ \frac{3+2n}{n+6} \right\} \subset \mathbb{R}$.

   (a) Is the set A bounded? Open? Closed? Compact?

   (b) Roughly sketch the set $A \times A$

2. (10 points) Consider the system of equations

$$\begin{cases} xyz^2 + x^3y + 4y^3z^3 - 2x = 4 \\ 2x^2yz + z^2 + 3xy + 5yx^3 = -5 \end{cases}$$

   (a) Are the function $x(z)$ and $y(z)$ defined around the point $A = (-1, 1, 1)$?

   (b) Find df/dz, where $f(z) = x(z)y(z)$

3. (10 points) Consider the functions $f(x, y) = x^2 + 2x + y^2 + 6y + 7$ and $g(x, y) = x^2 - 8x + y^2 - 10y + 9$. Find all the points where the gradients are parallel.

4. Find and classify all the local extrema of the function $g(x, y) = y^3 - 12y + x^2e^y$. Which of them are global?

5. (10 points) The level curves of the function $f(x, y)$ are given by the equation $y - x^2 = c$.

   Draw two level curves of the function $g(x, y) = f(|x| - 2, |y| + 1)$.

6. (10 points) Use the first-order differential to approximate $\sqrt[3]{4 \cdot 0.9^2 + 2.2^2}$.

7. (20 points) Let $u(c_t)$ be utility function of consumption c_t at time t which is discrete, $t \in \mathbb{N}$, ($\mathbb{N}$ − set of natural numbers). Function u is continuously differentiable and strictly concave for $c > 0$, $u(0) = 0$, $u'(c) > 0$, $\lim_{c \to 0+} u'(c) = +\infty$.

   (a) (5 points) Consider maximization problem: $\sum_{t=1}^{T} u(c_t) \to \max$ subject to $\sum_{t=1}^{T} c_t = s$, $c_t \geq 0$, where the parameter s is positive. Let $T = 2$. Show that if (c_1^*, c_2^*) is the optimal bundle then $c_1^* = c_2^*$.

   (b) (7 points) Generalize this result for any natural T. You may refer to the Lagrange method.

   (c) (8 points) Let $(c_1^*, c_2^*, c_3^*, \ldots, c_T^*)$ be the optimal bundle. Find the limit of $\sum_{t=1}^{T} u(c_t^*)$ as $T \to \infty$ or show that it does not exist.

8. (20 points) In the method of least squares the straight line $a + bx$ is fit to the data $\{(x_i, y_i), \ \ i \in 1, 2, \ldots, n\}$, by minimizing the sum $S = \sum_{i=1}^{n} (y_i - (a + bx_i))^2$ with respect to a and b.

   (a) (15 points) Using first-order conditions find optimal a and b. Under what conditions does the solution for a and b exist?

   (b) (5 points) Show that the sufficient conditions are met.