Name, group no:

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1. (10 points) Consider the function $f(x, y) = x^3 + 3ay^3 + 2axy$ where a is a parameter. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:

. . . .

2. (10 points) Consider the system

$$\begin{cases} 3x^3 + u(y) + u(z) = 5 \\ x + x^3 + 2u(y^3 x) = 4 \end{cases},$$

where $u(x)$ is a differentiable function with $u(1) = 1$.

(a) Clearly state conditions sufficient to guarantee that the system defines the functions $z(y)$ and $x(y)$ at a point $(1, 1, 1)$.

(b) Find $z'(y)$ provided the conditions are met.

Name, group no:

. . . .

3. (10 points) Consider the function $f(x, y) = xy^3$ and the point $A = (-1, -2)$.

   Find the direction of the maximal rate of change of the function and this maximal rate.

Name, group no:

. . . .

4. (10 points) For $x > 0$, $y > 0$ find the limit

$$u(x,y) = \lim_{t \to 0} \left(x^{\frac{t-1}{t}} + y^{\frac{t-1}{t}} \right)^{\frac{t}{t-1}}$$

Name, group no:

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5. (10 points) Provide an explicit example of a non-convergent sequence (x_n) in $\mathbb{R}^2$ such that the sequences $y_n = \|x_n\|$ and $z_n = \|x_n + 2x_{n+1}\|$ are convergent.

Name, group no:

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6. (10 points) Let's consider the sets $A_n = \{x \in \mathbb{R} \mid x^2 n = 1\}$.

   Describe the set $A = \cup_{n=1}^{\infty} A_n$: is it closed, open, bounded, compact?

Name, group no:

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7. A curve represented by the equation $(x^2 + y^2)^2 = x^2 - y^2$ is called lemniscate.

   (a) (5 points) While solving for $y = y(x)$ implicit function defined by this equation is it possible to use IFT in the neighborhood of the point $(0, 0)$?

   (b) (5 points) Show that in the first quadrant $\{(x, y) \mid x > 0, y > 0\}$ such implicit function $y = y(x)$ exists. Justify your reasoning.

   (c) (10 points) Find $y'(0)$ if it exists.

   Hint: for c) it is convenient to change Cartesian coordinates to polar coordinates following the formulas $x = r\cos\phi$ and $y = r\sin\phi$.

Name, group no:

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8. Let u be a composite function $u = g(x^2 + y^2)$, where $g(t) \in C^2$ for $t > 0$.

   (a) (5 points) Is the formula
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   for the total differential valid? Provide a clear argument.

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   $$d^2u = g''(x^2 + y^2)(2xdx + 2ydy)^2$$
   Does this method work? Justify your answer.

   (c) (10 points) Let $g(t) = \sqrt{t}$. Prove that for the function $u(x, y) = \sqrt{x^2 + y^2}$ the second-order differential is non-negative.

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1. (10 points) Consider the function $f(x, y) = x^3 + 4ay^3 + 2axy$ where a is a parameter. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:

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2. (10 points) Consider the system

$$\begin{cases} 3x^3 + u(y) + u(z) = 5 \\ x + x^3 + 3u(y^3x) = 5 \end{cases},$$

where $u(x)$ is a differentiable function with $u(1) = 1$.

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6. (10 points) Let's consider the sets $A_n = \{x \in \mathbb{R} \mid x^2 n = 2\}$.

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$$\begin{cases} 3x^3 + u(y) + u(z) = 5 \\ x + x^3 + 4u(y^3x) = 6 \end{cases},$$

where $u(x)$ is a differentiable function with $u(1) = 1$.

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Name, group no:

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3. (10 points) Consider the function $f(x, y) = xy^3$ and the point $A = (-1, -4)$.

   Find the direction of the maximal rate of change of the function and this maximal rate.

Name, group no:

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