Name, group no:

. . . .

1. (10 points) Solve the equation

$$z^3 + iz^2 - \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) z - i\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 0$$

in complex numbers if one of the roots is $-i$.

Name, group no:

. . . .

2. (10 points) The function $f(x, y, z)$ is homogeneous of degree 9. Consider two functions, $h(x, y, z) = (x+3y+5z)\frac{\partial^2 f}{\partial x \partial y}$ and $q(x, y, z) = h(x, y, z)+(x^3+3xyz)\frac{\partial f}{\partial x}$. Check the homogeneity of these functions, for homogeneous functions state the degree.

Name, group no:

. . . .

3. (10 points) Find the minimal value of $x_1 + 2x_2 + 3x_3$ for non-negative values of all x_i given that $x_1 + 2x_2 + 2x_3 \geq 3$ and $3x_1 + 2x_2 + x_3 \geq 4$.

Name, group no:

....

4. (10 points) Bill and John are relaxing in a pub. Bill decided to spend 10 dollars, John decided to spend 2 dollars. Money is infinitely divisible. They love the same music. Each can spend money on music or on drinks. The utility of each player is $u_i = (m_1 + m_2) \cdot d_i$, where $(m_1 + m_2)$ — is the total sum of money spent on music by both players and d_i — the personal expenses on drinks.

   Find all Nash Equilibria in pure strategies.

Name, group no:

. . . .

5. (10 points) Find the global maximum of $x + 2y + 3z$ given that $x^2 + y^2 + z^2 \leq 6$.

Name, group no:

. . . .

6. (10 points) Find the general solution of the difference equation $y_{t+3} - 2y_{t+2} + y_{t+1} - 2y_t = 2^t$.

Name, group no:

. . . .

7. In exchange economy of two agents and two goods all Pareto-optimal allocations can be found by solving the maximization problem

$$\begin{cases} u_1 = 2\sqrt{x_1} + y_1 \to \max \\ u_2 = 2x_2 + y_2 \geq \bar{u}_2; \\ x_1 + x_2 = 1; \\ y_1 + y_2 = 1, \end{cases}$$

where $\bar{u}_2$ is a nonnegative parameter and all the amounts of goods x_1, x_2, y_1, y_2 are consumed by agents in nonnegative quantities.

(a) (5 points) Write the Kuhn-Tucker Lagrangian of the problem and set the system of first-order Kuhn-Tucker conditions;

(b) (5 points) Solve it in particular case when all goods are consumed in positive quantities;

(c) (10 points) Solve the system completely and find all corner solutions.

Name, group no:

. . . .

8. (20 points)

Consider the system of differential equations

$$\begin{cases} \dot{x} = x - y + \frac{\sin t + \cos t}{2 \sin t}; \\ \dot{y} = 2x - y. \end{cases}$$

(a) (4 points) Reduce the system to a single equation for $y(t)$.

(b) (16 points) By applying the variation of parameters method or otherwise find general solution for $y(t)$.

Note: you don't need to find $x(t)$.

Name, group no:

. . . .

1. (10 points) Solve the equation

$$z^3 + iz^2 - \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) z - i\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 0$$

in complex numbers if one of the roots is $-i$.

Name, group no:

. . . .

2. (10 points) The function $f(x, y, z)$ is homogeneous of degree 8. Consider two functions, $h(x, y, z) = (2x + 4y + 3z)\frac{\partial^2 f}{\partial x \partial y}$ and $q(x, y, z) = h(x, y, z) + (x^3 + 2xyz)\frac{\partial f}{\partial x}$. Check the homogeneity of these functions, for homogeneous functions state the degree.

Name, group no:

. . . .

3. (10 points) Find the minimal value of $x_1 + 2x_2 + 4x_3$ for non-negative values of all x_i given that $x_1 + 2x_2 + 2x_3 \geq 3$ and $3x_1 + 2x_2 + x_3 \geq 4$.

Name, group no:

. . . .

4. (10 points) Bill and John are relaxing in a pub. Bill decided to spend 20 dollars, John decided to spend 4 dollars. Money is infinitely divisible. They love the same music. Each can spend money on music or on drinks. The utility of each player is $u_i = (m_1 + m_2) \cdot d_i$, where $(m_1 + m_2)$ — is the total sum of money spent on music by both players and d_i — the personal expenses on drinks.

   Find all Nash Equilibria in pure strategies.

Name, group no:
....

5. (10 points) Find the global maximum of $2x + y + 3z$ given that $x^2 + y^2 + z^2 \leq 6$.

Name, group no:

. . . .

6. (10 points) Find the general solution of the difference equation $y_{t+3} + 2y_{t+2} + y_{t+1} + 2y_t = (-2)^t$.

Name, group no:

....

7. In exchange economy of two agents and two goods all Pareto-optimal allocations can be found by solving the maximization problem

$$\begin{cases} u_1 = 2\sqrt{x_1} + y_1 \to \max \\ u_2 = 2x_2 + y_2 \geq \bar{u}_2; \\ x_1 + x_2 = 1; \\ y_1 + y_2 = 1, \end{cases}$$

where $\bar{u}_2$ is a nonnegative parameter and all the amounts of goods x_1, x_2, y_1, y_2 are consumed by agents in nonnegative quantities.

(a) (5 points) Write the Kuhn-Tucker Lagrangian of the problem and set the system of first-order Kuhn-Tucker conditions;

(b) (5 points) Solve it in particular case when all goods are consumed in positive quantities;

(c) (10 points) Solve the system completely and find all corner solutions.

Name, group no:

. . . .

8. (20 points)

   Consider the system of differential equations

$$\begin{cases} \dot{x} = x - y + \frac{\sin t + \cos t}{2 \sin t}; \\ \dot{y} = 2x - y. \end{cases}$$

   (a) (4 points) Reduce the system to a single equation for $y(t)$.

   (b) (16 points) By applying the variation of parameters method or otherwise find general solution for $y(t)$.

   Note: you don't need to find $x(t)$.

Name, group no:

. . . .

1. (10 points) Solve the equation

$$z^3 - iz^2 - \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) z + i\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 0$$

in complex numbers if one of the roots is i.

Name, group no:

. . . .

2. (10 points) The function $f(x, y, z)$ is homogeneous of degree 7. Consider two functions, $h(x, y, z) = (4x + 3y + 5z)\frac{\partial^2 f}{\partial x \partial y}$ and $q(x, y, z) = h(x, y, z) + (5x^3 + 3xyz)\frac{\partial f}{\partial x}$. Check the homogeneity of these functions, for homogeneous functions state the degree.

Name, group no:

. . . .

3. (10 points) Find the minimal value of $x_1 + 2x_2 + 5x_3$ for non-negative values of all x_i given that $x_1 + 2x_2 + 2x_3 \geq 3$ and $3x_1 + 2x_2 + x_3 \geq 4$.

Name, group no:

. . . .

4. (10 points) Bill and John are relaxing in a pub. Bill decided to spend 30 dollars, John decided to spend 6 dollars. Money is infinitely divisible. They love the same music. Each can spend money on music or on drinks. The utility of each player is $u_i = (m_1 + m_2) \cdot d_i$, where $(m_1 + m_2)$ — is the total sum of money spent on music by both players and d_i — the personal expenses on drinks.

   Find all Nash Equilibria in pure strategies.

Name, group no:

. . . .

5. (10 points) Find the global maximum of $3x + 2y + z$ given that $x^2 + y^2 + z^2 \leq 6$.

Name, group no:

. . . .

6. (10 points) Find the general solution of the difference equation $2y_{t+3} + y_{t+2} + 2y_{t+1} + y_t = (-1/2)^t$.

Name, group no:

. . . .

7. In exchange economy of two agents and two goods all Pareto-optimal allocations can be found by solving the maximization problem

$$\begin{cases} u_1 = 2\sqrt{x_1} + y_1 \to \max \\ u_2 = 2x_2 + y_2 \geq \bar{u}_2; \\ x_1 + x_2 = 1; \\ y_1 + y_2 = 1, \end{cases}$$

where $\bar{u}_2$ is a nonnegative parameter and all the amounts of goods x_1, x_2, y_1, y_2 are consumed by agents in nonnegative quantities.

(a) (5 points) Write the Kuhn-Tucker Lagrangian of the problem and set the system of first-order Kuhn-Tucker conditions;

(b) (5 points) Solve it in particular case when all goods are consumed in positive quantities;

(c) (10 points) Solve the system completely and find all corner solutions.

Name, group no:

. . . .

8. (20 points)

Consider the system of differential equations

$$\begin{cases} \dot{x} = x - y + \frac{\sin t + \cos t}{2 \sin t}; \\ \dot{y} = 2x - y. \end{cases}$$

(a) (4 points) Reduce the system to a single equation for $y(t)$.

(b) (16 points) By applying the variation of parameters method or otherwise find general solution for $y(t)$.

Note: you don't need to find $x(t)$.

Name, group no:

....

1. (10 points) Solve the equation

$$z^3 - iz^2 - \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) z + i\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 0$$

in complex numbers if one of the roots is i.

Name, group no:

. . . .

2. (10 points) The function $f(x, y, z)$ is homogeneous of degree 6. Consider two functions, $h(x, y, z) = (4x - 3y + 5z)\frac{\partial^2 f}{\partial x \partial y}$ and $q(x, y, z) = h(x, y, z) + (7x^3 + 3xyz)\frac{\partial f}{\partial x}$. Check the homogeneity of these functions, for homogeneous functions state the degree.

Name, group no:

. . . .

3. (10 points) Find the minimal value of $x_1 + 2x_2 + 6x_3$ for non-negative values of all x_i given that $x_1 + 2x_2 + 2x_3 \geq 3$ and $3x_1 + 2x_2 + x_3 \geq 4$.

Name, group no:

. . . .

4. (10 points) Bill and John are relaxing in a pub. Bill decided to spend 40 dollars, John decided to spend 8 dollars. Money is infinitely divisible. They love the same music. Each can spend money on music or on drinks. The utility of each player is $u_i = (m_1 + m_2) \cdot d_i$, where $(m_1 + m_2)$ — is the total sum of money spent on music by both players and d_i — the personal expenses on drinks.

   Find all Nash Equilibria in pure strategies.

Name, group no:

. . . .

5. (10 points) Find the global maximum of $3x + y + 2z$ given that $x^2 + y^2 + z^2 \leq 6$.

Name, group no:

. . . .

6. (10 points) Find the general solution of the difference equation $2y_{t+3} - y_{t+2} + 2y_{t+1} - y_t = (1/2)^t$.

Name, group no:

. . . .

7. In exchange economy of two agents and two goods all Pareto-optimal allocations can be found by solving the maximization problem

$$\begin{cases} u_1 = 2\sqrt{x_1} + y_1 \to \text{max} \\ u_2 = 2x_2 + y_2 \geq \bar{u}_2; \\ x_1 + x_2 = 1; \\ y_1 + y_2 = 1, \end{cases}$$

where $\bar{u}_2$ is a nonnegative parameter and all the amounts of goods x_1, x_2, y_1, y_2 are consumed by agents in nonnegative quantities.

(a) (5 points) Write the Kuhn-Tucker Lagrangian of the problem and set the system of first-order Kuhn-Tucker conditions;

(b) (5 points) Solve it in particular case when all goods are consumed in positive quantities;

(c) (10 points) Solve the system completely and find all corner solutions.

Name, group no:

....

8. (20 points)

Consider the system of differential equations

$$\begin{cases} \dot{x} = x - y + \frac{\sin t + \cos t}{2\sin t}; \\ \dot{y} = 2x - y. \end{cases}$$

(a) (4 points) Reduce the system to a single equation for $y(t)$.

(b) (16 points) By applying the variation of parameters method or otherwise find general solution for $y(t)$.

Note: you don't need to find $x(t)$.