Assignment 3 (September 28 – October 3)

1. Prove that any intersection of closed sets is closed.

2. For each of the following subsets of $\mathbb{R}^2$,

   - Sketch the set.
   - Determine whether or not it is **open**, **closed** or **compact**. *Hint: a set is closed if and only if its complement is open.*
   - Give reasons for your negative answers to the previous part.

   (a) $\{(x, y)\colon x = 0, y \geqslant 0\}$,

   (b) $\{(x, y)\colon 1 \leqslant x^2 + y^2 \leqslant 2\}$,

   (c) $\{(x, y)\colon 1 \leqslant x \leqslant 2\}$,

   (d) $\{(x, y)\colon x = 0 \text{ or } y = 0, \text{ but not both}\}$.

3. Sketch level sets for each of the following functions from $\mathbb{R}^3$ to $\mathbb{R}^1$:

   (a) $f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$,

   (b) $f(x_1, x_2, x_3) = x_1^2 + x_2^2$,

   (c) $f(x_1, x_2, x_3) = x_1^2 - x_2 - x_3$,

   (d) $f(x_1, x_2, x_3) = x_1 + 2x_2 + 3x_3$.

4. Does the following limit exist: $\lim\limits_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$?

5. Find points of discontinuity of the following functions:

   (a) $u = \sin \frac{1}{xy}$,

   (b) $u = \ln \frac{1}{\sqrt{(x-a)^2+(y-b)^2+(z-c)^2}}$.