Variant 1. Section A. Problems 1 and 2 out of 8

1. At the beginning James Bond is located at the point $(1, 1)$. To choose his new location he calculates the gradient of the function $f(x, y) = x^2 + y^2 - 3xy + x$ from his current location and moves in the direction given by the gradient by its length. Where he will be after two movements?

2. Consider the system of equations

$$\begin{cases} x^4 + y^4 + z^4 = 3 \\ x + x^3 + y + 2y^3 + z + 3z^3 = 5 \end{cases}$$

   (a) Are the function $x(z)$ and $y(z)$ defined around the point $A = (-1, 1, 1)$?

   (b) Find dx/dz and dy/dz

Name, group:
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Variant 1. Section A. Problems 3 and 4 out of 8

3. Find and classify unconstrained extrema of the function $f(x, y) = x^4 + y^8 - 2xy$
4. Find and classify constrained extrema of the function $f(x, y) = xy$ subject to $x^2 + 4y^2 = 9$

Name, group:

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Variant 1. Section A. Problems 5 and 6 out of 8

5. The function u is defined by the equation $u^3(t) + u(t) = f(x, y)$, where $x = 2 - t$ and $y = 1 + 2t$ and f is in C^2. Find du/dt and d^2u/dt^2

6. Consider the function $f(x) = h(x) - ax$, where the function h is twice differentiable and $h''(x) < 0$ for all x. The global maximum of f is denoted by $x^*(a)$.

   (a) Find dx^*/da

   (b) It is known that for $a = 1$ the optimal point is $x^* = 3$ and the value of maximum is 2015. What is the approximate value of maximum for $a = 1.01$?

Name, group:

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Variant 1. Section B. Problems 7 and 8 can be solved separately.

7. A risk-averse Alex possesses w dollars of wealth in money and property. His house worth $L < w$ dollars can be completely destroyed by a landslide with the probability p, $0 < p < 1$. Let's denote his wealth if landslide occurs by x_L and x_{NL} otherwise. Then his expected utility can be calculated by $E(u) = p \ln x_L + (1-p) \ln x_{NL}$, where $x_L, x_{NL} > 0$.

   (a) Show that $E(u)$ is a concave function in its domain.

   (b) Show that the set in (x_L, x_{NL}) plane defined by the inequality $E(u) \geq const$ is convex.

8. In order to reduce risk Alex buys insurance from a perfectly competitive company. By doing that he maximizes his expected utility $E(u)$ with respect to (x_L, x_{NL}) subject to constraint imposed by the company $p(w - L - x_L) + (1-p)(w - x_{NL}) = 0$.

   (a) Find his optimal bundle (x_L^*, x_{NL}^*). Use bordered Hessian to check sufficiency. Is Alex better-off with the insurance? Explain.

   (b) Let $E(u)^*$ be the maximum value of $E(u)$ with insurance. By applying Envelope Theorem find $\partial E(u)^* / \partial p$. Express your answer in terms of p, w and L alone.

Name, group:

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Variant 1. Section A. Problems 1 and 2 out of 8

1. At the beginning James Bond is located at the point $(1,1)$. To choose his new location he calculates the gradient of the function $f(x,y) = x^2 + y^2 - 3xy + 2x$ from his current location and moves in the direction given by the gradient by its length. Where he will be after two movements?

2. Consider the system of equations

$$\begin{cases} x^4 + y^4 + z^4 = 3 \\ x + x^3 + y + 2y^3 + 2z + 3z^3 = 6 \end{cases}$$

   (a) Are the function $x(z)$ and $y(z)$ defined around the point $A = (-1, 1, 1)$?

   (b) Find dx/dz and dy/dz

Name, group: ....

Variant 2. Section A. Problems 3 and 4 out of 8

3. Find and classify unconstrained extrema of the function $f(x, y) = x^8 + 16y^4 - 4xy$
4. Find and classify constrained extrema of the function $f(x, y) = 2xy$ subject to $x^2 + 4y^2 = 9$

Name, group:

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Variant 2. Section A. Problems 5 and 6 out of 8

5. The function u is defined by the equation $u^3(t) + u(t) = f(x, y)$, where $x = 4 - 3t$ and $y = -2 + 2t$ and f is in C^2. Find du/dt and d^2u/dt^2

6. Consider the function $f(x) = h(x) - 2ax$, where the function h is twice differentiable and $h''(x) < 0$ for all x. The global maximum of f is denoted by $x^*(a)$.

   (a) Find dx^*/da

   (b) It is known that for $a = 1$ the optimal point is $x^* = 3$ and the value of maximum is 2015. What is the approximate value of maximum for $a = 1.01$?

Name, group: ....

Variant 2. Section B. Problems 7 and 8 can be solved separately.

7. A risk-averse Alex possesses w dollars of wealth in money and property. His house worth $L < w$ dollars can be completely destroyed by a landslide with the probability p, $0 < p < 1$. Let's denote his wealth if landslide occurs by x_L and x_{NL} otherwise. Then his expected utility can be calculated by $E(u) = p \ln x_L + (1-p) \ln x_{NL}$, where $x_L, x_{NL} > 0$.

   (a) Show that $E(u)$ is a concave function in its domain.

   (b) Show that the set in (x_L, x_{NL}) plane defined by the inequality $E(u) \geq const$ is convex.

8. In order to reduce risk Alex buys insurance from a perfectly competitive company. By doing that he maximizes his expected utility $E(u)$ with respect to (x_L, x_{NL}) subject to constraint imposed by the company $p(w - L - x_L) + (1-p)(w - x_{NL}) = 0$.

   (a) Find his optimal bundle (x_L^*, x_{NL}^*). Use bordered Hessian to check sufficiency. Is Alex better-off with the insurance? Explain.

   (b) Let $E(u)^*$ be the maximum value of $E(u)$ with insurance. By applying Envelope Theorem find $\partial E(u)^* / \partial p$. Express your answer in terms of p, w and L alone.

Name, group: ....

Variant 3. Section A. Problems 1 and 2 out of 8

1. At the beginning James Bond is located at the point $(1, 1)$. To choose his new location he calculates the gradient of the function $f(x, y) = x^2 + y^2 - 3xy + 2x$ from his current location and moves in the direction given by the gradient by its length. Where he will be after two movements?

2. Consider the system of equations

$$\begin{cases} x^4 + y^4 + z^4 = 3 \\ x + x^3 + y + 2y^3 + 3z + 3z^3 = 7 \end{cases}$$

   (a) Are the function $x(z)$ and $y(z)$ defined around the point $A = (-1, 1, 1)$?

   (b) Find dx/dz and dy/dz

Name, group:

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Variant 3. Section A. Problems 3 and 4 out of 8

3. Find and classify unconstrained extrema of the function $f(x, y) = 16x^4 + y^8 - 4xy$
4. Find and classify constrained extrema of the function $f(x, y) = 3xy$ subject to $x^2 + 4y^2 = 9$

Name, group:
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Variant 3. Section A. Problems 5 and 6 out of 8

5. The function u is defined by the equation $u^3(t) + u(t) = f(x, y)$, where $x = 2 + 3t$ and $y = 1 + 3t$ and f is in C^2. Find du/dt and d^2u/dt^2

6. Consider the function $f(x) = h(x) - 3ax$, where the function h is twice differentiable and $h''(x) < 0$ for all x. The global maximum of f is denoted by $x^*(a)$.

   (a) Find dx^*/da

   (b) It is known that for $a = 1$ the optimal point is $x^* = 3$ and the value of maximum is 2015. What is the approximate value of maximum for $a = 1.01$?

Name, group:

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Variant 3. Section B. Problems 7 and 8 can be solved separately.

7. A risk-averse Alex possesses w dollars of wealth in money and property. His house worth $L < w$ dollars can be completely destroyed by a landslide with the probability p, $0 < p < 1$. Let's denote his wealth if landslide occurs by x_L and x_{NL} otherwise. Then his expected utility can be calculated by $E(u) = p \ln x_L + (1-p) \ln x_{NL}$, where $x_L, x_{NL} > 0$.

   (a) Show that $E(u)$ is a concave function in its domain.

   (b) Show that the set in (x_L, x_{NL}) plane defined by the inequality $E(u) \geq const$ is convex.

8. In order to reduce risk Alex buys insurance from a perfectly competitive company. By doing that he maximizes his expected utility $E(u)$ with respect to (x_L, x_{NL}) subject to constraint imposed by the company $p(w - L - x_L) + (1-p)(w - x_{NL}) = 0$.

   (a) Find his optimal bundle (x_L^*, x_{NL}^*). Use bordered Hessian to check sufficiency. Is Alex better-off with the insurance? Explain.

   (b) Let $E(u)^*$ be the maximum value of $E(u)$ with insurance. By applying Envelope Theorem find $\partial E(u)^* / \partial p$. Express your answer in terms of p, w and L alone.

Name, group:

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Variant 4. Section A. Problems 1 and 2 out of 8

1. At the beginning James Bond is located at the point $(1, 1)$. To choose his new location he calculates the gradient of the function $f(x, y) = x^2 + y^2 - 3xy + 4x$ from his current location and moves in the direction given by the gradient by its length. Where he will be after two movements?

2. Consider the system of equations

$$\begin{cases} x^4 + y^4 + z^4 = 3 \\ x + x^3 + y + 2y^3 + 4z + 3z^3 = 8 \end{cases}$$

   (a) Are the function $x(z)$ and $y(z)$ defined around the point $A = (-1, 1, 1)$?

   (b) Find dx/dz and dy/dz

Name, group: ....

Variant 4. Section A. Problems 3 and 4 out of 8

3. Find and classify unconstrained extrema of the function $f(x, y) = x^8 + y^4 - 2xy$
4. Find and classify constrained extrema of the function $f(x, y) = 4xy$ subject to $x^2 + 4y^2 = 9$

Name, group:

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Variant 4. Section A. Problems 5 and 6 out of 8

5. The function u is defined by the equation $u^3(t) + u(t) = f(x, y)$, where $x = 2 + t$ and $y = 1 - 2t$ and f is in C^2. Find du/dt and d^2u/dt^2

6. Consider the function $f(x) = h(x) - 4ax$, where the function h is twice differentiable and $h''(x) < 0$ for all x. The global maximum of f is denoted by $x^*(a)$.

   (a) Find dx^*/da

   (b) It is known that for $a = 1$ the optimal point is $x^* = 3$ and the value of maximum is 2015. What is the approximate value of maximum for $a = 1.01$?

Name, group:

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Variant 4. Section B. Problems 7 and 8 can be solved separately.

7. A risk-averse Alex possesses w dollars of wealth in money and property. His house worth $L < w$ dollars can be completely destroyed by a landslide with the probability p, $0 < p < 1$. Let's denote his wealth if landslide occurs by x_L and x_{NL} otherwise. Then his expected utility can be calculated by $E(u) = p \ln x_L + (1-p) \ln x_{NL}$, where $x_L, x_{NL} > 0$.

   (a) Show that $E(u)$ is a concave function in its domain.

   (b) Show that the set in (x_L, x_{NL}) plane defined by the inequality $E(u) \geq \textit{const}$ is convex.

8. In order to reduce risk Alex buys insurance from a perfectly competitive company. By doing that he maximizes his expected utility $E(u)$ with respect to (x_L, x_{NL}) subject to constraint imposed by the company $p(w - L - x_L) + (1-p)(w - x_{NL}) = 0$.

   (a) Find his optimal bundle (x_L^*, x_{NL}^*). Use bordered Hessian to check sufficiency. Is Alex better-off with the insurance? Explain.

   (b) Let $E(u)^*$ be the maximum value of $E(u)$ with insurance. By applying Envelope Theorem find $\partial E(u)^*/\partial p$. Express your answer in terms of p, w and L alone.

Name, group:

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