Assignment 10 (Due on the week November 23– 28)

1. Which of the following functions on $\mathbb{R}^n$ are concave or convex?

   (a) $f(x) = 3e^x + 5x^4 - \ln x$,

   (b) $f(x, y) = -3x^2 + 2xy - y^2 + 3x - 4y + 1$,

   (c) $f(x, y, z) = 3e^x + 5y^4 - \ln z$,

   (d) $f(x, y, z) = Ax^\alpha y^\beta z^\gamma$, $\alpha, \beta, \gamma > 0$.

2. Graph each of the following sets, and indicate whether it is convex:

   (a) $\{(x, y) \mid y = e^x\}$,

   (b) $\{(x, y) \mid y \geqslant e^x\}$,

   (c) $\{(x, y) \mid y \leqslant 13 - x^2\}$,

   (d) $\{(x, y) \mid xy \geqslant 1; x > 0,\ y > 0\}$.

   Find critical points using the first-order conditions. To check whether a critical point is the optimal solution try the Weierstrass theorem where applicable.

3. $z = \dfrac{x}{a} + \dfrac{y}{b}$, if $x^2 + y^2 = 1$,

4. $z = x^2 + 12xy + 2y^2$, if $4x^2 + y^2 = 25$,

5. Maximize $u(x, y, z) = xy^2z^3$ subject to $x + 2y + 3z = a$, where x, y, z, $a > 0$.