Variant 1. 2017-03-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find indefinite integrals
   (a) $\int e^{3x} \sin 2x \, dx$;
   (b) $\int \frac{x+3}{x^2-6x+9} \, dx$.

2. Solve the differential equation
$$y''' - y'' + 6y' - 6 = 42 \sin(x\sqrt{6}).$$

3. Solve the difference equation
$$y_{t+2} - 6y_{t+1} + 9y_t = 5t.$$

4. The function $f(x, y)$ is non-constant and homogeneous. It is also known that $h(x, y) = f'_x(x, y) + 3x^2y$ is homogeneous of degree 3. Find the value of $\frac{xf'_x(x,y)+yf'_y(x,y)}{f(x,y)}$.

5. Solve the following linear programming problem:
$$\begin{cases} 2x_1 + 2x_2 + 3x_3 \to \min \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ 3x_1 + 5x_2 + x_3 \geq 8 \\ 5x_1 + 3x_2 + x_3 \geq 9 \end{cases}.$$

6. Maximize the function
$$11 + 10x_1 - x_1^2 - 3x_2 + 8x_3 - x_3^2$$
subject to constraints $2x_1 - x_2 + 4x_3 \leq 10$ and $x_2 \leq 100$.

SECTION B

7. Consider the second-order differential equation
$$xy'' - y' - 4x^3y = 0.$$
. It can be solved by an appropriate change of the variable $t = \phi(x)$.
   (a) (10 points) Find this function $\phi(x)$ by setting the task of cancellation the term with $\frac{dy}{dt}$.
   (b) (5 points) After the substitution the transformed equation has constant coefficients. Find its general solution.
   (c) (5 points) Solve the original equation.

8. Three players play the following game. Simulteneously each of them chooses one possible bet: either 1\$ or 2\$. A player is declared winner if his bet is unique and wins the amount of his bet. For example, if players have chosen 1, 2 and 1 then their corresponding payoffs are 0, 2 and 0.
   (a) Find all Nash equilibria in pure strategies.
   (b) Find symmetric Nash equilibrium in mixed strategies.

Variant 2. 2017-03-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find indefinite integrals

   (a) $\int e^{-x} \sin 5x \, dx$;

   (b) $\int \frac{x-3}{x^2+6x+9} \, dx$.

2. Solve the differential equation

$$y''' - y'' + 5y' - 5 = 30\cos(x\sqrt{5}).$$

3. Solve the difference equation

$$y_{t+2} + 6y_{t+1} + 9y_t = -3t.$$

4. The function $f(x, y)$ is non-constant and homogeneous. It is also known that $h(x, y) = f'_x(x, y) + 3x^2y^2$ is homogeneous of degree 4. Find the value of $\frac{xf'_x(x,y)+yf'_y(x,y)}{f(x,y)}$.

5. Solve the following linear programming problem:

$$\begin{cases} 2x_1 + 2x_2 + 3x_3 \to \min \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ 3x_1 + 5x_2 + x_3 \geq 9 \\ 5x_1 + 3x_2 + x_3 \geq 8 \end{cases}.$$

6. Maximize the function

$$22 + 10x_1 - x_1^2 - 3x_2 + 8x_3 - x_3^2$$

   subject to constraints $2x_1 - x_2 + 4x_3 \leq 10$ and $x_2 \leq 200$.

SECTION B

7. Consider the second-order differential equation

$$xy'' - y' - 4x^3y = 0.$$

   . It can be solved by an appropriate change of the variable $t = \phi(x)$.

   (a) (10 points) Find this function $\phi(x)$ by setting the task of cancellation the term with $\frac{dy}{dt}$.

   (b) (5 points) After the substitution the transformed equation has constant coefficients. Find its general solution.

   (c) (5 points) Solve the original equation.

8. Three players play the following game. Simulteneously each of them chooses one possible bet: either 1$ or 2$. A player is declared winner if his bet is unique and wins the amount of his bet. For example, if players have chosen 1, 2 and 1 then their corresponding payoffs are 0, 2 and 0.

   (a) Find all Nash equilibria in pure strategies.

   (b) Find symmetric Nash equilibrium in mixed strategies.

Variant 3. 2017-03-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find indefinite integrals
   (a) $\int e^{-2x} \cos 3x \, dx$;
   (b) $\int \frac{x-2}{x^2+4x+4} \, dx$.
2. Solve the differential equation
$$y''' - y'' + 3y' - 3 = 12 \sin(x\sqrt{3}).$$
3. Solve the difference equation
$$y_{t+2} - 4y_{t+1} + 4y_t = -2t.$$
4. The function $f(x, y)$ is non-constant and homogeneous. It is also known that $h(x, y) = f'_x(x, y) + 7x^2 y$ is homogeneous of degree 3. Find the value of $\frac{xf'_x(x,y)+yf'_y(x,y)}{f(x,y)}$.
5. Solve the following linear programming problem:
$$\begin{cases} 4x_1 + 4x_2 + 6x_3 \to \min \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ 3x_1 + 5x_2 + x_3 \geq 8 \\ 5x_1 + 3x_2 + x_3 \geq 9 \end{cases}.$$
6. Maximize the function
$$33 + 10x_1 - x_1^2 - 3x_2 + 8x_3 - x_3^2$$
subject to constraints $2x_1 - x_2 + 4x_3 \leq 10$ and $x_2 \leq 300$.

SECTION B

7. Consider the second-order differential equation
$$xy'' - y' - 4x^3 y = 0.$$
. It can be solved by an appropriate change of the variable $t = \phi(x)$.
   (a) (10 points) Find this function $\phi(x)$ by setting the task of cancellation the term with $\frac{dy}{dt}$.
   (b) (5 points) After the substitution the transformed equation has constant coefficients. Find its general solution.
   (c) (5 points) Solve the original equation.
8. Three players play the following game. Simulteneously each of them chooses one possible bet: either 1$ or 2$. A player is declared winner if his bet is unique and wins the amount of his bet. For example, if players have chosen 1, 2 and 1 then their corresponding payoffs are 0, 2 and 0.
   (a) Find all Nash equilibria in pure strategies.
   (b) Find symmetric Nash equilibrium in mixed strategies.

Variant 4. 2017-03-28. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find indefinite integrals
   (a) $\int e^{2x} \cos 2x \, dx$;
   (b) $\int \frac{x+2}{x^2-4x+4} \, dx$.

2. Solve the differential equation
$$y''' - y'' + 2y' - 2 = 6\cos(x\sqrt{2}).$$

3. Solve the difference equation
$$y_{t+2} + 4y_{t+1} + 4y_t = 3t.$$

4. The function $f(x,y)$ is non-constant and homogeneous. It is also known that $h(x,y) = f'_x(x,y) + 7x^2y^2$ is homogeneous of degree 4. Find the value of $\frac{xf'_x(x,y)+yf'_y(x,y)}{f(x,y)}$.

5. Solve the following linear programming problem:
$$\begin{cases} 4x_1 + 4x_2 + 6x_3 \to \min \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \\ 3x_1 + 5x_2 + x_3 \geq 9 \\ 5x_1 + 3x_2 + x_3 \geq 8 \end{cases}.$$

6. Maximize the function
$$44 + 10x_1 - x_1^2 - 3x_2 + 8x_3 - x_3^2$$
subject to constraints $2x_1 - x_2 + 4x_3 \leq 10$ and $x_2 \leq 400$.

SECTION B

7. Consider the second-order differential equation
$$xy'' - y' - 4x^3y = 0.$$
. It can be solved by an appropriate change of the variable $t = \phi(x)$.
   (a) (10 points) Find this function $\phi(x)$ by setting the task of cancellation the term with $\frac{dy}{dt}$.
   (b) (5 points) After the substitution the transformed equation has constant coefficients. Find its general solution.
   (c) (5 points) Solve the original equation.

8. Three players play the following game. Simulteneously each of them chooses one possible bet: either 1$ or 2$. A player is declared winner if his bet is unique and wins the amount of his bet. For example, if players have chosen 1, 2 and 1 then their corresponding payoffs are 0, 2 and 0.
   (a) Find all Nash equilibria in pure strategies.
   (b) Find symmetric Nash equilibrium in mixed strategies.