Assignment 12 (Due on the week December 7– 12)

1. Use the Lagrange multiplier method to write the first-order conditions for the maximum of the function $f(x, y) = \sqrt{x} + \sqrt{y}$, subject to $ax + y = 1$, where a is a real parameter. For what values of a solution exists? Check sufficiency condition.

2. Find the points of relative optimum and classify them using second-order conditions: $u = x^2 + y + 2z \to extr$, s.t. $x^3yz^2 = w$, where w is a real parameter.

3. Find the points of relative optimum and classify them using second-order conditions: $u = (x + z)y \to extr$, s.t. $x^2 + y^2 = 2$, $y + z = 2$, where all the variables are positive.

4. A firm with the smooth production function $Q(x, y)$ wants to find the least-cost input combination for a production of a specified level output Q_0 representing, say, a customer's special order. Show that at the point of optimal input combination, the input-price-marginal-product ratio must be the same for each input.

5. Show that the function $z = (1+e^y)\cos x - ye^y$ has an infinite number of points of maximum and no point of minimum.