

1. Find the local maxima and minima of the function  $f(x, y) = x^4 + 2y^4 - xy$ . Determine whether the extrema you have found are global or local.
2. Consider the difference equation  $y_{t+1}(2 + 3y_t) = 4y_t$  with initial condition  $y_0 = 2/3$ .
  - (a) Using the substitution  $z_t = 1/y_t$  solve the difference equation.
  - (b) What is the limit of  $y_t$  as  $t \rightarrow \infty$ ?
3. Solve the system of differential equations:

$$\begin{cases} \dot{x} = x - y \\ \dot{y} = 2x - y \end{cases}$$

4. Consider the implicit function  $y(x)$  given by the equation

$$y^3 + y + 3x^3 + x^2 = 14.$$

- (a) Does this equation define the implicit function  $y(x)$  in the neighborhood of the point  $(1, 2)$ ?
  - (b) If the implicit function is defined find the Taylor series for  $y(x)$  up to the second order term.
5. Find the gradient of the function  $h(x, y) = f(x, y) \cdot g(x, y)$  at the point  $A = (1, 7)$ . It is known that at the point  $A$ :  $\text{grad } f = (1, 1)$ ,  $\text{grad } g = (3, 3)$ ,  $f(A) = 4$ ,  $g(A) = 5$ .