Variant 1. 2017-01-20. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x,y) = \cos(e^{2x}-1) - \cos(e^{3y}-1)$ at a point $x = 0$, $y = 0$.

2. Find the limit or prove that it does not exist
$$\lim_{x\to 0, y\to 0} \frac{x^2y^2}{x^4+3y^4}$$

3. Consider the sphere given by $x^2+y^2+z^2 = 1$. Find the equation of the tangent plane to the sphere at the point $x = 1/\sqrt{3}$, $y = 1/\sqrt{3}$, $z = -1/\sqrt{3}$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x,y,z) = 2x+3y+9z$ subject to $x^2+y^2+4z^2 = 1$.

5. Consider the set A on the plane (x,y) given by the inequality
$$\frac{(x^2+y^2-3)(x^2+y^2-10)}{x^2+y^2-10} \geq 0$$
   (a) Is the set A closed? open? bounded? convex? compact?
   (b) If possible represent the set A in the form $A = B_1 \times B_2$ where each set $B_i \subset \mathbb{R}$.

6. Find and classify the critical poinst of the function $f(x,y) = \exp(-x^2-6y^2+2xy+2y)$. Check whether these local extrema are the global ones.

SECTION B

7. Short-run total costs of a firm are given by
$$STC(q,K) = q^2 + 3qK + 4K^2 - K + \frac{1}{16},$$
where q is the output and K is the amount of capital fixed in the short-run. In the long-run the firm can always adjust the capital in order to minimize costs. Use the appropriate envelope theorem to find $MC = (TC)'$ long-run marginal costs.

8. Solve the constrained minimization problem in two variables: $x^2+y^2 \to \min$ subject to constraint $(x-1)^3 = y^2$. Check firstly whether the method of Lagrange multipliers is valid to apply.

Variant 2. 2017-01-20. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Find the second order Taylor expansion of the function $f(x,y) = \cos(e^{2x}-1) - \cos(e^{3y}-1)$ at a point $x=0$, $y=0$.

2. Find the limit or prove that it does not exist
$$\lim_{x\to 0, y\to 0} \frac{x^2y^2}{x^4+5y^4}$$

3. Consider the sphere given by $x^2+y^2+z^2=1$. Find the equation of the tangent plane to the sphere at the point $x=1/\sqrt{3}$, $y=-1/\sqrt{3}$, $z=-1/\sqrt{3}$.

4. Using Lagrange multiplier method find and classify the constrained extrema of $f(x,y,z)=2x+3y+3z$ subject to $x^2+y^2+4z^2=1$.

5. Consider the set A on the plane (x,y) given by the inequality
$$\frac{(x^2+y^2-5)(x^2+y^2-10)}{x^2+y^2-10} \geq 0$$
   (a) Is the set A closed? open? bounded? convex? compact?
   (b) If possible represent the set A in the form $A = B_1 \times B_2$ where each set $B_i \subset \mathbb{R}$.

6. Find and classify the critical poinst of the function $f(x,y) = \exp(-x^2-4y^2+2xy+2y)$. Check whether these local extrema are the global ones.

SECTION B

7. Short-run total costs of a firm are given by
$$STC(q,K) = q^2 + 3qK + 4K^2 - K + \frac{1}{16},$$
where q is the output and K is the amount of capital fixed in the short-run. In the long-run the firm can always adjust the capital in order to minimize costs. Use the appropriate envelope theorem to find $MC = (TC)'$ long-run marginal costs.

8. Solve the constrained minimization problem in two variables: $x^2+y^2 \to \min$ subject to constraint $(x-1)^3 = y^2$. Check firstly whether the method of Lagrange multipliers is valid to apply.