Assignment 8 (Due on the week November 9 – 14)

1. Find d^2u, if $u = x^3 + y^3 - 3xy(x - y)$.

2. Find dz and d^2z, if $xyz = x + y + z$.

3. Express the quadratic approximation of the following functions:

   (a) $f(x_1, x_2) = e^{x_1 x_2 - 1}$ around the point $a = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$;

   (b) $f(x, y) = \dfrac{x}{y}$ around the point $a = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

4. Determine the definiteness of the following symmetric matrices:

   a) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} -3 & 4 \\ 4 & -5 \end{pmatrix}$ c) $\begin{pmatrix} -3 & 4 \\ 4 & -6 \end{pmatrix}$ d) $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}$

   e) $\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{pmatrix}$ f) $\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ g) $\begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 5 \\ 3 & 0 & 4 & 0 \\ 0 & 5 & 0 & 6 \end{pmatrix}$

5. Express the quadratic approximation of the function $f(x, y) = \tan^{-1} \dfrac{1 + x + y}{1 - x + y}$ around the point $a = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.