1. (10 points) Find the gradient of $f(x,y) = \frac{x^2 y}{\sqrt{x^2+y^2}}$ at the point $M(1,3)$. Compute the derivative of f at M in direction of the vector $\{-1,1\}$.

2. Find all stationary points of $f(x,y) = x^4 + y^4 - 4xy + 1$. Classify them as local minimum, maximum or saddle point.

3. (10 points) Consider the function $u(x,y) = x^2 - 4xy + ay^2 - \ln(xy)$ for $x > 0$ and $y > 0$. For which values of a the function u is convex?

4. (10 points) Find the second order Taylor approximation of a function $f(x,y) = x^5 y^3 + 3x^2 y$ at a point $x = 1, y = -1$.

5. (10 points) Use Lagrange multipliers to find the height and radius of a cylinder with the maximal volume among those with a surface $S = 20\pi$. Make sure you check the second order condition for maximisation.

6. (10 points) Let $h(x,y) = kx^2 + 6xy + 14y^2 + 4y + 10$.

   (a) Find the minimal value of the function h for $k = 3$.

   (b) Using envelope theorem find approximate minimal value of h for $k = 2.98$.

7. We wish to build a picnic zone for the travellers along a highway. The picnic zone should be rectangular with an area of 1000 m^2 and should have a fence on the three sides not adjacent to the highway. The price of one meter of fence is equal to \$ 20.

   (a) Find the dimensions of the picnic area that minimize the fencing costs.

   (b) Using hessian or otherwise check that you have found the costs-minimizing solution.

   (c) Using the Envelope theorem estimate the change in the costs if we increase the area of the picnic zone by 1 m^2.

8. A monopolistic firm with the cost function $TC(Q) = 30 + 15Q + Q^2$ sells a single product in two separate markets. The demand functions for these markets are given by $Q_1 = 25 - P_1$, $Q_2 = 29 - P_2$.

   (a) Find the optimal quantities Q_1, Q_2 to be supplied to the respective markets in order to maximize the profit. Using hessian or otherwise check the second order condition.

   (b) Calculate the point elasticity of demand for each of the three markets. Is it true that the optimal price is negatively related to the absolute value of elasticity at the optimal levels of output?

   (c) Using the Envelope theorem estimate the change in the optimal profit if the demand on the second market changes to $Q_2 = 29.2 - 1.1P_2$.