Name, group no:

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1. (10 points) Ded Moroz considers the function $f(x,y) = (1 + 2x + 3y)^{2022}$.

   Please, help him to find the second order Taylor approximation of this function at a point $x = 0$, $y = 0$.

Name, group no:

....

2. (10 points) Consider the function

$$f(x,y) = \int_0^x 3e^{u^2}\,du + \int_0^y 2\cos(u^2)\,du.$$

Find the gradient $\operatorname{grad} f$ at the point $(0,0)$.

Name, group no:

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3. (10 points) Consider the system

$$\begin{cases} x + y + z = 2 \\ 2x^2 + 2y^2 = z^2 \end{cases} .$$

   (a) Are the functions $x(z)$ and $y(z)$ defined in a neighborhood of the point $A(x = 1, y = -1, z = 2)$?

   (b) Find dx/dz at the point A if possible.

Name, group no:

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4. (10 points) The set S is defined by $S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq 2 - x^2\}$. Two rectangles one on the top of the other are inscribed in S, thus they have the common side and the upper vertices lie on this parabola. Let $A_1 + A_2$ be the sum of their areas, where $A_1 > 0$, $A_2 > 0$.

   Consider the maximization problem $A_1 + A_2 \to \max$.

   (a) Solve the maximization problem or show that the maximum does not exist.

   (b) Check whether the Weierstrass theorem is applicable.

Name, group no:

. . . .

5. (10 points) An implicit function is defined by equation $x^2 + y^2 + xy + 2x + 4y = 0$. Find all the point(s) (x^*, y^*) on the curve represented by this equation, where

   (a) the tangent line to the curve is horizontal with the equation $y = c$;

   (b) conditions of the implicit function theorem are satisfied.

Name, group no:

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6. (10 points) Consider the function $g(x) = \sqrt{x - 1 + \sqrt{x - 1 + \sqrt{x - 1 + \sqrt{\cdots}}}}$ defined for $x > 1$.

   (a) Express $g^2(x)$ through the linear combination of $g(x)$ and $x - 1$.

   (b) Find $g'(2)$.

Name, group no:

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7. (20 points) The Mean Value Theorem in Calculus claims that given a continuously differentiable function $f(x)$ on the closed segment $[x_0; x_0 + \Delta x]$, there exists $0 < \theta < 1$ such that $f(x_0 + \Delta x) - f(x_0) = f'(x_0 + \theta\Delta x)\Delta x$.

   Prove the version of this theorem for a function of the two variables $z = f(x, y) \in C^1$ in the following form: there exists $0 < \theta < 1$ such that

$$f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0) = \frac{\partial f}{\partial x}(x_0 + \theta\Delta x, y_0 + \theta\Delta y)\Delta x + \frac{\partial f}{\partial y}(x_0 + \theta\Delta x, y_0 + \theta\Delta y)\Delta y.$$

   Hint. Join the 2 points (x_0, y_0) and $(x_0 + \Delta x, y_0 + \Delta y)$ with the segment of a straight line passing through these points and apply the chain rule to the composite function $f(x_0 + t\Delta x, y_0 + t\Delta y)$ where $t \in [0; 1]$.

Name, group no:

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8. An economist solves a problem of the minimization of expenses of an individual whose utility function is $u(x,y) = \sqrt{x} + \sqrt{y}$. Expenses are calculated by the formula $E = 3x + 4y$ and the consumer would prefer to fix the value of the utility at 7 utiles.

   (a) (5 points) Formulate the problem of the expenses minimization and form a Lagrangian of this problem.

   (b) (5 points) Using first-order conditions find the minimizing bundle (x^*, y^*).

   (c) (5 points) Using bordered Hessian or otherwise check the sufficiency condition.

   (d) (5 points) A consumer decided to improve her welfare by adding additional 0,1 to 7 utiles. How this decision will affect the expenditure value $E^* = 3x^* + 4y^*$, where the bundle (x^*, y^*) corresponds to a greater utility value? Use appropriate Envelope Theorem to estimate E^*.

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Name, group no:

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2. (10 points) Consider the function

$$f(x, y) = \int_0^x 3e^{u^2}\, du + \int_0^y 5\cos(u^2)\, du.$$

Find the gradient grad f at the point $(0, 0)$.

Name, group no:

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3. (10 points) Consider the system

$$\begin{cases} x + y + z = 2 \\ 2x^2 + 2y^2 = z^2 \end{cases}.$$

(a) Are the functions $x(z)$ and $y(z)$ defined in a neighborhood of the point $A(x = -1, y = 1, z = 2)$?

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Name, group no:

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5. (10 points) An implicit function is defined by equation $x^2 + y^2 + xy - 2x - 4y = 0$. Find all the point(s) (x^*, y^*) on the curve represented by this equation, where

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Name, group no:
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Name, group no:

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   Hint. Join the 2 points (x_0, y_0) and $(x_0 + \Delta x, y_0 + \Delta y)$ with the segment of a straight line passing through these points and apply the chain rule to the composite function $f(x_0 + t\Delta x, y_0 + t\Delta y)$ where $t \in [0; 1]$.

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   (a) (5 points) Formulate the problem of the expenses minimization and form a Lagrangian of this problem.

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   (d) (5 points) A consumer decided to improve her welfare by adding additional 0,1 to 7 utiles. How this decision will affect the expenditure value $E^* = 4x^* + 3y^*$, where the bundle (x^*, y^*) corresponds to a greater utility value? Use appropriate Envelope Theorem to estimate E^*.

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Name, group no:

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Find the gradient grad f at the point $(0,0)$.

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Name, group no:

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Name, group no:
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