Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x_1,x_2,x_3\to 0} \frac{x_1^2 + 4x_1x_2 - x_2^2 + 6x_3x_2}{x_1^2 + x_2^2 + x_3^2}$$

Name, group no:

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2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 + 2y^2$ subject to $x^2 + y^2 = 16$. Check sufficiency conditions.

Name, group no:

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3. (10 points) Consider the function $u(x, y) = x^2 + x^4 + axy + y^2$. For which values of a the function u is convex? concave?

Name, group no:

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4. (10 points) Using Envelope theorem find the approximate minimum of the function

$$f(x,y) = x^4 + 0.001(x^2 + y + y^2) + (y-1)^4.$$

Name, group no:

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5. (10 points) Find and classify all the local extrema of the function $f = x^2 - 4xy + y^3 + 4y$.

Name, group no:

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6. (10 points) Using optimization techniques prove for $x > 0$, $y > 0$ the inequality

$$\frac{x+y}{2} \geq \frac{2}{x^{-1}+y^{-1}}$$

Name, group no:

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7. Consider a problem $f(x_1, x_2, \alpha) \to \max$ subject to $g(x_1, x_2) = 0$. The function f is maximized with respect to x_1, x_2 and α is a real parameter. Both functions f and g are twice continuous differentiable. Let (x_1^*, x_2^*) be a solution to this problem depending on α and $\phi(\alpha)$ be the value function of this problem.

   (a) (5 points) Formulate the envelope theorem that provides the value of $d\phi/da$.

   (b) (5 points) Introduce the function $F = f(x_1, x_2, \alpha) - \phi(\alpha)$. Clearly state the second-order sufficiency condition applicable to F that guarantees the optimality of (x_1^*, x_2^*).

   (c) (10 points) The SOC condition stated in b) should justify inequality $\frac{\partial^2 f}{\partial x_1 \partial \alpha}\frac{dx_1^*}{d\alpha} + \frac{\partial^2 f}{\partial x_2 \partial \alpha}\frac{dx_2^*}{d\alpha} > 0$. Show this.

Name, group no:

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8. Consider a function
$$f(x) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j}{\sum_{i=1}^{n} x_i^2}$$
in $\mathbb{R}^n$ defined everywhere except at the origin. Here a_{ij} are entries of the symmetric matrix A.

   (a) (5 points) Show that $A = cI$ implies that $f(x) = c$. Here c is a real number and I is identity matrix.

   (b) (5 points) Let A be a matrix other than in a). Show that then f is discontinuous at the origin with the irremovable discontinuity. *Hint: you may show this using part c) or otherwise.*

   (c) (10 points) In order to find the points of extremum of f the optimal problem is reformulated as follows:
$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j \to \text{max / min} \\ \sum_{i=1}^{n} x_i^2 = 1 \end{cases}$$
   Solve it by Lagrangian and find the maximum and minimum values of f in terms of eigenvalues of matrix A.

Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x_1,x_2,x_3\to 0} \frac{x_1^2+4x_1x_2+4x_2^2+6x_3x_2}{x_1^2+x_2^2+x_3^2}$$

Name, group no:

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2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 + 3y^2$ subject to $x^2 + y^2 = 16$. Check sufficiency conditions.

Name, group no:

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3. (10 points) Consider the function $u(x, y) = x^2 + x^4 + axy + 2y^2$. For which values of a the function u is convex? concave?

Name, group no:

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4. (10 points) Using Envelope theorem find the approximate minimum of the function

$$f(x, y) = x^4 + 0.001(x^2 + 3y + y^2) + (y - 1)^4.$$

Name, group no:

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5. (10 points) Find and classify all the local extrema of the function $f = x^2 - 4xy + y^3 + 4y + 5$.

Name, group no:

. . . .

6. (10 points) Using optimization techniques prove for $x > 0$, $y > 0$ the inequality

$$\frac{x+y}{2} \geq \frac{2}{x^{-1}+y^{-1}}$$

Name, group no:

. . . .

7. Consider a problem $f(x_1, x_2, \alpha) \to \max$ subject to $g(x_1, x_2) = 0$. The function f is maximized with respect to x_1, x_2 and α is a real parameter. Both functions f and g are twice continuous differentiable. Let (x_1^*, x_2^*) be a solution to this problem depending on α and $\phi(\alpha)$ be the value function of this problem.

   (a) (5 points) Formulate the envelope theorem that provides the value of $d\phi/da$.

   (b) (5 points) Introduce the function $F = f(x_1, x_2, \alpha) - \phi(\alpha)$. Clearly state the second-order sufficiency condition applicable to F that guarantees the optimality of (x_1^*, x_2^*).

   (c) (10 points) The SOC condition stated in b) should justify inequality $\frac{\partial^2 f}{\partial x_1 \partial \alpha}\frac{dx_1^*}{d\alpha} + \frac{\partial^2 f}{\partial x_2 \partial \alpha}\frac{dx_2^*}{d\alpha} > 0$. Show this.

Name, group no:

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8. Consider a function

$$f(x) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j}{\sum_{i=1}^{n} x_i^2}$$

in $\mathbb{R}^n$ defined everywhere except at the origin. Here a_{ij} are entries of the symmetric matrix A.

(a) (5 points) Show that $A = cI$ implies that $f(x) = c$. Here c is a real number and I is identity matrix.

(b) (5 points) Let A be a matrix other than in a). Show that then f is discontinuous at the origin with the irremovable discontinuity. *Hint: you may show this using part c) or otherwise.*

(c) (10 points) In order to find the points of extremum of f the optimal problem is reformulated as follows:

$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j \to \text{max / min} \\ \sum_{i=1}^{n} x_i^2 = 1 \end{cases}$$

Solve it by Lagrangian and find the maximum and minimum values of f in terms of eigenvalues of matrix A.

Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x_1,x_2,x_3\to 0} \frac{x_1^2 + 9x_1x_2 - x_2^2 + 6x_3x_2}{x_1^2 + x_2^2 + x_3^2}$$

Name, group no:

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2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 + 4y^2$ subject to $x^2 + y^2 = 16$. Check sufficiency conditions.

Name, group no:

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3. (10 points) Consider the function $u(x, y) = x^2 + x^4 + axy + 5y^2$. For which values of a the function u is convex? concave?

Name, group no:

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4. (10 points) Using Envelope theorem find the approximate minimum of the function

$$f(x,y) = x^4 + 0.001(x^2 + 4y + y^2) + (y-1)^4.$$

Name, group no:

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5. (10 points) Find and classify all the local extrema of the function $f = x^2 - 4xy + y^3 + 4y + 9$.

Name, group no:

. . . .

6. (10 points) Using optimization techniques prove for $x > 0$, $y > 0$ the inequality

$$\frac{x+y}{2} \geq \frac{2}{x^{-1} + y^{-1}}$$

Name, group no:

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7. Consider a problem $f(x_1, x_2, \alpha) \to \max$ subject to $g(x_1, x_2) = 0$. The function f is maximized with respect to x_1, x_2 and α is a real parameter. Both functions f and g are twice continuous differentiable. Let (x_1^*, x_2^*) be a solution to this problem depending on α and $\phi(\alpha)$ be the value function of this problem.

   (a) (5 points) Formulate the envelope theorem that provides the value of $d\phi/da$.

   (b) (5 points) Introduce the function $F = f(x_1, x_2, \alpha) - \phi(\alpha)$. Clearly state the second-order sufficiency condition applicable to F that guarantees the optimality of (x_1^*, x_2^*).

   (c) (10 points) The SOC condition stated in b) should justify inequality $\frac{\partial^2 f}{\partial x_1 \partial \alpha} \frac{dx_1^*}{d\alpha} + \frac{\partial^2 f}{\partial x_2 \partial \alpha} \frac{dx_2^*}{d\alpha} > 0$. Show this.

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8. Consider a function

$$f(x) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j}{\sum_{i=1}^{n} x_i^2}$$

in $\mathbb{R}^n$ defined everywhere except at the origin. Here a_{ij} are entries of the symmetric matrix A.

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Solve it by Lagrangian and find the maximum and minimum values of f in terms of eigenvalues of matrix A.

Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x_1,x_2,x_3\to 0} \frac{7x_1^2+4x_1x_2-x_2^2+6x_3x_2}{x_1^2+x_2^2+x_3^2}$$

Name, group no:

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2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x,y) = x^2 + 5y^2$ subject to $x^2 + y^2 = 16$. Check sufficiency conditions.

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Name, group no:

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5. (10 points) Find and classify all the local extrema of the function $f = x^2 - 4xy + y^3 + 4y - 11$.

Name, group no:

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6. (10 points) Using optimization techniques prove for $x > 0$, $y > 0$ the inequality

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Solve it by Lagrangian and find the maximum and minimum values of f in terms of eigenvalues of matrix A.