Variant 1. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = x^5 + 2xyz - 3z^3$. Using the total differential find the approximate value of $f(1.02, 0.99, 1)$.

2. Consider the function $f(x, y, z) = x^4 + (x + y)^2 + (x + z)^3$.

   (a) Find the Hesse matrix. Clearly state the Young theorem even if you don't use it.

   (b) Find all the points where the Hesse matrix is positive definite.

3. Consider the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{7x^2+3y^2}}, & \text{if } (x, y) \neq (0, 0) \\ a, & \text{if } (x, y) = (0, 0) \end{cases}$$

   Find all the values of a such that the function f will be continuous.

4. The microbe Veniamin is staying on the surface of the ellipsoid $4x^2 + y^2 + z^2 = 9$ at the point $(1, 2, 1)$. All coordinates are measured in centimetres. He digs into the ellipsoid perdpendicularly to the surface by 0.02 cm. Find new approximate coordinates of Veniamin.

5. Consider the function $u(x, y) = 2f(r)$ where $r = \sqrt{x^2 + y^2}$. Is it possible to represent the function $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ as a function of r alone, i.e. $g(r)$? If yes, then find $g(r)$.

6. Given the system

$$\begin{cases} u^2 - w^2 + x^2 + y^2 = 0 \\ uw + xy = 0 \end{cases}$$

   (a) Define a sufficient condition for functions $u(x, y)$ and $w(x, y)$ to be differentiable

   (b) Find $\frac{\partial u}{\partial x}$

SECTION B

7. (20 points) Let the demand and supply for an ice-cream on the sunny day be $q_D = D(p, T, d)$ and $q_S = S(p, T)$ correspondingly. Here p is the price, T is the temperature on this day, d — distance of the selling place from the center of the park, $D_p < 0$, $D_T > 0$, $S_p > 0$, $S_T < 0$, $D_d < 0$.

   (a) Find analytically how the equilibrium price p^* changes with the increase of T. How does it change with the increase of d?

   (b) Let q^* be the equilibrium supply quantity. Find $\frac{\partial q^*}{\partial T}$. Find the condition when $\frac{\partial q^*}{\partial T} < 0$.

8. (20 points) Consider the utility function

$$U(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

   where $\sigma \geq 0$ is a parameter.

   (a) Sketch the indifference curves corresponding to $\sigma = 2$.

   (b) Find the limiting utility function and sketch the corresponding indifference curves for two cases: $\sigma \to \infty$ and $\sigma \to 0$

Variant 2. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = -3x^5 + 3xyz - z^3$. Using the total differential find the approximate value of $f(1.02, 0.99, 1)$.

2. Consider the function $f(x, y, z) = x^4 + (x + y)^2 + (x + z)^3$.

   (a) Find the Hesse matrix. Clearly state the Young theorem even if you don't use it.

   (b) Find all the points where the Hesse matrix is positive definite.

3. Consider the function
$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{4x^2+7y^2}}, & \text{if } (x, y) \neq (0, 0) \\ a, & \text{if } (x, y) = (0, 0) \end{cases}$$
   Find all the values of a such that the function f will be continuous.

4. The microbe Kharlampiy is staying on the surface of the ellipsoid $3x^2 + y^2 + 2z^2 = 9$ at the point $(1, 2, 1)$. All coordinates are measured in centimetres. He digs into the ellipsoid perdpendicularly to the surface by 0.02 cm. Find new approximate coordinates of Kharlampiy.

5. Consider the function $u(x, y) = 3f(r)$ where $r = \sqrt{x^2 + y^2}$. Is it possible to represent the function $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ as a function of r alone, i.e. $g(r)$? If yes, then find $g(r)$.

6. Given the system
$$\begin{cases} u^2 - w^2 + x^2 + y^2 = 0 \\ uw + xy = 0 \end{cases}$$

   (a) Define a sufficient condition for functions $u(x, y)$ and $w(x, y)$ to be differentiable

   (b) Find $\frac{\partial u}{\partial y}$

SECTION B

7. (20 points) Let the demand and supply for an ice-cream on the sunny day be $q_D = D(p, T, d)$ and $q_S = S(p, T)$ correspondingly. Here p is the price, T is the temperature on this day, d — distance of the selling place from the center of the park, $D_p < 0$, $D_T > 0$, $S_p > 0$, $S_T < 0$, $D_d < 0$.

   (a) Find analytically how the equilibrium price p^* changes with the increase of T. How does it change with the increase of d?

   (b) Let q^* be the equilibrium supply quantity. Find $\frac{\partial q^*}{\partial T}$. Find the condition when $\frac{\partial q^*}{\partial T} < 0$.

8. (20 points) Consider the utility function
$$U(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$
   where $\sigma \geq 0$ is a parameter.

   (a) Sketch the indifference curves corresponding to $\sigma = 2$.

   (b) Find the limiting utility function and sketch the corresponding indifference curves for two cases: $\sigma \to \infty$ and $\sigma \to 0$

Variant 3. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = 2x^5 + 2xyz - z^3$. Using the total differential find the approximate value of $f(1.02, 0.99, 1)$.

2. Consider the function $f(x, y, z) = x^4 + (x + z)^2 + (x + y)^3$.

   (a) Find the Hesse matrix. Clearly state the Young theorem even if you don't use it.

   (b) Find all the points where the Hesse matrix is positive definite.

3. Consider the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{5x^2+3y^2}}, & \text{if } (x, y) \neq (0, 0) \\ a, & \text{if } (x, y) = (0, 0) \end{cases}$$

   Find all the values of a such that the function f will be continuous.

4. The microbe Vassissualiy is staying on the surface of the ellipsoid $2x^2 + y^2 + 3z^2 = 9$ at the point $(1, 2, 1)$. All coordinates are measured in centimetres. He digs into the ellipsoid perdpendicularly to the surface by 0.02 cm. Find new approximate coordinates of Vassissualiy.

5. Consider the function $u(x, y) = 4f(r)$ where $r = \sqrt{x^2 + y^2}$. Is it possible to represent the function $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ as a function of r alone, i.e. $g(r)$? If yes, then find $g(r)$.

6. Given the system

$$\begin{cases} u^2 - w^2 + x^2 + y^2 = 0 \\ uw + xy = 0 \end{cases}$$

   (a) Define a sufficient condition for functions $u(x, y)$ and $w(x, y)$ to be differentiable

   (b) Find $\frac{\partial w}{\partial x}$

SECTION B

7. (20 points) Let the demand and supply for an ice-cream on the sunny day be $q_D = D(p, T, d)$ and $q_S = S(p, T)$ correspondingly. Here p is the price, T is the temperature on this day, d — distance of the selling place from the center of the park, $D_p < 0$, $D_T > 0$, $S_p > 0$, $S_T < 0$, $D_d < 0$.

   (a) Find analytically how the equilibrium price p^* changes with the increase of T. How does it change with the increase of d?

   (b) Let q^* be the equilibrium supply quantity. Find $\frac{\partial q^*}{\partial T}$. Find the condition when $\frac{\partial q^*}{\partial T} < 0$.

8. (20 points) Consider the utility function

$$U(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

   where $\sigma \geq 0$ is a parameter.

   (a) Sketch the indifference curves corresponding to $\sigma = 2$.

   (b) Find the limiting utility function and sketch the corresponding indifference curves for two cases: $\sigma \to \infty$ and $\sigma \to 0$

Variant 4. Please, don't forget to write you variant number. Sections A and B will make up 60% and 40% of the exam grade, respectively. Total duration of the exam is 120 min. Good luck! :)

SECTION A

1. Consider the function $f(x, y, z) = 2x^5 + xyz - 2z^3$. Using the total differential find the approximate value of $f(1.02, 0.99, 1)$.

2. Consider the function $f(x, y, z) = x^4 + (x + z)^2 + (x + y)^3$.

   (a) Find the Hesse matrix. Clearly state the Young theorem even if you don't use it.

   (b) Find all the points where the Hesse matrix is positive definite.

3. Consider the function
$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{4x^2+6y^2}}, & \text{if } (x, y) \neq (0, 0) \\ a, & \text{if } (x, y) = (0, 0) \end{cases}$$
   Find all the values of a such that the function f will be continuous.

4. The microbe Afanasiy is staying on the surface of the ellipsoid $x^2 + y^2 + 4z^2 = 9$ at the point $(1, 2, 1)$. All coordinates are measured in centimetres. He digs into the ellipsoid perdpendicularly to the surface by 0.02 cm. Find new approximate coordinates of Afanasiy.

5. Consider the function $u(x, y) = 5f(r)$ where $r = \sqrt{x^2 + y^2}$. Is it possible to represent the function $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ as a function of r alone, i.e. $g(r)$? If yes, then find $g(r)$.

6. Given the system
$$\begin{cases} u^2 - w^2 + x^2 + y^2 = 0 \\ uw + xy = 0 \end{cases}$$

   (a) Define a sufficient condition for functions $u(x, y)$ and $w(x, y)$ to be differentiable

   (b) Find $\frac{\partial w}{\partial y}$

SECTION B

7. (20 points) Let the demand and supply for an ice-cream on the sunny day be $q_D = D(p, T, d)$ and $q_S = S(p, T)$ correspondingly. Here p is the price, T is the temperature on this day, d — distance of the selling place from the center of the park, $D_p < 0$, $D_T > 0$, $S_p > 0$, $S_T < 0$, $D_d < 0$.

   (a) Find analytically how the equilibrium price p^* changes with the increase of T. How does it change with the increase of d?

   (b) Let q^* be the equilibrium supply quantity. Find $\frac{\partial q^*}{\partial T}$. Find the condition when $\frac{\partial q^*}{\partial T} < 0$.

8. (20 points) Consider the utility function
$$U(x_1, x_2) = \left(x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$
   where $\sigma \geq 0$ is a parameter.

   (a) Sketch the indifference curves corresponding to $\sigma = 2$.

   (b) Find the limiting utility function and sketch the corresponding indifference curves for two cases: $\sigma \to \infty$ and $\sigma \to 0$