Name, group no:

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1. [10 points] Check whether the function $f(x, y) = 4x^4 + y^2 + y^4 + 4x^2 + xy$ is concave, convex or neither.

Name, group no:

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2. [10 points] Find and classify the local extrema of $f(x, y) = 4 + x^3 + y^3 - 3xy$.

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3. [10 points] Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 5x + 4y + 8z$ subject to $x^2 + y^2 + z^2 = 1$.

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4. [10 points] Microbe Veniamin lives on the (x, y) plane. Veniamin likes to hop and likes the function $f(x, y) = 5x^2 + 2y^4$. From the point (x_t, y_t) he hops into the point

$$(x_{t+1}, y_{t+1}) = (x_t, y_t) - 0.001 \cdot \operatorname{grad} f(x_t, y_t)$$

Veniamin starts hopping from the point $(x = 1, y = 2)$.

a. What are the exact coordinates of Veniamin after one hop?

b. Where he may find himself after 10^{2017} hops?

Name, group no:

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5. [10 points] Consider the function $p(x_1, x_2) = h(x_1 + x_2 a)$, where $h(t) = \exp(t)/(1 + \exp(t))$ and a is a fixed parameter. Find the second order Taylor expansion of p at $(x_1 = 0, x_2 = 0)$.

Name, group no:

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6. [10 points] Consider the function f defined for $x > 0$:

$$f(x) = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \dots}}}$$

a. Simplify the expression $f(x) - \frac{1}{f(x)}$;

b. Using implicit function theorem find $f'(1)$.

Name, group no:

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7. Let $f(x_1, x_2)$ be twice continuously differentiable function whose Hessian is negative definite. Consider long-run profit maximization problem

$$f(x_1, x_2) - w_1 x_1 - w_2 x_2 \to \max_{x_1, x_2},$$

where $w_1, w_2 > 0$ are factor prices. The optimal bundle of factors consists of x_1^L, x_1^L which are called demand on factors.

a. [10 points] Write down first-order conditions for the problem and check that IFT is applicable here in order to find x_1^L, x_1^L.

b. [10 points] Prove that $\frac{\partial x_1^L}{\partial w_1} < 0$.

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8. The previous problem is stated in the long-run. In the short-run the quantity of x_2 is fixed, i.e. $x_2 = b > 0$. The value function $\pi_L^*(w_1,\, w_2)$ for long-run problem is called profit function. It is clear that $\pi_L^*(w_1,\, w_2) \geqslant \pi_S^*$, where $\pi_S^*(w_1,\, w_2)$ is the profit function for the new short-run problem

$$f(x_1,\, b) - w_1 x_1 - w_2 b \to \max_{x_1}$$

a. [10 points] Let $g(x_1, x_2) \in C^2$ be an arbitrary function that takes the minimum value at $(\tilde{x}_1, \tilde{x}_2)$. Provide the argument justifying that $\frac{\partial^2 g}{\partial x_1^2} \geqslant 0$.

b. [5 points] Let $z = \pi_L^* - \pi_S^*$. Explain why $\frac{\partial^2 z}{\partial w_1^2} \geqslant 0$.

c. [5 points] Using Envelope Theorem show that $\frac{\partial x_1^L}{\partial w_1} \leqslant \frac{\partial x_1^S}{\partial w_1}$, where x_1^L and x_1^S are factor demands in different periods.

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Name, group no:

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3. [10 points] Using Lagrange multiplier method find and classify the constrained extrema of $f(x, y, z) = 7x + 2y + 9z$ subject to $x^2 + y^2 + z^2 = 1$.

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