Variant 1. Section A. Problems 1 and 2 out of 8

- 1. Solve differential equation $y'' + 2y' + y = 2x + 3e^{-x}\sqrt{x+1}$.
- 2. Given the difference equation $3y_{t+2} + 2y_{t+1} + \gamma y_t = 5$ with the real parameter γ , find all the values of γ for whose the time path of this equation is convergent. Choose some value of γ beyond the found range and show that the corresponding time path is divergent.

| Name, group: | | |
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Variant 1. Section A. Problems 3 and 4 out of 8

3. Find all mixed Nash equilibria of the following game

| | d | e | f |
|--------------|------|------|------|
| a | 0;0 | 0;-1 | 6;-2 |
| b | 0;0 | -1;1 | 7;0 |
| \mathbf{c} | -1;5 | -1;4 | 5;9 |

4. For non-negative y_i minimize the function $11y_1 + 5y_2 + 4y_3 + 4y_4$ subject to constraints $y_1 + y_2 + y_3 + 0.1y_4 \ge 2$ and $2y_1 + y_2 + 0.1y_3 + y_4 \ge 7$.

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Variant 1. Section A. Problems 5 and 6 out of 8

- 5. Use the Lagrange multiplier method to find the maximum value of $f(x, y, z) = (7+x)^2(1+y)^2(7+z)^2$ among positive numbers subject to $x^2 + 49y^2 + z^2 = 100$.
- 6. James Bond added $x^3 + 7x 8y 6y^3$ to the unknown homogeneous function f(x, y). The new function was homogeneous once again! Please help the Secret Service agent recover the function f if f(1,1) = 1.

| Name, group: | | |
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Variant 1. Section B. Problems 7 and 8 out of 8

7. Solve initial-value problem for the difference equation

$$y_{t+3} - 4y_{t+2} + 5y_{t+1} - 2y_t = 1$$

with the initial values $y_0 = y_1 = y_2 = 0$.

Hint: one of the characteristic roots is 1.

8. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = 2x_1 - x_2 - 3x_3$ subject to constraints: $3(x_1 - 1)^2 + (x_2 + 1)^2 + 2x_3^2 \le 4$ and $x_1, x_2, x_3 \ge 0$. Find the point(s) of maximum and maximum value of f.

| Name, group: | | |
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Variant 2. Section A. Problems 1 and 2 out of 8

- 1. Solve differential equation $y'' 2y' + y = -x + 3e^x\sqrt{x+1}$.
- 2. Given the difference equation $5y_{t+2} + 2y_{t+1} + \gamma y_t = 8$ with the real parameter γ , find all the values of γ for whose the time path of this equation is convergent. Choose some value of γ beyond the found range and show that the corresponding time path is divergent.

| Name, group: | | |
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Variant 2. Section A. Problems 3 and 4 out of 8

3. Find all mixed Nash equilibria of the following game

| | d | e | f |
|--------------|------|------|------|
| a | 0;0 | 0;-1 | 6;-2 |
| b | 0;0 | -1;2 | 7;1 |
| \mathbf{c} | -1;4 | -1;3 | 5;8 |

4. For non-negative y_i minimize the function $12y_1 + 5y_2 + 4y_3 + 4y_4$ subject to constraints $y_1 + y_2 + y_3 + 0.1y_4 \ge 2$ and $2y_1 + y_2 + 0.1y_3 + y_4 \ge 5$.

| Name, group: | |
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Variant 2. Section A. Problems 5 and 6 out of 8

- 5. Use the Lagrange multiplier method to find the maximum value of $f(x, y, z) = (6+x)^2(1+y)^2(6+z)^2$ among positive numbers subject to $x^2 + 36y^2 + z^2 = 100$.
- 6. James Bond added $2x^3 + 5x 6y 6y^3$ to the unknown homogeneous function f(x, y). The new function was homogeneous once again! Please help the Secret Service agent recover the function f if f(1,1) = 1.

| Name, group: | |
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Variant 2. Section B. Problems 7 and 8 out of 8

7. Solve initial-value problem for the difference equation

$$y_{t+3} - 4y_{t+2} + 5y_{t+1} - 2y_t = 1$$

with the initial values $y_0 = y_1 = y_2 = 0$.

Hint: one of the characteristic roots is 1.

8. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -3x_1 + 2x_2 - x_3$ subject to constraints: $2x_1^2 + 3(x_2 - 1)^2 + (x_3 + 1)^2 \le 4$ and $x_1, x_2, x_3 \ge 0$. Find the point(s) of maximum and maximum value of f.

| Name, group: | |
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Variant 3. Section A. Problems 1 and 2 out of 8

- 1. Solve differential equation $y'' 4y' + 4y = x + 3e^{2x}\sqrt{x-1}$.
- 2. Given the difference equation $3y_{t+2} + 4y_{t+1} + \gamma y_t = 3$ with the real parameter γ , find all the values of γ for whose the time path of this equation is convergent. Choose some value of γ beyond the found range and show that the corresponding time path is divergent.

| Name, group: | | |
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Variant 3. Section A. Problems 3 and 4 out of 8

3. Find all mixed Nash equilibria of the following game

| | d | e | f |
|--------------|------|------|------|
| a | 0;0 | 0;-1 | 7;-2 |
| b | 0;0 | -1;3 | 7;1 |
| \mathbf{c} | -1;5 | -1;4 | 5;9 |

4. For non-negative y_i minimize the function $13y_1 + 5y_2 + 4y_3 + 4y_4$ subject to constraints $y_1 + y_2 + y_3 + 0.1y_4 \ge 2$ and $2y_1 + y_2 + 0.1y_3 + y_4 \ge 4$.

| Name, group: | |
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Variant 3. Section A. Problems 5 and 6 out of 8

- 5. Use the Lagrange multiplier method to find the maximum value of $f(x, y, z) = (8+x)^2(1+y)^2(8+z)^2$ among positive numbers subject to $x^2 + 64y^2 + z^2 = 100$.
- 6. James Bond added $3x^3 + 2x 3y 6y^3$ to the unknown homogeneous function f(x, y). The new function was homogeneous once again! Please help the Secret Service agent recover the function f if f(1,1) = 1.

| Name, group: | |
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Variant 3. Section B. Problems 7 and 8 out of 8

7. Solve initial-value problem for the difference equation

$$y_{t+3} - 4y_{t+2} + 5y_{t+1} - 2y_t = 1$$

with the initial values $y_0 = y_1 = y_2 = 0$.

Hint: one of the characteristic roots is 1.

8. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -3x_1 - x_2 + 2x_3$ subject to constraints: $2x_1^2 + (x_2 + 1)^2 + 3(x_3 - 1)^2 \le 4$ and $x_1, x_2, x_3 \ge 0$. Find the point(s) of maximum and maximum value of f.

| Name, group: | | |
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Variant 4. Section A. Problems 1 and 2 out of 8

- 1. Solve differential equation $y'' + 4y' + 4y = -x + 3e^{-2x}\sqrt{x-2}$.
- 2. Given the difference equation $5y_{t+2} + 4y_{t+1} + \gamma y_t = 7$ with the real parameter γ , find all the values of γ for whose the time path of this equation is convergent. Choose some value of γ beyond the found range and show that the corresponding time path is divergent.

| Name, group: | | |
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Variant 4. Section A. Problems 3 and 4 out of 8

3. Find all mixed Nash equilibria of the following game

| | d | e | f |
|--------------|------|-------|------|
| a | 0;0 | 0;-1 | 6;-2 |
| b | 0;0 | -1;4 | 7;1 |
| \mathbf{c} | -1;9 | -1;-3 | 5;8 |

4. For non-negative y_i minimize the function $14y_1 + 5y_2 + 4y_3 + 4y_4$ subject to constraints $y_1 + y_2 + y_3 + 0.1y_4 \ge 2$ and $2y_1 + y_2 + 0.1y_3 + y_4 \ge 3$.

| Name, group: | |
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Variant 4. Section A. Problems 5 and 6 out of 8

- 5. Use the Lagrange multiplier method to find the maximum value of $f(x, y, z) = (9+x)^2(1+y)^2(9+z)^2$ among positive numbers subject to $x^2 + 81y^2 + z^2 = 100$.
- 6. James Bond added $4x^3 + 2x 3y 6y^3$ to the unknown homogeneous function f(x, y). The new function was homogeneous once again! Please help the Secret Service agent recover the function f if f(1,1) = 1.

| Name, group: | |
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Variant 4. Section B. Problems 7 and 8 out of 8

7. Solve initial-value problem for the difference equation

$$y_{t+3} - 4y_{t+2} + 5y_{t+1} - 2y_t = 1$$

with the initial values $y_0 = y_1 = y_2 = 0$.

Hint: one of the characteristic roots is 1.

8. Using Lagrange multipliers maximize the function $f(x_1, x_2, x_3) = -x_1 - 3x_2 + 2x_3$ subject to constraints: $(x_1 + 1)^2 + 2x_2^2 + 3(x_3 - 1)^2 \le 4$ and $x_1, x_2, x_3 \ge 0$. Find the point(s) of maximum and maximum value of f.

| Name, group: | | |
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