Name, group no:

....

1. (10 points) Consider the function $f(x, y) = x^3 + y^3 + 2xy$. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:

. . . .

2. (10 points) Consider the system

$$\begin{cases} x^3 + y^3 + z^2 = 3 \\ x + x^3 + 2y^3 x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

(b) Find $z'(y)$ if possible.

Name, group no:

. . . .

3. (10 points) Consider the function $f(x, y, z) = x^2 + 9y^2 + 2xy + \alpha z^2$.

   (a) Find the Hesse matrix. Clearly state the Young theorem if you use it.

   (b) For each value of α find the definiteness of Hesse matrix.

Name, group no:

. . . .

4. (10 points) Consider the function $u(x) = f(a, b, c)$, where $a = \alpha(q, r)$, $b = \beta(x)$, $c = \gamma(x, q)$, $q = x^2$ and $r = x^3$. All the functions are differentiable. Find $u'(x)$.

Name, group no:

. . . .

5. (10 points) Consider the function $f(x, y) = x^2 + y^2 + 4y$. The microbe Veniamin is standing at $(1, 1)$ and is moving according to a simple rule. From a point (a, b) he jumps into the point $(a, b) - 0.01 \operatorname{grad} f(a, b)$.

   (a) Where Veniamin will be after two jumps?

   (b) What will be the approximate location of Veniamin after 2018 jumps?

Name, group no:

. . . .

6. (10 points) Let $h(a,b) = \int_a^b \exp(-t^2) \cdot dt$. Find the grad $h(1,2)$.

Name, group no:

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7. The domain of the function $z = xy - \frac{2}{3}x\sqrt{x} - \frac{1}{3}y^3 + 5x + 3y$ is the nonnegative quadrant $\{x \geq 0, y \geq 0\}$.

   (a) (10 points) Find the equation of the tangent plane to the graph of z at $(1, 1, 8)$.

   (b) (10 points) Let grad $z(1, 1) = c$. Find all such points that grad $z(x, y) = c$.

Name, group no:

....

8. Two drivers on a lonely island get utility from fast driving and money. Let $0 \leq x_1 \leq 1$ be the speed of the first car and $0 \leq x_2 \leq 1$ be the speed of the second car, respectively. They have the same amount of wealth $I > 1$. Utilities of the drivers are $U_1(x_1, x_2) = x_1 + I \cdot (1 - x_1 x_2)$ and $U_2(x_1, x_2) = \ln x_2 + I \cdot (1 - x_1 x_2)$.

   (a) (7 points) On (x_1, x_2)-plane draw the solutions of the equations $\frac{\partial U_1}{\partial x_1} = 0$ and $\frac{\partial U_2}{\partial x_2} = 0$.

   (b) (10 points) Let $(x_1^*, x_2^*) = (1, 1/I)$. Show that the system of inequalities hold $U_1(x_1^*, x_2^*) \geq U_1(x_1, x_2^*)$ and $U_2(x_1^*, x_2^*) \geq U_2(x_1^*, x_2)$.

   (c) (3 points) Explain why even the small bribe offered by the second driver will stop the first driver from using his car?

Name, group no:

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1. (10 points) Consider the function $f(x, y) = x^3 + y^3 + 3xy$. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:
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2. (10 points) Consider the system

$$\begin{cases} x^3 + y^3 + 2z^2 = 4 \\ x + x^3 + 2y^3 x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

(b) Find $z'(y)$ if possible.

Name, group no:

. . . .

3. (10 points) Consider the function $f(x, y, z) = x^2 + 10y^2 + 2xy + \alpha z^2$.

   (a) Find the Hesse matrix. Clearly state the Young theorem if you use it.

   (b) For each value of α find the definiteness of Hesse matrix.

Name, group no:

. . . .

4. (10 points) Consider the function $u(x) = f(a, b, c)$, where $a = \alpha(q, r)$, $b = \beta(x)$, $c = \gamma(x, q)$, $q = x^2$ and $r = -x^3$. All the functions are differentiable. Find $u'(x)$.

Name, group no:

. . . .

5. (10 points) Consider the function $f(x, y) = x^2 + y^2 + 6y$. The microbe Veniamin is standing at $(1, 1)$ and is moving according to a simple rule. From a point (a, b) he jumps into the point $(a, b) - 0.01 \operatorname{grad} f(a, b)$.

   (a) Where Veniamin will be after two jumps?

   (b) What will be the approximate location of Veniamin after 2018 jumps?

Name, group no:

. . . .

6. (10 points) Let $h(a, b) = \int_a^b \exp(-2t^2) \cdot dt$. Find the grad $h(1, 2)$.

Name, group no:

....

7. The domain of the function $z = xy - \frac{2}{3}x\sqrt{x} - \frac{1}{3}y^3 + 5x + 3y$ is the nonnegative quadrant $\{x \geq 0, y \geq 0\}$.

   (a) (10 points) Find the equation of the tangent plane to the graph of z at $(1, 1, 8)$.

   (b) (10 points) Let grad $z(1, 1) = c$. Find all such points that grad $z(x, y) = c$.

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   (a) (7 points) On (x_1, x_2)-plane draw the solutions of the equations $\frac{\partial U_1}{\partial x_1} = 0$ and $\frac{\partial U_2}{\partial x_2} = 0$.

   (b) (10 points) Let $(x_1^*, x_2^*) = (1, 1/I)$. Show that the system of inequalities hold $U_1(x_1^*, x_2^*) \geq U_1(x_1, x_2^*)$ and $U_2(x_1^*, x_2^*) \geq U_2(x_1^*, x_2)$.

   (c) (3 points) Explain why even the small bribe offered by the second driver will stop the first driver from using his car?

Name, group no:

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1. (10 points) Consider the function $f(x, y) = x^3 + y^3 + 4xy$. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:

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2. (10 points) Consider the system

$$\begin{cases} x^3 + y^3 + 3z^2 = 5 \\ x + x^3 + 2y^3 x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

(b) Find $z'(y)$ if possible.

Name, group no:

. . . .

3. (10 points) Consider the function $f(x, y, z) = x^2 + 11y^2 + 2xy + \alpha z^2$.

   (a) Find the Hesse matrix. Clearly state the Young theorem if you use it.

   (b) For each value of α find the definiteness of Hesse matrix.

Name, group no:

. . . .

4. (10 points) Consider the function $u(x) = f(a, b, c)$, where $a = \alpha(q, r)$, $b = \beta(x)$, $c = \gamma(x, q)$, $q = -x^2$ and $r = x^3$. All the functions are differentiable. Find $u'(x)$.

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   (a) (7 points) On (x_1, x_2)-plane draw the solutions of the equations $\frac{\partial U_1}{\partial x_1} = 0$ and $\frac{\partial U_2}{\partial x_2} = 0$.

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Name, group no:

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2. (10 points) Consider the system

$$\begin{cases} x^3 + y^3 + 4z^2 = 6 \\ x + x^3 + 2y^3 x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

(b) Find $z'(y)$ if possible.

Name, group no:

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3. (10 points) Consider the function $f(x, y, z) = x^2 + 12y^2 + 2xy + \alpha z^2$.

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