1. (10 points) Consider the function
$$f(x, y) = 4x^2 + 9y^2 + 4cxy + 3d.$$
Find all values of c and d such that the function f is convex everywhere.

2. (10 points) Minimize the function
$$f(x, y, z) = x^2 + 3y^2 + 5z^2, \text{ subject to } x + y + z \leq -23.$$
Using any method check sufficiency conditions.

3. (10 points) Two points on the complex plane are given, $z_1 = 8i$, $z_2 = 6$. Consider the set of all points $z \in \mathbb{C}$ equidistant (having the same distance) from z_1 and z_2.

   Find an equation of this set in terms of z and $\bar{z}$.

4. (10 points) Solve the following differential equation
$$y'' - 7y' + 6y = x \exp(6x).$$

5. (10 points) Solve the linear programming problem
$$\begin{cases} 5x_1 + 5x_2 + 30x_3 \to \min \\ x_1 + 3x_2 + 2x_3 \geq 10 \\ 2x_1 + x_2 + 3x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$
Find the minimal value and the optimal point.

6. (10 points) Special request by Alla Fridman :) Consider one variable minimization problem
$$f(x) = x^2 + 6x + 8, \quad x \in [-2; 5].$$
   (a) Carefully check NDCQ.
   (b) Introduce two Lagrange multipliers and write down first order conditions.
   (c) Which equations or inequalities are called «complementary slackness conditions»?

   Note: You are NOT required to solve FOC nor to find the optimal point.

7. (20 points) Consider $(n \times n)$ matrix A_n with 2 on the main diagonal and 1 just above and below it,

$$A_n = \begin{pmatrix} 2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 2 & 1 & 0 \\ 0 & \cdots & 0 & 1 & 2 & 1 \\ 0 & \cdots & 0 & 0 & 1 & 2 \end{pmatrix}.$$

Let $d_n = \det A_n$.

(a) State the difference equation on d_n, d_{n-1} and d_{n-2}.

(b) Find d_1, d_2 and finally d_n.

8. (20 points) Consider the second order differential equation with constant coefficients

$$y'' + ay' + by = 0.$$

Find necessary and sufficient conditions on a and b that guarantee that

(a) every solution $y(t)$ is bounded for all $t \in \mathbb{R}$;

(b) every solution $y(t)$ tends to zero as $t \to +\infty$.