Name, group no:

. . . .

1. (10 points) Consider the function $f(x, y) = x^3 - 3y^3 + 2xy$. Using the total differential find the approximate value of $f(1.98, 0.99)$.

Name, group no:

. . . .

2. (10 points) Consider the system

$$\begin{cases} 3x^3 + y^3 + z^2 = 5 \\ x + x^3 + 2y^3x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

(b) Find $z'(y)$ if possible.

Name, group no:

. . . .

3. (10 points) Consider the function $h(b) = f(f(f(b \cdot f(b))))$. Find dh/db for $b = 1$ if it is known that $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f'(1) = 3$, $f'(3) = 2$, $f'(2) = 1$.

Name, group no:

. . . .

4. (10 points) Consider the function $f(x, y) = xyz^3$, the vector $v = (1, 2)$ and the point $A = (-1, -1)$.

   (a) Find the gradient of f at the point A.

   (b) Find the directional derivative of f at the point A in the direction given by v.

Name, group no:

. . . .

5. (10 points) Provide an explicit example of a sequence in $\mathbb{R}^2$ that is unbounded and has exactly two accumulation points.

Name, group no:

. . . .

6. Two identical firms compete in a labor market with the supply function $w(L) = w_0 + aL$, where $w_0 > 0$, $a > 0$ and L is the labor amount supplied at the wage rate w.

   In order to find equilibrium one has to solve the system of equations

$$\begin{cases} f(L_1) - ME_1 = 0 \\ f(L_2) - ME_2 = 0 \end{cases},$$

   where $f'(L) < 0$ for all $L > 0$ and ME_1, ME_2 are marginal expenses which are found by differentiation, $ME_i = \partial(w(L)L_i)/\partial L_i$ for $i \in \{1, 2\}$ and $L = L_1 + L_2$.

   Suppose the equilibrium exists.

   (a) (10 points) Prove that $L_1^* = L_2^*$.

   (b) (15 points) Find $\partial L_1^*/\partial w_0$.

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Name, group no:

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2. (10 points) Consider the system

$$\begin{cases} -3x^3 + y^3 + z^2 = -1 \\ x + x^3 + 2y^3x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

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Name, group no:

. . . .

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$$\begin{cases} 5x^3 + y^3 + z^2 = 7 \\ x + x^3 + 2y^3x = 4 \end{cases}$$

(a) Check whether the functions $z(y)$ and $x(y)$ are defined at a point $(1, 1, 1)$;

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