Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x,y\to 0} \frac{1-\cos(x+2y)}{\sin(xy)}$$

Name, group no:

. . . .

2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 + 4xy + y^2$ subject to $x^2 + 2y^2 = 16$.

Name, group no:

. . . .

3. (10 points) Consider the function $u(x, y) = x^2 - 4xy + ay^2 - \ln(xy)$ for $x > 0$ and $y > 0$. For which values of a the function u is convex?

Name, group no:

. . . .

4. (10 points) Find the second order Taylor approximation of a function $f(x, y) = x^5y^3 + 3x^2y$ at a point $x = 1$, $y = 2$.

Name, group no:

. . . .

5. (10 points) Use Lagrange multipliers to find the height and radius of a cylinder with the maximal volume among those with a surface $S = 10\pi$. Make sure you check the second order condition for maximisation.

Name, group no:

. . . .

6. (10 points) Let $h(x, y) = kx^2 + 6xy + 14y^2 + 4y + 10$.

   (a) Find the minimal value of the function h for $k = 2$.

   (b) Using envelope theorem find approximate minimal value of h for $k = 1.98$.

Name, group no:

....

7. This is a road construction costs minimization problem. Let the terrain profile be represented by the function $y(t) = \begin{cases} 3 - 3|t|, \text{ if } |t| \leq 1 \\ 0, \text{ otherwise} \end{cases}$. A road works start from the east of this hill (on the negative half-axis). Excavation costs can be found by the formula

$$I(a,b) = \int_{-b/a}^{0} (at + b - y(t))^2 \, dt$$

where $at + b$ is the road profile we need to find with the constants $a > 0$, $b > 0$, and $b/a \geq 1$.

(a) (15 points) Find the Hessian matrix of $I(a, b)$ and check its sign-definitness.

(b) (5 points) Let (a^*, b^*) be the solution of the first-order conditions for the minimization problem. Justify your choice for the (a^*, b^*) values.

Name, group no:

. . . .

8. (continuation of problem 7) (20 points)

   Allowable grade of the road satisfies constraint $a \leq 1$. Under this constraint solve the problem $I(a, b) \to \min$ with respect to b.

Name, group no:

. . . .

1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x,y\to 0} \frac{1-\cos(x+3y)}{\sin(xy)}$$

Name, group no:

. . . .

2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 + 6xy + y^2$ subject to $x^2 + 2y^2 = 16$.

Name, group no:

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3. (10 points) Consider the function $u(x, y) = x^2 - 6xy + ay^2 - \ln(xy)$ for $x > 0$ and $y > 0$. For which values of a the function u is convex?

Name, group no:

. . . .

4. (10 points) Find the second order Taylor approximation of a function $f(x,y) = x^5y^3 + 4x^2y$ at a point $x = 1, y = 2$.

Name, group no:

. . . .

5. (10 points) Use Lagrange multipliers to find the height and radius of a cylinder with the maximal volume among those with a surface $S = 12\pi$. Make sure you check the second order condition for maximisation.

Name, group no:

. . . .

6. (10 points) Let $h(x, y) = kx^2 + 4xy + 14y^2 + 4y + 10$.

   (a) Find the minimal value of the function h for $k = 2$.

   (b) Using envelope theorem find approximate minimal value of h for $k = 1.98$.

Name, group no:

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where $at + b$ is the road profile we need to find with the constants $a > 0$, $b > 0$, and $b/a \geq 1$.

(a) (15 points) Find the Hessian matrix of $I(a, b)$ and check its sign-definitness.

(b) (5 points) Let (a^*, b^*) be the solution of the first-order conditions for the minimization problem. Justify your choice for the (a^*, b^*) values.

Name, group no:

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8. (continuation of problem 7) (20 points)

   Allowable grade of the road satisfies constraint $a \leq 1$. Under this constraint solve the problem $I(a, b) \to \min$ with respect to b.

Name, group no:

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1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x,y\to 0}\frac{1-\cos(x+4y)}{\sin(xy)}$$

Name, group no:

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Name, group no:

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3. (10 points) Consider the function $u(x, y) = x^2 - 8xy + ay^2 - \ln(xy)$ for $x > 0$ and $y > 0$. For which values of a the function u is convex?

Name, group no:

. . . .

4. (10 points) Find the second order Taylor approximation of a function $f(x, y) = x^5y^3 + 7x^2y$ at a point $x = 1$, $y = 2$.

Name, group no:

. . . .

5. (10 points) Use Lagrange multipliers to find the height and radius of a cylinder with the maximal volume among those with a surface $S = 14\pi$. Make sure you check the second order condition for maximisation.

Name, group no:

. . . .

6. (10 points) Let $h(x, y) = kx^2 + 6xy + 12y^2 + 4y + 10$.
   (a) Find the minimal value of the function h for $k = 2$.
   (b) Using envelope theorem find approximate minimal value of h for $k = 1.98$.

Name, group no:

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Name, group no:
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8. (continuation of problem 7) (20 points)

   Allowable grade of the road satisfies constraint $a \leq 1$. Under this constraint solve the problem $I(a, b) \to \min$ with respect to b.

Name, group no:

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1. (10 points) Find the limit or prove that it does not exist

$$\lim_{x,y \to 0} \frac{1 - \cos(x + 5y)}{\sin(xy)}$$

Name, group no:

. . . .

2. (10 points) Using Lagrange multipliers find the extrema of the function $f(x, y) = x^2 - 4xy + y^2$ subject to $x^2 + 2y^2 = 16$.

Name, group no:

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Name, group no:

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Name, group no:

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Name, group no:

. . . .

8. (continuation of problem 7) (20 points)

   Allowable grade of the road satisfies constraint $a \leq 1$. Under this constraint solve the problem $I(a, b) \to \min$ with respect to b.