

Name, group no:

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1. (10 points) Consider the function

$$f(x, y) = 2x^2 + \frac{9}{2}y^2 + 2cxy + d.$$

Find all values of c and d such that the function f is convex everywhere.

Name, group no:

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2. (10 points) Minimize the function

$$f(x, y, z) = x^2 + 3y^2 + 5z^2, \text{ subject to } x + y + z \leq -23.$$

Using any method check sufficiency conditions.

Name, group no:

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3. (10 points) Two points on the complex plane are given, $z_1 = 8i$, $z_2 = 6$. Consider the set of all points $z \in \mathbb{C}$ equidistant (having the same distance) from z_1 and z_2 .

Find an equation of this set in terms of z and \bar{z} .

Name, group no:

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4. (10 points) Solve the following differential equation

$$y'' - 7y' + 6y = x \exp(6x).$$

Name, group no:

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5. (10 points) Solve the linear programming problem

$$\begin{cases} 5x_1 + 5x_2 + 30x_3 \rightarrow \min \\ x_1 + 3x_2 + 2x_3 \geq 10 \\ 2x_1 + x_2 + 3x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Find the minimal value and the optimal point.

Name, group no:

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6. (10 points) Special request by Alla Fridman :) Consider one variable minimization problem

$$f(x) = x^2 + 6x + 8, \quad x \in [-2; 5].$$

- (a) Carefully check NDCQ.
- (b) Introduce two Lagrange multipliers and write down first order conditions.
- (c) Which equations or inequalities are called «complementary slackness conditions»?

Note: You are NOT required to solve FOC nor to find the optimal point.

Name, group no:

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7. (20 points) Consider $(n \times n)$ matrix A_n with 2 on the main diagonal and 1 just above and below it,

$$A_n = \begin{pmatrix} 2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 2 & 1 & 0 \\ 0 & \cdots & 0 & 1 & 2 & 1 \\ 0 & \cdots & 0 & 0 & 1 & 2 \end{pmatrix}.$$

Let $d_n = \det A_n$.

- (a) State the difference equation on d_n , d_{n-1} and d_{n-2} .
- (b) Find d_1 , d_2 and finally d_n .

Name, group no:

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8. (20 points) Consider the second order differential equation with constant coefficients

$$y'' + ay' + by = 0.$$

Find necessary and sufficient conditions on a and b that guarantee that

- (a) every solution $y(t)$ is bounded for all $t \in \mathbb{R}$;
- (b) every solution $y(t)$ tends to zero as $t \rightarrow +\infty$.

Name, group no:

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1. (10 points) Consider the function

$$f(x, y) = \frac{3}{2}x^2 + \frac{27}{2}y^2 + 2cxy + d.$$

Find all values of c and d such that the function f is convex everywhere.

Name, group no:

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2. (10 points) Minimize the function

$$f(x, y, z) = 3x^2 + y^2 + 5z^2, \text{ subject to } x + y + z \leq -23.$$

Using any method check sufficiency conditions.

Name, group no:

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3. (10 points) Two points on the complex plane are given, $z_1 = 6i$, $z_2 = 4$. Consider the set of all points $z \in \mathbb{C}$ equidistant (having the same distance) from z_1 and z_2 .

Find an equation of this set in terms of z and \bar{z} .

Name, group no:

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4. (10 points) Solve the following differential equation

$$y'' - 5y' + 4y = x \exp(4x).$$

Name, group no:

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5. (10 points) Solve the linear programming problem

$$\begin{cases} 5x_1 + 5x_2 + 30x_3 \rightarrow \min \\ x_1 + 3x_2 + 3x_3 \geq 10 \\ 2x_1 + x_2 + 2x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Find the minimal value and the optimal point.

Name, group no:

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6. (10 points) Special request by Alla Fridman :) Consider one variable minimization problem

$$f(x) = x^2 + 6x + 9, \quad x \in [-2; 5].$$

- (a) Carefully check NDCQ.
- (b) Introduce two Lagrange multipliers and write down first order conditions.
- (c) Which equations or inequalities are called «complementary slackness conditions»?

Note: You are NOT required to solve FOC nor to find the optimal point.

Name, group no:

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7. (20 points) Consider $(n \times n)$ matrix A_n with 2 on the main diagonal and 1 just above and below it,

$$A_n = \begin{pmatrix} 2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 2 & 1 & 0 \\ 0 & \cdots & 0 & 1 & 2 & 1 \\ 0 & \cdots & 0 & 0 & 1 & 2 \end{pmatrix}.$$

Let $d_n = \det A_n$.

- (a) State the difference equation on d_n , d_{n-1} and d_{n-2} .
- (b) Find d_1 , d_2 and finally d_n .

Name, group no:

.....

8. (20 points) Consider the second order differential equation with constant coefficients

$$y'' + ay' + by = 0.$$

Find necessary and sufficient conditions on a and b that guarantee that

- (a) every solution $y(t)$ is bounded for all $t \in \mathbb{R}$;
- (b) every solution $y(t)$ tends to zero as $t \rightarrow +\infty$.

Name, group no:

.....

1. (10 points) Consider the function

$$f(x, y) = 8x^2 + 2y^2 + 2cxy + d.$$

Find all values of c and d such that the function f is convex everywhere.

Name, group no:

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2. (10 points) Minimize the function

$$f(x, y, z) = 5x^2 + 3y^2 + z^2, \text{ subject to } x + y + z \leq -23.$$

Using any method check sufficiency conditions.

Name, group no:

.....

3. (10 points) Two points on the complex plane are given, $z_1 = 4i$, $z_2 = 6$. Consider the set of all points $z \in \mathbb{C}$ equidistant (having the same distance) from z_1 and z_2 .

Find an equation of this set in terms of z and \bar{z} .

Name, group no:

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4. (10 points) Solve the following differential equation

$$y'' - 9y' + 8y = x \exp(8x).$$

Name, group no:

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5. (10 points) Solve the linear programming problem

$$\begin{cases} 6x_1 + 6x_2 + 36x_3 \rightarrow \min \\ x_1 + 3x_2 + 2x_3 \geq 10 \\ 2x_1 + x_2 + 3x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Find the minimal value and the optimal point.

Name, group no:

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6. (10 points) Special request by Alla Fridman :) Consider one variable minimization problem

$$f(x) = x^2 + 6x - 7, \quad x \in [-2; 5].$$

- (a) Carefully check NDCQ.
- (b) Introduce two Lagrange multipliers and write down first order conditions.
- (c) Which equations or inequalities are called «complementary slackness conditions»?

Note: You are NOT required to solve FOC nor to find the optimal point.

Name, group no:

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7. (20 points) Consider $(n \times n)$ matrix A_n with 2 on the main diagonal and 1 just above and below it,

$$A_n = \begin{pmatrix} 2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 2 & 1 & 0 \\ 0 & \cdots & 0 & 1 & 2 & 1 \\ 0 & \cdots & 0 & 0 & 1 & 2 \end{pmatrix}.$$

Let $d_n = \det A_n$.

- (a) State the difference equation on d_n , d_{n-1} and d_{n-2} .
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Name, group no:

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- (a) every solution $y(t)$ is bounded for all $t \in \mathbb{R}$;
- (b) every solution $y(t)$ tends to zero as $t \rightarrow +\infty$.

Name, group no:

.....

1. (10 points) Consider the function

$$f(x, y) = 2x^2 + \frac{25}{2}y^2 + 2cxy + d.$$

Find all values of c and d such that the function f is convex everywhere.

Name, group no:

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2. (10 points) Minimize the function

$$f(x, y, z) = 5x^2 + y^2 + 3z^2, \text{ subject to } x + y + z \leq -23.$$

Using any method check sufficiency conditions.

Name, group no:

.....

3. (10 points) Two points on the complex plane are given, $z_1 = 2i$, $z_2 = 6$. Consider the set of all points $z \in \mathbb{C}$ equidistant (having the same distance) from z_1 and z_2 .

Find an equation of this set in terms of z and \bar{z} .

Name, group no:

.....

4. (10 points) Solve the following differential equation

$$y'' - 4y' + 3y = x \exp(3x).$$

Name, group no:

.....

5. (10 points) Solve the linear programming problem

$$\begin{cases} 6x_1 + 6x_2 + 36x_3 \rightarrow \min \\ x_1 + 3x_2 + 3x_3 \geq 10 \\ 2x_1 + x_2 + 2x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}.$$

Find the minimal value and the optimal point.

Name, group no:

.....

6. (10 points) Special request by Alla Fridman :) Consider one variable minimization problem

$$f(x) = x^2 + 6x + 2, \quad x \in [-2; 5].$$

- (a) Carefully check NDCQ.
- (b) Introduce two Lagrange multipliers and write down first order conditions.
- (c) Which equations or inequalities are called «complementary slackness conditions»?

Note: You are NOT required to solve FOC nor to find the optimal point.

Name, group no:

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Name, group no:

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