Home Assignment 1 – Burst Image Deblurring

Burst mode is a modality available in virtually all modern digital cameras and it is a powerful and fun "hidden" feature in the phone native camera apps allowing to continuously capture multiple photos. It is especially useful for capturing the perfect moment when your subjects are moving. It is also a good idea to use it when your hand or camera is moving – in order to give you the best chance of getting a sharp shot with good composition. In addition that is the way that is used for catching pictures through videos and detection like car detection. Many other applications can be useful for this mode such as taking space pictures and etc. But, videos and bursts captured with hand-held cameras often suffer from a significant amount of blur, mainly caused by the inevitable natural tremor of the photographer's hand.

In this assignment we are going to simulate a very basic burst mode and a camera motion blur due to the hand movement synthesizing an images burst (sequence) from a source image.

The settings in the assignment will be simplified – we will assume that the camera is moving and the hand is shaking in one plane, parallel to the image plane and all objects are at the same depth relative to the camera. Then we are going to take those blurred images burst and reproduce the original image that should be as close as possible to the original one.

As the imaging setting we assume a static planar scene parallel to the image sensor (i.e., all objects are at the same depth relative to the camera) being captured in a burst mode, i.e., a rapid sequence of N frames is captured at a frame-rate of $f = 1msec^{-1}$, while the camera moves due to the natural shaking of the hand.

i.e. only to the sides.

The camera motion is assumed to be only parallel to the image plane (i.e., no off-plane rotation or translation. You can imagine that all the pictures in the burst are taken while the camera is moving on the same flat plane with lenses parallel to the image).

The motion trajectory (path) of the camera shake movement is given as the location of the sensor center $o(t)$ at time t in image plane units.

only one channel

Image sensor has monochrome pixels arranged in an orthogonal grid with a $1\,\mu m$ step in each direction. Each pixel can be assumed to have an ideal box response of size $1\,\mu m \times 1\,\mu m$ (that is, it integrates light in the said box centered around the pixel location).

So if the grid size is 3×3 for example, one pixel will look on the grid as $1\mu m \times 1\mu m$ rectangle, like shown in the figure below:

Summary:

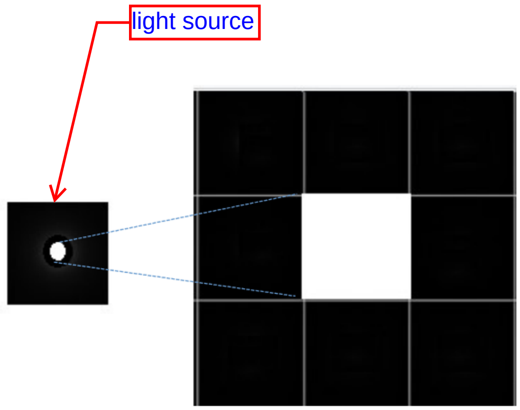
N - num of frames per burst

f - frame rate of burst [1msec]^-1

o(t) - location of sensor center in time t. [image planes]

pixel - 1um x 1um - ideal box response.

shutter time - 1msec i.e. 1/f



The camera optics can be assumed an ideal anti-aliasing filter, so that the imaged scenes are band-limited according to the sampling rate of the pixels.

The camera has a global shutter that raises instantly to 1 when the exposure starts and drops immediately to 0 when the exposure finishes. The pixel integrates light in this interval. The same integration time $T = 1\,msec$ can be assumed for all frames in the burst. **Imaging noise can be considered negligible.**

1 Theoretical questions

1. Let $f(\mathbf{x})$ be the image that would be formed on the image plane if the camera were absolutely still. The actually formed digital image of the k-th frame can be described as

Point-Spread Function (PSF) or impulse response. The PSF is the output of the imaging system for an input point source.

window function

$$g_k[\mathbf{n}] = (f * h_k)(\mathbf{n}),$$

where $h_k(\mathbf{x})$ is the effective point spread function corresponding to the k-th frame. Express h_k in terms of the camera motion. Camera path is given by o(t)

2. Express the Fourier transform of h_k.

3. Show that the action of the camera can be expressed fully in the digital domain as

$$G_k[\boldsymbol{\omega}] = F[\boldsymbol{\omega}] \cdot P_k[\boldsymbol{\omega}] \,\mathrm{sinc}\,(\boldsymbol{\omega})\,.$$

Write an expression for the frequency response of the discrete kernel P_k.

4. Write an upper bound on $|P_k[\boldsymbol{\omega}]|$.

5. Assume that on the interval $\tau \in [k, k+1)$, the camera moves with constant velocity v in the horizontal direction, $\mathbf{o}(\tau) = (\tau - k - 0.5)\, v\, \mathbf{e}_1$, where $\mathbf{e}_1 = (1, 0, 0)^{\mathrm{T}}$. Express $P_k[\boldsymbol{\omega}]$ in this case. How does $|P_k[\boldsymbol{\omega}]|$ depend on v?

6. Generalize the previous result to the case where on the interval $\tau \in [k, k+1)$, $\mathbf{o}(\tau) = \mathbf{q} + \tau\,\mathbf{v}$.

7. Suppose that N frames have been acquired such that in the k-th frame the camera was moving with some *unknown* but constant velocity $\mathbf{v}_k$. Given the frames $g_1, \ldots, g_N$ that have been pre-aligned, suggest a way to estimate f.

2 Practical questions

To do this exercise, please download the following materials:

- The source image: DIPSourceHW1.jpg
- The file with 100 trajectories (100_motion_paths.mat). Each trajectory k ($k = 0, \ldots, 99$) corresponds to the exposure time of the k^{th} frame and is sampled at 256 points $(X_k(i), Y_k(i) \mid i = 0 \ldots 256)$. Note that all the trajectories are aligned in the sense that every trajectory has a zero mean.

Part I: Forward problem synthesis

- Plot the trajectories resulted by the handshake
- Generate and display the discrete point spread functions corresponding to each trajectory.
- Generate the blurred frame for each trajectory.

Part II: Solving the inverse problem

Now assume that you do not have the source image, and do not know the PSFs. You only have the blurred images burst that were generated in the previous assignment. Now your mission is to restore the source image - to get a new image that will be as close as possible to the original one.

Your measurement for the quality of your restoration will be the PSNR (peak signal to noise ratio) with respect to the source image (that is assumed known only for PSNR evaluation!).

Use the results of the theoretical part to produce an estimate of the source image given the first k frames in the burst. Plot the PSNR as a function of the number of frames.

3 Deliverables

- **For Part I:** Write the answers for questions 1-7 in a digital file.

- **For Part II:**
  - The code
  - 100 PSF matrices
  - 100 blurred images
  - 100 deblurred images
  - 100 PSNR values

Submit all the above mentioned deliverables by **19th December, 11:55PM**.

Submission email will be uploaded soon.

Thanks and enjoy!
Alex and Sanketh