Optimization – Dry

Question 1

Let a,b be two points on the surface C. since C is a convex set, the line that connects the two points lies entirely within C, and f1 and f2 are convex on every point of that line. We will show that g(x) is convex by definition. Let , then we need to show that

Let’s take a look at the two expressions. Since f1(x) is convex in this line:

From symmetry this is true for the other expression inside the maximum. Hence we get:

So g is convex in C.

Question 2

Let . It implies that . We will show that any point on the line between points a and b is within the set L. For every choice of , the point . Let’s take a look at the value of the function f in such a point.

Because f is convex:

Question 3 –

Let y be a vector of the appropriate size. We will show the positive semidefiniteness of the hessian directly.

Ay is a vector, and since hessian of f is positive semidefinite:

In conclusion, g has a positive semidefinite hessian which means g is convex.