MACHINE LEARNING

FRACTAL-3 ASSIGNMENT

Problem 1: Perceptron

Following training samples are given

x_1	x_2	class
1	1	+1
-1	-1	-1
0	0.5	-1
0.1	0.5	-1
0.2	0.2	+1
0.9	0.5	+1

Assuming weight vector & initial decision boundary $w^T x = 0$ as $w = [1, 1]$, Solve the following:

1. In how many steps perceptron learning algorithm will converge.
2. What will be the final decision boundary? Show step-wise-step update & weight vector using computation as well as hand-drawn plot.

Soln:

Given:

Assuming weight vector of initial decision boundary $W^T x = 0$ as $w = [1, 1]$

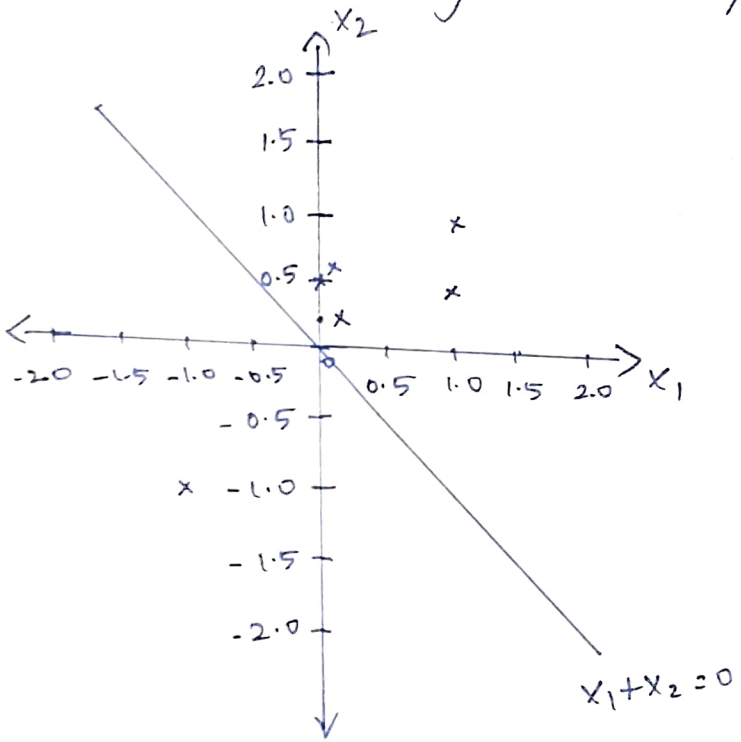
$\Rightarrow x_1 + x_2 = 0$

$b = 0$

$y_{in} = w_i^T x_i + b$

$= w_1 x_1 + w_1 x_2 + b$

Initial decision boundary and samples



Assuming learning rate as 1.

$$y = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$$

$\Delta w_1 = \alpha t x_1$

$\Delta w_2 = \alpha t x_2$

$\Delta b = \alpha t$

Iteration 1:

x_1	x_2	dan (t)	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	2	+1	0	0	0	1	1	0
-1	-1	-1	-2	-1	0	0	0	1	1	0
0	0.5	-1	0.5	+1	0	-0.5	-1	1	0.5	-1
0.1	0.5	-1	-0.65	-1	0	0	0	1	0.5	-1
0.2	0.2	+1	-0.7	-1	0.2	0.2	1	1.2	0.7	0
0.9	0.5	+1	1.43	+1	0	0	0	1.2	0.7	0

Iteration 2:

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.9	+1	0	0	0	1.2	0.7	0
-1	-1	-1	-1.9	-1	0	0	0	1.2	0.7	0
0	0.5	-1	0.35	+1	0	-0.5	-1	1.2	0.2	-1
0.1	0.5	-1	-0.78	-1	0	0	0	1.2	0.2	-1
0.2	0.2	+1	-0.72	-1	0.2	0.2	1	1.4	0.4	0
0.9	0.5	+1	1.46	+1	0	0	0	1.4	0.4	0

Iteration 3:

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.8	+1	0	0	0	1.4	0.4	0
-1	-1	-1	-1.8	-1	0	0	0	1.4	0.4	0
0	0.5	-1	0.2	+1	0	-0.5	-1	1.4	-0.1	-1
0.1	0.5	-1	-0.81	-1	0	0	0	1.4	-0.1	-1
0.2	0.2	+1	-0.74	-1	0.2	0.2	1	1.6	0.1	0
0.9	0.5	+1	1.49	+1	0	0	0	1.6	0.1	0

Iteration 4:

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.7	+1	0	0	0	1.6	0.1	0
-1	-1	-1	-1.7	-1	0	0	0	1.6	0.1	0
0	0.5	-1	0.05	+1	0	-0.5	-1	1.6	-0.4	-1
0.1	0.5	-1	-1.04	-1	0	0	0	1.6	-0.4	-1
0.2	0.2	+1	-0.76	-1	0.2	0.2	1	1.8	-0.2	0
0.9	0.5	+1	1.52	+1	0	0	0	1.8	-0.2	0

Iteration 5:

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.6	+1	0	0	0	1.8	-0.2	0
-1	-1	-1	-1.6	-1	0	0	0	1.8	-0.2	0
0	0.5	-1	-0.1	-1	0	0	0	1.8	-0.2	0
0.1	0.5	-1	0.08	+1	-0.1	-0.5	-1	1.7	-0.7	-1
0.2	0.2	+1	-0.8	-1	0.2	0.2	1	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

Iteration 6:

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.4	+1	0	0	0	1.9	-0.5	0
-1	-1	-1	-1.4	-1	0	0	0	1.9	-0.5	0
0	0.5	-1	-0.25	-1	0	0	0	1.9	-0.5	0
0.1	0.5	-1	-0.06	-1	0	0	0	1.9	-0.5	0
0.2	0.2	+1	0.28	+1	0	0	0	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

The perceptron learning algorithm has converged in 6 Iterations.

The final weight vector of the decision boundary is $w = [1.9, -0.5]$

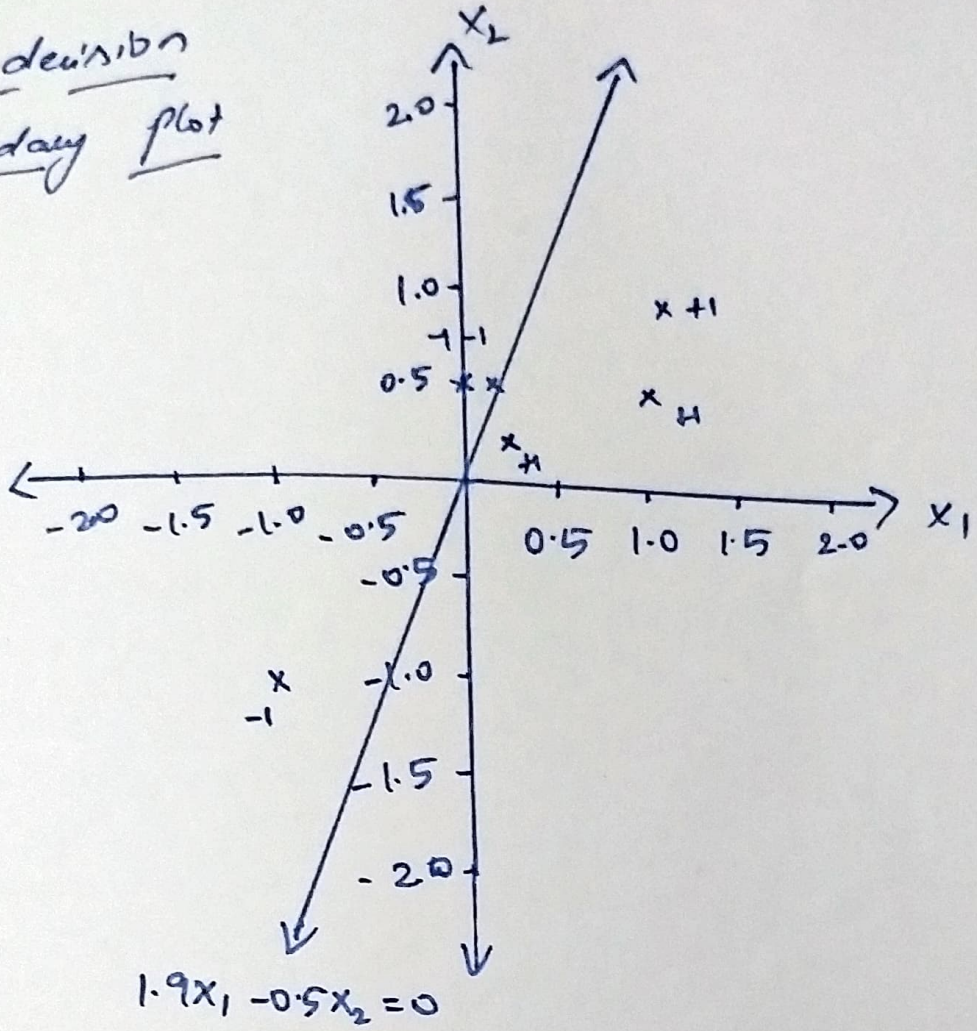
$$1.9 x_1 + (-0.5) x_2 = 0$$

$$\Rightarrow 1.9 x_1 - 0.5 x_2 = 0$$

Plot of the final decision boundary.

we can see that $1.9 x_1 - 0.5 x_2 = 0$ line seperates the two classes correctly.

Final decision boundary plot



Neural network corresponding to the perceptron