A Proof of Theorem 1

In this section, we prove that with Algorithm 1, Theorem 1 is true. The assumption of Algorithm 1 is that no pause frame is ever corrupted or loss. Same as Section IV-C, we denote the variables at the n-th time slot with a subscript n. Symbols involved in the proof are listed int Table A1. Note that there are several restrictions. First, the total buffer reserved for the queue H should be configured as $H \geq \Delta + 2R_sT$. Second, without loss of generality, RTT is divisible by T so that $N = \frac{RTT}{T}$. Third, as the discussion in Section III-B, without loss of generality, $\tilde{R}_n \leq R_s$.

Table A1. Symbols

Symbol	Description
L_n	queue length at the end of n-th time slot
F_n	in-flight bytes of downstream port at the end of n-th time cycle
c_n	granted bytes calculated at the end of n-th time slot
$\tilde{R}_n$	average draining rate during the n-th time slot
R_s	full sending rate
H	total buffer reserved for the queue
T	time slot duration
Δ	BDP of long distance link
N	number of time slots in one RTT

According to the Algorithm 1, the transition of variables in time slot n is shown in Equation 1. Since the calculation of c_n involves a branch, we will discuss them in both cases.

$$L_n = L_{n-1} + c_{n-(N+1)} - \tilde{R}_n T \tag{1a}$$
$$c_n = \min(R_sT, H - L_n - F_{n-1}) \tag{1b}$$
$$F_n = F_{n-1} - c_{n-(N+1)} + c_n \tag{1c}$$

Case 1: when $R_sT \leq H - L_n - F_{n-1}$, we have $c_n = R_sT$, and,

$$\begin{aligned} L_n &\leq H - R_sT - F_{n-1} \\ &= H - R_sT - (F_n + c_{n-(N+1)} - c_n) \\ &= H - R_sT - F_n - c_{n-(N+1)} + R_sT \\ &= H - c_{n-(N+1)} - F_n \end{aligned} \tag{2}$$

With Equation 2, $L_n + F_n \leq H - c_{n-(N+1)} \leq H$.
On the other hand, by adding Equation 1a and 1c, we have,

$$\begin{aligned} L_n + F_n &= L_{n-1} + F_{n-1} + c_n - \tilde{R}_n T \\ &= L_{n-1} + F_{n-1} + (R_s - \tilde{R}_n)T \\ &\geq L_{n-1} + F_{n-1} \end{aligned} \tag{3}$$

With mathematical induction, if

$$L_{n-1} + F_{n-1} \geq \Delta + R_sT \tag{4}$$

then $L_n + F_n \geq \Delta + R_sT$. Since $L_1 + F_1 \geq \Delta + R_sT$, we can infer that $\forall n \in N^+$, $L_n + F_n \geq \Delta + R_sT$.
Now, we conclude that when $R_sT \leq H - L_n - F_{n-1}$, $\Delta + R_sT \leq L_n + F_n \leq H$.
Case 2: when $R_sT > H - L_n - F_{n-1}$, we have $c_n = H - L_n - F_{n-1} < R_sT$. According to Equation 1c,

$$\begin{aligned} F_n &= F_{n-1} - c_{n-(N+1)} + c_n \\ &= F_{n-1} - c_{n-(N+1)} + H - L_n - F_{n-1} \\ &= H - c_{n-(N+1)} - L_n \end{aligned} \tag{5}$$

so that,

$$L_n + F_n = H - c_{n-(N+1)} \leq H \tag{6}$$

At the same time, we have configured that $H \geq \Delta + 2R_sT$, so that,

$$\begin{aligned} L_n + F_n &= H - c_{n-(N+1)} \\ &\geq \Delta + 2R_sT - c_{n-(N+1)} \\ &\geq \Delta + R_sT \end{aligned} \tag{7}$$

Now we can conclude that when $R_sT > H - L_n - F_{n-1}$, $\Delta + R_sT \leq L_n + F_n \leq H$.

To sum up, for both cases in Equation 1b, $\Delta + R_sT \leq L_n + F_n \leq H$ holds, which means Theorem 1 is true.