Exercise Number 12 Date: 26-03-2025

Multi Layer Perceptron

Aim:

To perform Multi Layer Perceptron on Linearly and Non-Linearly Separable Data

Algorithm

Model Overview

Multi-Layer Perceptron (MLP) Overview

The **Multi-Layer Perceptron (MLP)** is a class of feedforward artificial neural networks used for both binary and multi-class classification. It consists of an input layer, one or more hidden layers, and an output layer. Each layer is composed of neurons with non-linear activation functions that allow the network to model complex, non-linear relationships in data. The MLP is trained using backpropagation and gradient descent to minimize the prediction error by adjusting weights and biases across all layers.

Steps Involved

1. **Data Loading:**
   Two synthetic datasets are generated – one that is linearly separable and another that is non-linearly separable.

2. **Data Exploration:**
   The datasets are visualized using scatter plots to understand their underlying structure and complexity.

3. **Data Preprocessing:**
   - **Feature Scaling:** Standardization is applied to ensure that each input feature contributes equally to learning, improving convergence speed.
4. **Model Training:**
   An MLP model is implemented from scratch with at least one hidden layer. It is trained on both datasets using stochastic gradient descent (SGD) with backpropagation.

   The weight and bias updates follow the backpropagation algorithm, which includes:

   - Forward pass to compute predictions
   - Loss computation (e.g., mean squared error or cross-entropy)
   - Backward pass to compute gradients
   - Parameter updates using the rule:

   W_new = W_old - learning_rate * ∂Loss/∂W

5. **Model Evaluation:**
   The trained MLP is evaluated using accuracy and loss metrics on the test set. Decision boundaries are visualized to demonstrate the network's ability to separate classes, even in non-linear settings.

Code and Output

Linearly Separable Data

Import necessary libraries

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_moons
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
```

Load the dataset

In [2]:
```
X_linear, y_linear = make_classification(n_samples=500, n_features=2, n_informat
                                         n_redundant=0, n_clusters_per_class=1,
```

Preprocess the data

In [3]:
```
scaler = StandardScaler()
X_linear = scaler.fit_transform(X_linear)
```

Split the Data

In [4]:
```
X_train_lin, X_test_lin, y_train_lin, y_test_lin = train_test_split(X_linear, y_
```

Multi Layer Perceptron

In [5]:
```
class MultiLayerPerceptron:
    def __init__(self, input_dim, hidden_dim, output_dim=1, learning_rate=0.1, e
        self.learning_rate = learning_rate
        self.epochs = epochs
        self.W1 = np.random.randn(input_dim, hidden_dim) * 0.1
        self.b1 = np.zeros((1, hidden_dim))
        self.W2 = np.random.randn(hidden_dim, output_dim) * 0.1
        self.b2 = np.zeros((1, output_dim))

    def sigmoid(self, z):
        return 1 / (1 + np.exp(-z))

    def sigmoid_derivative(self, a):
        return a * (1 - a)

    def forward(self, X):
        self.Z1 = np.dot(X, self.W1) + self.b1
        self.A1 = self.sigmoid(self.Z1)
```

```
        self.Z2 = np.dot(self.A1, self.W2) + self.b2
        self.A2 = self.sigmoid(self.Z2)
        return self.A2

    def backward(self, X, y, output):
        m = X.shape[0]
        dZ2 = output - y.reshape(-1, 1)
        dW2 = (1 / m) * np.dot(self.A1.T, dZ2)
        db2 = (1 / m) * np.sum(dZ2, axis=0, keepdims=True)

        dA1 = np.dot(dZ2, self.W2.T)
        dZ1 = dA1 * self.sigmoid_derivative(self.A1)
        dW1 = (1 / m) * np.dot(X.T, dZ1)
        db1 = (1 / m) * np.sum(dZ1, axis=0, keepdims=True)

        self.W2 -= self.learning_rate * dW2
        self.b2 -= self.learning_rate * db2
        self.W1 -= self.learning_rate * dW1
        self.b1 -= self.learning_rate * db1

    def train(self, X, y):
        for _ in range(self.epochs):
            output = self.forward(X)
            self.backward(X, y, output)

    def predict(self, X):
        probs = self.forward(X)
        return (probs >= 0.5).astype(int).flatten()
```

Training the model

In [6]:
```
mlp_linear = MultiLayerPerceptron(input_dim=2, hidden_dim=5)
mlp_linear.train(X_train_lin, y_train_lin)
```

Evaluating the model

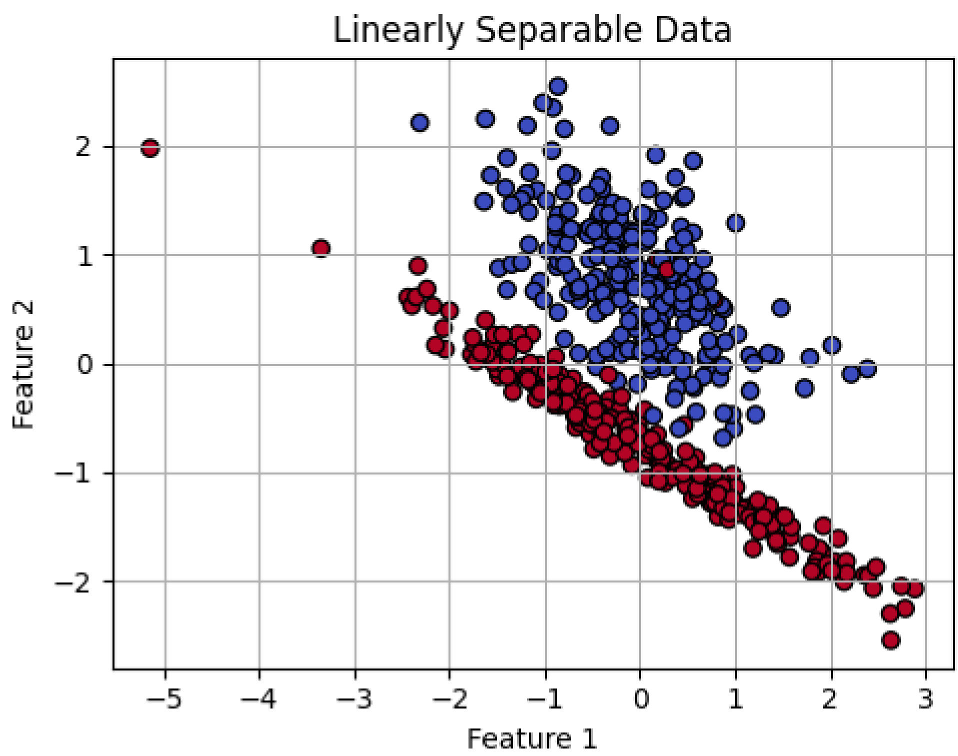
In [7]:
```
y_pred_lin = mlp_linear.predict(X_test_lin)
print("Accuracy (Linear Data):", accuracy_score(y_test_lin, y_pred_lin))
```

```
Accuracy (Linear Data): 0.96
```

Visualisation

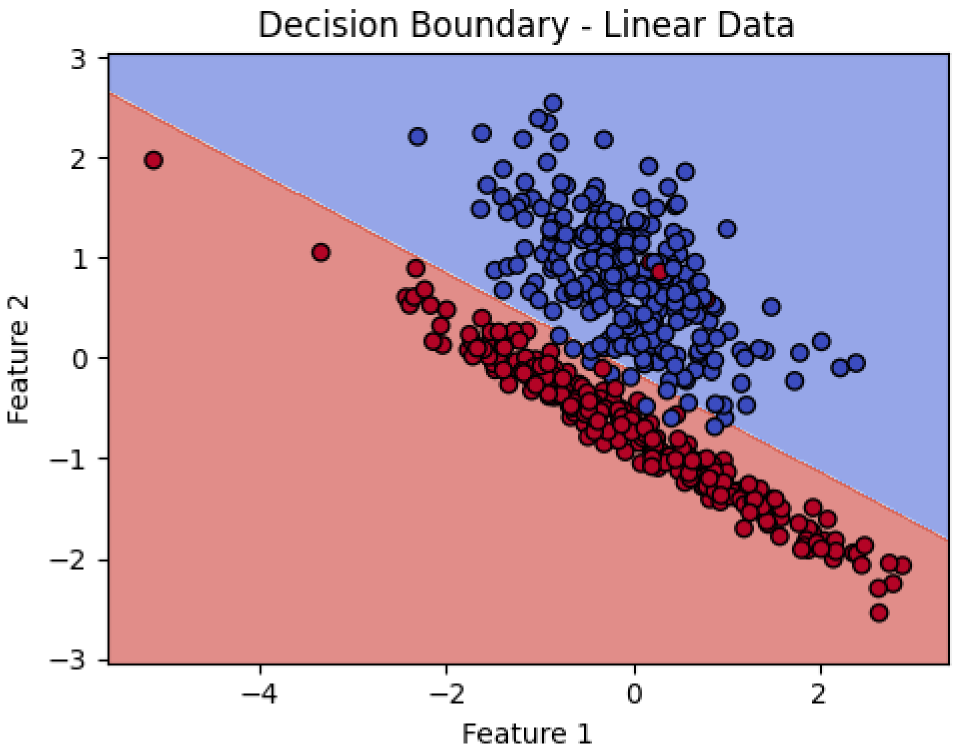
In [8]:
```
def plot_data(X, y, title):
    plt.figure(figsize=(5, 4))
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.coolwarm, edgecolor='k')
    plt.title(title)
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.grid(True)
    plt.tight_layout()
    plt.show()
```

In [9]:
```
plot_data(X_linear, y_linear, "Linearly Separable Data")
```



In [10]:
```
def plot_decision_boundary(model, X, y, title):
    h = 0.02
    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    grid = np.c_[xx.ravel(), yy.ravel()]
    preds = model.predict(grid)
    preds = preds.reshape(xx.shape)
    plt.figure(figsize=(5, 4))
    plt.contourf(xx, yy, preds, cmap=plt.cm.coolwarm, alpha=0.6)
    plt.scatter(X[:, 0], X[:, 1], c=y, edgecolor='k', cmap=plt.cm.coolwarm)
    plt.title(title)
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.tight_layout()
    plt.show()
```

In [11]:
```
plot_decision_boundary(mlp_linear, X_linear, y_linear, "Decision Boundary - Line
```



<u>Non-Linearly Separable Data</u>

Import necessary libraries

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_moons
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import accuracy_score
```

Load the dataset

In [13]:
```
X_nonlinear, y_nonlinear = make_moons(n_samples=500, noise=0.2, random_state=42)
```

Preprocess the data

In [14]:
```
scaler = StandardScaler()
X_nonlinear = scaler.fit_transform(X_nonlinear)
```

Splitting the data

In [15]:
```
X_train_non, X_test_non, y_train_non, y_test_non = train_test_split(X_nonlinear,
```

Multi Layer Perceptron

In [16]:
```
class MultiLayerPerceptron:
    def __init__(self, input_dim, hidden_dim, output_dim=1, learning_rate=0.1, e
        self.learning_rate = learning_rate
        self.epochs = epochs
        self.W1 = np.random.randn(input_dim, hidden_dim) * 0.1
```

```python
        self.b1 = np.zeros((1, hidden_dim))
        self.W2 = np.random.randn(hidden_dim, output_dim) * 0.1
        self.b2 = np.zeros((1, output_dim))

    def sigmoid(self, z):
        return 1 / (1 + np.exp(-z))

    def sigmoid_derivative(self, a):
        return a * (1 - a)

    def forward(self, X):
        self.Z1 = np.dot(X, self.W1) + self.b1
        self.A1 = self.sigmoid(self.Z1)
        self.Z2 = np.dot(self.A1, self.W2) + self.b2
        self.A2 = self.sigmoid(self.Z2)
        return self.A2

    def backward(self, X, y, output):
        m = X.shape[0]
        dZ2 = output - y.reshape(-1, 1)
        dW2 = (1 / m) * np.dot(self.A1.T, dZ2)
        db2 = (1 / m) * np.sum(dZ2, axis=0, keepdims=True)

        dA1 = np.dot(dZ2, self.W2.T)
        dZ1 = dA1 * self.sigmoid_derivative(self.A1)
        dW1 = (1 / m) * np.dot(X.T, dZ1)
        db1 = (1 / m) * np.sum(dZ1, axis=0, keepdims=True)

        self.W2 -= self.learning_rate * dW2
        self.b2 -= self.learning_rate * db2
        self.W1 -= self.learning_rate * dW1
        self.b1 -= self.learning_rate * db1

    def train(self, X, y):
        for _ in range(self.epochs):
            output = self.forward(X)
            self.backward(X, y, output)

    def predict(self, X):
        probs = self.forward(X)
        return (probs >= 0.5).astype(int).flatten()
```

Training the model

In [17]:
```python
mlp_nonlinear = MultiLayerPerceptron(input_dim=2, hidden_dim=10)
mlp_nonlinear.train(X_train_non, y_train_non)
```

Evaluating the model

In [18]:
```python
y_pred_non = mlp_nonlinear.predict(X_test_non)
print("Accuracy (Non-Linear Data):", accuracy_score(y_test_non, y_pred_non))
```

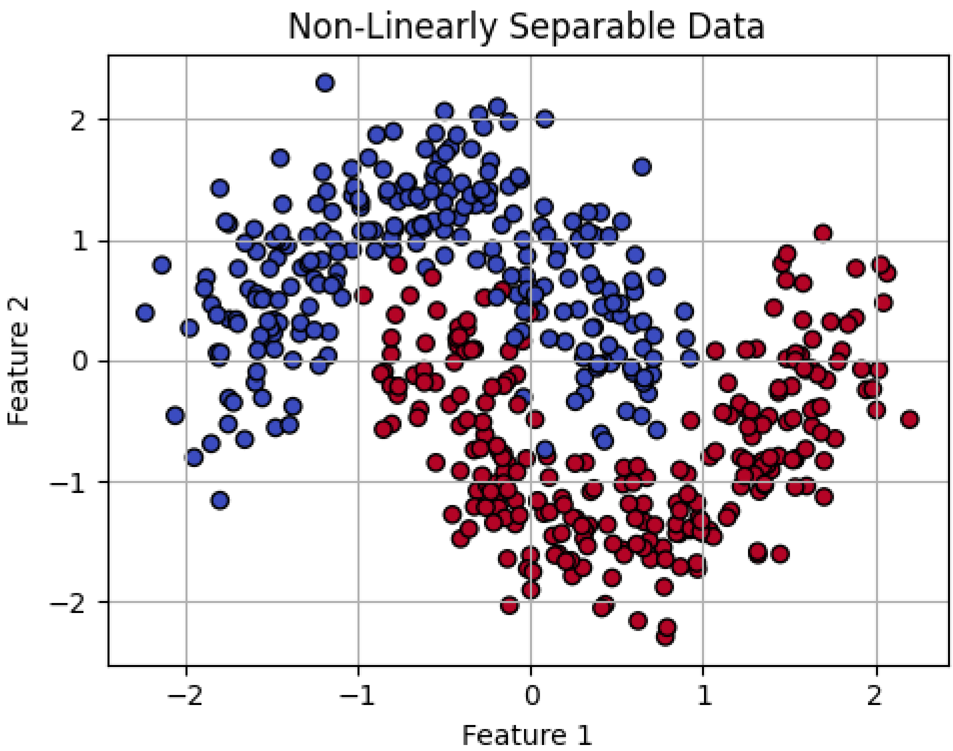
```
Accuracy (Non-Linear Data): 0.86
```

Visualisation

In [19]:
```python
def plot_data(X, y, title):
    plt.figure(figsize=(5, 4))
```

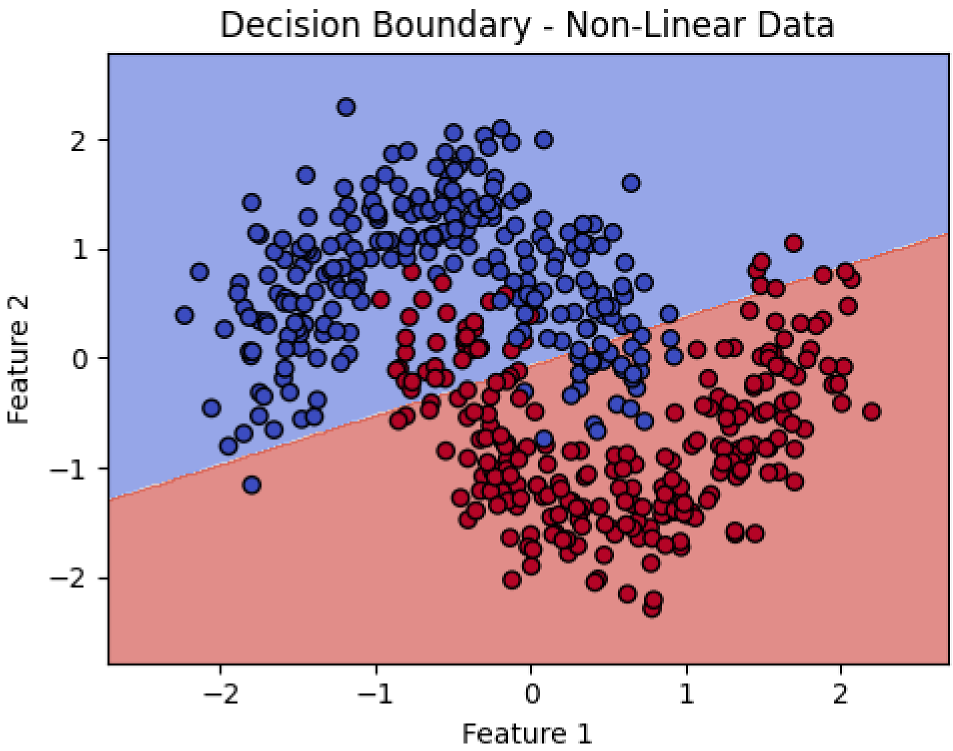
```python
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.coolwarm, edgecolor='k')
    plt.title(title)
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.grid(True)
    plt.tight_layout()
    plt.show()
```

In [20]:
```python
plot_data(X_nonlinear, y_nonlinear, "Non-Linearly Separable Data")
```



In [21]:
```python
def plot_decision_boundary(model, X, y, title):
    h = 0.02
    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    grid = np.c_[xx.ravel(), yy.ravel()]
    preds = model.predict(grid)
    preds = preds.reshape(xx.shape)
    plt.figure(figsize=(5, 4))
    plt.contourf(xx, yy, preds, cmap=plt.cm.coolwarm, alpha=0.6)
    plt.scatter(X[:, 0], X[:, 1], c=y, edgecolor='k', cmap=plt.cm.coolwarm)
    plt.title(title)
    plt.xlabel("Feature 1")
    plt.ylabel("Feature 2")
    plt.tight_layout()
    plt.show()
```

In [22]:
```python
plot_decision_boundary(mlp_nonlinear, X_nonlinear, y_nonlinear, "Decision Bounda
```



Result:

The outputs were verified successfully
