

# Learning local behavioral sequences to better infer non-local properties in real multi-robot systems

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**Abstract**—When members of a multi-robot team follow regular motion rules sensitive to robots and other environmental factors within sensing range, the team itself may become an informational fabric for gaining situational awareness without explicit signalling among robots. In our previous work [1], we used machine learning to develop a scalable module, trained only on data from 3-robot teams, that could predict the positions of all robots in larger multi-robot teams based only on observations of the movement of a robot’s nearest neighbor. Not only was this approach scalable from 3-to-many robots, but it did not require knowledge of the control laws of the robots under observation, as would a traditional observer-based approach. However, performance was only tested in simulation and could only be a substitute for explicit communication for short periods of time or in cases of very low sensing noise. In this work, we apply more sophisticated machine learning methods to data from a physically realized robotic team to develop Remote Teammate Localization (RTL) modules that can be used in realistic environments. To be specific, we adopt Long-Short-Term-Memory (LSTM) [2] to learn the evolution of behaviors in a modular team, which has the effect of greatly reducing errors from regression outcomes. In contrast with our previous work in simulation, all of the experiments conducted in this work were conducted on the *Thymio* physical, two-wheeled robotic platform.

## I. INTRODUCTION

In multi-robot systems including swarms, each robot typically has the capability to observe only a subset of its team members to determine its next action according to relatively simple motion rules. Consequently, these multi-robot systems are fundamentally distributed, having long-term outcomes that are all coupled together but with no simple whole-team coordination mechanism that can be taken for granted. One approach to facilitating coordination in such distributed systems is to establish a distinguished leader with a singular influence on all team members, e.g., [3]–[5]. However, if others in the team in potentially influential positions could indirectly infer the state and intentions of the leader, complementary actions could extend the leader’s direct influence and improve the performance of the collective. For example, recognition by one robot at the end of a formation of a non-trivial and informative formation deviation by a robot at the other end could trigger motions that allow *both* robots to work together to appropriately move the collective [1].

In our previous work [1], we used a machine-learning method to solve the RTL problem where a robot (*Tail*) at one



Fig. 1: Illustration of our proposed pipeline in a snapshot example of 5 robots at time  $t$ . Each robot has a limited view and a motion rule dependent on its neighbors except the *Head* robot leading the team at the front. *Tail* uses recent observations on its neighbor, *Follower 3*, which is denoted as  $O(t-1, t)$ . A sequence of historical poses,  $h$ , is also encoded for the model to make a final prediction on the unseen teammates.

end of a line formation of a multi-robot team is to predict positions of all other teammates only using local observations about a single nearby teammate. Because each robot has a limited sensor radius and a relatively simple motion rule that depends on the position of its nearest neighbors, the *Tail* has to be able to learn the regularity of the observed motions of its neighbor to finally infer the poses of all other robots. We introduced a repetitive prediction scheme to use predictions about nearer teammates to make predictions on farther ones until the prediction reached the *Head* robot at the other end in line formation. In a multi-robot simulation, we showed the feasibility of using the method in an example caging scenario in which the *Tail* could recognize the early stages of a caging action of *Head* and promote a proactive maneuver to better assist in coordinating the team to quickly enclose an encountered object in the environment.

Figure 1 illustrates our proposed pipeline in which a deep neural network is to learn to synthesize the observations of its neighbor with knowledge about historical positions of all the teammates. As shown in Fig. 2, a LSTM layer is deployed to encode the historical sequence input, which could learn robot dynamics and the probable evolution of the team shape over time under physical constraints. Furthermore, the learned sequence encoding could help filter out impossible solution candidates that the model might produce if it only utilized the most recent observations on the neighbor, as in our previous work [1]. In addition to the more powerful deep-learning pipeline, we also make use of a more realistic robotic platform than in our previous work. Previously, we conducted all demonstrations on computer simulations, but

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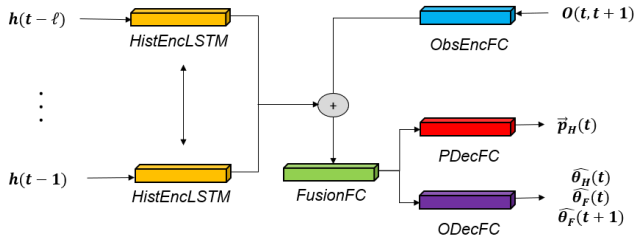


Fig. 2: Structure of our proposed deep neural network. This is an snapshot example when applied to a focused module of 3 robots, called *Tail*, *Follower*, *Head* within it, at time  $t + 1$ . The Encoder–Decoder structure encodes: 1) historical positions and orientations of *Follower* and *Head* until  $t - 1$ , and 2) the observed positions of the *Follower* at  $t$  and  $t + 1$ . The decoder part learns to estimate: 1) the position of *Head* at  $t$ , and 2) orientations of *Follower* at  $t$  and  $t + 1$  and *Head* at  $t$ .

in this work, we implement our approach in a realistic and reproducible physical environment realized by a widely available two-wheeled robotic platform, *Thymio* [6].

This paper is organized as follows. In Section II, we explore related literature and the distinction of our work. Section III explains more details about our setting of RTL problem. Then, we introduce our proposed deep-learning solution method in Section IV. In Section V, we provide details about experiments performed on real robots, including data collection and hyperparameters used for learning. We then summarize experimental results in Section VI. Lastly, we summarize our research and discuss future directions in Section VII.

## II. RELATED WORK

In this section, we discuss results from the multi-robot systems literature similar to ours and elaborate on the distinct contribution of our work. We conclude this section with an elaboration of the differences between our results here and our previously presented work.

### A. Cooperative localization and tracking

Although the RTL problem is superficially similar to other known cooperative localization and tracking problems [7]–[11], such as cooperative SLAM, it is a distinct from general robotic localization. In RTL, the robot does not execute predictions on its own location but on its teammates using accessible information. Moreover, in contrast with cooperative localization approaches, robots in RTL are assumed to be communication free and thus not allowed to communicate with other members during the prediction of positions. Hence, in lieu of direct signalling, an underlying assumption of the RTL problem is that the robot behaviors are correlated with the state of the environment around them and thus contain cues about the state of their neighbors. In this sense, state observers in a networked robotic system are more similar to RTL than robotic localization [12], [13], but RTL does not depend upon knowledge of the structure of the underlying robotic controllers and does not require that robots move according to simplistic, analytically tractable dynamical models. Furthermore, we emphasize RTL solutions focused on scaling from training with small teams

to implementation in potentially much larger and possibly variably sized teams.

### B. Group state recognition

The RTL problem aims to infer the positional state of an entire multi-robot team using only locally obtainable information in the aspect of a robot member. This is because the knowledge about global configuration could help make a better decision for the sake of whole team. In this spirit, Brown and Goodrich [14] as well as Berger et al. [15] show that local interactions of robots within a swarm can be used to classify swarm-level macroscopic structures, such as particular swarm-level shapes like *flock* and *torus*. In contrast, our work is to estimate the pose of robots themselves, which are the microscopic elements of the multi-robot team. Consequently, we make inferences on a space with far more degrees of freedom and require a more powerful regression model.

### C. Behavioral cue interpretation

In the RTL problem, the pose of remote robots is to be inferred from the motions of nearby robots performing otherwise nominal behaviors. This approach allows information to flow around a multi-robot team without traditional communication modalities for explicit signalling, such as radio communication. Motivated by similar constraints on reducing the use of these modalities, Novitzky et al. [16] and Das et al. [17] showed how robots performing a special behavior, similar to a “waggle dance” of honeybees [18], could convey information visually or mechanically to remotely observing robots. Their approaches are different from ours in that we do not require robots to deviate from their normal behaviors for the purposes of explicit communication; we infer positions only from the latent information in nominal robot behavior and interactions.

### D. Robot dynamics learning

Byranvan and Fox [19] proposed a deep learning approach to predict the next visual frame given both a current visual frame as well as knowledge of a force acting on an object within the frame. One of the motivations for this work was to understand the dynamics of robotic arms and the relationship with control commands possibly executed. In the RTL problem described in Section I, the *Tail* robot may have accumulated a global shape of robot team over time, and it has to be able to predict the future formation as a new observation on its neighbor is provided, which could be viewed as gaining knowledge of an applied force on the team. Such a similarity inspired the architecture of our neural network model, but Byranvan and Fox focused on learning motions of rigid objects, while a chain of robots in our work can present much flexibility in team shape. In addition, the positional information about the nearest neighbor is only loosely analogous to the perfect knowledge of force used in the frame-prediction example. Consequently, our approach is a significant deviation from the one proposed by Byranvan and Fox [19].

### E. Contribution beyond past work

We first presented the RTL problem in our previous work [1]. To solve the problem with little robot-to-robot communication, the *Tail* robot is designed to use a repetitive strategy of inference with which the predictions on a closer robot are used as input to prediction for more distant team members. Such an approach enables scaling the estimation capability from training on small teams to implementation in larger teams without further training. We use the same repetitive prediction scheme here; however, our focus is now on improving the regression engine to broaden its applicability from simulation to physically implemented robotic teams using commercially available, off-the-shelf robotic platforms.

### III. RTL PROBLEM & SYSTEM DESIGN

Here, we build on the problem and approach we introduced in our previous work [1]. In particular, we consider a team of  $n \in \{3, 4, \dots\}$  robots designed to move in a line formation. Each robot maintains a programmed proximity with both the robot ahead of it and behind it, except for the *Head* robot that leads the convoy and the *Tail* that only follows its single neighbor ahead of it. Each robot in between *Head* and *Tail* are referred to as *Follower*  $i$ , where  $i \in \{1, 2, \dots, n-2\}$  represents the position relative to the *Tail* (i.e., *Follower* 1 is closest to the *Tail*).

Every robot has the same sensory and motion capabilities and constraints, although they may take different actions according to motion rules depending on their roles. This leads to the *Follower* and *Tail* robots to behave differently based on the current positions of their close neighbors. For simplicity, we assume that every robot has an ability to accelerate fast enough to always keep the neighbors within their sensory range.

The RTL is to localize all teammates from the view of *Tail* only using locally observable information of the position of *Follower* 1 at every time step. We assume that until a specific time instant  $\tau$ , all the information about positions and orientations of all robots have been shared reliably with the *Tail* robot, possibly via global communication, but continued information sharing is unavailable after time  $\tau$  thus requiring *Tail* to use this localization technique to extrapolate from the previously known reliable positions.

Formally, at the time instant  $\tau$ , the following set of poses of all teammates is available:

$$\{\vec{p}_{r@t}, \theta_{r@t}\} \quad (1)$$

where  $\vec{p}_{r@t} \triangleq (x_{r@t}, y_{r@t})$  is the position of robot  $r$  at time  $t$  and  $\theta_{r@t}$  is the orientation of robot  $r$  at time  $t$ , with  $r \in \{T, F_1, F_2, \dots, F_{n-2}, H\}$  and  $t \leq \tau$ . The RTL problem is thus, at each time  $t > \tau$ , to observe the position of  $F_1$ ,  $\vec{p}_{F_1@t}$ , and use all available information to predict the pose set in Eq. (1) for time  $t'$  where  $\tau < t' \leq t$ .

### IV. PROPOSED MODEL

Here, we first briefly review our scalable 3-to-many prediction approach for training with a small team and implementation on a larger team without additional training [1].

Then in Section IV-B, we present our improved regression approach that uses not only current nearest-neighbor observations but also sequences of past team formations.

#### A. Scalable, modular prediction approach

The scalable RTL implementation we previously introduced [1] iteratively applies predictions of farther and farther robots within focal 3-robot teams, starting with the nearest *Follower* robots and eventually leading to prediction of the *Head*. In particular, *Tail* initially uses all available information about *Follower* 1 at time  $t > \tau$  and  $t+1$  to predict  $\vec{p}_{F_2@t}$  of *Follower* 2. Then, at  $t+2$ , *Tail* may repeat this approach centered on *Follower* 2 to predict  $\vec{p}_{F_3@t}$  of *Follower* 3. Thus, after  $n-1$  iterations of this approach, the *Tail* will eventually have an estimate of the position  $\vec{p}_{H@t}$  of *Head* at time  $t$ .

Because each inference is only applied in 3-robot teams (i.e., the *Tail*, a focal *Follower*, and the robot immediately ahead of that *Follower*), the predictor can be trained simply with a 3-robot team and used for a larger size time without additional training for the larger-team case. However, this iterative modular-prediction approach brings about a time delay in making a prediction for a far robot. Furthermore, any estimation errors tend to accumulate, making the use of this approach limited either to short chains of robots or short estimation time periods. Thus, in the next section, we describe a regressor significantly improved over our previous version [1] that increases the feasibility of using this approach in realistic multi-robot teaming scenarios.

#### B. Improved regression model

Because the learned regressor is designed to work in a modular team of 3 robots, notations for robot  $r \in \{H, F, T\}$  only indicate the identities of *Head*, *Follower*, and *Tail* and not any additional intermediate *Follower* robots. In addition, all positions noted here are assumed to be expressed in the reference frame of the *Tail* robot because, in practice, the *Tail* has to understand positions of others by projection onto the local coordinate system centered at itself. Though a similar conversion may be considered for orientation, we use absolute direction in this work (i.e., assume that all robots agree on compass directions) for simplicity.

In this work, we use a significantly different deep neural network architecture than our previous RTL work [1]. As shown in Fig. 2, our proposed model uses not only the current observation of the follower as an input but also a historical sequence of poses. We deconstruct the architecture here.

- We use a fully connected(FC) layer  $f_{obs}$  of size  $k \in \mathbb{N}$  to encode observations into a feature vector  $o \in \mathbb{R}^k$ . That is,

$$o = f_{obs}(X_{obs}) \quad (2)$$

where  $X_{obs} = (\vec{p}_{F@t}, \vec{p}_{F@t+1})$  is the observed positions of *Follower* at times  $t$  and  $t+1$ .

- A historical sequence of poses is encoded by a bi-directional LSTM layer [20],

$$h = f_{hist}(X_{hist}) \quad (3)$$

where  $f_{hist}$  is a LSTM layer,  $X_{hist} = (\vec{p}_{H@t-\ell:t-1}, \theta_{H@t-\ell:t-1}, \vec{p}_{F@t-\ell:t-1}, \theta_{F@t-\ell:t-1})$ , and  $h \in \mathbb{R}^{2 \times m}$  is the encoded history feature where  $\ell$  is the length of historical sequence, and  $m$  is the size of LSTM layer.

- The  $o$  and  $h$  feature vectors are synthesized by a layer  $\phi$ , which is passed as input to two separate final regressors,  $g_p$  and  $g_\theta$ , such that

$$\begin{aligned} Y_p &= g_p(\phi(o, h)), \\ Y_\theta &= g_\theta(\phi(o, h)) \end{aligned} \quad (4)$$

where  $Y_p = \hat{p}_{H@t}$  and  $Y_\theta = (\hat{\theta}_{F@t}, \hat{\theta}_{H@t}, \hat{\theta}_{F@t+1})$ .

For fusion of  $o$  and  $h$  features, layer  $\phi$  in Eq. (4) could be implemented by any type of layer. During our experiments, we built a FC layer of size  $d \in \mathbb{N}$  to find a nonlinear relationship between the input features and achieved a satisfactory performance.

Furthermore, orientation estimate  $\hat{\theta}_{F@t+1}$  gained with  $\hat{\theta}_{F@t}$  is estimated again when  $\hat{\theta}_{F@t+2}$  is estimated at the next prediction step. Although our model keeps the later estimate only, we discovered that involving it in both steps can regulate the regressor during learning to produce a model that achieves a better score in validation.

To find a best combination of model parameters mentioned above, we performed an extensive random search with choices of other learning parameters and finally set  $k = 80$ ,  $m = 160$ , and  $d = 160$ .

## V. EXPERIMENTAL METHODOLOGY

### A. Robotic platform

To demonstrate the effectiveness of our method, we employ a commercially available, two-wheeled physical robotics platform, *Thymio* [6], which allows to execute a team of small two-wheeled mobile robots. Although each *Thymio* robot has sensing and computation capabilities, we used a central computer connected with a overhead camera to simulate better proximity sensors, more powerful computing power, and a GPS system that would be generalizable to other robotic platforms run in a similar physical scenario. Specifically, the central system is configured to detect the locations of robots in real time using off-the-shelf computer vision packages and communicate with a *Raspberry Pi* board [21] mounted on each robot, which implements control laws based on the received positional information about neighboring robots. The *Tail* and each *Follower* robots were configured, as in our previous simulation work [1], to regulate distance with the robot ahead of it. Regulation of the distance behind each *Follower* robot was achieved through stopping (as opposed to backward motion), which reduced higher frequency oscillations in individual robot motion. Additional details about the physical constraints of the laboratory testbed are provided in a supplementary video.

### B. Data collection

We collected the pose data from two robot teams, one of 3 robots and one of 5 robots, that ran separately in

	Duration	Num. of Samples	Num. of Instances
3 robots	100.6 minutes	6,975	465
5 robots	45.0 minutes	8,736	208

TABLE I: Description of data collected from executions of 3-robot and 5-robot teams.

an arena of  $2.5m \times 1.9m$ . The location detection was performed at 4 frames per second at each of which a new command was received by each robot. Also, all pose data was collected at the rate of 2 frames per second, which was not necessarily synchronized with the command timing. We set the length of history to 5 seconds (10 time steps in data recording) and the time window for prediction to the next 8 seconds (16 time steps). Table I provides details about the collected data, where a *Sample* refers to a set of coordinates and orientations in a 3-robot group with which a prediction can be performed, and an *Instance* is set of all available samples for 13 seconds from the entire team. To reduce temporal autocorrelation to help ensure independence of instances, we clustered recordings so that instances are separated by at least 7 seconds. All the collected data is open to the public<sup>1</sup> to encourage more future works on RTL.

### C. Model training and performance evaluation

Our model is implemented in the *Tensorflow Python* library<sup>2</sup> to realize the entire pipeline, and it was trained to minimize loss functions such as Euclidean distance and mean absolute errors for position and orientation estimation, respectively. 60% data from the 3-robot team is used to train our model, and another 10% was set aside for validation. At each epoch of training, a validation followed so that the learned weights that achieved the best validation performance were saved. The rest of 20% data and all the data from 5-robot were used to test the model. We compare our proposed model to two different alternative approaches:

- *2X Heuristic*: The prediction on *Head* within a modular subteam is performed by doubling the vector  $\vec{p}_F - \vec{p}_T$ .
- *FC*: Two fully connected layers run in the predictor without historical information, which is based on our previous proof-of-concept work [1].

## VI. RESULTS

### A. Overall performance

In Fig. 3, we compare the averaged error for position predictions of the three models in both the 3-robot and 5-robot team cases. In the case of 3 robots, the average error is calculated only on the prediction for the *Head* robot, while in the 5-robot case, it involves all predictions for *Follower 2*, *Follower 3*, and *Head*. Moreover, for every model except the *2X Heuristic*, the mean performance of 5 separate learning sessions is reported with the standard deviation.

Figure 3 shows that the machine learning methods outperform the *2X Heuristic* in any case. In particular, the performance gap between *2X Heuristic* and *FC* becomes

<sup>1</sup><https://github.com/ctyeong/rtl-lstm>

<sup>2</sup><https://www.tensorflow.org/>

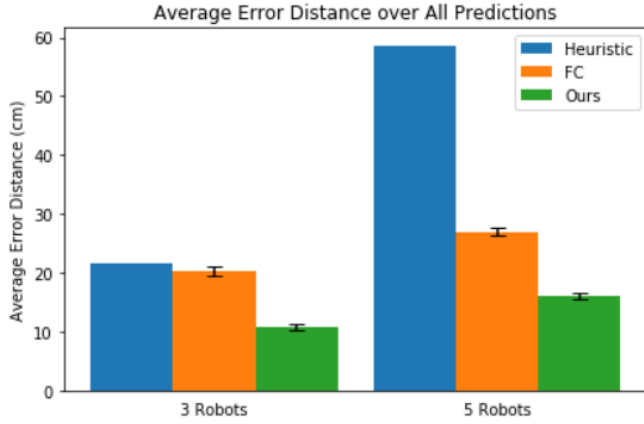


Fig. 3: Average accumulated error of each model in two different sizes of robot team. For each of machine learning methods, the mean performance of 5 separate training sessions is reported with a error bar to visualize the performance variation.

much larger in 5-robot case suggesting that realistic stochasticity presents a significant challenge to the straightforward *2X Heuristic* case. Furthermore, our model clearly shows the performance improvement over the *FC* model, since the average error was reduced by 47% and 40% in the 3-robot and 5-robot case, respectively. In addition, considering the diameter of a *Thymio* robot is 12 cm, as only three robots are deployed, the average distance between the prediction and the true position is shorter than the length of the robot body. This overall result proves the effectiveness of encoding a sequence of historical behaviors as a feature input over the model fed only with very recent observation.

### B. Microscopic analysis

In this section, we analyze the performance of the machine-learning based models from a more fine-grained perspective. For each model, Fig. 4 visualizes the step-wise error for different target robots (i.e., *Follower 2* and *Head*) in 5-robot case, which is calculated by averaging the errors of each step across instances. By doing so, we can better understand the approximate time frame within which estimation error stays below an acceptable error level. During this evaluation, we also had 5 separate training sessions to gain the mean and the variability of the performance.

Figure 4 shows that for any target robot, the error increases over time due to the design of the repetitive prediction method where previous errors would negatively impact the prediction power for the following steps. In a similar sense, each model brings about larger error on the *Head* prediction than on the *Follower 2*. However, at every time step, our approach causes less error than the *FC* model for any robot, and the visualized error bars confirm that the performance improvement of our approach is significant in every case. Especially for first 4 seconds, the prediction on *Head* from our model appears more accurate than the prediction on *Follower 2* from the *FC* implying that until then, our method also has less error for the estimate of *Follower 3* as well. Having said that, the more rapid increase in error of our

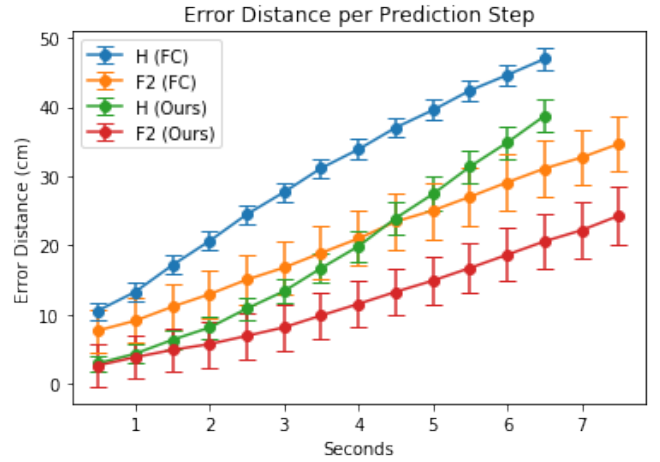


Fig. 4: Average step-wise error for different target robots in 5-robot team. For each model, all the prediction errors at different time steps are averaged for a specific robot. The error bars represent the standard deviation of 5 separate models in terms of the performance metric. For the sake of visualization, the error for *Follower 3* is omitted, but note that in any model, the error ranges between the two visualized errors at every step.

method for the position of *Head* may highlight the liability of relying more on prior information for future predictions. Nevertheless, these results demonstrate the potential of our approach to reduce the communication bandwidth necessary for a multi-robot team to perform localization.

### C. Qualitative results

Figure 5 shows images of three different instances of 5-robot team in which both the true positions of all robots and the predictions on them are represented. The triangles indicate the true positions and orientations, where the yellow triangle is the *Tail* and the red triangle is the *Head*. The circles indicate the estimated positions of the two farthest *Follower* robots as well as the *Head*. Predictions until 4 seconds are shown to be reliable, although there are some errors while the group of robots is turning with a high angle and when the *Head* changes its destination. Predictions tend to be biased more toward the inside of the curve the team is moving on. Unexpectedly, at any instant of time, the maximum position error is often at intermediate *Follower* robots and not at the most distant *Head* robot. Such corrections may be the result of using historical information to constrain the possible locations of farther robots, as in data fusion techniques that use formal dynamical models to physically constrain estimator variance. However, our approach does not make use of explicit formal dynamical models.

## VII. SUMMARY & FUTURE WORK

By formally incorporating historical data into the design of a scalable, localization regression module, we have improved upon our previous work [1] in estimating the positions of robots in a multi-robot team using only observations of a single, nearest neighbor robot. As in our previous model, the regression module we design can be trained on small





Fig. 5: Sample video frames with prediction results of our proposed model for three different instances of 5 robot team. Each row presents four frames of an instance for first 4 seconds. The yellow robot is *Tail*, the pink is *Head*, and in-between are *Followers* robots. The colored circles are predicted position outputs, in each of which a black arrow is drawn to indicate the predicted orientation as well. The colored triangles with a black line through the center are the results of localization detection used in data collection stage. A small red dot on the arena in each frame is the random destination the *Head* is moving toward, which is re-sampled at intervals.

teams of 3 robots and generalize to larger teams with no additional training. However, our new approach greatly reduces estimator error, giving it the potential to greatly reduce communication bandwidth in other localization schemes that otherwise require continuous communication among robots.

To test our approach, we utilized a commercially available robotic platform, *Thymio*, to collect datasets from 3-robot and 5-robot teams. Through empirical experiments, we showed that the proposed machine learning model offers more accurate estimation than other approaches. We analyzed our estimator performance over time and demonstrated its improved performance for short horizons but potential added liabilities for longer time horizons. Lastly, we overlaid visualizations of the prediction outcomes on images of the actual robotic system to illustrate specific cases where low estimation error was provided for far-away robots despite high error on more nearby robots.

In the future work, we could more deeply investigate the factors in model accuracy such as the length of history or types of team behavior that might favor the prediction scheme. Also, because the LSTM layer is exposed to various evolutions of team shape during training, the vector representation of it may be examined to characterize team states and ultimately detect large-scale group-shape abnormalities that may be important to detect for situational awareness. Beyond these approaches, we are currently working toward Bayesian implementations that make better use of uncertainty information from sensors and provide confidence estimates

for the position of each robot in the team. Such approaches, when incorporated into a framework such as partially observable Markov decision processes, could lead to approaches for using actuation (as opposed to communication) to reduce estimator error.

## REFERENCES

- [1] T. Choi, T. P. Pavlic, and A. W. Richa, “Automated synthesis of scalable algorithms for inferring non-local properties to assist in multi-robot teaming,” in *2017 13th IEEE Conference on Automation Science and Engineering (CASE)*. IEEE, 2017, pp. 1522–1527.
- [2] S. Hochreiter and J. Schmidhuber, “Long short-term memory,” *Neural computation*, vol. 9, no. 8, pp. 1735–1780, 1997.
- [3] K. Elamvazhuthi and S. Berman, “Scalable formation control of multi-robot chain networks using a PDE abstraction,” in *Proc. of the 12th International Symposium on Distributed Autonomous Robotic Systems*, 2016, pp. 357–369.
- [4] J. J. Daymude, R. Gmyr, A. W. Richa, C. Scheideler, and T. Strothmann, “Improved leader election for self-organizing programmable matter,” in *International Symposium on Algorithms and Experiments for Sensor Systems, Wireless Networks and Distributed Robotics*. Springer, 2017, pp. 127–140.
- [5] R. E. Stern, S. Cui, M. L. Delle Monache, R. Bhadani, M. Bunting, M. Churchill, N. Hamilton, H. Pohlmann, F. Wu, B. Piccoli, *et al.*, “Dissipation of stop-and-go waves via control of autonomous vehicles: Field experiments,” *Transportation Research Part C: Emerging Technologies*, vol. 89, pp. 205–221, 2018.
- [6] J. Shin, R. Siegwart, and S. Magnenat, “Visual programming language for thymio ii robot,” in *Conference on Interaction Design and Children (IDC’14)*. ETH Zürich, 2014.
- [7] B. Grocholsky, V. Kumar, and H. Durrant-Whyte, “Anonymous cooperation in robotic sensor networks,” in *Proc. of the AAAI-04 Workshop on Sensor Networks*, 2004.
- [8] L. Luft, T. Schubert, S. I. Roumeliotis, and W. Burgard, “Recursive decentralized collaborative localization for sparsely communicating

- robots.” in *Robotics: Science and Systems*. New York, NY, USA, 2016.
- [9] A. Franchi, P. Stegagno, M. Di Rocco, and G. Oriolo, “Distributed target localization and encirclement with a multi-robot system,” in *Proc. of the 7th IFAC Symposium on Intelligent Autonomous Vehicles*, 2010, pp. 151–156.
  - [10] Y. Cheng and D. Xie, “Distributed observer design for bounded tracking control of leader–follower multi-agent systems in a sampled-data setting,” *International Journal of Control*, vol. 87, no. 1, pp. 41–51, 2014.
  - [11] O. De Silva, G. K. I. Mann, and R. G. Gosine, “Efficient distributed multi-robot localization: A target tracking inspired design,” in *Proc. of the 2015 IEEE International Conference on Robotics and Automation*, 2015, pp. 434–439.
  - [12] L. Xiao-Yuan, H. Na-Ni, and G. Xin-Ping, “Leader-following formation control of multi-agent networks based on distributed observers,” *Chinese Physics B*, vol. 19, no. 10, 2010.
  - [13] G. Antonelli, F. Arrichiello, F. Caccavale, and A. Marino, “A decentralized controller-observer scheme for multi-agent weighted centroid tracking,” *IEEE Transactions on Automatic Control*, vol. 58, no. 5, pp. 1310–1316, 2012.
  - [14] D. S. Brown and M. A. Goodrich, “Limited bandwidth recognition of collective behaviors in bio-inspired swarms,” in *Proceedings of the 2014 international conference on Autonomous agents and multi-agent systems*. International Foundation for Autonomous Agents and Multiagent Systems, 2014, pp. 405–412.
  - [15] M. Berger, L. M. Seversky, and D. S. Brown, “Classifying swarm behavior via compressive subspace learning,” in *2016 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2016, pp. 5328–5335.
  - [16] M. Novitzky, C. Pippin, T. R. Collins, T. R. Balch, and M. E. West, “Bio-inspired multi-robot communication through behavior recognition,” in *Proc. of the 2012 IEEE Conference on Robotics and Biomimetics (ROBIO)*, 2012, pp. 771–776.
  - [17] B. Das, M. S. Couceiro, and P. A. Vargas, “MRoCS: A new multi-robot communication system based on passive action recognition,” *Robotics and Autonomous Systems*, vol. 82, pp. 46–60, 2016.
  - [18] K. Von Frisch, *The Dance Language and Orientation of Bees*. Harvard University Press, 1967.
  - [19] A. Byravan and D. Fox, “Se3-nets: Learning rigid body motion using deep neural networks,” in *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017, pp. 173–180.
  - [20] Y. Wu, M. Schuster, Z. Chen, Q. V. Le, M. Norouzi, W. Macherey, M. Krikun, Y. Cao, Q. Gao, K. Macherey, *et al.*, “Google’s neural machine translation system: Bridging the gap between human and machine translation,” *arXiv preprint arXiv:1609.08144*, 2016.
  - [21] E. Upton and G. Halfacree, *Raspberry Pi user guide*. John Wiley & Sons, 2014.